CENG 384 - Signals and Systems for Computer Engineers
Spring 2021
Homework 2

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1. (a)

$$x(t) + -5y(t) - 6\int_{\tau=-\infty} y(\tau)\, d\tau = \dot{y}(t)$$
$$\dot{x}(t) - 5\dot{y}(t) - 6y(t) = \ddot{y}(t)$$
$$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = \dot{x}(t)$$

(b)

$$y_h(t) = C_1 e^{-5t} + C_2 e^{-t}$$
$$y_p(t) = K(e^{-t} + e^{-4t})u(t)$$
$$\dot{y}_p(t) = K(-e^{-t} - 4e^{-4t})u(t)$$
$$\ddot{y}_p(t) = K(e^{-t} + 16e^{-4t})u(t)$$
$$K(e^{-t} + 16e^{-4t}) + 5K(-e^{-t} - 4e^{-4t}) + 6K(e^{-t} + e^{-4t}) = (e^{-t} + e^{-4t})$$
$$2K(e^{-t} + e^{-4t}) = (e^{-t} + e^{-4t})$$
$$K = \frac{1}{2}$$
$$y_p(t) = Kx(t) = \frac{e^{-t} + e^{-4t}}{2}u(t)$$
$$y(t) = y_h(t) + y_p(t) = C_1 e^{-5t} + C_2 e^{-t} + \frac{e^{-t} + e^{-4t}}{2}u(t)$$
$$\dot{y}(t) = -5C_1 e^{-5t} - C_2 e^{-t} - \frac{e^{-t} + 4e^{-4t}}{2} \cdot u(t)$$
$$y(0) = 0, \dot{y}(0) = 0$$
$$y(0) = C_1 + C_2 + 1$$
$$\dot{y}(0) = -5C_1 - C_2 - \frac{5}{2} = 0$$
$$C_1 = \frac{-3}{8}, C_2 = \frac{-5}{8}$$
$$y(t) = \frac{-3}{8}e^{-5t} + \frac{-5}{8}e^{-t} + \frac{e^{-t} + e^{-4t}}{2}u(t)$$

2. (a)

$$x_1[n] = x[n] - x[n-2]$$

With the time invariance property $x[n-2] \Rightarrow y[n-2] = \delta[n-3]$.
With the superposition property $-x[n-2] \Rightarrow -y[n-2] = -\delta[n-3]$.

$$x_1[n] = x[n] - x[n-2] \Rightarrow y_1[n] = y[n] - y[n-2] = \delta[n-1] - \delta[n-3]$$

(b)

$$y[n] = x[n] * h[n]$$
$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k] \cdot h[k]$$
$$\delta[n-1] = \sum_{k=-\infty}^{\infty} (\delta[n-k] + \delta[n-1-k]) \cdot h[k]$$
$$\delta[n-1] = h[n] + h[n-1]$$
$$h[n] = \delta[n-1] - h[n-1]$$

$$h[0] = 0$$
$$h[1] = \delta[0] - h[0] = 1$$
$$h[2] = \delta[1] - h[1] = -1$$
$$h[3] = \delta[2] - h[2] = 1$$
$$h[n] = (-1)^n u[n-1]$$

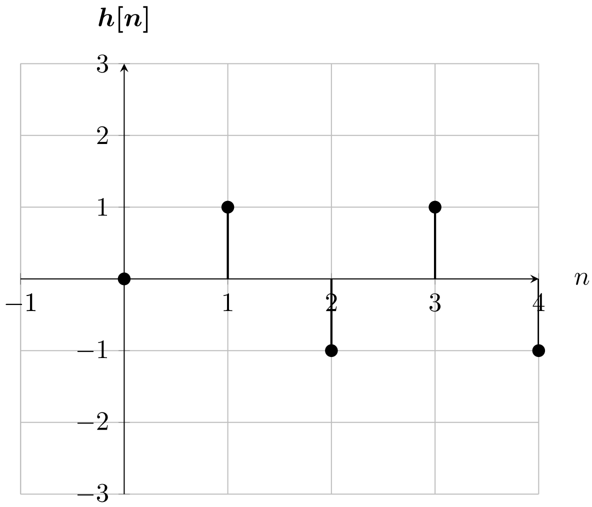
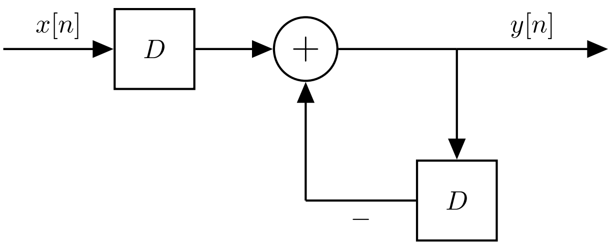


Figure 1: $h[n]$ - q2b

(c)

$$y[n] = x[n-1] - y[n-1]$$

(d)



