BBM402-Lecture 6: Turing Machines

Lecturer: Lale Özkahya

Resources for the presentation:
https://courses.engr.illinois.edu/cs373/fa2010/lectures
https://courses.engr.illinois.edu/cs498374/lectures.html

David Hilbert

- Early 1900s – crisis in math foundations
  - attempts to formalize resulted in paradoxes, etc.
- 1920, Hilbert's Program:

  "mechanize" mathematics
- Finite axioms, inference rules

  turn crank, determine truth

  needed: axioms *consistent* & *complete*

This slide recycled from Lecture 1

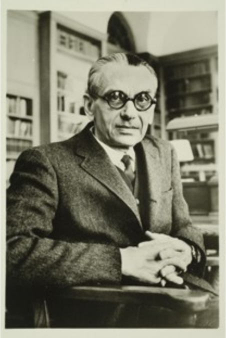
Kurt Gödel

- German logician, at age 25 (1931) proved:

"There are true statements that can't be proved"

  (i.e., "no" to Hilbert)

- Shook the foundations of
  - mathematics
  - philosophy
  - science
  - everything



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Alan Turing

- British mathematician
  - cryptanalysis during WWII
  - arguably, father of AI, Theory
  - several books, movies
- Defined "*computer*", "*program*"

and (1936) provided foundations for investigating fundamental question of what is computable, what is not computable.

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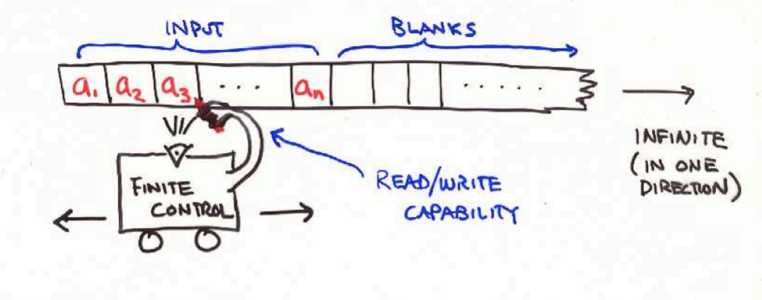
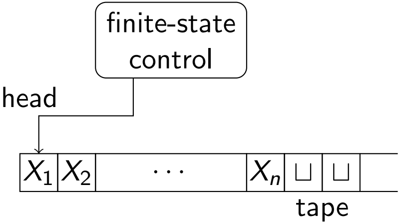


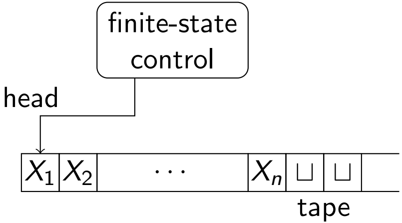
- DFA with (infinite) tape.
- One move: read, write, move, change state.

Turing Machines



- Unrestricted memory: an infinite tape

Turing Machines



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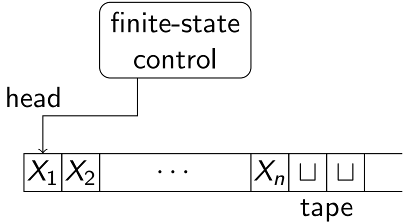

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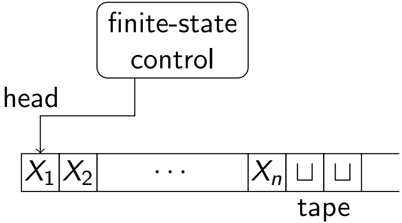

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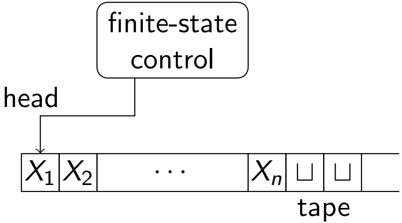

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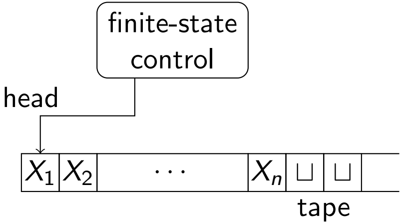

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- Initially, tape has input and the machine is reading (i.e., tape head is on) the leftmost input symbol.
- Transition (based on current state and symbol under head):
  - Change control state
  - Overwrite a new symbol on the tape cell under the head
  - Move the head left, or right.

Example

0 1 1 0 0 0 # 0 1 1 0 0 0 ␣ ...

x 1 1 0 0 0 # 0 1 1 0 0 0 ␣ ...

x 1 1 0 0 0 # x 1 1 0 0 0 ␣ ...

x 1 1 0 0 0 # x 1 1 0 0 0 ␣ ...

x x 1 0 0 0 # x 1 1 0 0 0 ␣ ...

x x x x x x # x x x x x x ␣ ...

accept

- Let M_1 be a Turing machine that tests if an input string is in the language B, where $B = \{w\#w | w \in \{0,1\}^*\}$.
- M_1 zig-zags across the tape: if no $\#$ is found, *reject*. Cross off symbols as they are checked to keep track.
- When all symbols to the left of the $\#$ have been crossed off, check for any remaining symbols to the right of the $\#$. If any symbol remained, *reject*, otherwise *accept*.

