```
Data: aststa::chicken
```

Monthly price of a pound of chicken Poultry (chicken), Whole bird spot price, Georgia docks, US cents per pound monthly, between 2001-2016 in time series format (from astsa vignette)

```
library(fUnitRoots)
library(astsa)
#library(forecast)

#storing my data and doing a basic plot to see:
chicken<-chicken
plot(chicken)
```

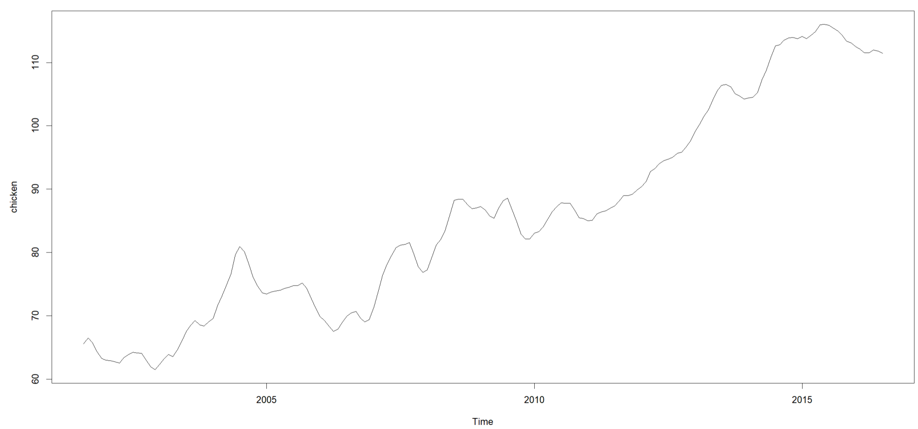

Figure 1: basic plot

We can see an upwards trend in prices, so to remove that I did differencing:

```
#plot differenced chicken
plot(diff(chicken))
```

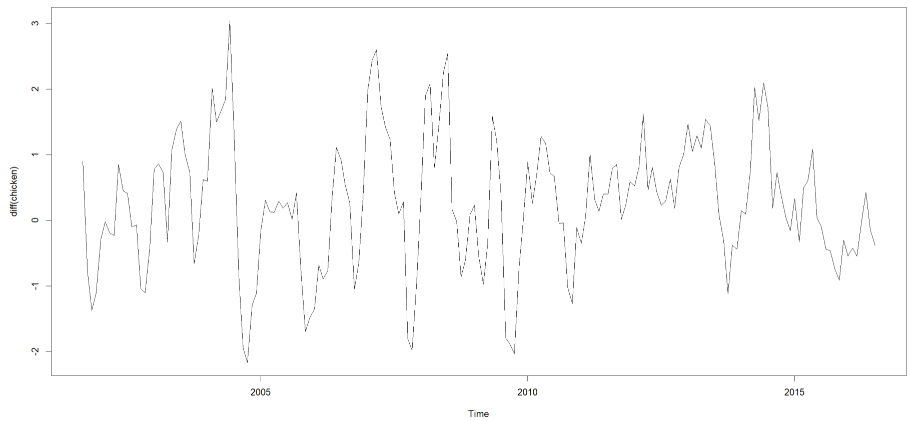

Figure 2: differenced chicken

I see the trend is removed but there is seasonality. I put in the years to be sure

```
plot(diff(chicken), xaxt = "n", xlab = "Year")
years <- floor(time(diff(chicken)))
axis(1, at=time(diff(chicken))[seq(1, length(time(diff(chicken))), by=12)],
labels=years[seq(1, length(years), by=12)])
```

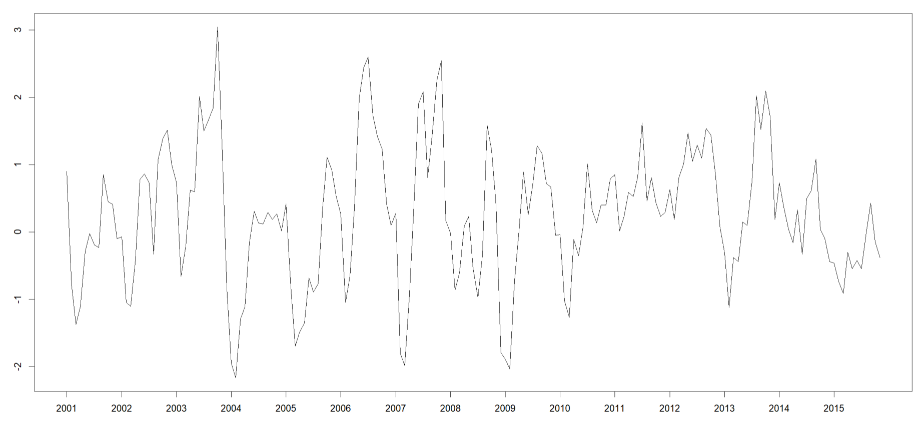

Figure 3: with years

Although some years seem to have 2 troughs I will go with lag-12

```
X<-diff(chicken, lag = 12)
X
plot(X)
```

```
mean(X)
abline(h = 0)
```

