CENG 384 - Signals and Systems for Computer Engineers
Spring 2021
Homework 1

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1.

$$e = \lim_{n \to 0} (1+n)^{\frac{1}{n}}$$

$$\frac{d}{dx} e^x = \lim_{\Delta x \to 0} \frac{e^{x+\Delta x} - e^x}{\Delta x}$$

$$\frac{d}{dx} e^x = e^x \cdot \lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x}$$

Some replacements for Δx below. Also note that as $\Delta x \to 0$, $n \to 0$.

$$n = e^{\Delta x} - 1 \quad n+1 = e^{\Delta x} \quad ln(n+1) = \Delta x$$

$$\frac{d}{dx} e^x = e^x \cdot \lim_{\Delta n \to 0} \frac{n}{ln(1+n)}$$

$$\frac{d}{dx} e^x = e^x \cdot \lim_{\Delta n \to 0} \frac{1}{\frac{1}{n} \cdot ln(1+n)}$$

$$\frac{d}{dx} e^x = e^x \cdot \lim_{\Delta n \to 0} \frac{1}{ln((1+n)^{\frac{1}{n}})}$$

$$\frac{d}{dx} e^x = e^x \cdot \frac{1}{ln(\lim_{\Delta n \to 0} (1+n)^{\frac{1}{n}})}$$

Using the definition we have written at the beginning we can prove that e^x's derivative equal to itself.

$$\frac{d}{dx} e^x = e^x \cdot \frac{1}{ln(e)}$$

$$\frac{d}{dx} e^x = e^x$$

2. (a)

$$z = x + jy, \bar{z} = x - jy$$

$$z - 3 = j - 2\bar{z}$$

$$z - 3 - j = -2\bar{z}$$

$$x + jy - 3 - j = -2(x - jy)$$

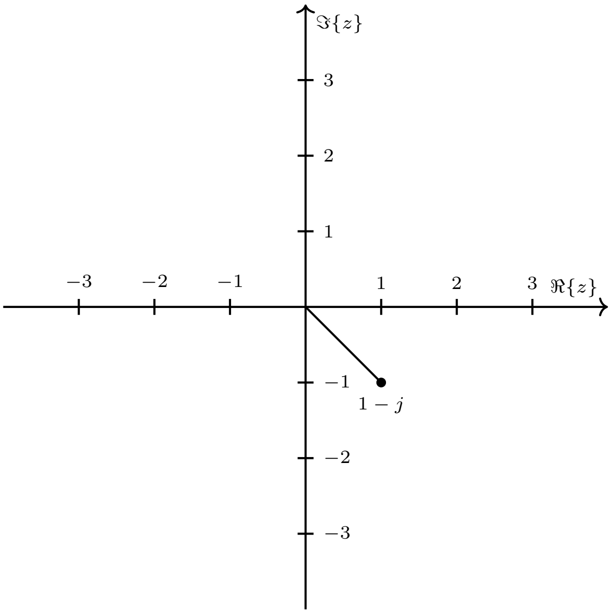
$$x + jy - 3 - j = -2x + 2jy$$

$$3x - jy = j + 3$$

$$x = 1, y = -1$$

i. $|z|^2 = x^2 + y^2 = 1^2 + (-1)^2 = 2$

ii.



(b)

$$z^4 = -81$$
$$z^4 + 81 = (z^2 + 9j) \cdot (z^2 - 9j) = 0$$
$$z^2 = 9j \text{ or } z^2 = -9j$$

$$z^2 = (a + bj)^2 = 9j$$
$$(a^2 - b^2) + 2abj = 0 + 9j$$
$$a = b, 2ab = 9, ab = \frac{9}{2}$$
$$a = b = \frac{3\sqrt{2}}{2}$$

$$z^2 = (a + bj)^2 = -9j$$
$$(a^2 - b^2) + 2abj = 0 + -9j$$
$$a = b, 2ab = -9, ab = \frac{-9}{2}$$
Since $a = b$, ab cannot be negative

$$r = \sqrt{a^2 + b^2} = \sqrt{\frac{9}{2} + \frac{9}{2}} = \frac{3}{2}$$
$$\frac{b}{a} = tan(\theta), \theta = \frac{\pi}{4}$$
$$z = \frac{3}{2} \cdot (cos(\frac{\pi}{4}) + j \cdot sin(\frac{\pi}{4}))$$

