Middle East Technical University Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall 2019-2020

Take Home Exam 2

Due date: October 29 2019, Tuesday, 23:55

Question 1

a) Given sets A, B, and C, express each of the following sets in terms of A, B, and C, using the symbols $\cup$, $\cap$, and $-$.

(i) $D = \{x \mid x \in A \wedge (x \in B \vee x \in C)\}$

(ii) $E = \{x \mid (x \in A \wedge x \in B) \vee x \in C\}$

(iii) $F = \{x \mid x \in A \wedge (x \in B \rightarrow x \in C)\}$

b) Prove or disprove the following

(i) $(A \times B) \times C = A \times (B \times C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

(iii) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

Note that $A \oplus B$ denotes the **symmetric difference** which is defined as the set containing those elements either in A or B but not in both A and B.
To prove $A = B$, show that $A \subseteq B$ and $B \subseteq A$, to disprove give a counterexample and/or explanation. You can use the laws given in the Table 1 (page 130) of the textbook with reference. You should give proofs for other set identities (if any) that you use.

Question 2

Let $f : A \rightarrow B$ be a function, $A_0 \subset A$, $B_0 \subset B$ and f^{-1} denote the **preimage** of B_0 under f defined by

$$f^{-1}(B_0) = \{a \mid f(a) \in B_0\}$$

a) Show that $A_0 \subseteq f^{-1}(f(A_0))$ and that equality holds if f is injective.

b) Show that $f(f^{-1}(B_0)) \subseteq B_0$ and that equality holds if f is surjective.

Question 3

Let A be a nonempty set. Show that the following are equivalent

(i) A is countable

(ii) There is a surjective function $f : \mathbb{Z}^+ \to A$

(iii) There is an injective function $f : A \to \mathbb{Z}^+$

Note: It is sufficient to show that (i) $\to$ (ii), (ii) $\to$ (iii), *and* (iii) $\to$ (i).

Question 4

A binary string is a sequence of 0s and 1s. A binary string is called **finite** if it is a finite sequence, and **infinite** if it is an infinite sequence of 0s and 1s.

a) Show that the set of finite binary strings is countable.

b) Show that the set of infinite binary strings is uncountable.

Question 5

a) Determine whether $\log n!$ is $\Theta(n \log n)$.

b) Which function grows faster, $n!$ or 2^n? Justify your answer algebraically.

Regulations

1. You have to write your answers to the provided sections of the template answer file given.
2. **Late Submission:** Not allowed.
3. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.
4. **Updates & Announces:** Possible updates will be announced via Odtuclass (odtuclass.metu.edu.tr).
5. **Evaluation:**Your latex file will be converted to pdf and evaluated by course assistants. The `.tex` file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

Submission

Submission will be done via Odtuclass. Download the given template answer file `the2.tex`. When you finish your exam upload the `.tex` file with the same name to Odtuclass.

Note: You cannot submit any other files. Make sure that your `.tex` file successfully compiles in Inek machines using the command below.

```
$ pdflatex the2.tex
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