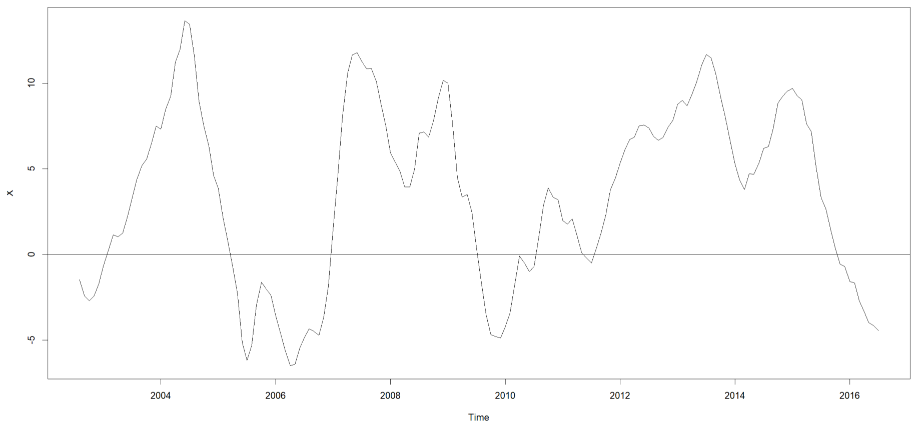

Figure 4: lag 12

then I did ACF to see seasonality better with lag 12:

```
acf(X, main = "ACF of Differenced Chicken Time Series")
```

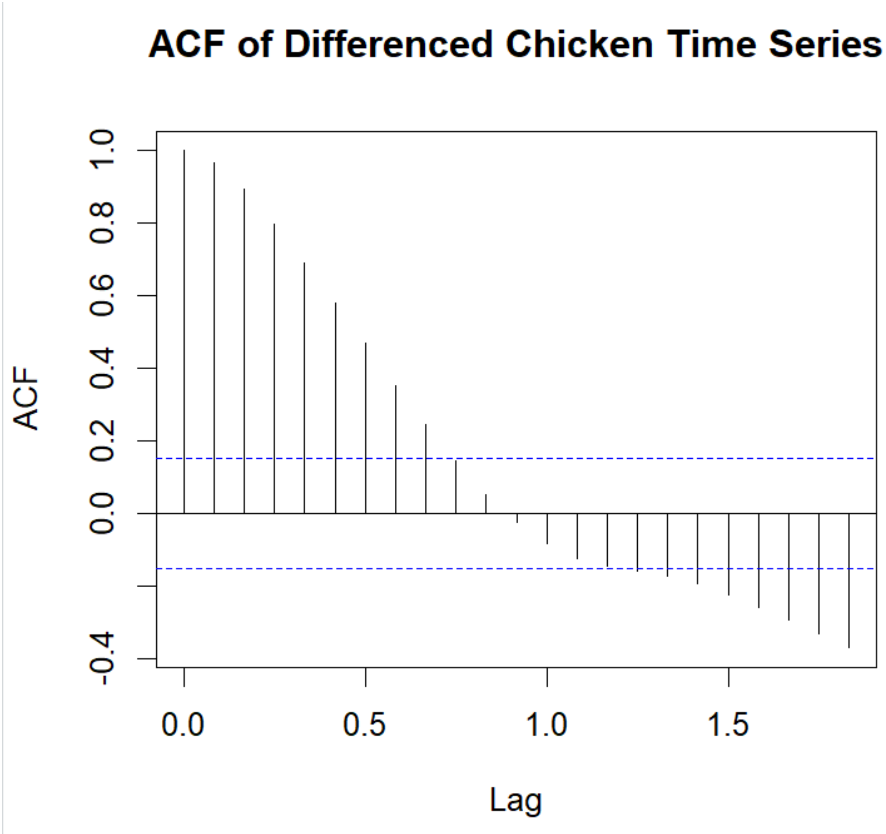

Figure 5: ACF lag 12

I am starting over from logging the data:

```
#log transformation
log_chicken <- log(chicken)
```

