Student Information

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Answer 1

a)

$$\begin{aligned}
E(X) &= \sum_x x \cdot f P_X(x) \\
&= 0 \cdot P_X(0) + 1 \cdot P_X(1) + 2 \cdot P_X(2) \\
&= 0 \cdot \frac{3}{12} + 1 \cdot \frac{6}{12} + 2 \cdot \frac{3}{12} \\
&= 0 + \frac{1}{2} + \frac{1}{2} \\
&= 1
\end{aligned}$$

$$\begin{aligned}
Var(X) &= E(X^2) - \mu^2 \\
&= E(X^2) - 1^2 \\
&= 0^2 \cdot P_X(0) + 1^2 \cdot P_X(1) + 2^2 \cdot P_X(2) - 1 \\
&= 0 \cdot \frac{3}{12} + 1 \cdot \frac{6}{12} + 4 \cdot \frac{3}{12} - 1 \\
&= \frac{1}{2} + 1 - 1 \\
&= \frac{1}{2}
\end{aligned}$$

b)

Let Z be $X + Y$.

$$
\begin{aligned}
P_Z(0) &= P(0,0) = \frac{1}{12} \\
P_Z(1) &= P(1,0) = \frac{4}{12} \\
P_Z(2) &= P(0,2) + P(2,0) = \frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4} \\
P_Z(3) &= P(1,2) = \frac{2}{12} = \frac{1}{6} \\
P_Z(4) &= P(2,2) = \frac{2}{12} = \frac{1}{6}
\end{aligned}
$$

c)

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

$$
\begin{aligned}
E(X) &= \sum_x x \cdot f P_X(x) \\
&= 0 \cdot P_X(0) + 1 \cdot P_X(1) + 2 \cdot P_X(2) \\
&= 0 \cdot \frac{3}{12} + 1 \cdot \frac{6}{12} + 2 \cdot \frac{3}{12} \\
&= 0 + \frac{1}{2} + \frac{1}{2} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
E(Y) &= \sum_y y \cdot f P_Y(y) \\
&= 0 \cdot P_Y(0) + 2 \cdot P_Y(2) \\
&= 0 \cdot \frac{6}{12} + 2 \cdot \frac{6}{12} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
E(XY) &= \sum_x \sum_y (xy) \cdot P(x,y) \\
&= 0 \cdot 0 \cdot \frac{1}{12} + 1 \cdot 0 \cdot \frac{4}{12} + 2 \cdot 0 \cdot \frac{1}{12} \\
&\quad + 0 \cdot 2 \cdot \frac{2}{12} + 1 \cdot 2 \cdot \frac{2}{12} + 2 \cdot 2 \cdot \frac{2}{12} \\
&= \frac{4}{12} + \frac{8}{12} = \frac{12}{12} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
Cov(X,Y) &= E(XY) - E(X)E(Y) \\
Cov(X,Y) &= 1 - 1 \cdot 1 \\
&= 0
\end{aligned}
$$

d)

Recall that $P(a,b) = P_A(a) \cdot P_B(b)$ if A and B are independent.

$$
\begin{aligned}
E(AB) &= \sum_a \sum_b (ab) \cdot P(a,b) \\
&= \sum_a \sum_b a \cdot P_A(a) \cdot b \cdot P_B(b) \\
&= \sum_a a \cdot P_A(a) \sum_b b \cdot P_B(b) \\
&= E(A)E(B)
\end{aligned}
$$

Since $Cov(A,B) = E(AB) - E(A)E(B)$, $Cov(A,B) = 0$.

e)

If A and B are independent, then $P(x,y) = P_X(x) \cdot P_Y(y)$.

$$
\begin{aligned}
P_Y(0) &= \frac{1}{12} + \frac{4}{12} + \frac{1}{12} = \frac{1}{2} \\
P_X(0) &= \frac{1}{12} + \frac{2}{12} = \frac{1}{4} \\
P(0,0) &= \frac{1}{12} \neq P_X(0) \cdot P_Y(0) = \frac{1}{8}
\end{aligned}
$$

As a result X and Y are dependent.

Answer 2

a)

We can calculate the result subtracting the probability of no broken pen, 1 broken pen, or 2 broken pen from 1. Let $P(x)$ be the function of the probability of x broken pen.

$$P(0) = (0.8)^{12} \cong 0.06871947$$
$$P(1) = (0.8)^{11} \cdot 0.2 \cdot C(12,1) \cong 0.20615843$$
$$P(2) = (0.8)^{10} \cdot (0.2)^2 \cdot C(12,2) \cong 0.2834678415$$

$$1 - P(0) - P(1) - P(2) \cong 0.441654251.$$

b)

There are 12 different pens. In order to be fifth pen we test second broken pen, the one of the four pens must be broken.

Thus the probability is $(0.8)^3 \cdot 0.2 \cdot C(4,1) \cdot 0.2 \cong 0.08192$.

c)

The probability of broken pen is 0.2. That means for every 5 pen, 1 pen is broken. Since we want to find 4 broken pens, we have to look $5 \cdot 4 = 20$ pen in average, to find 4 broken pens.

Answer 3

a)

If Bob gets a phone call every 4 hours, that means Bob gets a 0.25 phone call every hour. We can use Exponential Distribution to compute the probability.

$$\begin{aligned} P\{T \geq 2\} &= 1 - F(2) = 1 - 1 + e^{-\lambda \cdot 2} \\ &= e^{-\frac{1}{2}} \\ &\cong 0.6065306597 \end{aligned}$$

b)

If Bob gets a phone call every 4 hours, that means Bob gets a 0.25 phone call every hour, 2.5 phone call every 10 hours. We can use Poisson Distribution to compute the probability. Let X be the number of calls, and X has Poisson Distribution with parameter $\lambda = 2.5$. From table A3 from the textbook,

$$P\{X \leq 3\} = F_X(3) \cong 0.758$$

c)

If Bob gets a phone call every 4 hours, that means Bob gets a 0.25 phone call every hour, 2.5 phone call every 10 hours, 4 phone call every 16 hours. We can use Poisson Distribution to compute the probability.

Let X be the number of calls, and X has Poisson Distribution with parameter $\lambda = 2.5$. Let Y be the number of calls, and Y has Poisson Distribution with parameter $\lambda = 1.5$. Using the table A3 from the textbook,

$$P\{X > 3\} = 1 - F_X(3) = 1 - 0.758 = 0.242$$
$$P\{Y > 0\} = 1 - F_Y(0) = 1 - 0.223 = 0.777$$

$$P\{X > 2\} = 1 - F_X(3) = 1 - 0.544 = 0.456$$
$$P\{Y > 1\} = 1 - F_Y(0) = 1 - 0.558 = 0.442$$

$$P\{X > 1\} = 1 - F_X(3) = 1 - 0.287 = 0.713$$
$$P\{Y > 2\} = 1 - F_Y(0) = 1 - 0.809 = 0.191$$

$$P\{X > 0\} = 1 - F_X(3) = 1 - 0.082 = 0.918$$
$$P\{Y > 3\} = 1 - F_Y(0) = 1 - 0.934 = 0.066$$

$$0.242 \cdot 0.777 + 0.456 \cdot 0.442 + 0.713 \cdot 0.191 + 0.918 \cdot 0.066 \cong 0.586357$$