

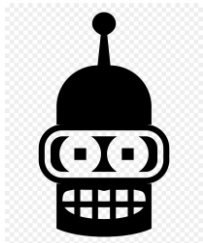
BANACH-TARSKI



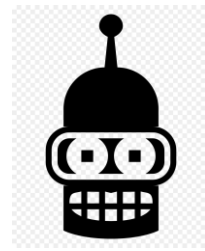
PARADOX

$$M = \sum_{n=0}^{\infty} 2^n \cdot \left[\frac{M_0}{2^{n(n+1)}} \right]$$



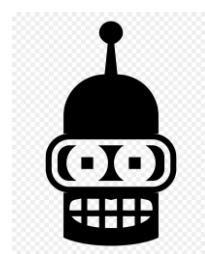


$$k = 0$$

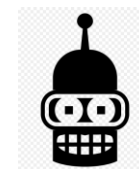
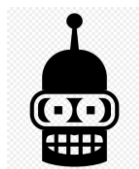


$$M_0$$

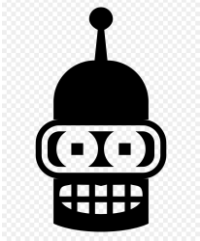
$$k = 0$$



$$M_0$$

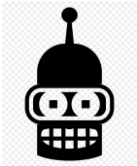


$$k = 0$$

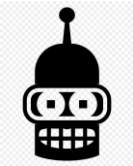


$$M_0$$

$$k = 1$$

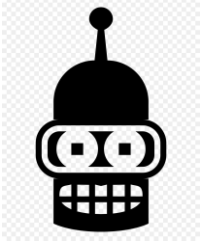


$$\frac{M_0}{2} \times \frac{1}{1+1}$$



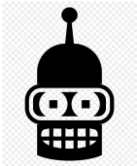
$$\frac{M_0}{2} \times \frac{1}{1+1}$$

$k = 0$

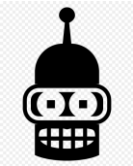


M_0

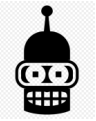
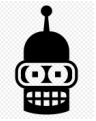
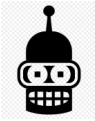
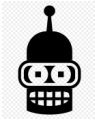
$k = 1$



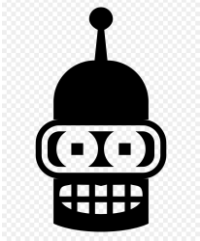
$$\frac{M_0}{2} \times \frac{1}{1+1}$$



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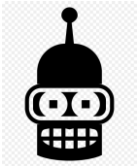


$k = 0$

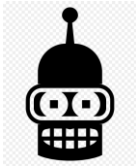


M_0

$k = 1$

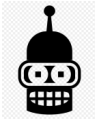


$$\frac{M_0}{2} \times \frac{1}{1+1}$$

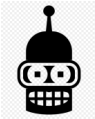


$$\frac{M_0}{2} \times \frac{1}{1+1}$$

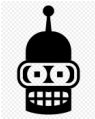
$k = 2$



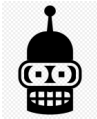
$$\frac{M_0}{4} \times \frac{1}{2+1}$$



$$\frac{M_0}{4} \times \frac{1}{2+1}$$

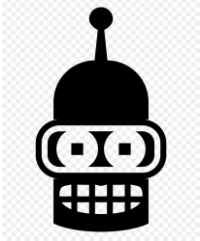


$$\frac{M_0}{4} \times \frac{1}{2+1}$$



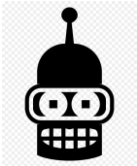
$$\frac{M_0}{4} \times \frac{1}{2+1}$$

$k = 0$

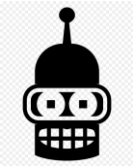


M_0

$k = 1$

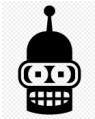


$$\frac{M_0}{2} \times \frac{1}{1+1}$$

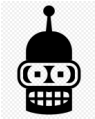


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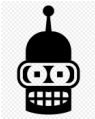
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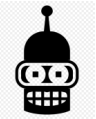
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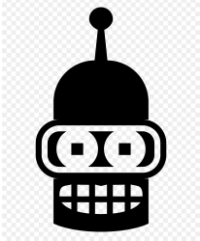
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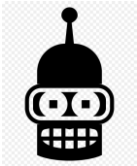