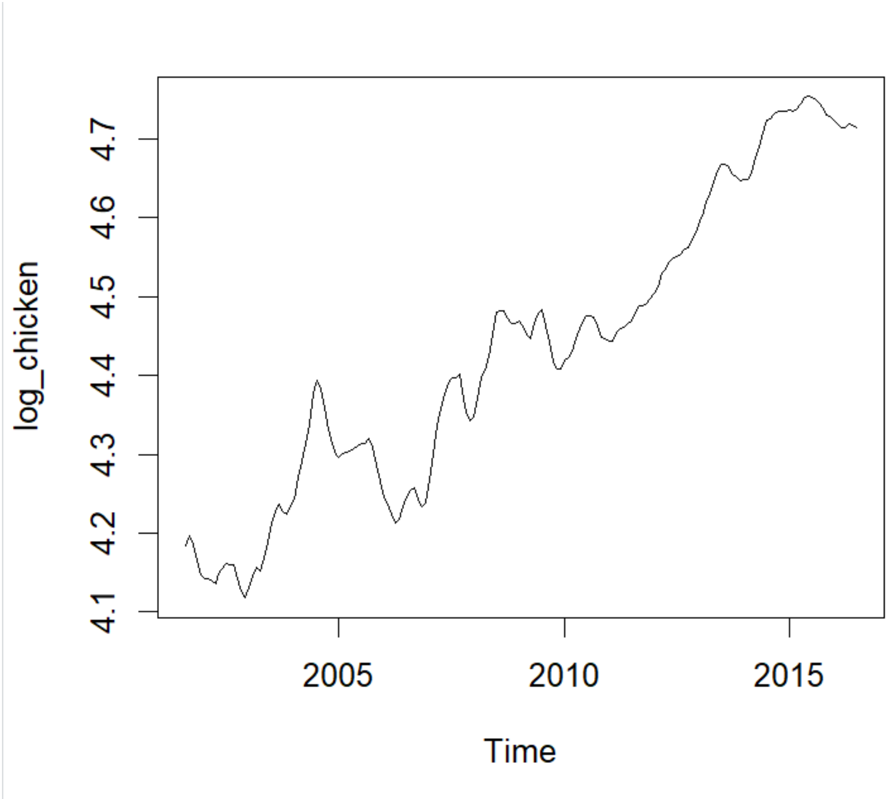

Figure 6: logged chicken

```
#differencing
log_diff_chicken <- diff(log_chicken)

plot(log_diff_chicken, main = "Differenced Log Chicken",
ylab = "Differenced Log(Chicken)", xlab = "Year")
```

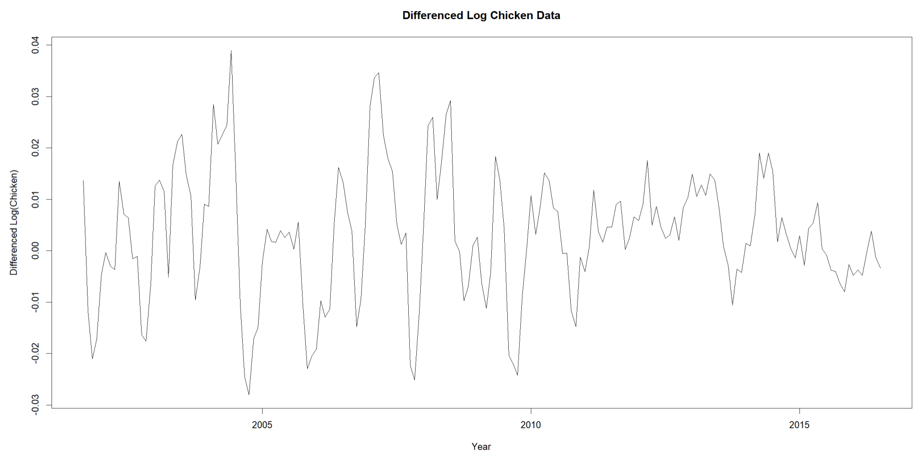

Figure 7: Differenced Log Chicken

```
log_diff_seasonal_chicken <- diff(log_diff_chicken, lag = 12)

# Plot the seasonally differenced data
plot(log_diff_seasonal_chicken, main = "Seasonally
Differenced Log Chicken Data",
ylab = "Seasonally Differenced Log(Chicken)",
xlab = "Year")
```