(c)

$$z = \frac{(\frac{1}{2} + \frac{1}{2}j)(1 - j)}{1 - \sqrt{3}j}$$
$$z = \frac{\frac{1}{2} - \frac{1}{2}j + \frac{1}{2}j + \frac{1}{2}}{1 - \sqrt{3}j}$$
$$z = a + bj = \frac{1}{4} + \frac{\sqrt{3}j}{4}$$

$$a = \frac{1}{4}, b = \frac{\sqrt{3}}{4}$$

$$r = \sqrt{a^2 + b^2} = \sqrt{\frac{1}{16} + \frac{3}{16}} = \frac{1}{2}$$

$$\theta = tan^{-1}(\frac{b}{a}) = tan^{-1}(\frac{\frac{\sqrt{3}}{4}}{\frac{1}{4}}) = tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

(d)

$$z = -\frac{3}{j} \cdot e^{j\pi/2}$$

$$r = -\frac{3}{j}, \theta = \frac{\pi}{2}$$

$$z = -\frac{3}{j} \cdot (cos(\frac{\pi}{2}) + j \cdot sin(\frac{\pi}{2}))$$

3. Q3

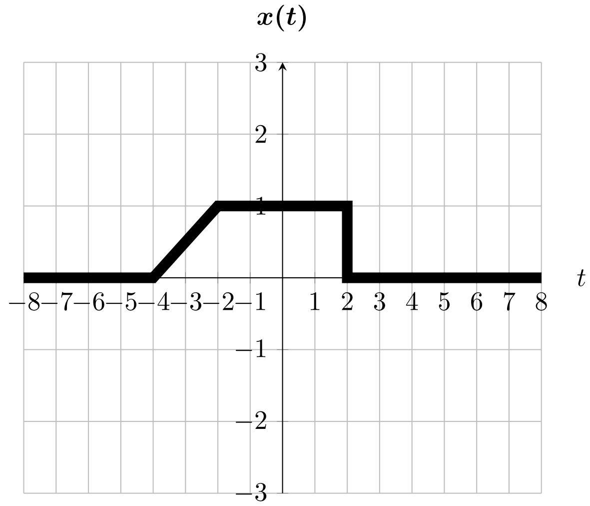


Figure 1: t vs. $x(\frac{t}{2})$.

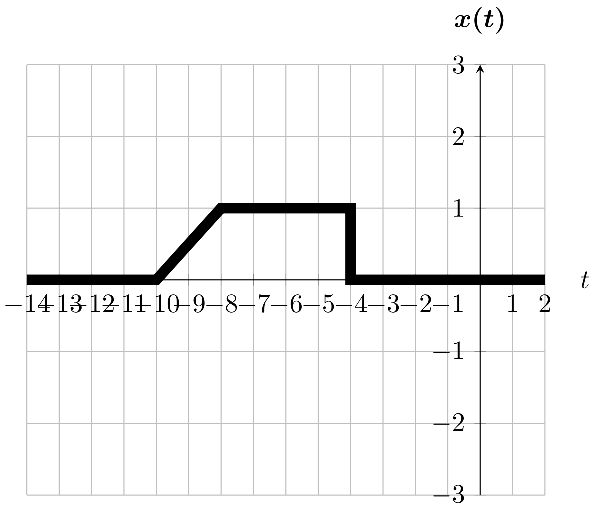


Figure 2: t vs. $x(\frac{t}{2} + 3)$.

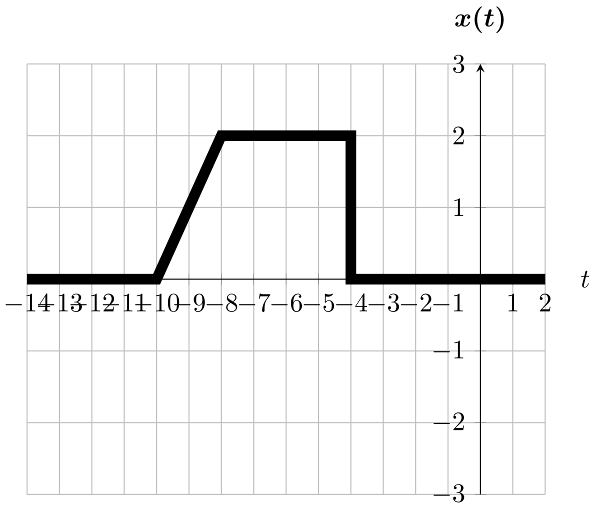


Figure 3: t vs. $y(t) = 2x(\frac{t}{2} + 3)$.