$k = 0$

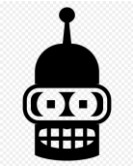


M_0

$k = 1$

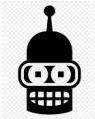


$$\frac{M_0}{2} \times \frac{1}{1+1}$$

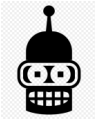


$$\frac{M_0}{2} \times \frac{1}{1+1}$$

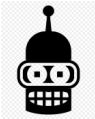
$k = 2$



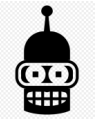
$$\frac{M_0}{4} \times \frac{1}{2+1}$$



$$\frac{M_0}{4} \times \frac{1}{2+1}$$



$$\frac{M_0}{4} \times \frac{1}{2+1}$$



$$\frac{M_0}{4} \times \frac{1}{2+1}$$

$k = 3$



$$\frac{M_0}{8} \times \frac{1}{3+1}$$



$$\frac{M_0}{8} \times \frac{1}{3+1}$$



$$\frac{M_0}{8} \times \frac{1}{3+1}$$



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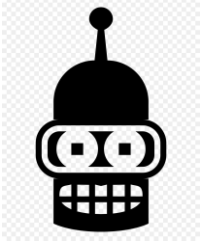


$$\frac{M_0}{8} \times \frac{1}{3+1}$$



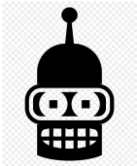
$$\frac{M_0}{8} \times \frac{1}{3+1}$$

$k = 0$

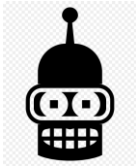


M_0

$k = 1$

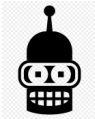


$$\frac{M_0}{2} \times \frac{1}{1+1}$$

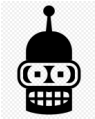


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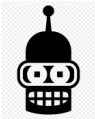
$k = 2$



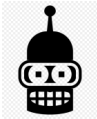
$$\frac{M_0}{4} \times \frac{1}{2+1}$$



$$\frac{M_0}{4} \times \frac{1}{2+1}$$



$$\frac{M_0}{4} \times \frac{1}{2+1}$$



$$\frac{M_0}{4} \times \frac{1}{2+1}$$

$k = 3$



$$\frac{M_0}{8} \times \frac{1}{3+1}$$



$$\frac{M_0}{8} \times \frac{1}{3+1}$$



$$\frac{M_0}{8} \times \frac{1}{3+1}$$



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Fórmula geral para a massa de cada Bender no k-esimo processo de duplicação

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$$M_k = \frac{1}{2^k} \frac{M_0}{k + 1} \quad (1)$$

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Massa total dos 2^k Benders no k-ésimo processo de duplicação

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$$M_k = \frac{1}{2^k} \frac{M_0}{k+1} \quad (1)$$

Massa total dos 2^k Benders no k-ésimo processo de duplicação

$$M = M_0 + 2M_1 + 4M_2 + 8M_3 + \cdots + 2^k M_k \quad (2)$$

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Massa total dos 2^k Benders no k-ésimo processo de duplicação

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Em notação de somatório

Fórmula geral para a massa de cada Bender no k-esimo processo de duplicação

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Em notação de somatório

$$M = \sum_{k=0}^n 2^k \frac{M_0}{2^k (k+1)} \quad (3)$$

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Simplificando, obtemos a famosa série Harmônica