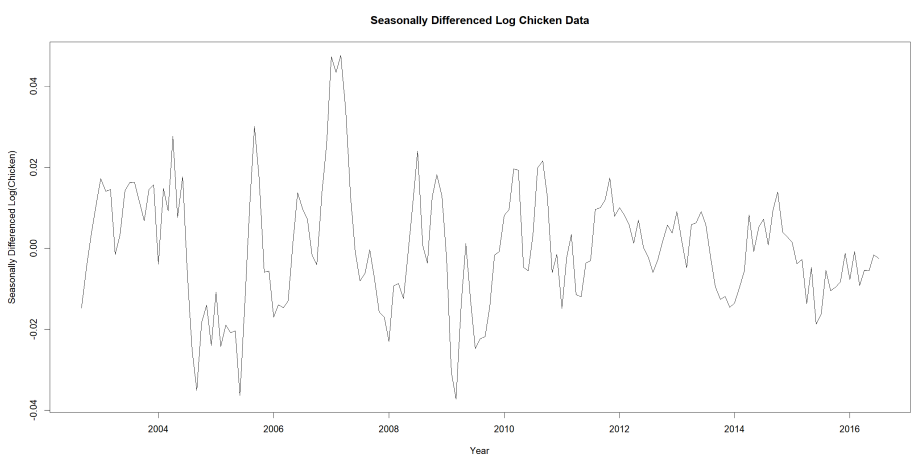

Figure 8: Seasonally Differenced Log(Chicken)

```
mean(log_diff_seasonal_chicken)
-9.815302e-05
```

```
I would say it is pretty close to zero.

Y<-log_diff_seasonal_chicken

# Run augmented Dickey--Fuller Test
(length(Y)-1)^(1/3)  #5.495865 so I take 5
adfTest(Y, ’nc’, lag = 5)
```

```
> adfTest(Y, 'nc', lag = 5)

Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 5
  STATISTIC:
    Dickey-Fuller: -4.2571
  P VALUE:
    0.01

Description:
 Mon Dec  2 04:01:49 2024 by user: ozozg
Warning message:
In adfTest(Y, "nc", lag = 5) : p-value smaller than printed p-value
```

Figure 9: ADF test

Also ran with constant

```
> adfTest(Y, 'c', lag = 5)

Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 5
  STATISTIC:
    Dickey-Fuller: -4.244
  P VALUE:
    0.01

Description:
 Mon Dec  2 04:04:27 2024 by user: ozozg
Warning message:
In adfTest(Y, "c", lag = 5) : p-value smaller than printed p-value
```

Figure 10: ADF with constant

So we reject the null hypothesis of an additional unit root.

```
# Look at ACF and PACF of the lag-12 differenced data
par(mfrow = c(1,2))
acf(Y, lag.max = 60)
pacf(Y, lag.max = 60)
```

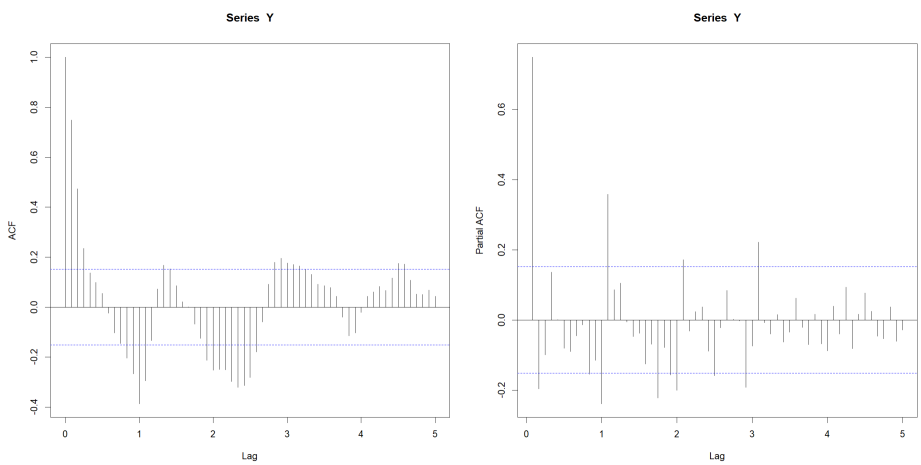

Figure 11: ACF PACF

It shows that I still haven't gotten rid of all the seasonal dependents. I can see "several" spikes outside blue lines. So I should be looking at a SARIMA model with a non-trivial seasonal autoregressive or moving average part. ACF shows decaying per about 12-14 lags, so my assumption is that it will be ARMA model.

p=1 from PACF
q= 2 or 3 from ACF
P=1
Q=1 Seasonal ACF shows a spike at lag 12.
s=12