4. (a) Q4a

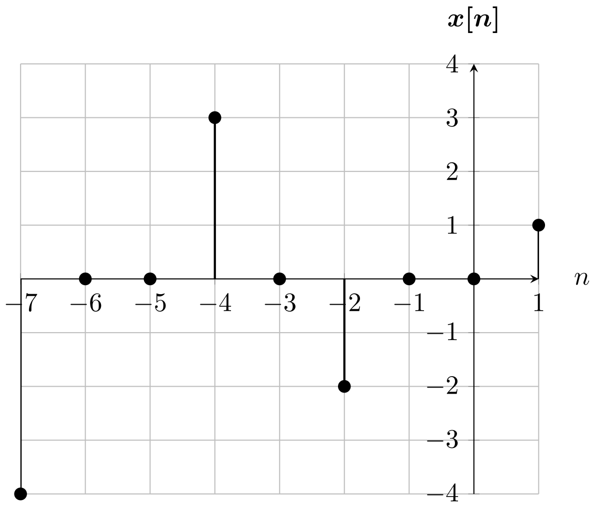


Figure 4: n vs. $x[-n]$.

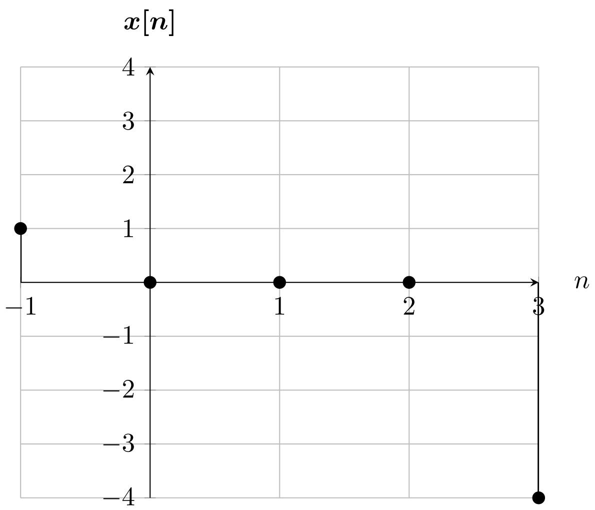


Figure 5: n vs. $x[2n+1]$.

   (b) $x[-n] + x[2n+1] = -4 \cdot \delta(n+7) + 3 \cdot \delta(n+4) - 2 \cdot \delta(n+2) + \delta(n+1) + \delta(n-1) - 4 \cdot \delta(n-3)$

5. (a) Yes.

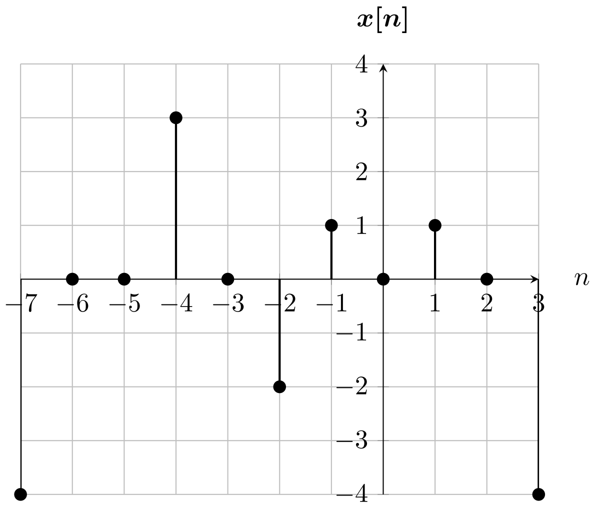


Figure 6: n vs. $x[-n] + x[2n+1]$.

$T_0 = \frac{2\pi}{7\pi} = \frac{2}{7}$

(b) No.

$T_0 = \frac{2}{4}\pi = \frac{1}{2}\pi$. Period is always a multiple of fundamental period. Since π is irrational, there is no T that is integer.

(c) Yes.

$T_{x0} = \frac{5 \cdot 2}{7} = \frac{10}{7}$

$T_{y0} = \frac{2 \cdot 2}{5} = \frac{4}{5}$

$T_0 = LCM(T_{x0}, T_{y0}) = \frac{20}{1} = 20$

6. (a) $x(t) = x(-t)$ when x is even. But $x(1.5) = 0 \neq 0.5 = x(-1.5)$. So x isn't even.