Fórmula geral para a massa de cada Bender no k-esimo processo de duplicação

$$M_k = \frac{1}{2^k} \frac{M_0}{k+1} \quad (1)$$

Massa total dos 2^k Benders no k-ésimo processo de duplicação

$$M = M_0 + 2M_1 + 4M_2 + 8M_3 + \cdots + 2^k M_k \quad (2)$$

Em notação de somatório

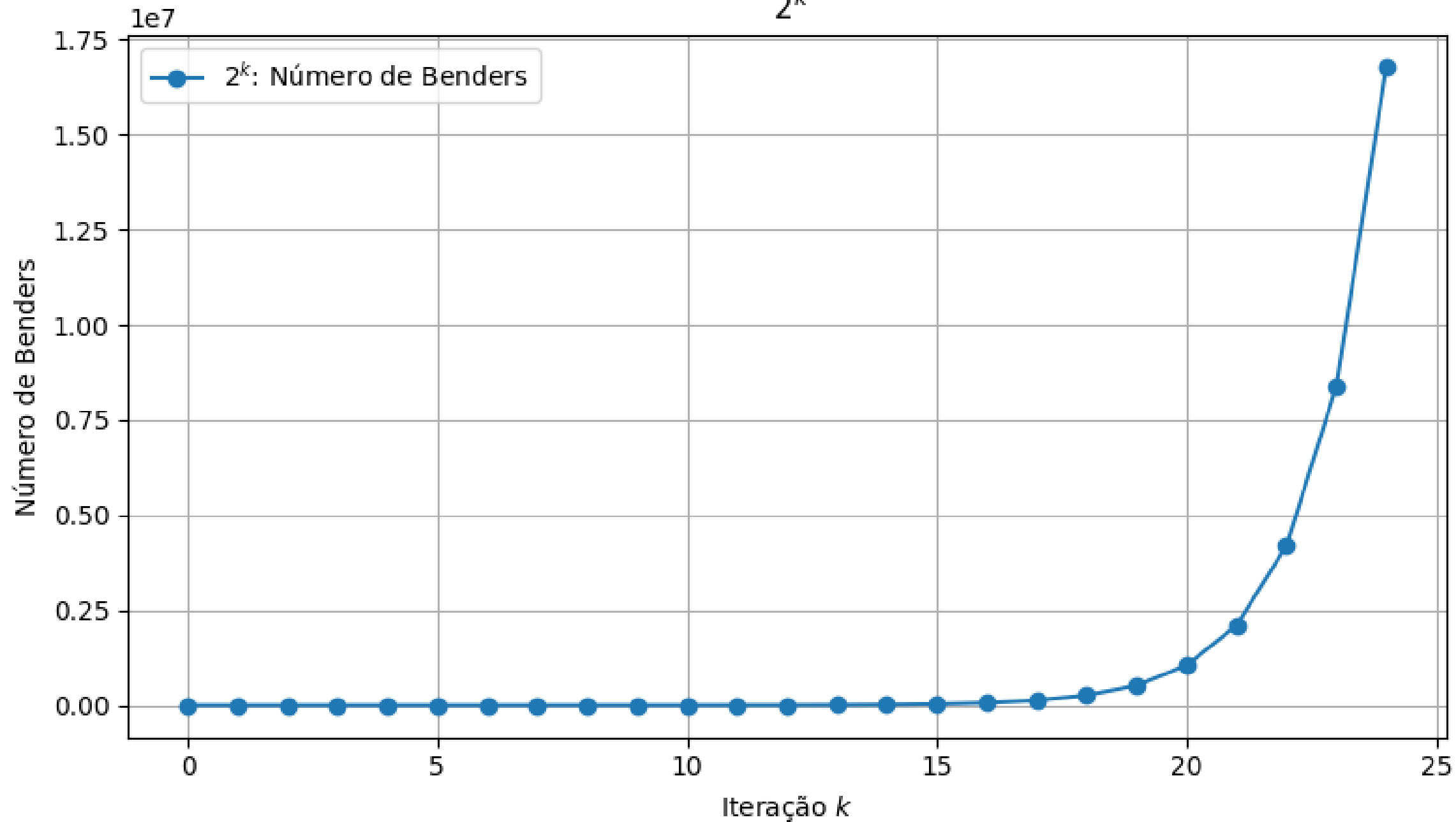
$$M = \sum_{k=0}^n 2^k \frac{M_0}{2^k (k+1)} \quad (3)$$