$$for\mathrm{ARMA}(p,q)\times(P,Q)_S$$

$$\mathrm{ARMA}(1,2)\times(1,0)_{12}$$

```
auto.arima(chicken, trace = TRUE, approximation = FALSE)

ARIMA(2,1,2)(1,0,1)[12] with drift         : Inf
 ARIMA(0,1,0)            with drift         : 512.1971
 ARIMA(1,1,0)(1,0,0)[12] with drift         : 357.5881
 ARIMA(0,1,1)(0,0,1)[12] with drift         : 399.618
 ARIMA(0,1,0)                               : 521.5318
```

```
ARIMA(1,1,0)            with drift         : 380.7635
ARIMA(1,1,0)(2,0,0)[12] with drift         : 356.8099
ARIMA(1,1,0)(2,0,1)[12] with drift         : Inf
ARIMA(1,1,0)(1,0,1)[12] with drift         : Inf
ARIMA(0,1,0)(2,0,0)[12] with drift         : 493.6446
ARIMA(2,1,0)(2,0,0)[12] with drift         : 348.8114
ARIMA(2,1,0)(1,0,0)[12] with drift         : 348.6757
ARIMA(2,1,0)            with drift         : 365.5143
ARIMA(2,1,0)(1,0,1)[12] with drift         : Inf
ARIMA(2,1,0)(0,0,1)[12] with drift         : 351.9621
ARIMA(2,1,0)(2,0,1)[12] with drift         : Inf
ARIMA(3,1,0)(1,0,0)[12] with drift         : 348.476
ARIMA(3,1,0)            with drift         : 363.5378
ARIMA(3,1,0)(2,0,0)[12] with drift         : 349.5943
ARIMA(3,1,0)(1,0,1)[12] with drift         : Inf
ARIMA(3,1,0)(0,0,1)[12] with drift         : 350.6821
ARIMA(3,1,0)(2,0,1)[12] with drift         : Inf
ARIMA(4,1,0)(1,0,0)[12] with drift         : 349.7064
ARIMA(3,1,1)(1,0,0)[12] with drift         : 349.2089
ARIMA(2,1,1)(1,0,0)[12] with drift         : 349.227
ARIMA(4,1,1)(1,0,0)[12] with drift         : 351.3947
ARIMA(3,1,0)(1,0,0)[12]                    : 348.0787
ARIMA(3,1,0)                               : 365.9982
ARIMA(3,1,0)(2,0,0)[12]                    : 348.724
ARIMA(3,1,0)(1,0,1)[12]                    : Inf
ARIMA(3,1,0)(0,0,1)[12]                    : 351.174
ARIMA(3,1,0)(2,0,1)[12]                    : Inf
ARIMA(2,1,0)(1,0,0)[12]                    : 347.8974
ARIMA(2,1,0)                               : 366.8595
ARIMA(2,1,0)(2,0,0)[12]                    : 347.6443
ARIMA(2,1,0)(2,0,1)[12]                    : Inf
ARIMA(2,1,0)(1,0,1)[12]                    : Inf
ARIMA(1,1,0)(2,0,0)[12]                    : 355.1769
ARIMA(2,1,1)(2,0,0)[12]                    : 349.1905
ARIMA(1,1,1)(2,0,0)[12]                    : 350.0006
ARIMA(3,1,1)(2,0,0)[12]                    : 351.228
```

```
 Best model: ARIMA(2,1,0)(2,0,0)[12]

Series: chicken
ARIMA(2,1,0)(2,0,0)[12]

Coefficients:
         ar1      ar2    sar1    sar2
      0.9137  -0.2325  0.2990  0.1176
s.e.  0.0735   0.0739  0.0735  0.0759

sigma^2 = 0.3883:  log likelihood = -168.65
AIC=347.3   AICc=347.64   BIC=363.23
```

Figure 12: arima model results

So the chosen model is ARIMA(2,1,0)(2,0,0) with seasonality 12.
Without approximation=FALSE:

```
auto.arima(chicken, trace = TRUE)
ARIMA(2,1,2)(1,0,1)[12] with drift         : Inf
 ARIMA(0,1,0)           with drift         : 512.1327
 ARIMA(1,1,0)(1,0,0)[12] with drift        : 357.8553
 ARIMA(0,1,1)(0,0,1)[12] with drift        : 399.0111
 ARIMA(0,1,0)                              : 521.4674
 ARIMA(1,1,0)           with drift         : 380.5252
 ARIMA(1,1,0)(2,0,0)[12] with drift        : 361.1736
 ARIMA(1,1,0)(1,0,1)[12] with drift        : Inf
 ARIMA(1,1,0)(0,0,1)[12] with drift        : 361.7262
 ARIMA(1,1,0)(2,0,1)[12] with drift        : Inf
 ARIMA(0,1,0)(1,0,0)[12] with drift        : 496.251
 ARIMA(2,1,0)(1,0,0)[12] with drift        : 349.6098
 ARIMA(2,1,0)           with drift         : 361.2204
 ARIMA(2,1,0)(2,0,0)[12] with drift        : 352.9452
 ARIMA(2,1,0)(1,0,1)[12] with drift        : Inf
 ARIMA(2,1,0)(0,0,1)[12] with drift        : 347.5637
 ARIMA(2,1,0)(0,0,2)[12] with drift        : 348.8669
 ARIMA(2,1,0)(1,0,2)[12] with drift        : Inf
 ARIMA(3,1,0)(0,0,1)[12] with drift        : 346.8076
 ARIMA(3,1,0)           with drift         : 359.6763
 ARIMA(3,1,0)(1,0,1)[12] with drift        : Inf
 ARIMA(3,1,0)(0,0,2)[12] with drift        : 348.5706
 ARIMA(3,1,0)(1,0,0)[12] with drift        : 350.3946
 ARIMA(3,1,0)(1,0,2)[12] with drift        : Inf
```

```
ARIMA(4,1,0)(0,0,1)[12] with drift         : 349.2894
ARIMA(3,1,1)(0,0,1)[12] with drift         : 347.6645
ARIMA(2,1,1)(0,0,1)[12] with drift         : 346.754
ARIMA(2,1,1)            with drift         : 359.2506
ARIMA(2,1,1)(1,0,1)[12] with drift         : Inf
ARIMA(2,1,1)(0,0,2)[12] with drift         : 348.4092
ARIMA(2,1,1)(1,0,0)[12] with drift         : 350.2436
ARIMA(2,1,1)(1,0,2)[12] with drift         : Inf
ARIMA(1,1,1)(0,0,1)[12] with drift         : 354.7731
ARIMA(2,1,2)(0,0,1)[12] with drift         : 348.0198
ARIMA(1,1,2)(0,0,1)[12] with drift         : 348.7836
ARIMA(3,1,2)(0,0,1)[12] with drift         : 349.5839
ARIMA(2,1,1)(0,0,1)[12]                    : 347.2775
```

```
 Now re-fitting the best model(s) without approximations...

 ARIMA(2,1,1)(0,0,1)[12] with drift         : 351.5017

 Best model: ARIMA(2,1,1)(0,0,1)[12] with drift

Series: chicken
ARIMA(2,1,1)(0,0,1)[12] with drift

Coefficients:
         ar1      ar2      ma1    sma1   drift
      1.2933  -0.5375  -0.4019  0.2756  0.2518
s.e.  0.2220   0.1542   0.2569  0.0692  0.1428

sigma^2 = 0.396:  log likelihood = -169.51
AIC=351.01   AICc=351.5   BIC=370.14
```