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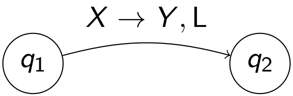
- $\delta : Q \times \Gamma \to Q \times \Gamma \times \{\mathsf{L}, \mathsf{R}\}$ is the transition function. Given the current state and symbol being read, the transition function describes the next state, symbol to be written and direction (left or right) in which to move the tape head.

Transition Function



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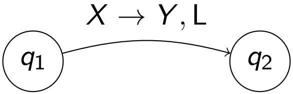

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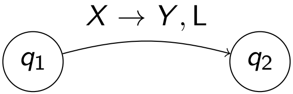

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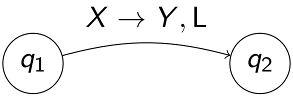

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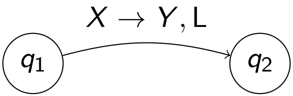

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- Transitions are deterministic
- Convention: if $\delta(q, X)$ is not explicitly specified, it is taken as leading to q_{rej}, i.e., say $\delta(q, X) = (q_{\mathrm{rej}}, \sqcup, \mathrm{R})$

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Single Step

Definition

We say one configuration (C_1) yields another (C_2), denoted as $\mathrm{C}_1 \vdash \mathrm{C}_2$, if one of the following holds.

- If $\delta(q, X_i) = (p, Y, \mathrm{L})$ then

$$X_1 X_2 \cdots X_{i-1} q X_i X_{i+1} \cdots X_n \vdash X_1 X_2 \cdots X_{i-2} p X_{i-1} Y X_{i+1} \cdots X_n$$

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Definition

A Turing machine M accepts w iff $q_0 w \vdash^* \alpha_1 q_{\text{acc}} \alpha_2$, where α_1, α_2 are some strings. In other words, the machine M when started in its intial state and with w as input, reaches the accept state.

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High level description

```
On input string w
    while there are unmarked 0s, do
        Mark the left most 0
        Scan right till the leftmost unmarked 1;
            if there is no such 1 then crash
        Mark the leftmost 1
    done
    Check to see that there are no unmarked 1s;
        if there are then crash
    accept
```

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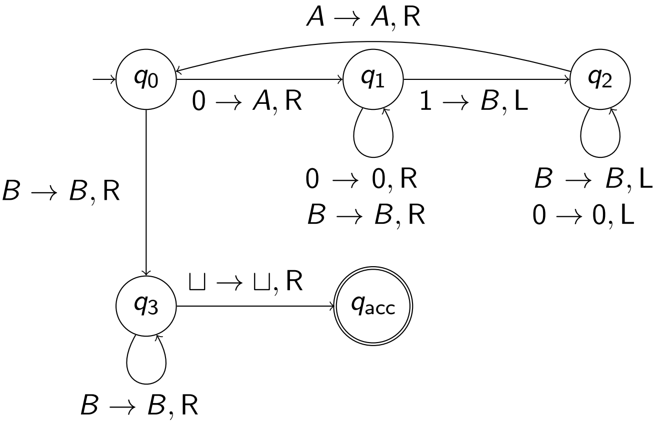

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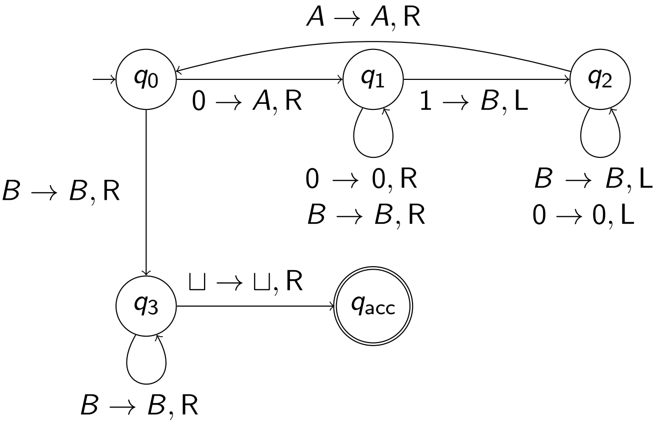