Simplificando, obtemos a famosa série Harmônica

$$M = \sum_{k=0}^n \frac{M_0}{k+1} \quad (4)$$

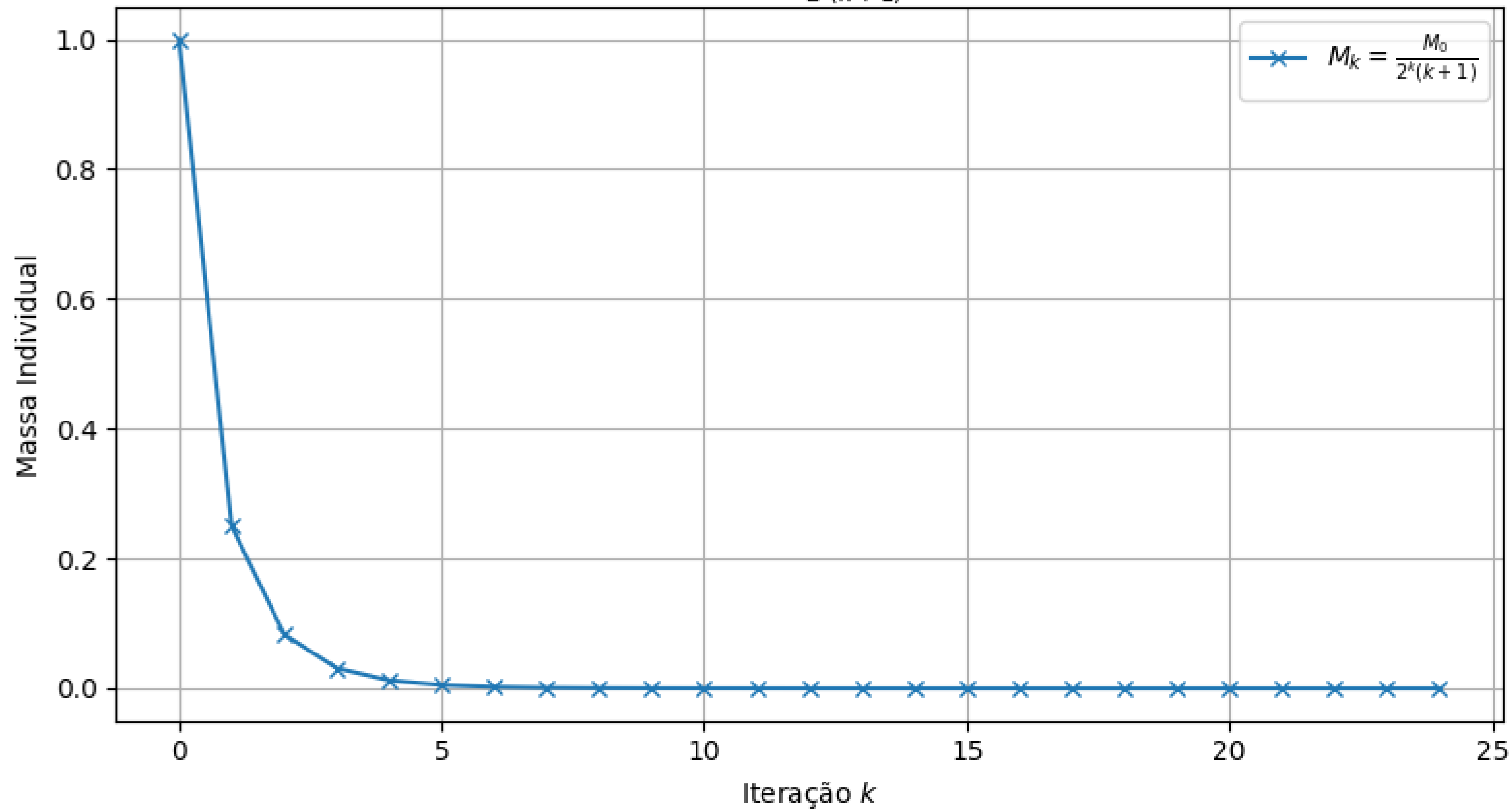
Duplicação de Benders por iteração

2^k



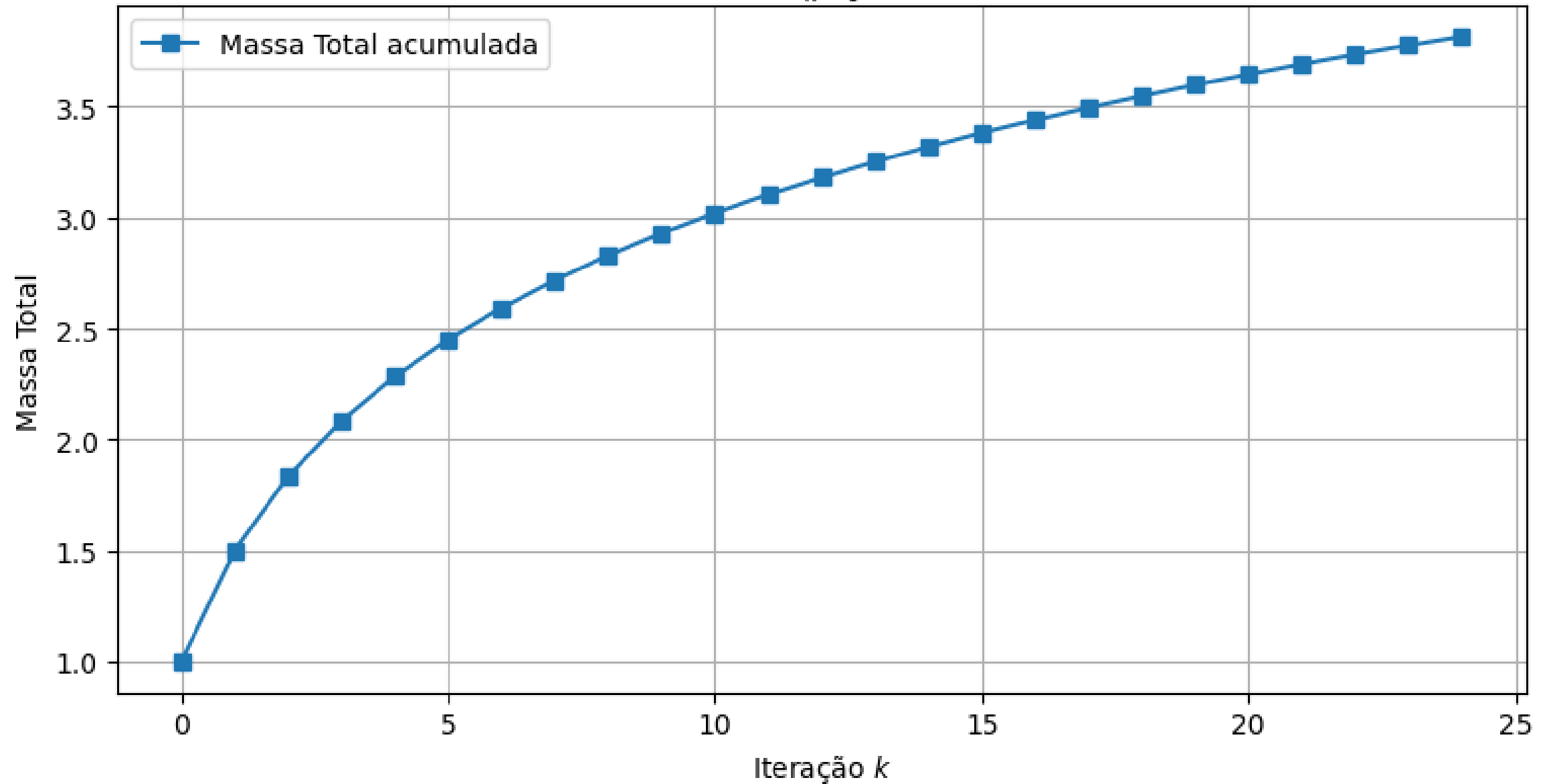
Massa individual de cada Bender

$$M_k = \frac{M_0}{2^k(k+1)}$$



Massa Total acumulada

$$M = \sum_{k=0}^n \frac{M_0}{k+1}$$





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