CENG 384 - Signals and Systems for Computer Engineers
Spring 2021
Homework 4

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1. (a)

$$4x(t) - 5y(t) + \int (x(t) - 6y(t))\,dt = \frac{dy(t)}{dt}$$
$$4x'(t) - 5y'(t) + x(t) - 6y(t) = y''(t)$$

(b)

$$4x'(t) - 5y'(t) + x(t) - 6y(t) = y''(t)$$
$$4x'(t) + x(t) = y''(t) + 5y'(t) + 6y(t)$$
$$H(j\omega) = \frac{4j\omega + 1}{(j\omega)^2 + 5j\omega + 6}$$

(c)

$$H(j\omega) = \frac{4j\omega + 1}{(j\omega)^2 + 5j\omega + 6}$$
$$H(j\omega) = \frac{-7}{j\omega + 2} + \frac{11}{j\omega + 3}$$
$$h(t) = -7e^{-2t}u(t) + 11e^{-3t}u(t)$$

(d)

$$Y(j\omega) = X(j\omega)H(j\omega)$$
$$Y(j\omega) = X(j\omega)(\frac{-7}{j\omega + 2} + \frac{11}{j\omega + 3})$$

if we take $x(t) = \frac{1}{4}e^{-t/4}u(t)$ then $X(jw) = \frac{\frac{1}{4}}{j\omega + \frac{1}{4}}$

$$Y(j\omega) = \frac{\frac{1}{4}}{j\omega + \frac{1}{4}}(\frac{-7}{j\omega + 2} + \frac{11}{j\omega + 3})$$
$$Y(j\omega) = \frac{1}{jw + 2} - \frac{1}{jw + 3}$$
$$y(t) = e^{-2t}u(t) - e^{-3t}u(t)$$

2. (a)

$$H(j\omega) = \frac{j\omega + 4}{-\omega^2 + 5j\omega + 6} = \frac{j\omega + 4}{(j\omega)^2 + 5j\omega + 6} = \frac{j\omega + 4}{(j\omega + 3)\cdot(j\omega + 2)}$$
$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$$

(b)

$$H(j\omega) = \frac{j\omega + 4}{(j\omega + 3)\cdot(j\omega + 2)} = \frac{2}{j\omega + 2} - \frac{1}{j\omega + 3}$$
$$h(t) = 2e^{-2t}u(t) - e^{-3t}u(t)$$

(c)

$$x(t) = e^{-4t}u(t) - te^{-4t}u(t)$$

$$Y(j\omega) = H(j\omega)X(j\omega) = (\frac{j\omega+4}{(j\omega+3)\cdot(j\omega+2)})(\frac{1}{j\omega+4} - \frac{1}{(j\omega+4)^2})$$

$$Y(j\omega) = \frac{1}{2(j\omega+2)} - \frac{1}{2(j\omega+4)}$$

(d)

$$y(t) = (\frac{1}{2}e^{-2t} + \frac{1}{2}e^{-4t})u(t)$$

3. (a)

using the property e from the week 10 notes table 2.

$$X(j\omega) = \frac{2}{\omega^2+1}$$

(b)

$$X(j\omega) = j\frac{d}{d\omega}X(j\omega)$$

$$X(j\omega) = j\cdot\frac{-4\omega}{(\omega^2+1)^2}$$

$$X(j\omega) = \frac{-4\omega j}{(\omega^2+1)^2}$$

(c)

$$X(j\omega) = \frac{-4\omega j}{(\omega^2+1)^2}$$

$$X(j\omega) = \frac{-4\omega j}{(-(j\omega)^2+1)^2}$$

4. (a)

$$y[n] = -\frac{y[n-2]}{8} + \frac{3y[n-1]}{4} + 2x[n]$$

$$y[n] - \frac{3y[n-1]}{4} + \frac{y[n-2]}{8} = 2x[n]$$

(b)

$$H(e^{j\omega}) = \frac{2}{1-\frac{3}{4}e^{-j\omega}+\frac{1}{8}e^{-2j\omega}}$$

