Middle East Technical University | Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall 2019-2020

Homework 5

Due date: 30 December 2019 23:55

Question 1 (15 pts)

Let S be a set containing the subsets of the set $\{0, 1, 2\}$ and R be a relation on $S \times S$, defined as

$$S = \{w \ : \ w \text{ is a set, } w \in P(\{0, 1, 2\})\}$$
$$R = \{(w_1, w_2) \ : \ w_1 \in S \text{ , } w_2 \in S \text{ and } w_1 \text{ is a subset of } w_2\}$$

where $P(\{0, 1, 2\})$ denotes the powerset of the set $\{0, 1, 2\}$.

a. Draw R as a directed graph. (For this part (only for a) you can draw by your hand or any tool you want and then include it as an image.) (2 pts)

b. Prove that (S, R) is a poset. (4 pts)

c. Is (S, R) a total order? Prove your answer. (3 pts)

d. Draw a Hasse diagram for (S, R). State the maximal and minimal elements. (4 pts)

e. Identify whether (S, R) constitutes a lattice or not. Explain your answer. (2 pts)

Question 2 (24 pts)

Given the directed graph G in Figure 1, answer the questions.

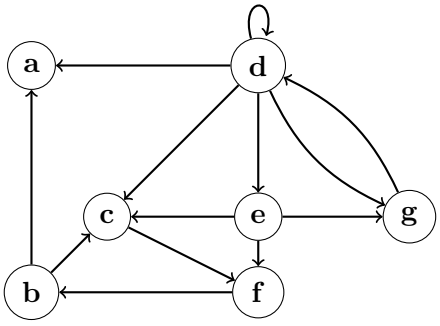

Figure 1: Graph G in Q2.

a. Provide an adjacency list representation of G. (3 pts)

b. Provide an adjaceny matrix representation of G. (3 pts)

c. Compute indegrees and outdegrees of every vertex in V. (3 pts)

d. List 6 different simple paths of length 4 in G. (3 pts)

e. List all simple circuits of length 3 in G. (3 pts)

f. Prove that G is weakly-connected. (3 pts)

g. Identify strongly-connected components of G. (3 pts)

h. How many different paths of length 3 exist from d to g in the subgraph H of G induced by the vertices $\{d, e, f, g\} \subset V$? (3 pts)

Question 3 (16 pts)

Given the undirected graph G in Figure 2, answer the following questions using clear formalism.

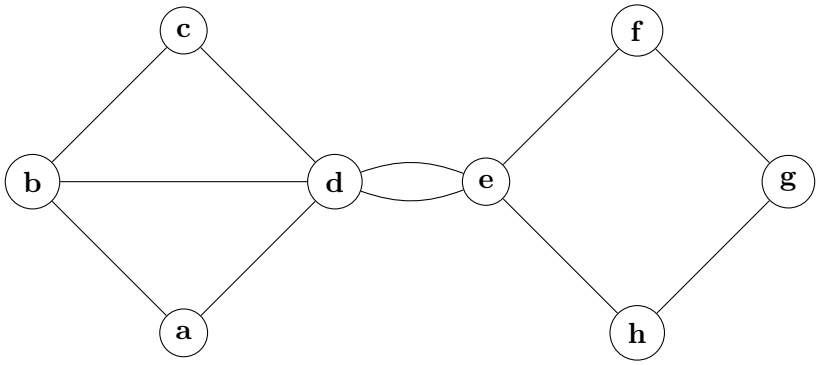

Figure 2: Graph G in Q3.

a. Prove whether G has a Euler path or not. (4 pts)

b. Prove whether G has a Euler circuit or not. (4 pts)

c. Prove whether G has a Hamiltonian path or not. (4 pts)

d. Prove whether G has a Hamiltonian circuit or not. (4 pts)

Question 4 (10 pts)

Determine if the following two graphs (G (Figure-3) and G' (Figure-4)) are isomorphic or not. Justify your answer.

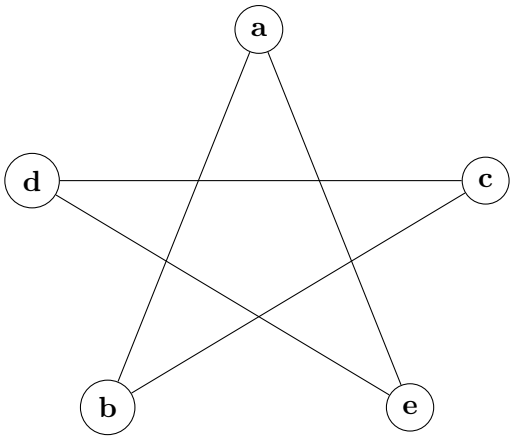

Figure 3: Graph G

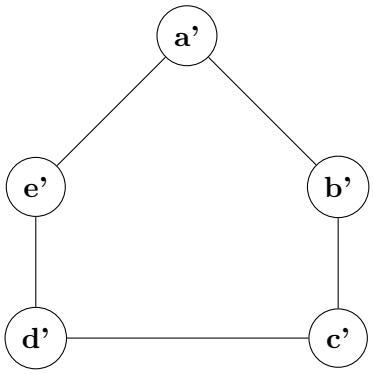

Figure 4: Graph G'

Question 5 (20 pts)

Given the undirected graph G in Figure 5, answer the questions.

