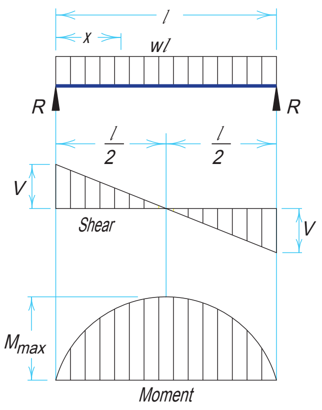
BEAM DIAGRAMS AND FORMULAS
For Various Static Loading Conditions

For meaning of symbols, see page 4-187

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . . $= wl$

$R = V$ . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

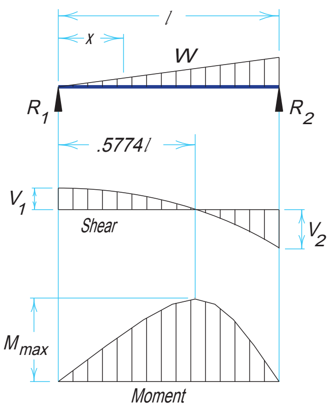
M_{max} (at center) . . . . $= \frac{wl^2}{8}$

M_x . . . . $= \frac{wx}{2}(l - x)$

Δ_{max} (at center) . . . . $= \frac{5wl^4}{384EI}$

Δ_x . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END



Total Equiv. Uniform Load . . . . . . $= \frac{16W}{9\sqrt{3}} = 1.0264W$

$R_1 = V_1$ . . . . $= \frac{W}{3}$

$R_2 = V_{2\ max}$ . . . . $= \frac{2W}{3}$

V_x . . . . $= \frac{W}{3} - \frac{Wx^2}{l^2}$

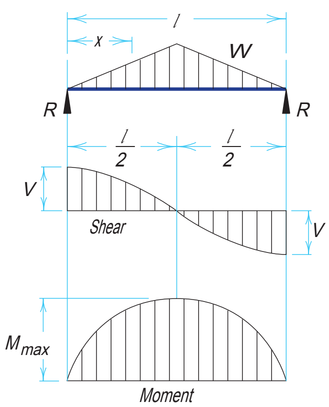
M_{max} (at $x = \frac{l}{\sqrt{3}} = .5774\,l$) . . . . $= \frac{2Wl}{9\sqrt{3}} = .1283\,Wl$

M_x . . . . $= \frac{Wx}{3l^2}(l^2 - x^2)$

Δ_{max} (at $x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = .5193\,l$) . . $= 0.01304\,\frac{Wl^3}{EI}$

Δ_x . . . . $= \frac{Wx}{180EIl^2}(3x^4 - 10l^2x^2 + 7l^4)$

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER



Total Equiv. Uniform Load . . . . . . $= \frac{4W}{3}$

$R = V$ . . . . $= \frac{W}{2}$

V_x (when $x < \frac{l}{2}$) . . . . $= \frac{W}{2l^2}(l^2 - 4x^2)$

M_{max} (at center) . . . . $= \frac{Wl}{6}$

M_x (when $x < \frac{l}{2}$) . . . . $= Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$

Δ_{max} (at center) . . . . $= \frac{Wl^3}{60EI}$

Δ_x (when $x < \frac{l}{2}$) . . . . $= \frac{Wx}{480EIl^2}(5l^2 - 4x^2)^2$

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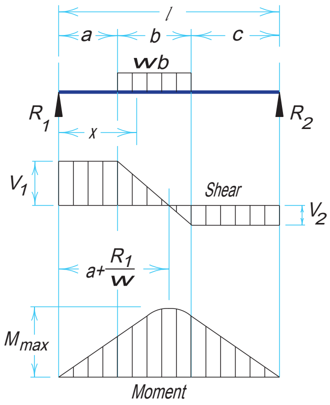
4. SIMPLE BEAM—UNIFORMLY LOAD PARTIALLY DISTRIBUTED



$R_1 = V_1$ (max. when $a < c$) . . . . $= \dfrac{wb}{2l}(2c + b)$

$R_2 = V_2$ (max. when $a > c$) . . . . $= \dfrac{wb}{2l}(2a + b)$

V_x (when $x > a$ and $< (a + b)$) . . . $= R_1 - w(x - a)$

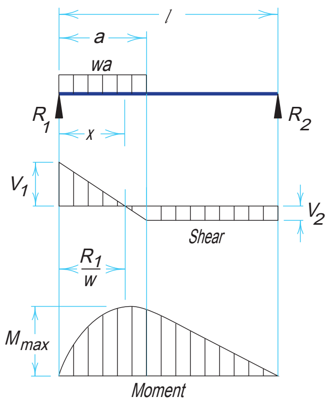
$M_{\max}$ $\left(\text{at } x = a + \dfrac{R_1}{w}\right)$ . . . . $= R_1\left(a + \dfrac{R_1}{2w}\right)$

M_x (when $x < a$) . . . . $= R_1 x$

M_x (when $x > a$ and $< (a + b)$) . . . $= R_1 x - \dfrac{w}{2}(x - a)^2$

M_x (when $x > (a + b)$) . . . . $= R_2(l - x)$

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



$R_1 = V_{1\ \max}$ . . . . $= \dfrac{wa}{2l}(2l - a)$

$R_2 = V_2$ . . . . $= \dfrac{wa^2}{2l}$

V_x (when $x < a$) . . . . $= R_1 - wx$

$M_{\max}$ $\left(\text{at } x = \dfrac{R_1}{w}\right)$ . . . . $= \dfrac{R_1^2}{2w}$