Figure 13: arima without approximation=FALSE

running it again without the drift

`auto.arima(chicken,allowdrift = FALSE, trace = TRUE)`

```
 ARIMA(3,1,0)(0,0,1)[12]                    : 351.174

 Best model: ARIMA(3,1,0)(0,0,1)[12]

Series: chicken
ARIMA(3,1,0)(0,0,1)[12]

Coefficients:
         ar1      ar2      ar3    sma1
      0.8982  -0.1416  -0.1255  0.2899
s.e.  0.0749   0.1005   0.0745  0.0676

sigma^2 = 0.3975:  log likelihood = -170.41
AIC=350.83   AICc=351.17   BIC=366.76
```

Figure 14: not allowing drift

```
#running auto.arima for logarithmically and seasonally differenced data
auto.arima(Y, allowdrift = FALSE, trace = TRUE, stationary = TRUE,
approximation = FALSE)
```

```
ARIMA(2,0,2)(1,0,1)[12] with non-zero mean : Inf
 ARIMA(0,0,0)            with non-zero mean : -934.2136
 ARIMA(1,0,0)(1,0,0)[12] with non-zero mean : -1100.37
 ARIMA(0,0,1)(0,0,1)[12] with non-zero mean : -1109.322
 ARIMA(0,0,0)            with zero mean     : -936.255
 ARIMA(0,0,1)            with non-zero mean : -1025.808
 ARIMA(0,0,1)(1,0,1)[12] with non-zero mean : -1107.377
 ARIMA(0,0,1)(0,0,2)[12] with non-zero mean : -1107.6
 ARIMA(0,0,1)(1,0,0)[12] with non-zero mean : -1056.714
 ARIMA(0,0,1)(1,0,2)[12] with non-zero mean : Inf
 ARIMA(0,0,0)(0,0,1)[12] with non-zero mean : -1017.156
 ARIMA(1,0,1)(0,0,1)[12] with non-zero mean : -1148.478
 ARIMA(1,0,1)            with non-zero mean : -1072.6
 ARIMA(1,0,1)(1,0,1)[12] with non-zero mean : -1146.36
 ARIMA(1,0,1)(0,0,2)[12] with non-zero mean : -1146.385
 ARIMA(1,0,1)(1,0,0)[12] with non-zero mean : -1104.631
 ARIMA(1,0,1)(1,0,2)[12] with non-zero mean : -1144.82
 ARIMA(1,0,0)(0,0,1)[12] with non-zero mean : -1143.414
 ARIMA(2,0,1)(0,0,1)[12] with non-zero mean : -1147.952
 ARIMA(1,0,2)(0,0,1)[12] with non-zero mean : -1150.304
 ARIMA(1,0,2)            with non-zero mean : -1077.053
 ARIMA(1,0,2)(1,0,1)[12] with non-zero mean : -1148.125
 ARIMA(1,0,2)(0,0,2)[12] with non-zero mean : -1148.125
 ARIMA(1,0,2)(1,0,0)[12] with non-zero mean : -1105.556
```

```
ARIMA(1,0,2)(1,0,2)[12] with non-zero mean : -1146.87
ARIMA(0,0,2)(0,0,1)[12] with non-zero mean : -1144.276
ARIMA(2,0,2)(0,0,1)[12] with non-zero mean : -1148.129
ARIMA(1,0,3)(0,0,1)[12] with non-zero mean : -1148.147
ARIMA(0,0,3)(0,0,1)[12] with non-zero mean : -1149.512
ARIMA(2,0,3)(0,0,1)[12] with non-zero mean : Inf
ARIMA(1,0,2)(0,0,1)[12] with zero mean     : -1152.38
ARIMA(1,0,2)            with zero mean     : -1079.163
ARIMA(1,0,2)(1,0,1)[12] with zero mean     : -1150.227
ARIMA(1,0,2)(0,0,2)[12] with zero mean     : -1150.227
ARIMA(1,0,2)(1,0,0)[12] with zero mean     : -1107.673
ARIMA(1,0,2)(1,0,2)[12] with zero mean     : -1148.991
ARIMA(0,0,2)(0,0,1)[12] with zero mean     : -1146.347
ARIMA(1,0,1)(0,0,1)[12] with zero mean     : -1150.514
ARIMA(2,0,2)(0,0,1)[12] with zero mean     : -1150.232
ARIMA(1,0,3)(0,0,1)[12] with zero mean     : -1150.25
ARIMA(0,0,1)(0,0,1)[12] with zero mean     : -1111.381
ARIMA(0,0,3)(0,0,1)[12] with zero mean     : -1151.594
ARIMA(2,0,1)(0,0,1)[12] with zero mean     : -1150.024
ARIMA(2,0,3)(0,0,1)[12] with zero mean     : Inf
```

```
 Best model: ARIMA(1,0,2)(0,0,1)[12] with zero mean

Series: Y
ARIMA(1,0,2)(0,0,1)[12] with zero mean

Coefficients:
         ar1     ma1     ma2     sma1
      0.4399  0.4672  0.2478  -0.824
s.e.  0.1306  0.1334  0.1080   0.063

sigma^2 = 5.205e-05:  log likelihood = 581.38
AIC=-1152.75   AICc=-1152.38   BIC=-1137.16
```