$x(t) = -x(-t)$ when x is odd. But $x(1.5) = 0 \neq -0.5 = -x(-1.5)$. So x isn't odd.

(b)

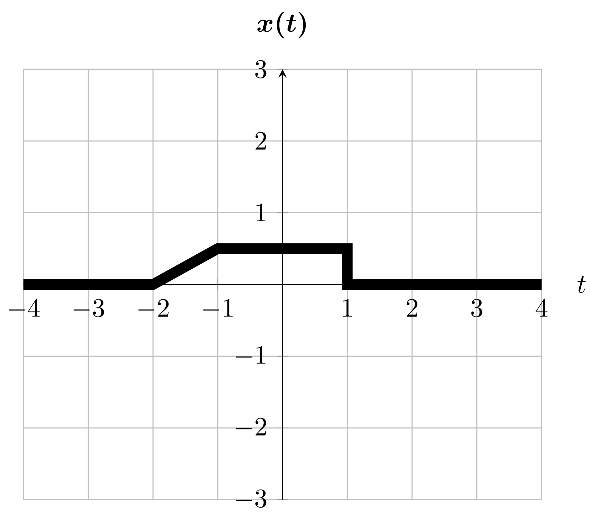


Figure 7: t vs. $\frac{1}{2} \cdot x(t)$.

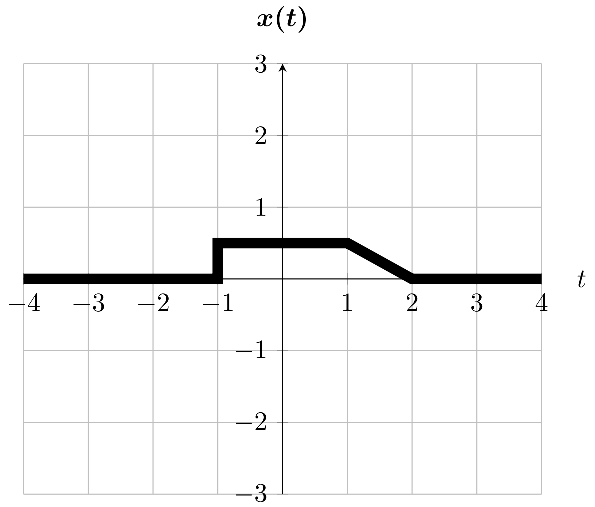


Figure 8: t vs. $\frac{1}{2} \cdot x(-t)$.

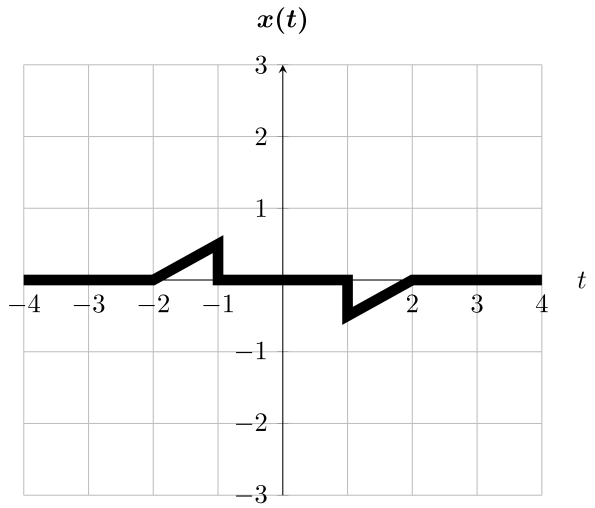


Figure 9: t vs. $Odd\{x(t)\} = \frac{1}{2}(x(t) - x(-t))$.