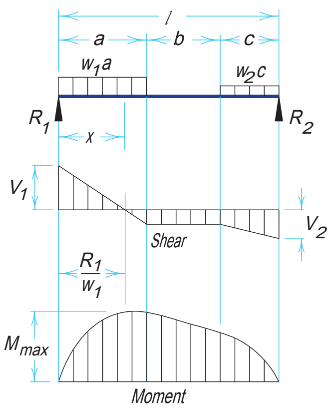
M_x (when $x < a$) . . . . $= R_1 x - \dfrac{wx^2}{2}$

M_x (when $x > a$) . . . . $= R_2\,(l - x)$

Δ_x (when $x < a$) . . . . $= \dfrac{wx}{24EIl}\left(a^2(2l - a)^2 - 2ax^2(2l - a) + lx^3\right)$

Δ_x (when $x > a$) . . . . $= \dfrac{wa^2(l - x)}{24EIl}(4xl - 2x^2 - a^2)$

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END



$R_1 = V_1$ . . . . $= \dfrac{w_1 a(2l - a) + w_2 c^2}{2l}$

$R_2 = V_2$ . . . . $= \dfrac{w_2 c(2l - c) + w_1 a^2}{2l}$

V_x (when $x < a$) . . . . $= R_1 - w_1 x$

V_x (when $x > a$ and $< (a + b)$) $= R_1 - w_1 a$

V_x (when $x > (a + b)$) . . . . $= R_2 - w_2\,(l - x)$

$M_{\max}$ $\left(\text{at } x = \dfrac{R_1}{w_1} \text{ when } R_1 < w_1 a\right)$ $= \dfrac{R_1^2}{2w_1}$

$M_{\max}$ $\left(\text{at } x = l - \dfrac{R_1}{w_2} \text{ when } R_2 < w_2 c\right)$ $= \dfrac{R_2^2}{2w_2}$

M_x (when $x < a$) . . . . $= R_1 x - \dfrac{w_1 x^2}{2}$

M_x (when $x > a$ and $< (a + b)$) . . . $= R_1 x - \dfrac{w_1 a}{2}(2x - a)$

M_x (when $x > (a + b)$) . . . . $= R_2(l - x) - \dfrac{w_2(l - x)^2}{2}$

BEAM DIAGRAMS AND FORMULAS
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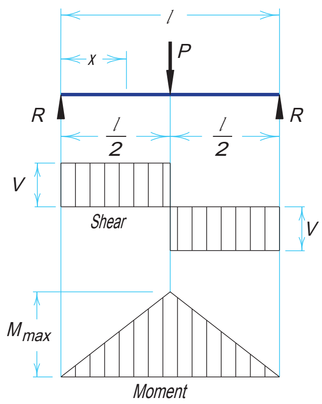
7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load . . . . $= 2P$

$R = V$ . . . . $= \dfrac{P}{2}$

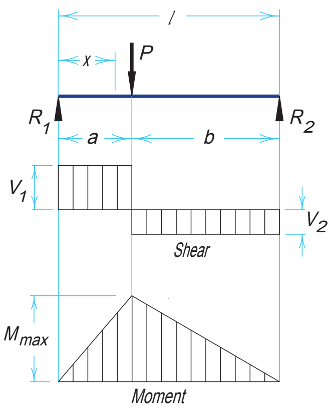
$M_{\max}$ (at point of load) . . . . $= \dfrac{Pl}{4}$

M_x $\left(\text{when } x < \dfrac{1}{2}\right)$ . . . . $= \dfrac{Px}{2}$

$\Delta_{\max}$ (at point of load) . . . . $= \dfrac{Pl^3}{48EI}$

Δ_x $\left(\text{when } x < \dfrac{1}{2}\right)$ . . . . $= \dfrac{Px}{48EI}(3l^2 - 4x^2)$

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load . . . . $= \dfrac{8Pab}{l^2}$

$R_1 = V_1$ (max when $a < b$) . . . . $= \dfrac{Pb}{l}$

$R_2 = V_2$ (max when $a > b$) . . . . $= \dfrac{Pa}{l}$

$M_{\max}$ (at point of load) . . . . $= \dfrac{Pab}{l}$

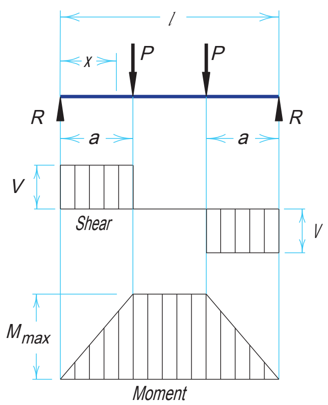
M_x (when $x < a$) . . . . $= \dfrac{Pbx}{l}$

$\Delta_{\max}$ $\left(\text{at } x = \sqrt{\dfrac{a(a+2b)}{3}} \text{ when } a > b\right)$ . . . $= \dfrac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$