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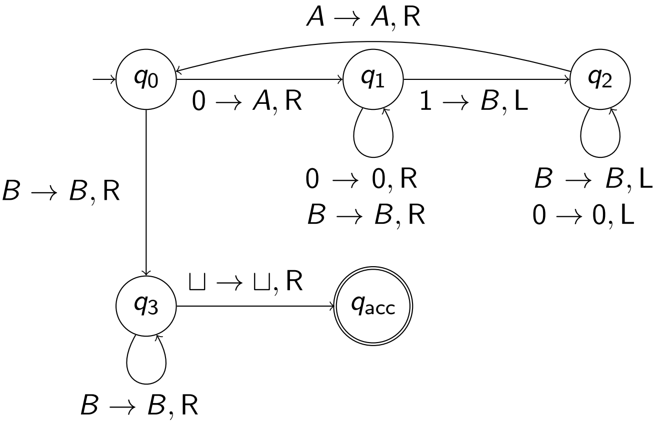

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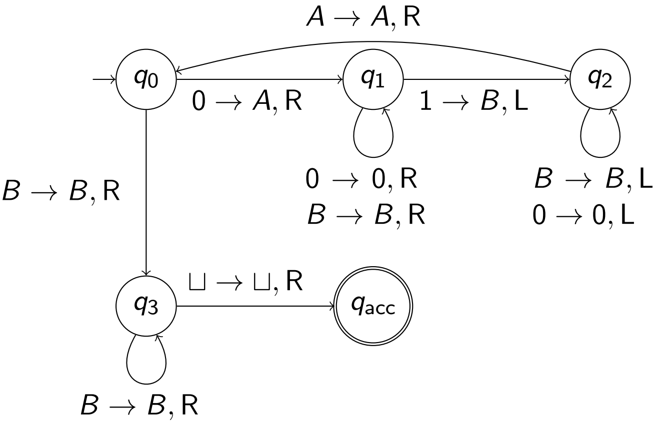
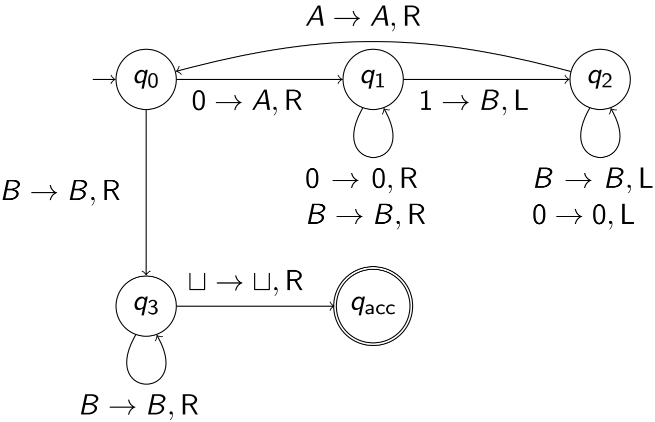


- Accepts input 0011: $q_0 0011 \vdash A q_1 011 \vdash A0 q_1 11 \vdash$

Example 1: TM for $\{0^n 1^n \mid n > 0\}$



- Accepts input 0011: $q_0 0011 \vdash Aq_1 011 \vdash A0q_1 11 \vdash Aq_2 0B1 \vdash$

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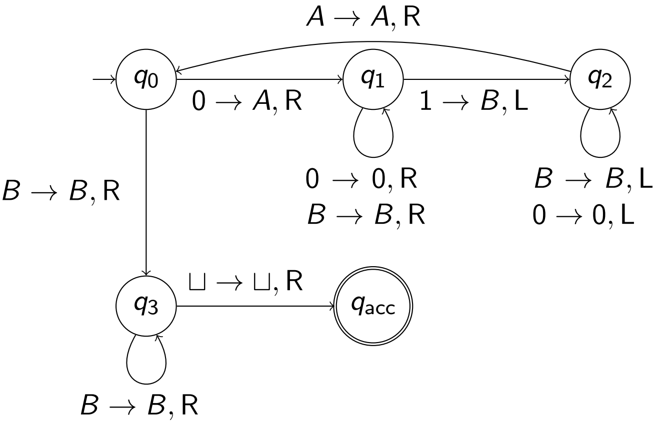

- Accepts input 0011: $q_0 0011 \vdash Aq_1 011 \vdash A0q_1 11 \vdash Aq_2 0B1 \vdash q_2 A0B1 \vdash$

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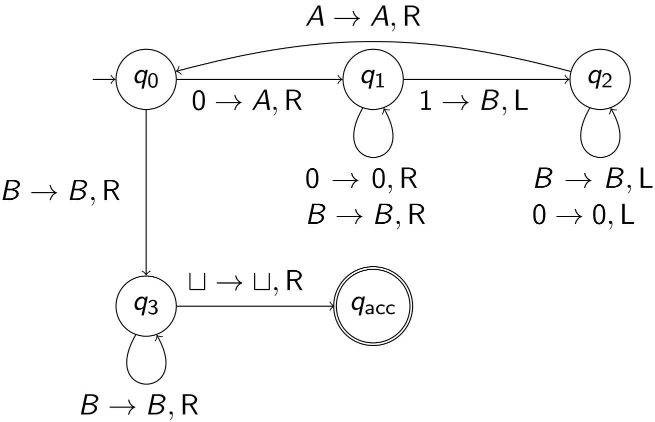