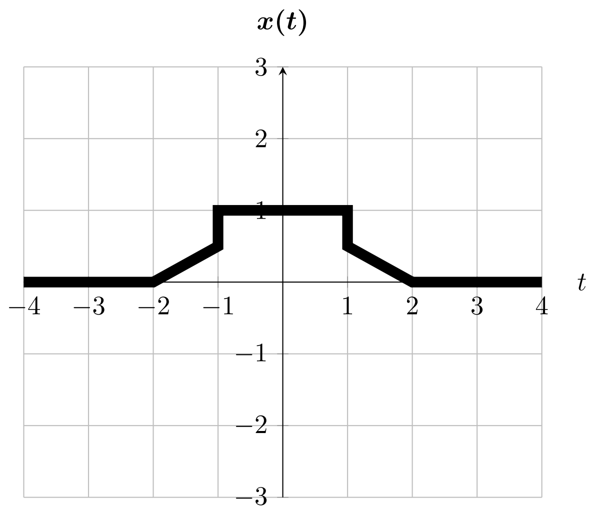


Figure 10: t vs. $Even\{x(t)\} = \frac{1}{2}(x(t) + x(-t))$.

7. (a) $x(t) = -3u(t-2) + 5u(t-3) - 3u(t-5)$

   (b) $\frac{du(t)}{dt} = \delta(t)$

   $\frac{dx(t)}{dt} = -3\delta(t-2) + 5\delta(t-3) - 3\delta(t-5)$

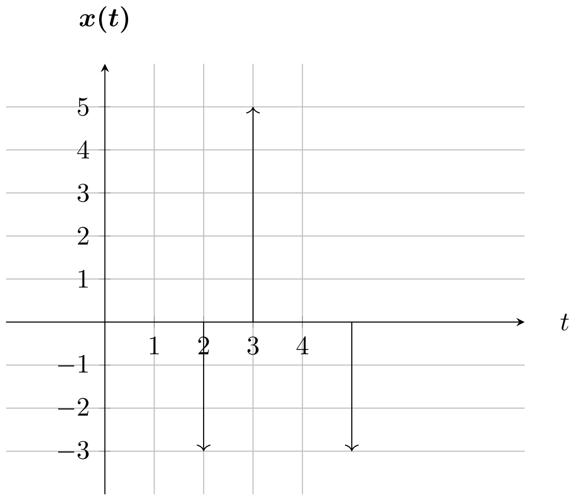


Figure 11: t vs. $\frac{dx(t)}{dt}$.

8. (a) i. Memory: Yes. $3n - 5 \neq n$
   ii. Stability: Yes, bounded inputs always generate bounded outputs.
   iii. Causality: No. When $n > 2$, it doesn't satisfy causality. $n = 3, 3 * (3) - 5 > (3)$
   iv. Linearity: Yes, it satisfies both superposition and homogeneity.
   v. Invertibility: Yes, for $h^{-1}[n] = \frac{n+5}{3}$.
   vi. Time-invariance: Yes, $y[n - n_0] = x[3(n - n_0) - 5]$.

   (b) i. Memory: Yes. $3t - 5 \neq t$
   ii. Stability: Yes, bounded inputs always generate bounded outputs.
   iii. Causality: No. When $t \geq 3$, it doesn't satisfy causality. $n = 3, 3 \cdot (3) - 5 > (3)$
   iv. Linearity: Yes, it satisfies both superposition and homogeneity.
   v. Invertibility: Yes, for $h^{-1}(t) = \frac{t+5}{3}$.
   vi. Time-invariance: Yes, $y(n - n_0) = x(3(n - n_0) - 5)$.

   (c) i. Memory: Yes. $t - 1 \neq t$
   ii. Stability: No, the bounded input does not always generate a bounded output.
   iii. Causality: Yes, $t - 1$ is less than t.
   iv. Linearity: Yes, it satisfies both superposition and homogeneity.
   v. Invertibility: No, for $t = 0$ the system is not invertible.
   vi. Time-invariance: No, $y(t - t_0) = (t - t_0) \cdot x((t - t_0) - 1) \neq t \cdot x((t - t_0) - 1)$.

   (d) i. Memory: Yes. $n - k \neq n$ when $k > 1$.
   ii. Stability: No, the sum is an infinity sum.
   iii. Causality: Yes, $n - k$ is always less than n.
   iv. Linearity: Yes, it satisfies both superposition and homogeneity.
   v. Invertibility: No.
   vi. Time-invariance: Yes, $y[n - n_0] = \sum_{k=1}^{\infty} x[(n - n_0) - k]$.

   Please note that:

   - Let $y(t) = h(x(t))$, $y_1(t) = h(x_1(t))$, and $y_2(t) = h(x_2(t))$.
   - Superposition means that $y_1(t) + y_2(t) = h(x_1(t) + x_2(t))$.
   - Homogeneity means that for a arbitrary k, $k \cdot y(t) = h(k \cdot x(t))$.