Δ_a (at point of load) . . . . $= \dfrac{Pa^2b^2}{3EIl}$

Δ_x (when $x < a$) . . . . $= \dfrac{Pbx}{6EIl}(l^2 - b^2 - x^2)$

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



Total Equiv. Uniform Load . . . . $= \dfrac{8Pa}{l}$

$R = V$ . . . . $= P$

$M_{\max}$ (between loads) . . . . $= Pa$

M_x (when $x < a$) . . . . $= Px$

$\Delta_{\max}$ (at center) . . . . $= \dfrac{Pa}{24EI}(3l^2 - 4a^2)$

Δ_x (when $x < a$) . . . . $= \dfrac{Px}{6EI}(3la - 3a^2 - x^2)$

Δ_x (when $x > a$ and $< (l - a)$) . . . . $= \dfrac{Pa}{6EI}(3lx - 3x^2 - a^2)$

BEAM DIAGRAMS AND FORMULAS
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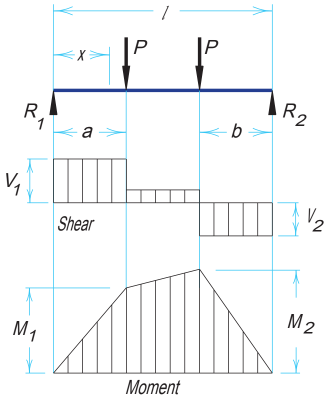
10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ (max. when $a < b$) . . . . $= \frac{P}{l}(l - a + b)$

$R_2 = V_2$ (max. when $a > b$) . . . . $= \frac{P}{l}(l - b + a)$

V_x (when $x > a$ and $< (l - b)$) . . . . . $= \frac{P}{l}(b - a)$

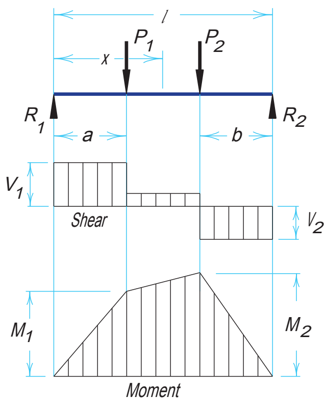
M_1 (max. when $a > b$) . . . . $= R_1 a$

M_2 (max. when $a < b$) . . . . $= R_2 b$

M_x (when $x < a$) . . . . $= R_1 x$

M_x (when $x > a$ and $< (l - b)$) . . . . . $= R_1 x - P(x - a)$

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ . . . . $= \frac{P_1(l - a) + P_2 b}{l}$

$R_2 = V_2$ . . . . $= \frac{P_1 a + P_2(l - b)}{l}$

V_x (when $x > a$ and $< (l - b)$) . . . . . $= R_1 - P_1$

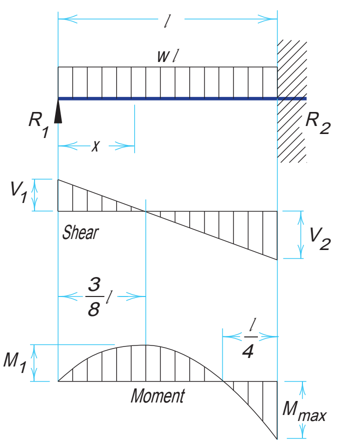
M_1 (max. when $R_1 < P_1$) . . . . $= R_1 a$

M_2 (max. when $R_2 < P_2$) . . . . $= R_2 b$

M_x (when $x < a$) . . . . $= R_1 x$

M_x (when $x > a$ and $< (l - b)$) . . . . . $= R_1 x - P(x - a)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= wl$

$R_1 = V_1$ . . . . $= \frac{3wl}{8}$

$R_2 = V_{2\,max}$ . . . . $= \frac{5wl}{8}$

V_x . . . . $= R_1 - wx$

M_{max} . . . . $= \frac{wl^2}{8}$

M_1 $\left(\text{at } x = \frac{3}{8}l\right)$ . . . . $= \frac{9}{128}wl^2$

M_x . . . . $= R_1 x - \frac{wx^2}{2}$

Δ_{max} $\left(\text{at } x = \frac{l}{16}(1 + \sqrt{33}) = .4215\, l\right)$ . . . $= \frac{wl^4}{185\,EI}$

Δ_x . . . . $\frac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$

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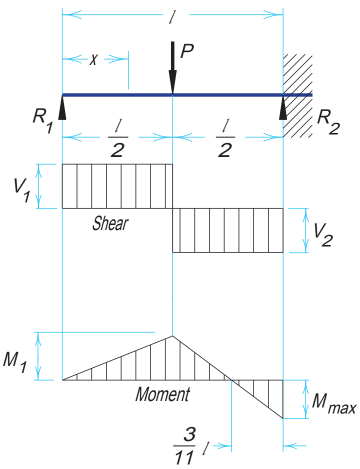
13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load . . . . $= \dfrac{3P}{2}$

$R_1 = V_1$ . . . . $= \dfrac{5P}{15}$

$R_2 = V_{2\ \max}$ . . . . $= \dfrac{11P}{16}$

$M_{\max}$ (at fixed end) . . . . $= \dfrac{3Pl}{16}$

M_1 (at point of load) . . . . $= \dfrac{5Pl}{32}$

M_x $\left(\text{when } x < \dfrac{l}{2}\right)$ . . . . $= \dfrac{5Px}{16}$

M_x $\left(\text{when } x > \dfrac{l}{2}\right)$ . . . . $= P\left(\dfrac{l}{2} - \dfrac{11x}{16}\right)$

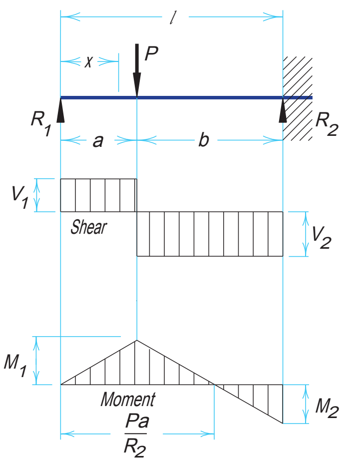
$\Delta_{\max}$ $\left(\text{at } x = l\sqrt{\dfrac{1}{5}} = .4472\,l\right)$ . . . . $= \dfrac{Pl^3}{48EI\sqrt{5}} = .009317\dfrac{Pl^3}{EI}$