- Accepts input 0011: $q_0 0011 \vdash Aq_1 011 \vdash A0q_1 11 \vdash Aq_2 0B1 \vdash q_2 A0B1 \vdash Aq_0 0B1 \vdash$

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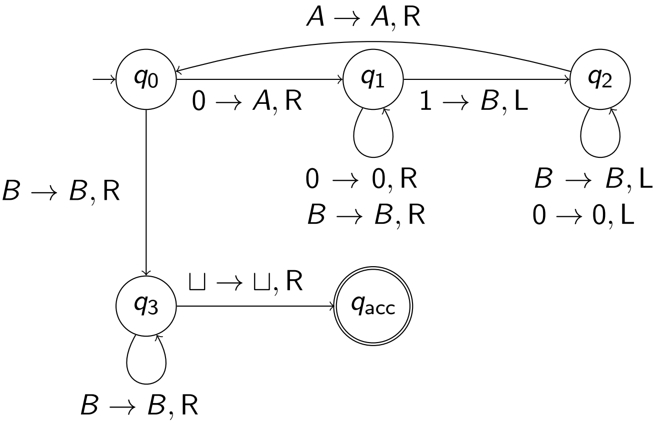

- Accepts input 0011: $q_0 0011 \vdash Aq_1 011 \vdash A0q_1 11 \vdash Aq_2 0B1 \vdash q_2 A0B1 \vdash Aq_0 0B1 \vdash AAq_1 B1 \vdash$

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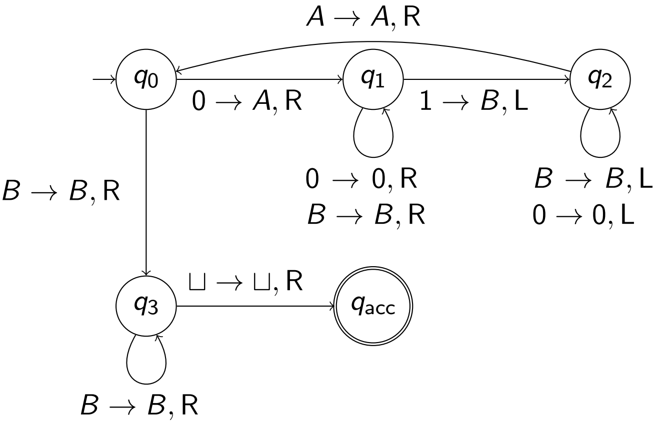

- Accepts input 0011: $q_0 0011 \vdash Aq_1 011 \vdash A0q_1 11 \vdash Aq_2 0B1 \vdash q_2 A0B1 \vdash Aq_0 0B1 \vdash AAq_1 B1 \vdash AABq_1 1 \vdash$

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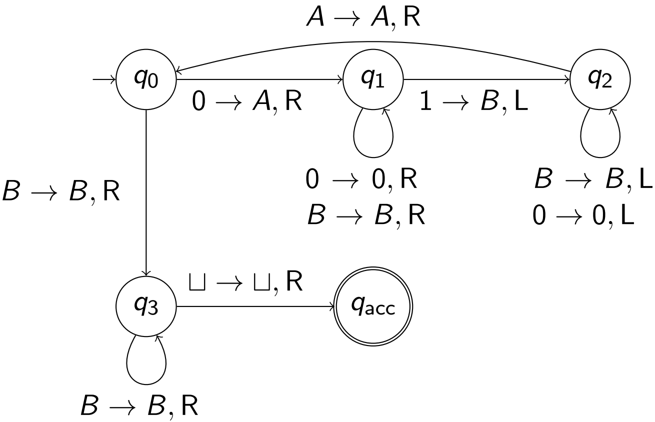

- Accepts input 0011: $q_0 0011 \vdash Aq_1 011 \vdash A0q_1 11 \vdash Aq_2 0B1 \vdash q_2 A0B1 \vdash Aq_0 0B1 \vdash AAq_1 B1 \vdash AABq_1 1 \vdash AAq_2 BB \vdash$

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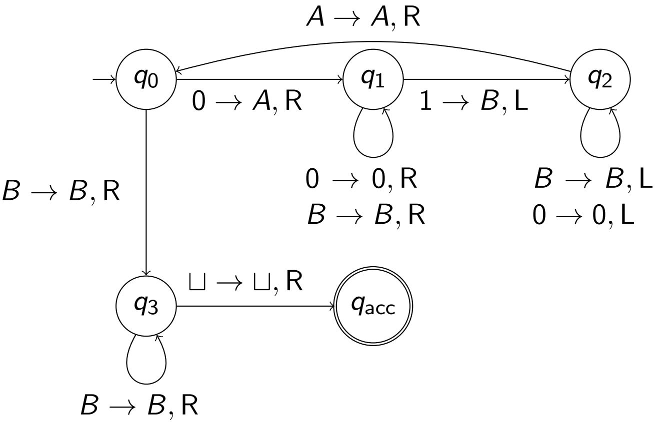

