Middle East Technical University | Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall '2019-2020

Take Home Exam 4

Due date: December 4 2019, Wednesday , 23:55

Question 1

Solve the following and explain your answers:

a) How many bit strings of length 9 are there such that every 1 is followed immediately by a 0?

b) How many bit strings of length 10 have at least eight 1s in them.

c) How many *onto* functions are there from a set with 4 elements to a set with 3 elements?

d) We have 5 Discrete Mathematics textbooks and 7 Signals and Systems textbooks at hand. In how many ways can you make a collection of 4 books from these 12 textbooks with the condition that at least one Discrete Mathematics textbook and at least one Signals and Systems textbook must be in the collection.

Question 2

Let a_n be the number of subsets of the set $\{1, 2, 3 \cdots n\}$ that do not contain two consecutive numbers.

a) Determine the recurrence relation for a_n.

b) Solve it by using generating functions.

Question 3

Solve the following recurrence relation with the given initial conditions:

$$a_n = 4a_{n-1} + a_{n-2} - 4a_{n-3}$$

with $a_0 = 4$, $a_1 = 8$, $a_2 = 34$.

Question 4

Let R be a binary relation on real numbers defined by $(x_1, y_1)\, R\, (x_2, y_2)$ iff $3x_1 - 2y_1 = 3x_2 - 2y_2$. Prove that R is an equivalence relation. Give a graphical representation of $[(2, 3)]$ and $[(2, -3)]$ in the Cartesian coordinate system, where $[(x, y)]$ denotes the equivalence class of (x, y) with respect to R.

1 Regulations

1. You have to write your answers to the provided sections of the template answer file given.
2. Do not write any extra stuff like question definitions to the answer file. Just give your solution to the question. Otherwise you will get 0 from that question.
3. **Late Submission:** Not allowed!
4. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.
5. **Newsgroup:** You must follow the newsgroup (cow.ceng.metu.edu.tr/c/courses-undergrad/ceng223) for discussions and possible updates on a daily basis.
6. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via odtuclass. Download the given template answer file "the4.tex". When you finish your exam upload the .tex file with the same name to odtuclass.
Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex the4.tex
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