Δ_x (at point of load) . . . . $= \dfrac{7PL^3}{768EI}$

Δ_x $\left(\text{when } x < \dfrac{l}{2}\right)$ . . . . $= \dfrac{Px}{96EI}(3l^2 - 5x^2)$

Δ_x $\left(\text{when } x > \dfrac{l}{2}\right)$ . . . . $= \dfrac{P}{96EI}(x - l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ . . . . $= \dfrac{Pb^2}{2l^3}(a + 2l)$

$R_2 = V_2$ . . . . $= \dfrac{Pa}{2l^3}(3l^2 - a^2)$

M (at point of load) . . . . $= R_1 a$

M_2 (at fixed end) . . . . $= \dfrac{Pab}{2l^2}(a + l)$

M_x (when $x < a$) . . . . $= R_1 x$

M_x (when $x > a$) . . . . $= R_1 x - P(x - a)$

$\Delta_{\max}$ $\left(\text{when } a < .414\,l \text{ at } x = l\dfrac{(l^2 + a^2)}{(3l^2 - a^2)}\right)$ $= \dfrac{Pa(l^2 + a^2)^3}{3EI(3l^2 - a^2)^2}$

$\Delta_{\max}$ $\left(\text{when } a > .414\,l \text{ at } x = l\sqrt{\dfrac{a}{2l + a}}\right)$ $= \dfrac{Pab^2}{6EI}\sqrt{\dfrac{a}{2l + a}}$

Δ_a (at point of load) . . . . $= \dfrac{Pa^2b^3}{12EIl^3}(3l + a)$

Δ (when $x < a$) . . . . $= \dfrac{Pb^2x}{12EIl^3}(3al^2 - 2lx^2 - ax^2)$

Δ_x (when $x > a$) . . . . $= \dfrac{Pa}{12EIl^2}(l - x)^2(3l^2x - a^2x - 2a^2l)$

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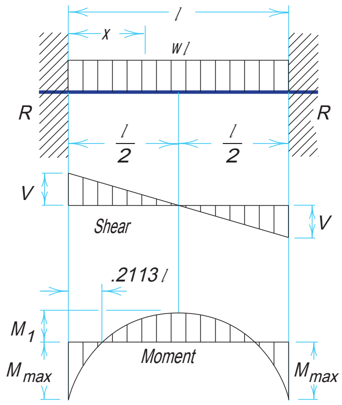
15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS



Total Equiv. Uniform Load . . . . $= \dfrac{2wl}{3}$

$R = V$ . . . . $= \dfrac{wl}{2}$

V_x . . . . $= w\left(\dfrac{l}{2} - x\right)$

$M_{\max}$ (at ends) . . . . $= \dfrac{wl^2}{12}$

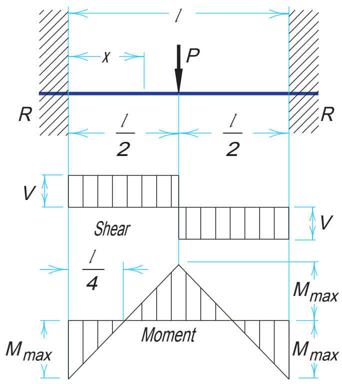
M_1 (at center) . . . . $= \dfrac{wl^2}{24}$

M_x . . . . $= \dfrac{w}{12}(6lx - l^2 - 6x^2)$

$\Delta_{\max}$ (at center) . . . . $= \dfrac{wl^4}{384EI}$

Δ_x . . . . $= \dfrac{wx^2}{24EI}(l - x)^2$

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load . . . . $= P$

$R = V$ . . . . $= \dfrac{P}{2}$

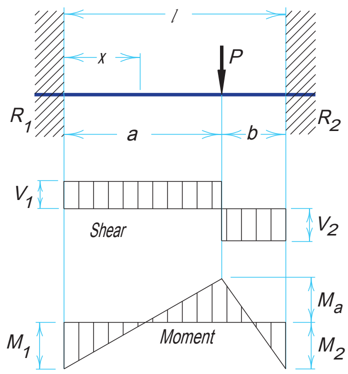
$M_{\max}$ (at center and ends) . . . . $= \dfrac{Pl}{8}$

M_x $\left(\text{when } x < \dfrac{l}{2}\right)$ . . . . $= \dfrac{P}{8}(4x - l)$

$\Delta_{\max}$ (at center) . . . . $= \dfrac{Pl^3}{192EI}$

Δ_x $\left(\text{when } x < \dfrac{l}{2}\right)$ . . . . $= \dfrac{Px^2}{48EI}(3l - 4x)$

17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ (max. when $a < b$) . . . . $= \dfrac{Pb^2}{l^3}(3a + b)$

$R_2 = V_2$ (max. when $a > b$) . . . . $= \dfrac{Pa^2}{l^3}(a + 3b)$

M_1 (max. when $a < b$) . . . . $= \dfrac{Pab^2}{l^2}$

M_2 (max. when $a > b$) . . . . $= \dfrac{Pa^2b}{l^2}$

M_a (at point of load) . . . . $= \dfrac{2Pa^2b^2}{l^3}$

M_x (when $x < a$) . . . . $= R_1x - \dfrac{Pab^2}{l^2}$

$\Delta_{\max}$ $\left(\text{when } a > b \text{ at } x = \dfrac{2al}{3a + b}\right)$ . . . . $= \dfrac{2Pa^3b^2}{3EI(3a + b)^2}$