- Accepts input 0011: $q_0 0011 \vdash Aq_1011 \vdash A0q_111 \vdash Aq_20B1 \vdash q_2A0B1 \vdash Aq_00B1 \vdash AAq_1B1 \vdash AABq_11 \vdash AAq_2BB \vdash Aq_2ABB \vdash$

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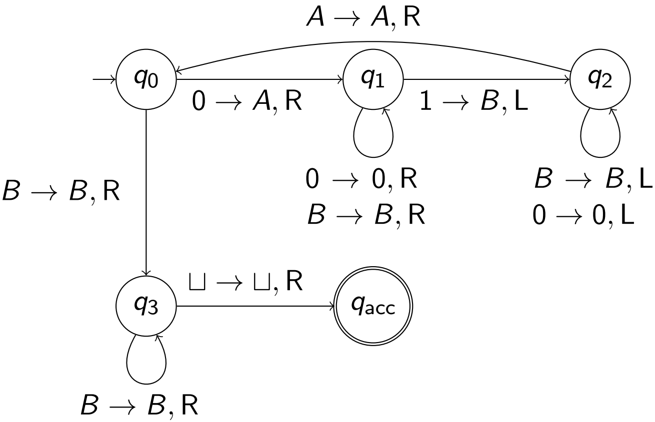

- Accepts input 0011: $q_0 0011 \vdash Aq_1 011 \vdash A0q_1 11 \vdash Aq_2 0B1 \vdash q_2 A0B1 \vdash Aq_0 0B1 \vdash AAq_1 B1 \vdash AABq_1 1 \vdash AAq_2 BB \vdash Aq_2 ABB \vdash AAq_0 BB \vdash$

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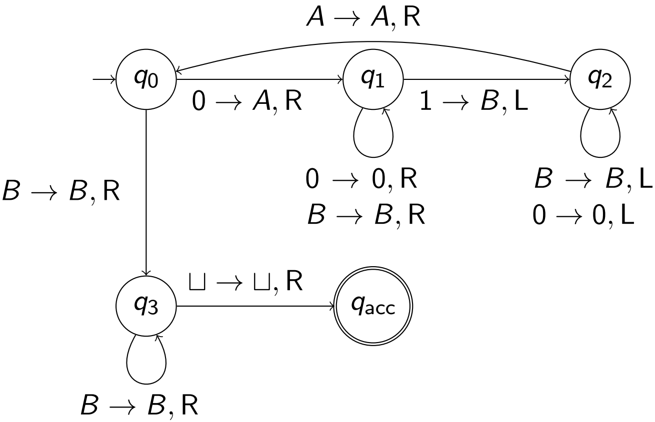

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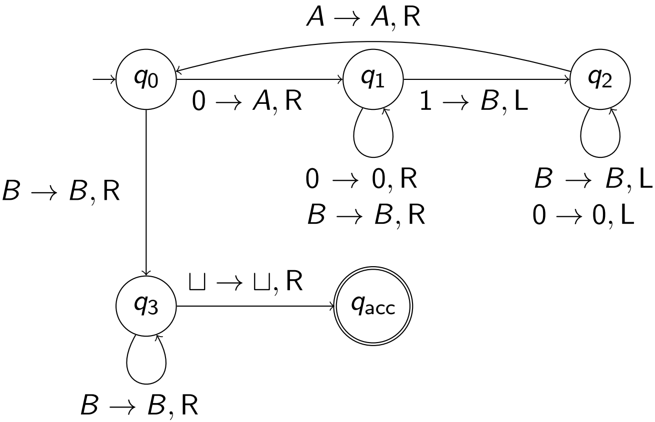

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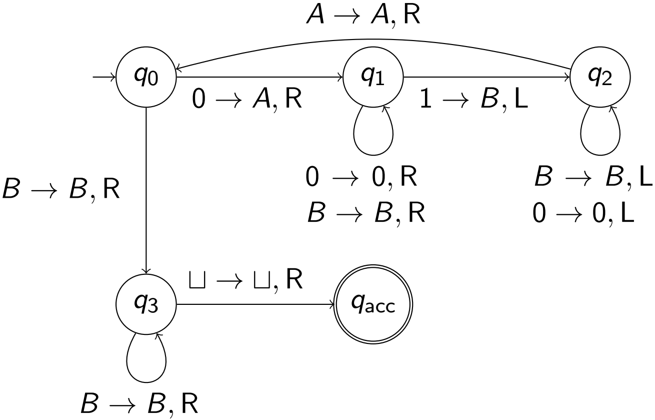

- Accepts input 0011: $q_0 0011 \vdash Aq_1 011 \vdash A0q_1 11 \vdash Aq_2 0B1 \vdash q_2 A0B1 \vdash Aq_0 0B1 \vdash AAq_1 B1 \vdash AABq_1 1 \vdash AAq_2 BB \vdash Aq_2 ABB \vdash AAq_0 BB \vdash AABq_3 B \vdash AABBq_3 \sqcup \vdash AABB\sqcup q_{acc} \sqcup$

