Oscar Led

Math Pre Session

1

1.1 $\frac{x^{32}}{x^9 \cdot x^2} \cdot \frac{x^7}{x^2} = \frac{x^{32}}{x^{11}} \cdot \frac{x^7}{x^2} = x^{21} \cdot x^5 = x^{26}$

1.2

$8^2 \cdot 4^x \cdot 2^x = 8^4$

$4^x \cdot 2^x = 8^2$

$2^{2x} \cdot 2^x = 8^2$

$2^{(3x)} = 8^2$

$3x = \log_2 16$

$3x = 4$

$x = 4/3$

1.3 $\frac{x}{y} = 3 \therefore x^{-4} y^4 = \ldots$

$x \cdot y^{-1} = 3$

$x^4 y^{-4} = 3^4$

$x^{-4} y^4 = 3^{-4}$

1.4 $\frac{\sqrt{4^{15}}}{\sqrt{4^7}} = \frac{4^{15/2}}{\sqrt{4^{2 \cdot 7}}} = \frac{4^{15/2}}{4^{14/2}} = \sqrt{4} = 2$

1.5 a) true, b) true, c) false, d) false

1.6 solution set $\ln(x) \geq e$

$e^{\ln(x)} \geq e^e$

$x \geq e^e$

$x \in [e^e, \infty)$

2

2.1 $0^\circ C = 32^\circ F$ $100^\circ C = 212^\circ F$ when are they sharing same number

$\beta = \frac{212 - 32}{100} = 1.8$

$y = \alpha + \beta x$

$y = 32 + 1.8x$

$x = -40$

2.2 $f(x) = 3x - 12$ find y if $f(y) = 0$

$3y - 12 = 0$

$y = \frac{12}{3} = 4$

2.3 find all x values

$9^{x^2 - 6x + 2} = 81$

$3^{2(x^2 - 6x + 2)} = 3^4$

$2x^2 - 12x = 4$

$x(2x - 12) = 0$

$x_1 = 0$ or $x_2 = 6$

2.4 Annual GDP = 3%
how long to triple?

$X \cdot 1.03^n = 3X$

$1.03^n = 3$

$n = \log_{1.03} 3 \approx 37.17$ years

2.5 calculate

$\log_\pi\left(\frac{1}{\pi^5}\right) =$

$\log_\pi(\pi)^{-5} = -5 \log_\pi \pi = -5$

3

3.1 calculate sum

$\sum_{i=0}^{\infty}\left(\frac{1}{5^i} + 0.3^i\right) = \sum_{i=0}^{\infty}\frac{1}{5^i} + \sum_{i=0}^{\infty} 0.3^i \approx -0.179$

3.2 $\lim_{x\to 5}\frac{(x^2-25)}{(x-5)} = \lim_{x\to 5}\frac{(x+5)(x-5)}{x-5} = \lim_{x\to 5}(x+5) = 10$

3.3 Find slope of function $f(x) = x^3 - 4$ at $(-2, -12)$

$f'(x) = 3x^2$

$f'(-2) = 3(-2)^2 = 12$

$f'(-12) = 3(-12)^2 = 432$ slopes

3.4 derivative of

$f(x) = \frac{x^5+3}{x^2-1} = \frac{5x^4(x^2-1) - (x^5+3)\,2x}{(x^2-1)^2}$

$= \frac{3x^6 - 5x^4 - 6x}{x^4 - 2x^2 + 1}$

3.5 second derivative of

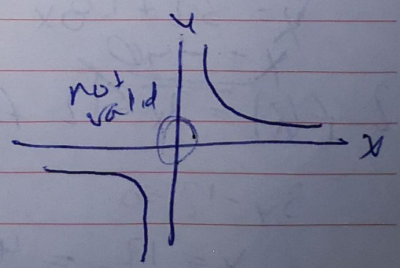
$f(x) = x^9 + 3$

$f'(x) = 9x^8$

$f''(x) = 72x^7$

3.6 is $f(x) = \frac{1}{x}$ continuous at 0? why.

you simply can't divide by zero!