Δ_a (at point of load) . . . . $= \dfrac{Pa^3b^3}{3EIl^3}$

Δ_x (when $x < a$) . . . . $= \dfrac{Pb^2x^2}{6EIl^2}(3al - 3ax - bx)$

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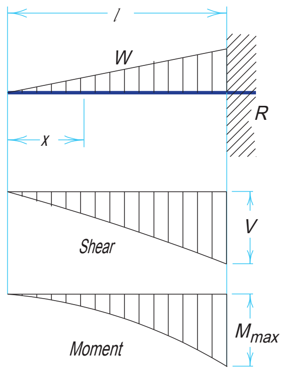
18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



Total Equiv. Uniform Load . . . . $= \frac{8}{3}W$

$R = V$ . . . . $= W$

V_x . . . . $= W\frac{x^2}{l^2}$

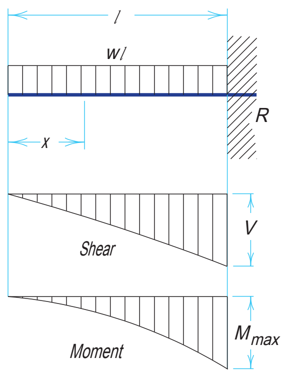
$M_{\max}$ (at fixed end) . . . . $= \frac{Wl}{3}$

M_x . . . . $= \frac{Wx^3}{3l^2}$

$\Delta_{\max}$ (at free end) . . . . $= \frac{Wl^3}{15EI}$

Δ_x . . . . $= \frac{W}{60EIl^2}(x^5 - 5l^4x + 4l^5)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= 4wl$

$R = V$ . . . . $= wl$

V_x . . . . $= wx$

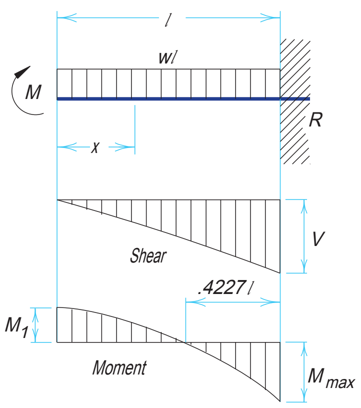
$M_{\max}$ (at fixed end) . . . . $= \frac{wl^2}{2}$

M_x . . . . $= \frac{wx^2}{2}$

$\Delta_{\max}$ (at free end) . . . . $= \frac{wl^4}{8EI}$

Δ_x . . . . $= \frac{w}{24EI}(x^4 - 4l^3x + 3l^4)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= \frac{8}{3}wl$

$R = V$ . . . . $= wl$

V_x . . . . $= wx$

$M_{\max}$ (at fixed end) . . . . $= \frac{wl^2}{3}$

M_x . . . . $= \frac{w}{6}(l^2 - 3x^2)$

$\Delta_{\max}$ (at deflected end) . . . . $= \frac{wl^4}{24EI}$

Δ_x . . . . $= \frac{w(l^2 - x^2)^2}{24EI}$

BEAM DIAGRAMS AND FORMULAS
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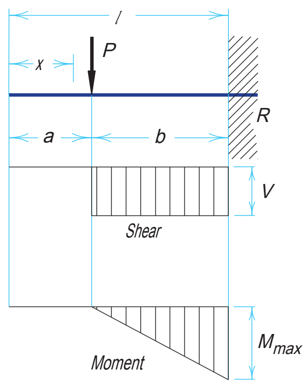
21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load . . . . $= \dfrac{8Pb}{l}$

$R = V$ . . . . $= P$

$M_{\max}$ (at fixed end) . . . . $= Pb$

M_x (when $x > a$) . . . . $= P(x - a)$

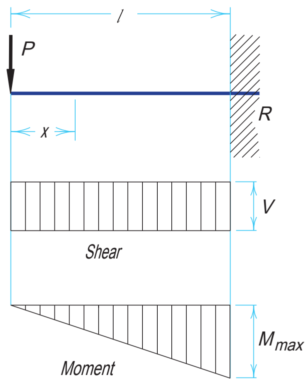
$\Delta_{\max}$ (at free end) . . . . $= \dfrac{Pb^2}{6EI}(3l - b)$

Δ_a (at point of load) . . . . $= \dfrac{Pb^3}{3EI}$

Δ_x (when $x < a$) . . . . $= \dfrac{Pb^2}{6EI}(3l - 3x - b)$

Δ_x (when $x > a$) . . . . $= \dfrac{P(l-x)^2}{6EI}(3b - l + x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



Total Equiv. Uniform Load . . . . $= 8P$

$R = V$ . . . . $= P$

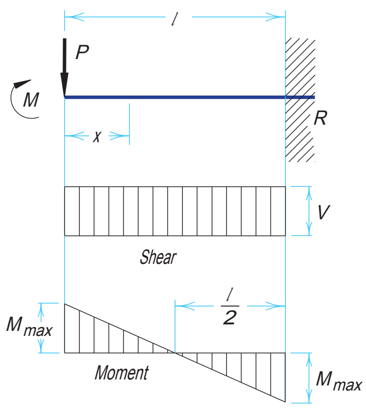
$M_{\max}$ (at fixed end) . . . . $= Pl$

