CENG 222

Statistical Methods for Computer Engineering

Spring 2019-2020 Homework 1 version 1

Due date: 01 03 2020, Sunday, 23:59

Introduction

In this assignment, there are 4 questions related to the 2^{nd} chapter and the beginning of the 3^{rd} chapter in your text book. While answering the questions, please **show your work** and the steps of your calculations. Give an explanation about what numbers mean in those steps. Otherwise, you may not get any point.

Questions

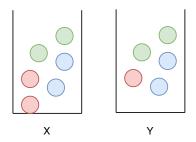


Figure 1: The boxes and balls mentioned in question 1.

Q1. (25 pts.) In this question we have two boxes named X and Y. There are 2 red balls, 2 blue balls and 2 green balls in box X and there are 1 red ball, 2 blue balls and 2 green balls in box Y. Figure 1 illustrates this.

We select a ball by following this procedure:

- 1. We randomly choose the box that we are going to pick from. The probability that we pick from the box X is 0.4.
- 2. We randomly pick one ball from the chosen box.

Considering the procedure above, please answer the following questions.

- a) (5 pts.) What is the probability that we pick a green ball given that the selected box is X?
- **b)** (8 pts.) What is the probability that we pick a red ball?
- c) (12 pts.) What is the probability that we had chosen the box Y given that the ball we picked is blue?

- Q2. (25 pts.) Suppose that A, B and C are events.
- a) (10 pts.) Prove or disprove the statement "A and B are mutually exclusive if and only if \overline{A} and \overline{B} are exhaustive".
- b) (15 pts.) Prove or disprove the statement "A, B and C are mutually exclusive if and only if \overline{A} , \overline{B} and \overline{C} are exhaustive".
- Q3. (25 pts.) Suppose that we are playing a game with 5 fair dice where each die has 6 possible outcomes: 1, 2, 3, 4, 5, 6. We roll all of the dice at once and when a die has a value greater than or equal to 5, than we say that that die is successful.

(Example: If the values of the dice are 4, 6, 1, 5, 4, then we have 2 successful dice (6 and 5).)

- a) (13 pts.) What is the probability that we have **exactly** 2 successful dice?
- b) (12 pts.) What is the probability that we have at least 2 successful dice?

Q4. (25 pts.) The joint probability table for binary (takes only two values, 0 or 1) random variables A, B and C is given in Table 1. Using that table, please do the followings.

a	\mathbf{b}	\mathbf{c}	P(A=a, B=b, C=c)
0	0	0	0.14
0	0	1	0.08
0	1	0	0.21
0	1	1	0.02
1	0	0	0.06
1	0	1	0.32
1	1	0	0.09
1	1	1	0.08

Table 1: Joint probability table of random variables A, B and C.

- a) (4 pts.) Calculate P(A=1, C=0).
- **b)** (4 pts.)Calculate P(B=1).
- c) (7 pts.) Show whether random variables A and B are independent or not.
- **d)** (10 pts.) Given C=1, show whether random variables A and B are conditionally independent or not.

Specifications

- You are expected to write your answers in LaTeX format. You can use the given template.
- Please do not skip the calculation steps. Show every step of your work.
- You have a total of 3 late days for this homework. For each day you have submitted late, you will lose 20 points. The homeworks you submit after 3 late days will not be graded.
- Cheating is forbidden. The violators will be punished according to the department regulations.
- Follow the course page on COW for any updates and clarifications. Please ask your questions on COW instead of e-mailing if they do not contain some part of the solution. If they contain, you can send an email to "artun@ceng.metu.edu.tr".

Submission

Submissions will be done via ODTUCLASS. If you do not have access to ODTUCLASS for some reason, please send an email to assistants about that. You are expected to submit a zip file named "hw1.zip" that contains both your latex source and also the compiled version of it in pdf format.