3. (a) $\delta[0] \cdot \delta[n] = \delta[n]$

$$\sum_{k=-\infty}^{\infty} (\delta[k-3] + 2\delta[k+1])(\delta[n-k-1] + 3\delta[n-k+2])$$
$$= (\delta[n-4] + 3\delta[n-1]) + 2(\delta[n] + 3\delta[n+3])$$
$$= 6\delta[n+3] + 2\delta[n] + 3\delta[n-1] + \delta[n-4]$$

(b) $u[0] \cdot u[n] = u[n]$

$$\sum_{k=-\infty}^{\infty} (u[k+3] - u[k])(u[n-k-1] - u[n-k-3])$$
$$= (u[n+2] - u[n]) - (u[n-1] - u[n-3])$$
$$= u[n+2] - u[n] - u[n-1] + u[n-3]$$

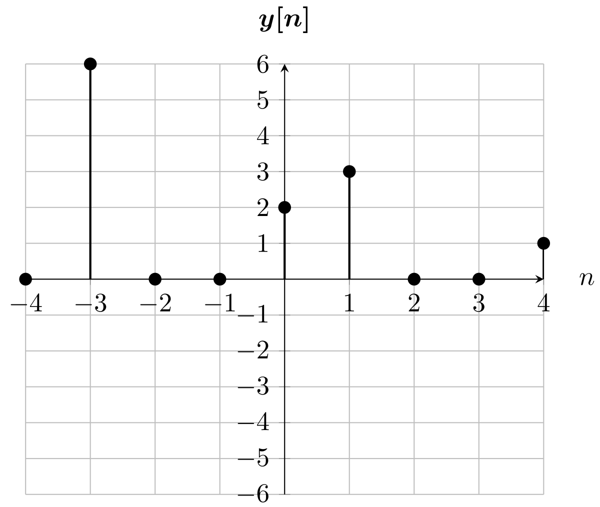


Figure 2: $y[n]$ - q3a

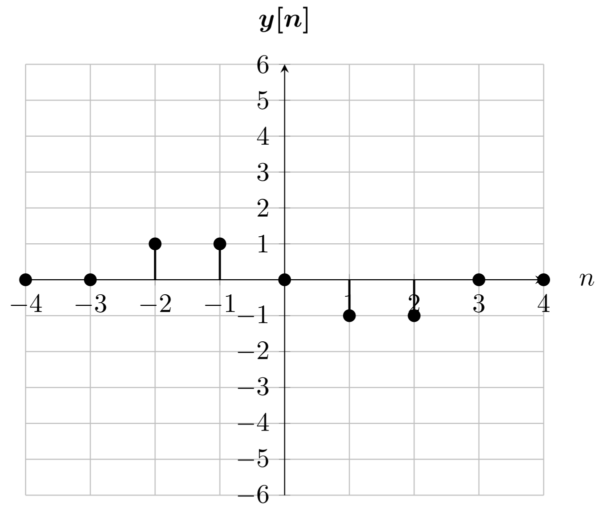


Figure 3: $y[n]$ - q3b

4. (a)

$$\int_{\tau=-\infty}^{\infty} e^{-3(t-\tau)}u(t-\tau)\cdot e^{-2\tau}u(\tau)\,d\tau$$

$$e^{-3t}\int_{\tau=0}^{t} e^{3\tau}\cdot e^{-2\tau}\,d\tau$$

$$e^{-3t}\int_{\tau=0}^{t} e^{\tau}\,d\tau$$

$$e^{-3t}\cdot(e^{t}-1)=e^{-2t}-e^{-3t}$$

(b)

$$\int_{\tau=-\infty}^{\infty} e^{2(t-\tau)}u(t-\tau)\cdot(u(\tau)-u(\tau-2))\,d\tau$$

$$e^{2t}\cdot\int_{\tau=-\infty}^{\infty} e^{-2\tau}u(t-\tau)u(\tau)-e^{-2\tau}u(t-\tau)u(\tau-2)\,d\tau$$

$$e^{2t}\cdot(\int_{\tau=0}^{t} e^{-2\tau}\,d\tau-\int_{\tau=2}^{t} e^{-2\tau}\,d\tau)$$

$$e^{2t}\cdot(\frac{1-e^{-2t}}{2}-\frac{e^{-4}-e^{-2t}}{2})$$

$$\frac{e^{2t}-1}{2}-\frac{e^{2t-4}-1}{2}$$

$$\frac{e^{2t}-e^{2t-4}}{2}$$

5. (a)

$$h[n] = s[n] - s[n-1] = n \cdot u[n] - (n-1) \cdot u[n-1] = u[n] \cdot (n - (n-1))$$
$$h[n] = u[n]$$