Figure 15: Seasonally Differenced Log Chicken Autoarima Results

And with the differenced data we have zero mean.
Trying ARIMA(1,0,2)(0,0,1)[12] on original data:

```
chickenmodel1<-sarima(chicken, 1,0,2,0,0,1, 12)
chickenmodel1$fit$aic
```

```
iter 100 value -0.419223
final  value -0.419223
stopped after 100 iterations
<><><><><><><><><><><><><><>

Coefficients:
      Estimate      SE  t.value p.value
ar1     0.9955  0.0050 197.6998       0
ma1     0.8291  0.0697  11.8993       0
ma2     0.4427  0.0561   7.8904       0
sma1    0.3063  0.0689   4.4448       0
xmean  86.6838 16.9580   5.1117       0

sigma^2 estimated as 0.4115548 on 175 degrees of freedom

AIC = 2.066098  AICc = 2.068013  BIC = 2.17253
```

Figure 16: sarima coefficients

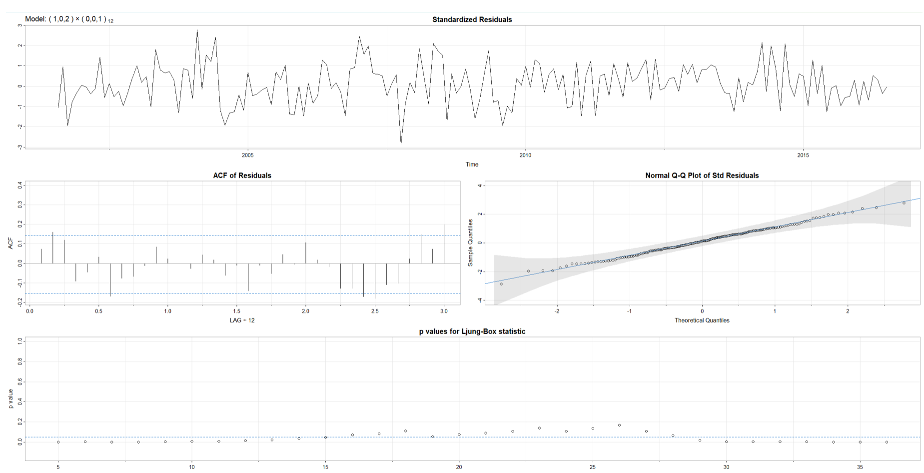

Figure 17: sarima plots on original data

Unfortunately most of Ljung Box values are under the target pvalue. Although ACF has little spikes this time. 4 lags seem to be above treshold but barely. Normal Q-Q plot shows normal distribution.
So, 1st model gave us 347.3, 2nd model 351.01, 3rd model 350.83, 4th model -1152.75 which is a bit surprising.