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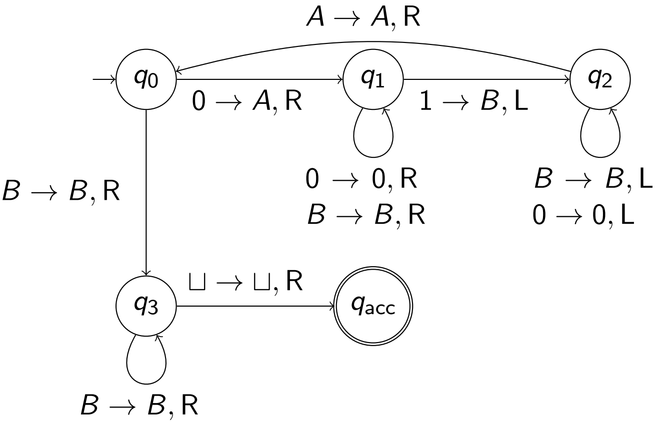

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- Rejects input 00: $q_0 00 \vdash Aq_1 0 \vdash A0q_1 \sqcup \vdash$

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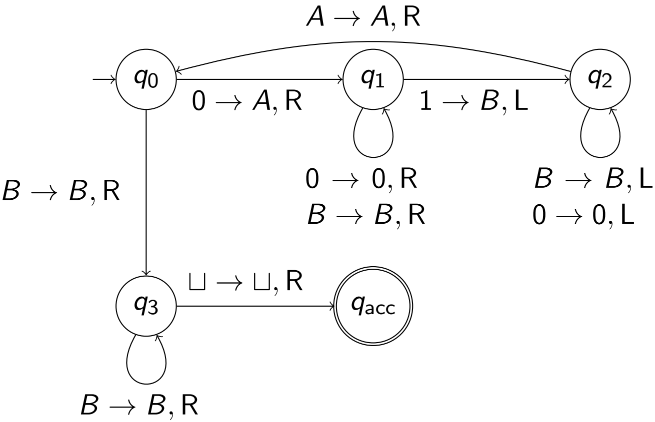

- Accepts input 0011: $q_0 0011 \vdash Aq_1 011 \vdash A0q_1 11 \vdash Aq_2 0B1 \vdash q_2 A0B1 \vdash Aq_0 0B1 \vdash AAq_1 B1 \vdash AABq_1 1 \vdash AAq_2 BB \vdash Aq_2 ABB \vdash AAq_0 BB \vdash AABq_3 B \vdash AABBq_3 \sqcup \vdash AABB\sqcup q_{acc} \sqcup$
- Rejects input 00: $q_0 00 \vdash Aq_1 0 \vdash A0q_1 \sqcup \vdash A0 \sqcup q_{rej} \sqcup$

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Formal Definition

The machine is $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$ where

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- $\Sigma = \{0, 1\}$, and $\Gamma = \{0, 1, A, B, \sqcup\}$

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- $\Sigma = \{0, 1\}$, and $\Gamma = \{0, 1, A, B, \sqcup\}$
- δ is given as follows

$$
\begin{array}{ll}
\delta(q_0, 0) = (q_1, A, R) & \delta(q_0, B) = (q_3, B, R) \\
\delta(q_1, 0) = (q_1, 0, R) & \delta(q_1, B) = (q_1, B, R) \\
\delta(q_1, 1) = (q_2, B, L) & \delta(q_2, B) = (q_2, B, L) \\
\delta(q_2, 0) = (q_2, 0, L) & \delta(q_2, A) = (q_0, A, R) \\
\delta(q_3, B) = (q_3, B, R) & \delta(q_3, \sqcup) = (q_{\text{acc}}, \sqcup, R)
\end{array}
$$

In all other cases, $\delta(q, X) = (q_{\text{rej}}, \sqcup, R)$.

The machine is $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$ where

- $Q = \{q_0, q_1, q_2, q_3, q_{acc}, q_{rej}\}$
- $\Sigma = \{0, 1\}$, and $\Gamma = \{0, 1, A, B, \sqcup\}$
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$$
\begin{array}{ll}
\delta(q_0, 0) = (q_1, A, R) & \delta(q_0, B) = (q_3, B, R) \\
\delta(q_1, 0) = (q_1, 0, R) & \delta(q_1, B) = (q_1, B, R) \\
\delta(q_1, 1) = (q_2, B, L) & \delta(q_2, B) = (q_2, B, L) \\
\delta(q_2, 0) = (q_2, 0, L) & \delta(q_2, A) = (q_0, A, R) \\
\delta(q_3, B) = (q_3, B, R) & \delta(q_3, \sqcup) = (q_{acc}, \sqcup, R)
\end{array}
$$

In all other cases, $\delta(q, X) = (q_{rej}, \sqcup, R)$. So for example, $\delta(q_0, 1) = (q_{rej}, \sqcup, R)$.