M_x . . . . $= Px$

$\Delta_{\max}$ (at free end) . . . . $= \dfrac{Pl^3}{3EI}$

Δ_x . . . . $= \dfrac{P}{6EI}(2l^3 - 3l^2x + x^3)$

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END



Total Equiv. Uniform Load . . . . $= 4P$

$R = V$ . . . . $= P$

$M_{\max}$ (at both ends) . . . . $= \dfrac{Pl}{2}$

M_x . . . . $= P\left(\dfrac{l}{2} - x\right)$

$\Delta_{\max}$ (at deflected end) . . . . $= \dfrac{pl^3}{12EI}$

Δ_x . . . . $= \dfrac{P(l-x)^2}{12EI}(l + 2x)$

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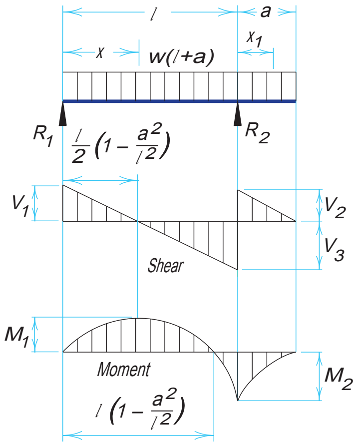
24. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD



$R_1 = V_1$ . . . . $= \dfrac{w}{2l}(l^2 - a^2)$

$R_2 = V_2 + V_3$ . . . . $= \dfrac{w}{2l}(l + a)^2$

V_2 . . . . $= wa$

V_3 . . . . $= \dfrac{w}{2l}(l^2 + a^2)$

V_x (between supports) . . . . . $= R_1 - wx$

V_{x_1} (for overhang) . . . . $= w(a - x_1)$

$M_1 \left(\text{at } x = \dfrac{l}{2}\left[1 - \dfrac{a^2}{l^2}\right]\right)$ . . . . . $= \dfrac{w}{8l^2}(l + a)^2(l - a)^2$

M_2 (at R_2) . . . . $= \dfrac{wa^2}{2}$

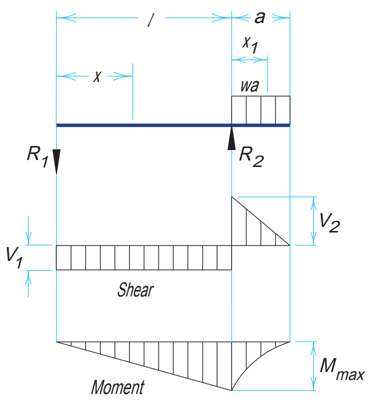
M_x (between supports) . . . . . $= \dfrac{wx}{2l}(l^2 - a^2 - xl)$

M_{x_1} (for overhang) . . . . $= \dfrac{w}{2}(a - x_1)^2$

Δ_x (between supports) . . . . . $= \dfrac{wx}{24EIl}(l^4 - 2l^2x^2 + lx^3 - 2a^2l^2 + 2a^2x^2)$

Δ_{x_1} (for overhang) . . . . $= \dfrac{wx_1}{24EI}(4a^2l - l^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)$

25. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD ON OVERHANG



$R_1 = V_1$ . . . . $= \dfrac{wa^2}{2l}$

$R_2 \quad V_1 + V_2$ . . . . $= \dfrac{wa}{2l}(2l + a)$

V_2 . . . . $= wa$

V_{x_1} (for overhang) . . . . $= w(a - x_1)$

M_{max} (at R_2) . . . . $= \dfrac{wa^2}{2}$

M_x (between supports) . . . . . $= \dfrac{wa^2x}{2l}$

M_{x_1} (for overhang) . . . . $= \dfrac{w}{2}(a - x_1)^2$

$\Delta_{max}\left(\text{between supports at } x = \dfrac{l}{\sqrt{3}}\right) = \dfrac{wa^2l^2}{18\sqrt{3}EI} = 0.03208\dfrac{wa^2l^2}{EI}$

Δ_{max} (for overhang at $x_1 = a$) . . . $= \dfrac{wa^3}{24EI}(4l + 3a)$

Δ_x (between supports) . . . . . $= \dfrac{wa^2x}{12EIl}(l^2 - x^2)$

Δ_{x_1} (for overhang) . . . . $= \dfrac{wx_1}{24EI}(4a^2l + 6a^2x_1 - 4ax_1^2 + x_1^3)$

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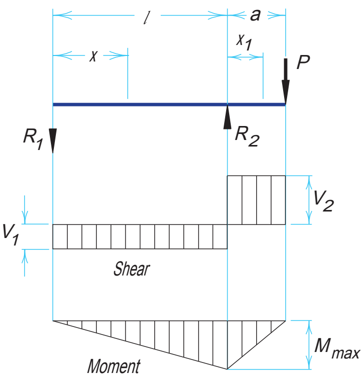
26. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT END OF OVERHANG



$R_1 = V_1$ . . . . $= \dfrac{Pa}{l}$

$R_2 = V_1 + V_2$ . . . . $= \dfrac{P}{l}(l + a)$

V_2 . . . . $= P$

M_{max} (at R_2) . . . . $= Pa$

M_x (between supports) . . . . $= \dfrac{Pax}{l}$

M_{x_1} (for overhang) . . . . $= P(a - x_1)$

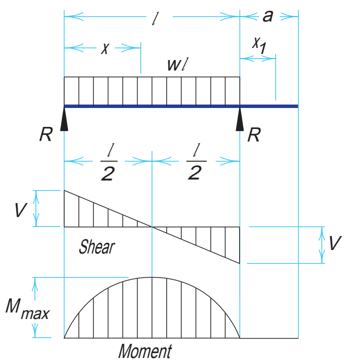
Δ_{max} $\left(\text{between supports at } x = \dfrac{l}{\sqrt{3}}\right)$ . . . . . $= \dfrac{Pal^2}{9\sqrt{3}\,EI} = .06415\dfrac{Pal^2}{EI}$