(c)

$$H(e^{j\omega}) = \frac{16}{(e^{-j\omega}-2)(e^{-j\omega}-4)}$$

Partial fraction expansion:

$$H(e^{j\omega}) = \frac{8}{e^{-j\omega}-4} - \frac{8}{e^{-j\omega}-2}$$

$$H(e^{j\omega}) = \frac{-2}{1-\frac{1}{4}e^{-j\omega}} + \frac{4}{1-\frac{1}{2}e^{-j\omega}}$$

$$h[n] = 4(\frac{1}{2})^n\cdot u[n] - 2(\frac{1}{4})^n\cdot u[n]$$

(d)

$$x[n] = (\frac{1}{4})^n u[n]$$

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) = (\frac{-2}{1-\frac{1}{4}e^{-j\omega}} + \frac{4}{1-\frac{1}{2}e^{-j\omega}})(\frac{1}{1-\frac{1}{4}e^{-j\omega}})$$

$$Y(e^{j\omega}) = \frac{4}{1-\frac{1}{4}e^{-j\omega}} - \frac{2}{(1-\frac{1}{4}e^{-j\omega})^2} + \frac{8}{1-\frac{1}{2}e^{-j\omega}}$$

$$y[n] = (-4(\frac{1}{4})^n - 2(n+1)(\frac{1}{4})^n + 8(\frac{1}{2})^n)u[n]$$

5.

$$H(e^{j\omega}) = \frac{5e^{-j\omega} - 12}{e^{-2j\omega} - 7e^{-j\omega} + 12}$$

$$H(e^{j\omega}) = \frac{5e^{-j\omega} - 12}{(e^{-j\omega} - 3)(e^{-j\omega} - 4)}$$

$$H(e^{j\omega}) = \frac{8}{e^{-j\omega} - 4} - \frac{3}{e^{-j\omega} - 3}$$

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{3}e^{-j\omega}} - \frac{2}{1 - \frac{1}{4}e^{-j\omega}}$$

$$h[n] = h_1[n] + h_2[n] = (\frac{1}{3})^n \cdot u[n] - 2(\frac{1}{4})^n \cdot u[n]$$

$$h_1[n] = (\frac{1}{3})^n \cdot u[n]$$

$$h_2[n] = -2(\frac{1}{4})^n \cdot u[n]$$

6. (a)

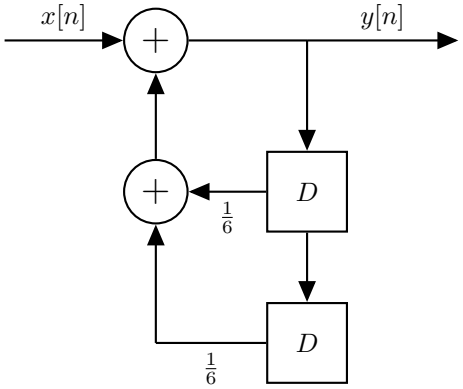
$$H(e^{j\omega}) = \frac{1}{1 - \frac{e^{-j\omega}}{6} - \frac{e^{-2j\omega}}{6}}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n]$$

(b)

$$y[n] = \frac{1}{6}y[n-1] + \frac{1}{6}y[n-2] + x[n]$$



(c)

$$H(e^{j\omega}) = \frac{1}{1 - \frac{e^{-j\omega}}{6} - \frac{e^{-2j\omega}}{6}} = \frac{6}{6 - e^{-j\omega} - e^{-2j\omega}}$$

$$H(e^{j\omega}) = \frac{6}{(3 + e^{-j\omega})(2 - e^{-j\omega})}$$

Partial fraction expansion:

$$H(e^{j\omega}) = \frac{2}{5} \cdot \frac{1}{1 + \frac{1}{3}e^{-j\omega}} + \frac{3}{5} \cdot \frac{1}{1 - \frac{1}{2}e^{-j\omega}}$$

$$h[n] = \frac{2}{5} \cdot (-\frac{1}{3})^n \cdot u[n] + \frac{3}{5} \cdot (\frac{1}{2})^n \cdot u[n]$$