3.7 local minima, maxima and inflection Points

$f(x) = 4x^3 - 12x$

$f'(x)\ 12x^2 - 12 = 0$

$x^2 = \frac{12}{12} \rightsquigarrow x^1 = 1$

$x^2 = 1 \rightsquigarrow x^2 = -1$

$f''(x) = 24x$

$f''(1) = 24$ convex ∴ min

$f''(-1) = -24$ concave ∴ max

$24x = 0$

$x = 0$ ≠ inflection point

3.8 $f(x,y) = x^3 - y^2$ calculate $f(2,3)$

$f(2,3) = 8 - 9 = -1$

3.9 $f(x,y) = \ln(x - 3y)$ combinations of x and y function defined in R^+

$x - 3y > 0 \rightarrow x > 3y$ x is bigger 3 times of y

3.10 $\frac{d}{dy}\left(x^5 y^7 + \frac{x^2}{y^3}\right) = 5x^4 y^7 + y^{-3} \cdot 2x$

3.11 local maxima or minima of $f(x,y) = \sqrt{xy} \rightarrow -y$

$\frac{df}{dx} = \frac{1}{2} \cdot \sqrt{y} \cdot \frac{1}{\sqrt{x}} - 1 = 0$

$\frac{df}{dy} = \frac{1}{2} \cdot \sqrt{x} \cdot \frac{1}{\sqrt{y}} - 1 = 0$

$\frac{\sqrt{y}}{2\sqrt{x}} = \frac{\sqrt{x}}{2\sqrt{y}}$

$\frac{y}{2} = \frac{x}{2}$

$x = y \rightarrow \frac{1}{2}\sqrt{x} \cdot \frac{1}{\sqrt{x}} \cdot 1 = 0 \quad \frac{1}{2} \cdot 1 \neq 0$

no minimum or maximum

3.12 Lagrange: $\max x^2 y^2$ s.t. $2x + y = 9$

$\alpha = x^2 y^2 - \lambda(2x + y - 9)$

$\frac{d\alpha}{dx} = 2xy^2 - 2\lambda = 0 \quad xy^2 = \lambda$

$\frac{d\alpha}{dy} = 2yx^2 - \lambda = 0 \quad 2yx^2 = \lambda$

$y = 2x$

$\frac{d\alpha}{d\lambda} = 2x + y - 9 = 0$

$2x + 2x - 9 = 0$

$4x = 9$

$x = 9/4$ ∴ $y = \frac{18}{4}$

4.1 $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 1 \\ 9 & 1 & 5 \end{bmatrix}$ $B \cdot A = \begin{bmatrix} 9 & 11 \\ 55 & 76 \end{bmatrix}$

4.2 $A \cdot B$

A \ B	0	-1	0
	2	1	2
5 3	46	23	6
0 1	2	1	2
1 2	12	6	4

4.3 $\begin{bmatrix} e & 93 & 4.7 \\ 2 & 6.1 & 4.22 \\ 4 & \pi & 0 \end{bmatrix}^T = \begin{bmatrix} e & 2 & 4 \\ 93 & 6.1 & \pi \\ 4.7 & 4.22 & 0 \end{bmatrix}$

4.4 Determinant $\begin{bmatrix} 2 & 6 \\ 2 & 8 \end{bmatrix}$ $2 \cdot 8 - 6 \cdot 2 = 4$

5

5.1 Ω of throwing dice twice

$\Omega = \{(1,1),(2,1),(3,1),(4,1),(5,1),(6,1),$
$(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),$
$(1,3),(2,3),(3,3),(4,3),(5,3),(6,3),$
$(1,4),(2,4),(3,4),(4,4),(5,4),(6,4),$
$(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),$
$(1,6),(2,6),(3,6),(4,6),(5,6),(6,6)\}$

5.2 drug user 0.1% $P(D_1)$ — non users 99.9% $P(D_2)$

	pos	neg
drug user	98% $P(A/D_1)$	2%
non users	0.3% $P(A/D_2)$	99.7%

$P(A) = .1\% \cdot 98\% + 99.9 \cdot 0.3 = 39.77$

Pos result is of a drug user

$P(D_1|A) = \frac{.1 \cdot 98}{39.77} = .246$

5.3 toss Dice 20 times
How many times do you get 5?

$E = n \cdot p = 20 \cdot \frac{1}{6} = \frac{20}{6} = \boxed{\frac{10}{3}}$