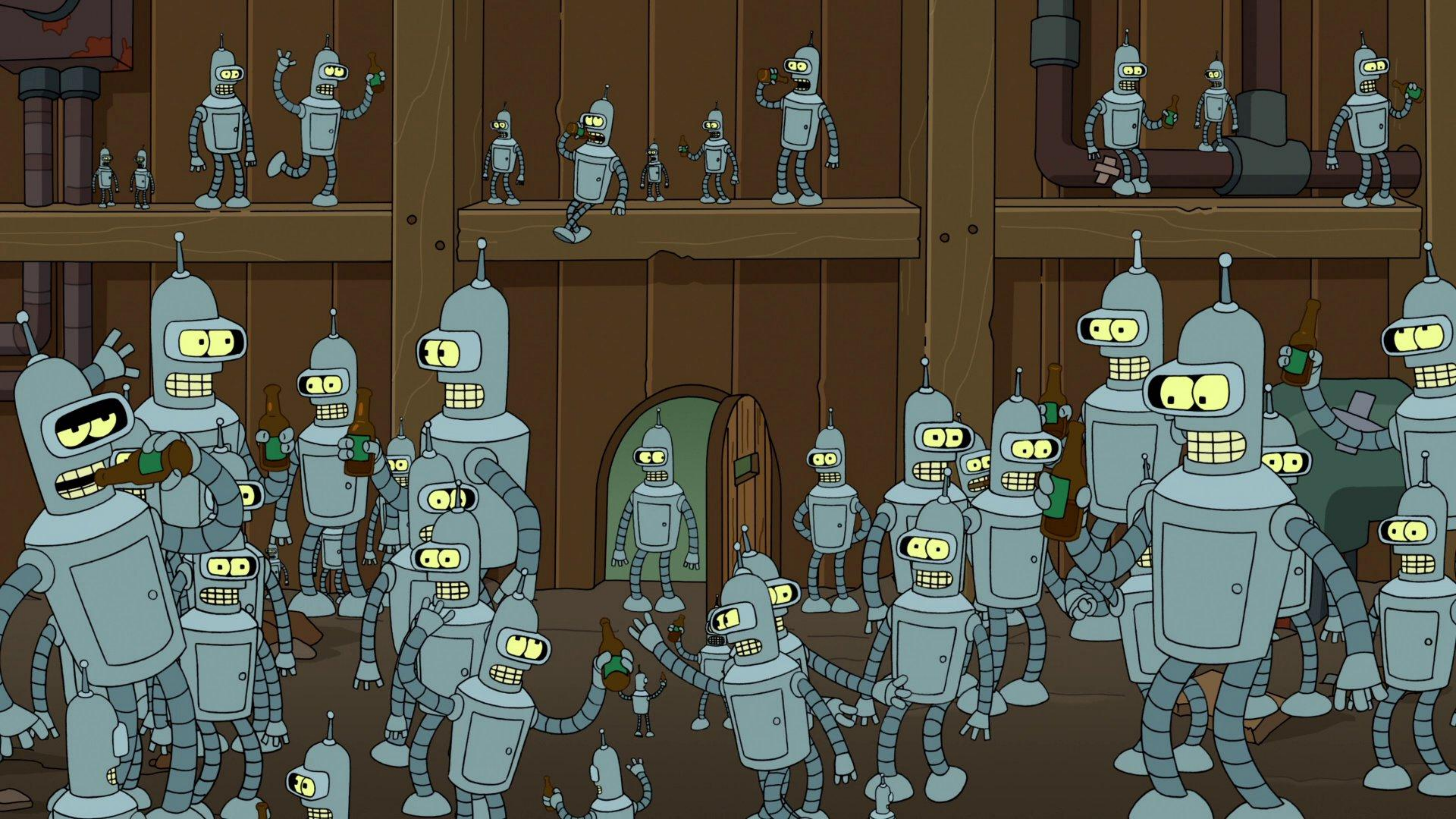
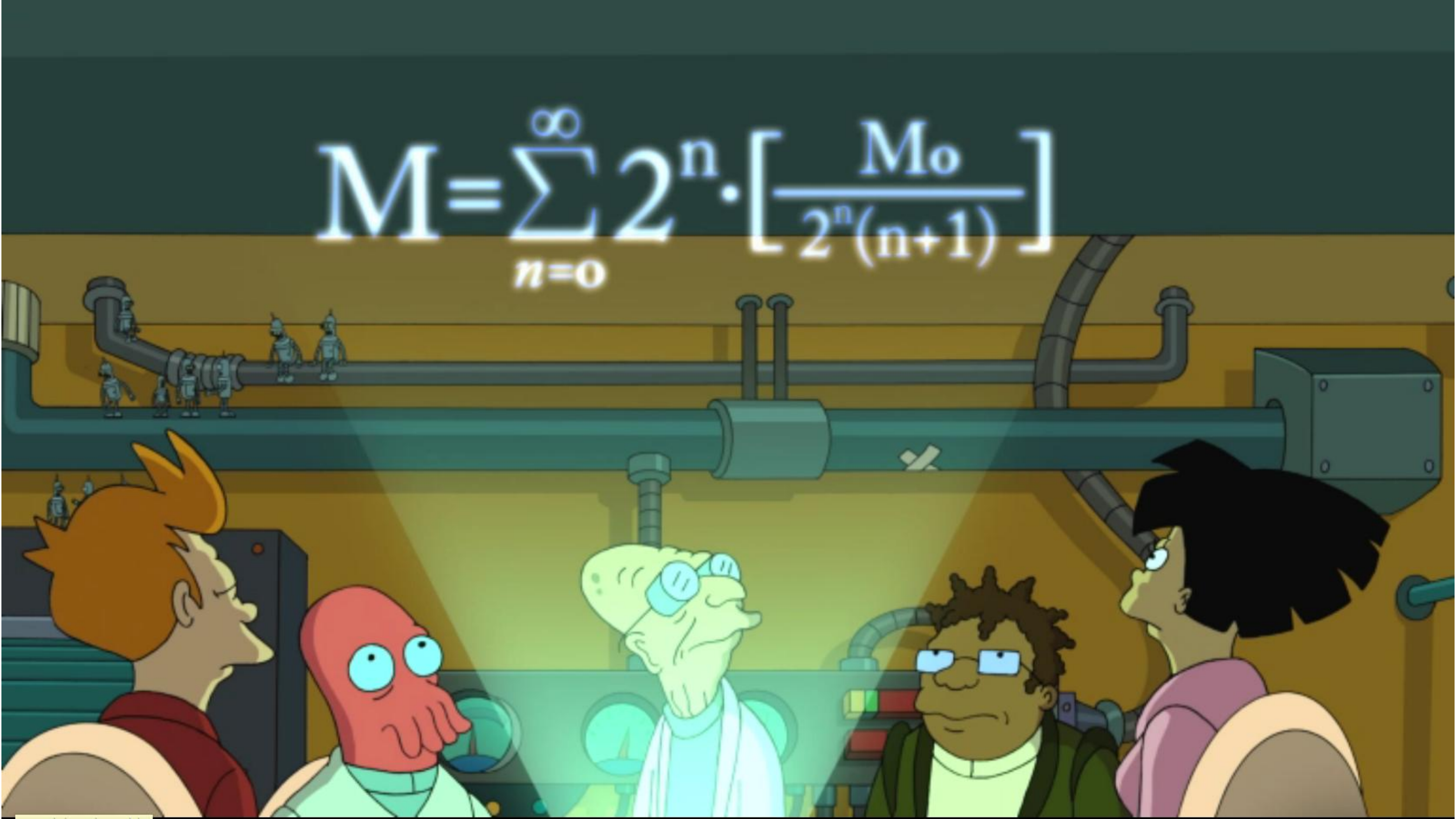
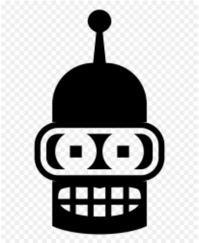

