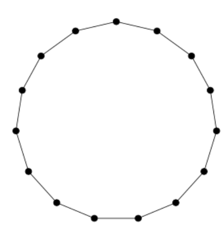
BBM462 Exercises for Weeks 2-3

21 February 2022

1. With the goal of representing real world networks:
   (a) What are the disadvantages of the $G(n,p)$ model (Erdős - Rényi model)? In which ways is it successful in representing them?
   (b) What is the advantage of the small-world model (Watts - Strogatz) compared to the $G(n,p)$ model?
   (c) What are the disadvantages of the grid graph? In which way is it successful in representing them?

2. In real-world networks, what is a reasonable estimation for
   (a) the *average* length of all shortest paths? (of the order of 10^1 or 10^2 or higher?)
   (b) the diameter of the graph as a function of n (say $n-1$ or $\sqrt{n}$ or...)?

3. A network consists of a cycle on n nodes, where n is odd. Show that all nodes have the same closeness centrality as a function of n.

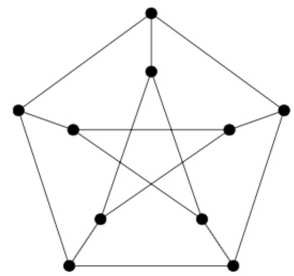


4. Consider an undirected tree of n nodes. Suppose that a particular node in the tree has degree k, so that its removal would divide the tree into k disjoint subtrees, and suppose that the orders (node number) of those subtrees are $n_1, \ldots, n_k$. Show that the unnormalized betweenness centrality x of this node is given by the expression below.

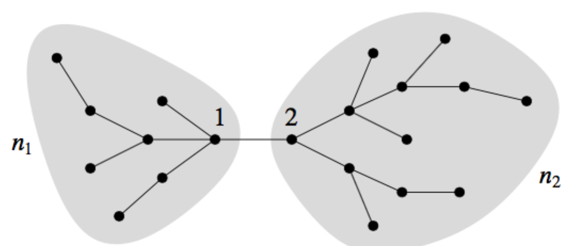
$$x = n^2 - \sum_{m=1}^{k} n_m^2.$$

5. Use the previous question to calculate the unnormalized betweenness centrality of the ith node from the end of a path of n nodes.

6. Calculate the closeness centrality of each node in the following network. (Note: This is a special graph, called Petersen graph.)



7. Consider an undirected tree of n nodes. A particular edge in the tree joins nodes 1 and 2 and divides the tree into two disjoint subgraphs of n_1 and n_2 nodes as sketched below. Show that the closeness centralities C_1 and C_2 of the two nodes are related by



$$\frac{1}{C_1} + \frac{n_1}{n} = \frac{1}{C_2} + \frac{n_2}{n}.$$