Δ_{max} (for overhang at $x_1 = a$) . . . . $= \dfrac{Pa^2}{3EI}(l + a)$

Δ_x (between supports) . . . . $= \dfrac{Pax}{6EIl}(l^2 - x^2)$

Δ_{x_1} (for overhang) . . . . $= \dfrac{Px_1}{6EI}(2al + 3ax_1 - x_1^2)$

27. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS



Total Equiv. Uniform Load . . . . $= wl$

$R = V$ . . . . $= \dfrac{wl}{2}$

V_x . . . . $= w\left(\dfrac{l}{2} - x\right)$

M_{max} (at center) . . . . $= \dfrac{wl^2}{8}$

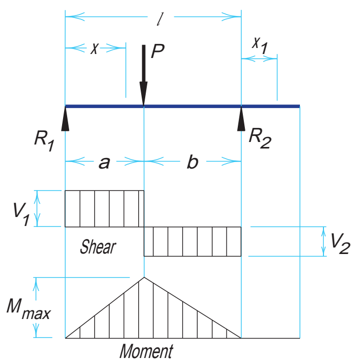
M_x . . . . $= \dfrac{wx}{2}(l - x)$

Δ_{max} (at center) . . . . $= \dfrac{5wl^4}{384EI}$

Δ_x . . . . $= \dfrac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

Δ_{x_1} . . . . $= \dfrac{wl^3x_1}{24EI}$

28. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS



Total Equiv. Uniform Load . . . . $= \dfrac{8Pab}{l^2}$

$R_1 = V_1$ (max. when $a < b$) . . . . $= \dfrac{Pb}{l}$

$R_2 = V_2$ (max. when $a > b$) . . . . $= \dfrac{Pa}{l}$

M_{max} (at point of load) . . . . $= \dfrac{Pab}{l}$

M_x (when $x < a$) . . . . $= \dfrac{Pbx}{l}$

Δ_{max} $\left(\text{at } x = \sqrt{\dfrac{a(a + 2b)}{3}} \text{ when } a > b\right)$ . . . $= \dfrac{Pab(a + 2b)\sqrt{3a(a + 2b)}}{27EIl}$

Δ_a (at point of load) . . . . $= \dfrac{Pa^2b^2}{3EIl}$

Δ_x (when $x < a$) . . . . $= \dfrac{Pbx}{6EIl}(l^2 - b^2 - x^2)$

Δ_x (when $x > a$) . . . . $= \dfrac{Pa(l - x)}{6EIl}(2lx - x^2 - a^2)$

Δ_{x_1} . . . . $= \dfrac{Pabx_1}{6EIl}(l + a)$

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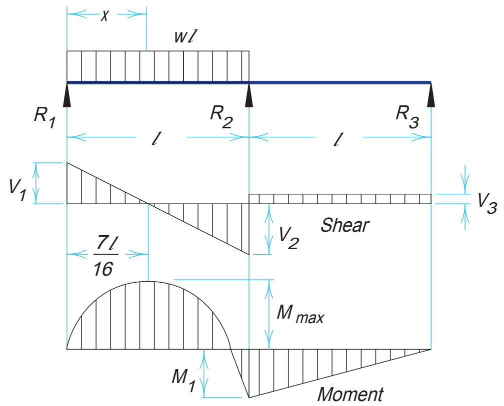
29. CONTINUOUS BEAM—TWO EQUAL SPANS—UNIFORM LOAD ON ONE SPAN



Total Equiv. Uniform Load $= \frac{49}{64} wl$

$R_1 = V_1$ . . . . $= \frac{7}{16} wl$

$R_2 = V_2 + V_3$ . . . . $= \frac{5}{8} wl$

$R_3 = V_3$ . . . . $= -\frac{1}{16} wl$

V_2 . . . . $= \frac{9}{16} wl$

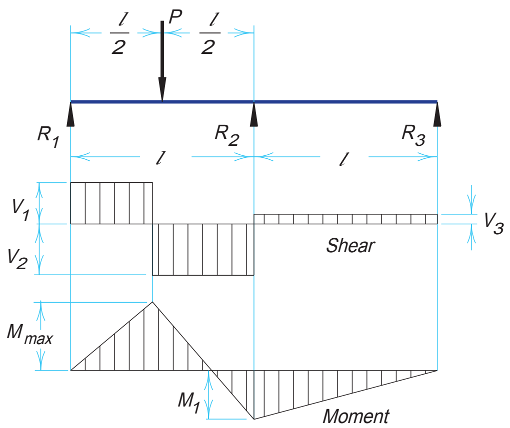
$M_{max} \left(\text{at } x = \frac{7}{16} l\right)$ . . . . . $= \frac{49}{512} wl^2$