I also wanted to see ARIMA(2,1,0)(2,0,0)[12] on chicken to see if it is a better fit although it is not zero mean which is expected since it is the original/non transformed data:

```
chickenmodel2<-sarima(chicken, 2,1,0,2,0,0, 12)
```

```
iter   9 value -0.479489
final  value -0.479489
converged
<><><><><><><><><><><><><><>

Coefficients:
         Estimate     SE t.value p.value
ar1        0.9116 0.0733 12.4393  0.0000
ar2       -0.2393 0.0741 -3.2279  0.0015
sar1       0.2911 0.0739  3.9372  0.0001
sar2       0.1089 0.0765  1.4238  0.1563
constant   0.2244 0.2204  1.0182  0.3100

sigma^2 estimated as 0.3778902 on 174 degrees of freedom

AIC = 1.945939  AICc = 1.947876  BIC = 2.052778
```

Figure 18: sarima(2,1,0)(2,0,0)[12]

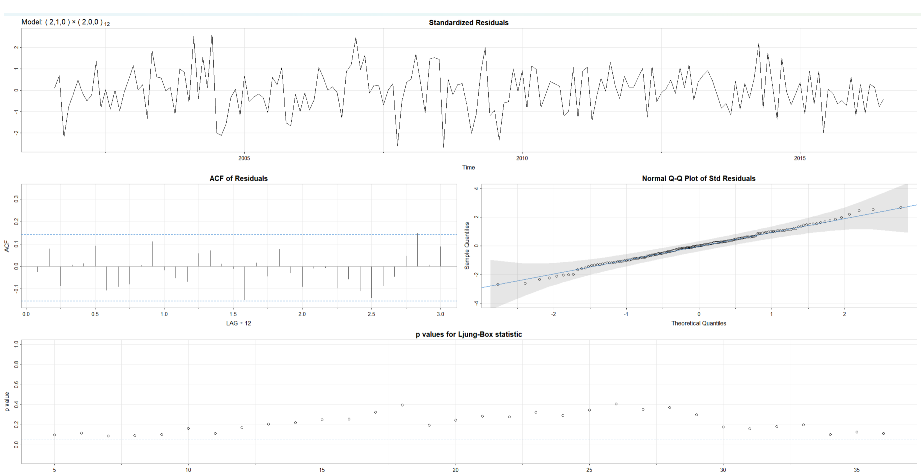

Figure 19: sarima(2,1,0)(2,0,0)[12] plots

All the p values are above the treshold, QQ plot is similar, ACF only has 1 over the line, barely.

Trying ARIMA(1,0,2)(0,0,1)[12] on differenced data:

```
chickenmodel3<-sarima(Y, 1,0,2,0,0,1, 12)
```

```
final  value -4.900463
converged
<><><><><><><><><><><><><><>

Coefficients:
      Estimate     SE  t.value p.value
ar1     0.4404 0.1305   3.3741  0.0009
ma1     0.4666 0.1334   3.4977  0.0006
ma2     0.2475 0.1080   2.2908  0.0233
sma1   -0.8242 0.0631 -13.0616  0.0000
xmean  -0.0001 0.0005  -0.2725  0.7856

sigma^2 estimated as 5.077663e-05 on 162 degrees of freedom

AIC = -6.891192  AICc = -6.88896  BIC = -6.779168
```

Figure 20: sarima on differenced chicken

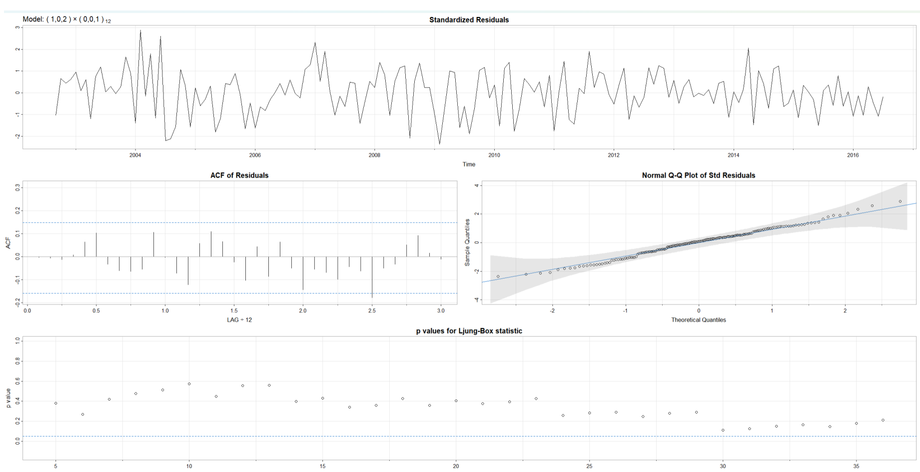

Figure 21: sarima on differenced chicken plots

pvalues in Ljung Box are all over the blue line. I have one small lag in ACF that is over the blue line. Residuals seem good. With these values, my fitted model is

$$\nabla_{12} X_t = (1 - 0.4404B)(1 + 0.4666B + 0.2475B^2)(1 + 0.8242B^{12})W_t,$$