Example 2: TM for $\{0^n 1^n 2^n \mid n > 0\}$

Design a TM to accept the language $L = \{0^n 1^n 2^n \mid n > 0\}$

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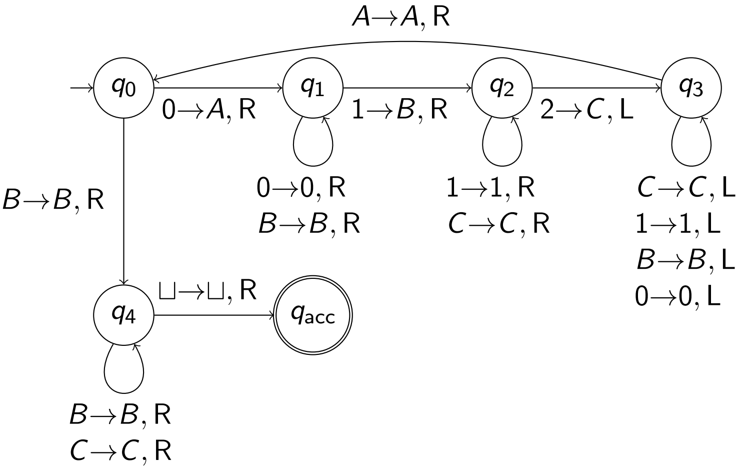
High level description

```
On input string w
    while there are unmarked 0s, do
        Mark the left most 0
        Scan right to reach the leftmost unmarked 1;
            if there is no such 1 then crash
        Mark the leftmost 1
        Scan right to reach the leftmost unmarked 2;
            if there is no such 2 then crash
        Mark the leftmost 2
    done
    Check to see that there are no unmarked 1s or 2s;
        if there are then crash
    accept
```

Example 2: TM for $\{0^n1^n2^n \mid n > 0\}$



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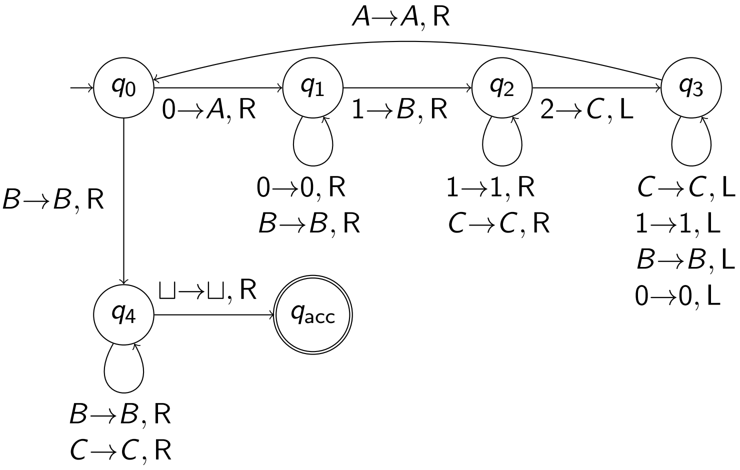

e.g.: $q_0001122 \vdash^* A0Bq_31C2$

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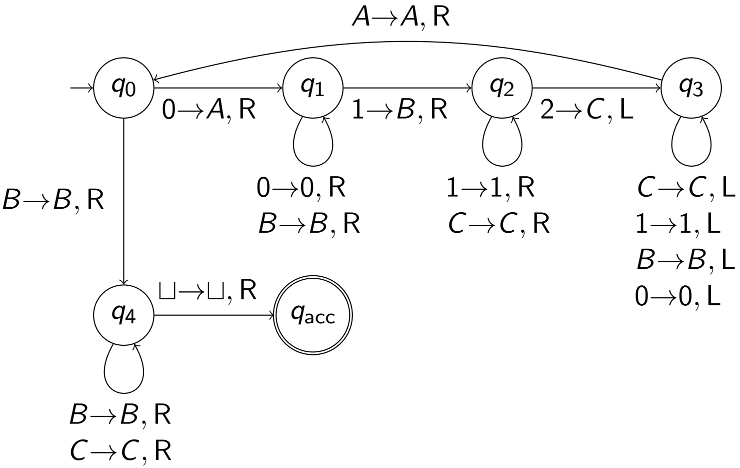