M_1 (at support R_2) . . . . $= \frac{1}{16} wl^2$

M_x (when $x < l$) . . . . . $= \frac{wx}{16}(7l - 8x)$

Δ_{max} (at 0.472 l from R_1) . . $= .0092\, wl^4 / EI$

30. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT CENTER OF ONE SPAN



Total Equiv. Uniform Load $= \frac{13}{8} P$

$R_1 = V_1$ . . . . $= \frac{13}{32} P$

$R_2 = V_2 + V_3$ . . . . $= \frac{11}{16} P$

$R_3 = V_3$ . . . . $= -\frac{3}{32} P$

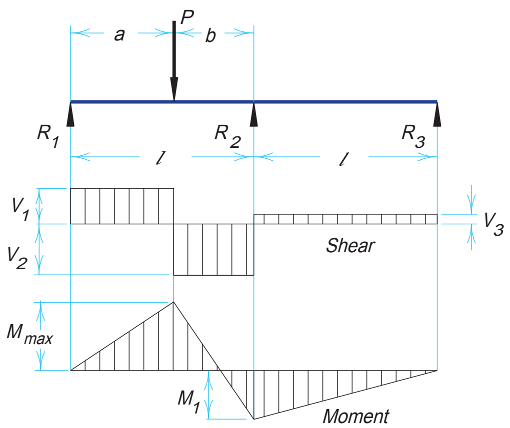
V_2 . . . . $= \frac{19}{32} P$

M_{max} (at point of load) . . . $= \frac{13}{64} Pl$

M_1 (at support R_2) . . . . $= \frac{3}{32} Pl$

Δ_{max} (at 0.480 l from R_1) . . $= .015\, Pl^3 / EI$

31. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ . . . . $= \frac{Pb}{4l^3}(4l^2 - a(l + a))$

$R_2 = V_2 + V_3$ . . . . $= \frac{Pa}{2l^3}(2l^2 + b(l + a))$

$R_3 = V_3$ . . . . $= -\frac{Pab}{4l^3}(l + a)$

V_2 . . . . $= \frac{Pa}{4l^3}(4l^2 + b(l + a))$

M_{max} (at point of load) . . . $= \frac{Pab}{4l^3}(4l^2 - a(l + a))$

M_1 (at support R_2) . . . . $= \frac{Pab}{4l^2}(l + a)$

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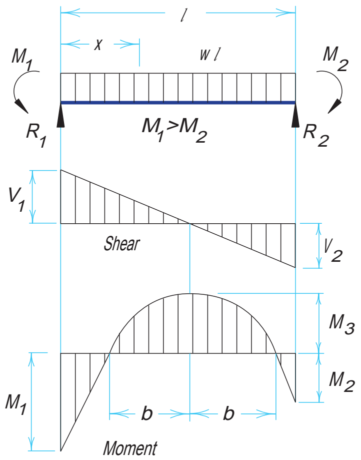
32. BEAM—UNIFORMLY DISTRIBUTED LOAD AND VARIABLE END MOMENTS



$$R_1 = V_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad = \frac{wl}{2} + \frac{M_1 - M_2}{l}$$

$$R_2 = V_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad = \frac{wl}{2} - \frac{M_1 - M_2}{l}$$

$$V_x \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad = w\left(\frac{l}{2} - x\right) + \frac{M_1 - M_2}{l}$$

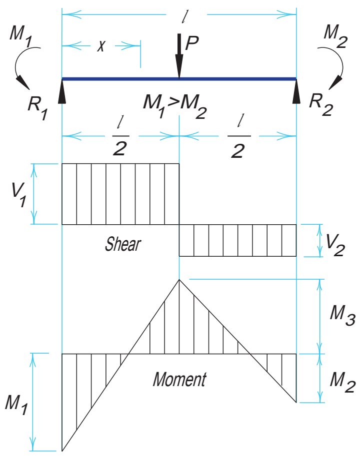
$$M_3 \left(\text{at } x = \frac{l}{2} + \frac{M_1 - M_2}{wl}\right) \; . \; . \quad = \frac{wl^2}{8} - \frac{M_1 + M_2}{2} + \frac{(M_1 - M_2)^2}{2wl^2}$$

$$M_x \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad = \frac{wx}{2}(l - x) + \left(\frac{M_1 - M_2}{l}\right)x - M_1$$

$$b \quad \text{(to locate inflection points)} = \sqrt{\frac{l^2}{4} - \left(\frac{M_1 + M_2}{w}\right) + \left(\frac{M_1 - M_2}{wl}\right)^2}$$

$$\Delta_x = \frac{wx}{24EI}\left[x^3 - \left(2l + \frac{4M_1}{wl} - \frac{4M_2}{wl}\right)x^2 + \frac{12M_1}{w}x + l^3 - \frac{8M_1 l}{w} - \frac{4M_2 l}{w}\right]$$

33. BEAM—CONCENTRATED LOAD AT CENTER AND VARIABLE END MOMENTS



$$R_1 = V_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad = \frac{P}{2} + \frac{M_1 - M_2}{l}$$

$$R_2 = V_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad = \frac{P}{2} - \frac{M_1 - M_2}{l}$$

$$M_3 \text{ (at center)} \; . \; . \; . \; . \; . \; . \; . \; . \quad = \frac{Pl}{4} - \frac{M_1 + M_2}{2}$$

$$M_x \left(\text{when } x < \frac{l}{2}\right) \; . \; . \; . \; . \; . \; . \quad = \left(\frac{P}{2} + \frac{M_1 - M_2}{l}\right)x - M_1$$

$$M_x \left(\text{when } x > \frac{l}{2}\right) \; . \; . \; . \; . \; . \; . \quad = \frac{P}{2}(l - x) + \frac{(M_1 - M_2)x}{l} - M_1$$

$$\Delta_x \left(\text{when } x < \frac{l}{2}\right) = \frac{Px}{48EI}\left(3l^2 - 4x^2 - \frac{8(l - x)}{Pl}\left[M_1(2l - x) + M_2(l + x)\right]\right)$$