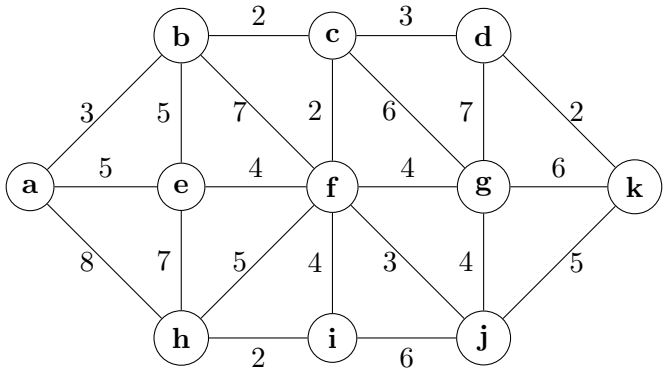

Figure 5: Graph G in Q5.

a. Find the shortest path from a to j using Dijkstra's algorithm. Clearly show each step. (10 pts)

b. Find a minimum spanning tree with root as vertex "a" using Prim's algorithm in Section 11.5 of the textbook. Explicitly show every step of computation and draw the resulting spanning tree. (10 pts)

Question 6 (15 pts)

Answer options **a-g** using the binary tree T in Figure 6. Vertices of T are marked with `<identifier:key>` annotations. Note that T has the vertex "a" as its root. Use the notational conventions in your textbook to decide whether a vertex is left or right child of some vertex whenever applicable.

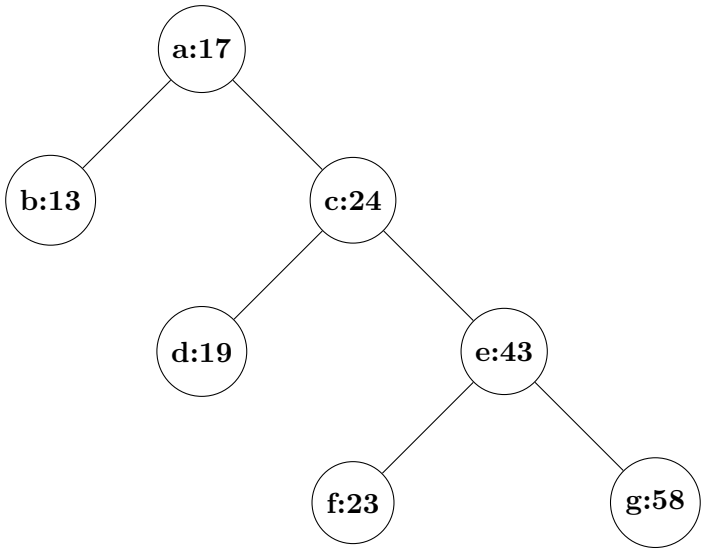

Figure 6: Tree T in Q6 options a, b, c, d, e, f, g.

a. What are the number of vertices, the number of edges and the height of T? (1 pt)

b. Carry out a preorder traversal of T and write down the order in which vertices are visited. (1 pt)

c. Carry out a postorder traversal of T and write down the order in which vertices are visited. (1 pt)

d. Carry out an inorder traversal of T and write down the order in which vertices are visited. (1 pt)

e. Is T a full binary tree? Justify your answer. (1 pt)

f. Is T a complete binary tree? Justify your answer. (1 pt)

g. Is T a binary search tree using provided keys under comparison with respect to the $\leq$ relation defined on $\mathbb{Z} \times \mathbb{Z}$? Justify your answer. (1 pt)

h. What is the minimum number of nodes for a full binary tree with height 5 ? (2 pt)

i. Construct a complete binary search tree by using the following set of integer keys {1, 2, 3, 4, 5, 6} employing the $\leq$ relation defined on $\mathbb{Z} \times \mathbb{Z}$. (2 pts)

j. Using the binary search tree in **i**, give sequences of vertices that are probed in order to find vertices with key values 1 and 6, repsectively. (1 pt)

k. Construct a spanning tree for the directed graph G in Figure 1 via breadth-first search under the assumption that unvisited vertices are selected for expansion in reverse alphabetic order of vertex identifiers. (2 pts)

l. What is the maximum height to create a binary search tree containing k vertices. Justify your answer.(1 pts)

1 Regulations

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.
2. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.
3. **Late Submission: Not allowed**
4. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.
5. **Newsgroup:** You must follow the odtuclass discussions (https://odtuclass.metu.edu.tr) for discussions and possible updates on a daily basis.
6. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via odtuclass. Download the given template file, "hw5.tex", when you finish your exam upload the .tex file with the same name to odtuclass.
Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex hw5.tex
```