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2 \vdash^* q_3 A0B1C2$

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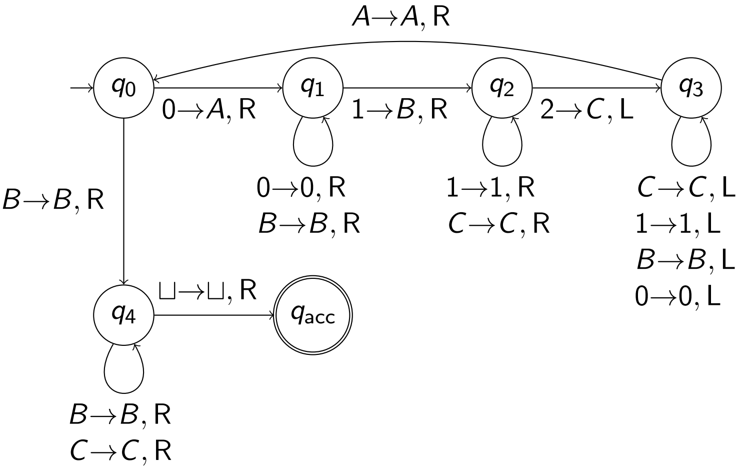

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2 \vdash^* q_3 A0B1C2 \vdash Aq_0 0B1C2$

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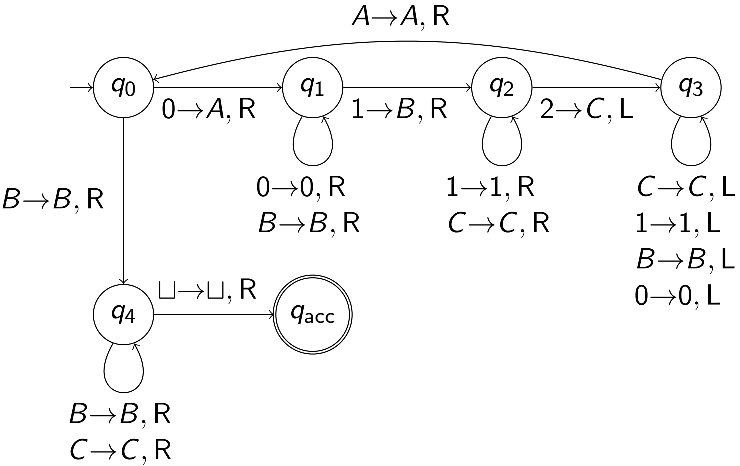

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2 \vdash^* q_3 A0B1C2 \vdash Aq_0 0B1C2$
$\vdash^* AAq_0 BBCC$

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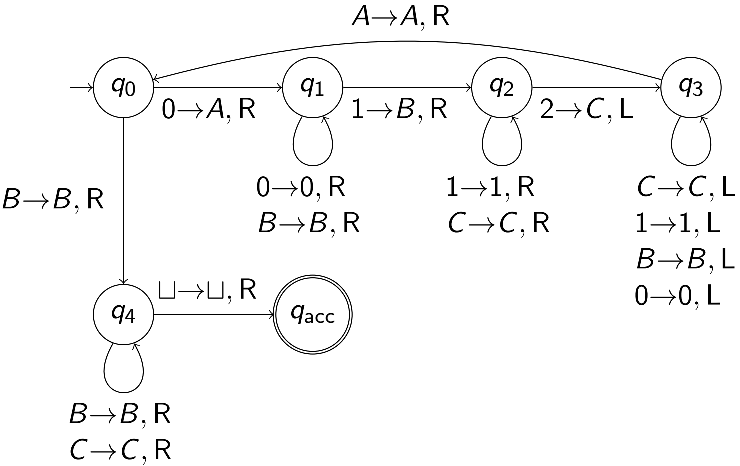

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2 \vdash^* q_3 A0B1C2 \vdash Aq_0 0B1C2$
$\vdash^* AAq_0 BBCC \vdash^* AABBCCq_4 \sqcup$

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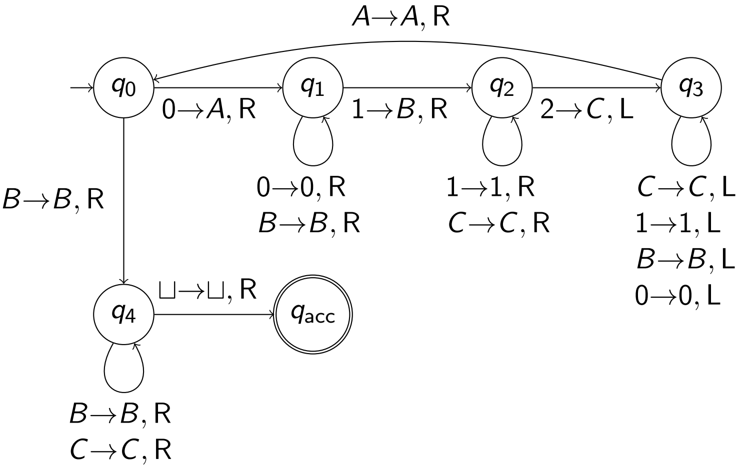

e.g.: $q_0 001122 \vdash^* A0Bq_3 1C2 \vdash^* q_3 A0B1C2 \vdash Aq_0 0B1C2$
$\vdash^* AAq_0 BBCC \vdash^* AABBCCq_4 \sqcup \vdash AABBCC\sqcup q_{acc} \sqcup$