(b)

$$y[n] = x[n] * h[n]$$
$$x[n] = h^{-1}[n] * y[n]$$
$$h[n] * h^{-1}[n] = \delta[n]$$
$$\sum_{k=-\infty}^{\infty} h[n-k] \cdot h^{-1}[k] = \delta[n]$$
$$\sum_{k=-\infty}^{\infty} u[n-k] \cdot \delta[k] = \delta[n]$$
$$h^{-1}[n] = \delta[n] - \delta[n-1]$$
$$x[n] = \sum_{k=-\infty}^{\infty} y[n-k] \cdot h^{-1}[k]$$
$$x[n] = \sum_{k=-\infty}^{\infty} (\delta[n-k] - \delta[n-1-k]) \cdot (\delta[k] - \delta[k-1])$$
$$x[n] = \delta[n] - \delta[n-1] - \delta[n-1] + \delta[n-2]$$

(c)

$$x[n] = \delta[n] - \delta[n-1] - \delta[n-1] + \delta[n-2]$$
$$y[n] = \delta[n] - \delta[n-1]$$
$$y[n-1] = \delta[n-1] - \delta[n-2]$$
$$-y[n-1] = -\delta[n-1] + \delta[n-2]$$
$$x[n] = y[n] - y[n-1]$$

6.

$$h(t) = \frac{ds(t)}{dt} = t \cdot u(t)$$

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{\tau=-\infty}^{\infty} x(t-\tau) \cdot h(\tau)\, d\tau$$

$$y(t) = \int_{\tau=-\infty}^{\infty} (e^{-(t-\tau)} \cdot u(t-\tau)) \cdot (\tau \cdot u(\tau))\, d\tau$$

$$y(t) = e^{-t} \int_{\tau=0}^{t} e^{\tau} \cdot \tau\, d\tau$$

$$y(t) = e^{-t} \cdot (e^{t} \cdot (t-1) + 1)$$

$$y(t) = t - 1 + e^{-t}$$

7. (a)

$$h(t) = u(t) \cdot (\delta(t-3) - \delta(t-5)) = u(t-3) - u(t-5)$$

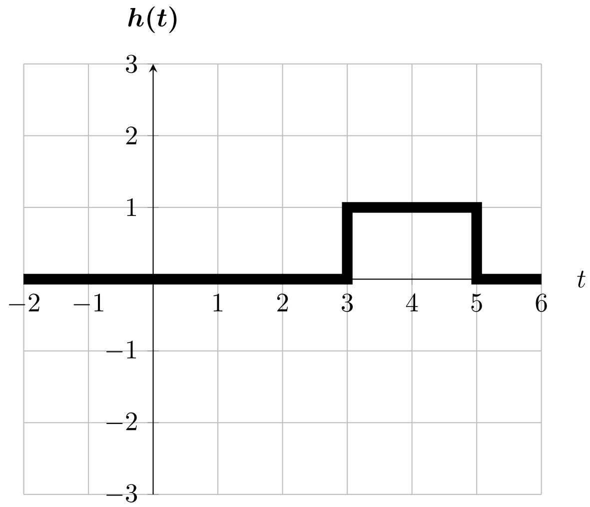


Figure 4: Impulse response of the system

(b)

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{\tau=-\infty}^{\infty} e^{-3(t-\tau)}u(t-\tau)\cdot(\delta(\tau-3)-\delta(\tau-5))u(\tau)\,d\tau$$

$$y(t) = \int_{\tau=-\infty}^{\infty} e^{-3(t-\tau)}u(t-\tau)\cdot u(\tau-3)\,d\tau - \int_{\tau=-\infty}^{\infty} e^{-3(t-\tau)}u(t-\tau)\cdot u(\tau-5)\,d\tau$$

$$y(t) = \int_{\tau=3}^{t} e^{-3(t-\tau)}\,d\tau - \int_{\tau=5}^{t} e^{-3(t-\tau)}\,d\tau$$

$$y(t) = e^{-3t}(\int_{\tau=3}^{t} e^{3\tau}\,d\tau - \int_{\tau=5}^{t} e^{3\tau}\,d\tau)$$

$$y(t) = e^{-3t}(\frac{e^{3t}-e^{9}}{3} - \frac{e^{3t}-e^{15}}{3})$$

$$y(t) = e^{-3t}\frac{e^{15}-e^{9}}{3}$$

(c)

$$\frac{dh(t)}{d(t)} = \delta(t-3)-\delta(t-5)$$

$$g(t) = \int_{\tau=-\infty}^{\infty} (\delta(\tau-3)-\delta(\tau-5))\cdot e^{-3(t-\tau)}u(t-\tau)\,d\tau$$

$$g(t) = \int_{\tau=-\infty}^{\infty} (\delta(\tau-3)-\delta(\tau-5))\cdot e^{-3(t-\tau)}u(t-\tau)\,d\tau$$

$$g(t) = e^{-3t}(\int_{\tau=-\infty}^{t} \delta(\tau-3)\cdot e^{3\tau}\,d\tau - \int_{\tau=-\infty}^{t} \delta(\tau-5)\cdot e^{3\tau}\,d\tau)$$

$$g(t) = e^{-3t}(e^{9}\cdot(u(t-3)-u(t-5)))$$

$$g(t) = e^{-27t}(u(t-3)-u(t-5))$$