BANACH-TARSKI
DUPLA-SHRINKER



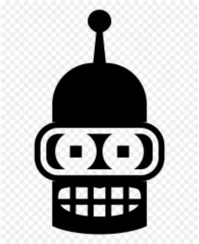

$M = \sum_{n=0}^{\infty} 2^n \cdot \left[\frac{Mo}{2^n(n+1)} \right]$

BANACH-TARSKI

PARADOX

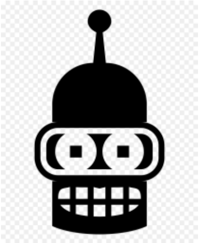


$k = 0$

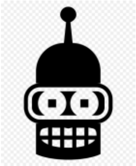


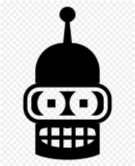
M_0

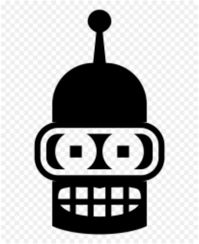
$k = 0$



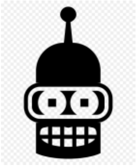
M_0

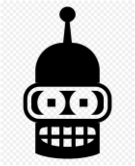




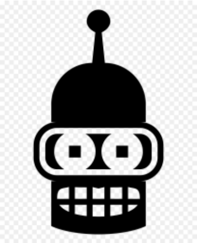


$k = 0$ $\quad M_0$

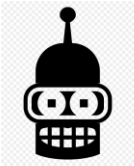


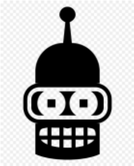


$k = 1$ $\quad \frac{M_0}{2} \times \frac{1}{1+1} \qquad \frac{M_0}{2} \times \frac{1}{1+1}$



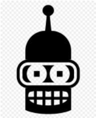
$k = 0$ $\quad M_0$

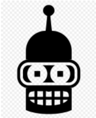


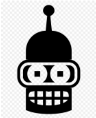


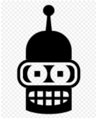
$k = 1$

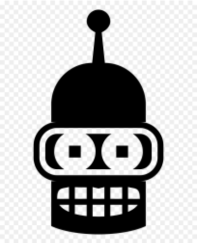
$$\frac{M_0}{2} \times \frac{1}{1+1} \qquad \frac{M_0}{2} \times \frac{1}{1+1}$$











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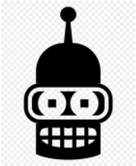
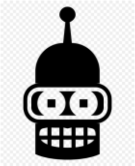
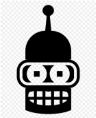
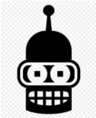
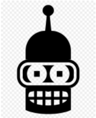
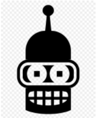
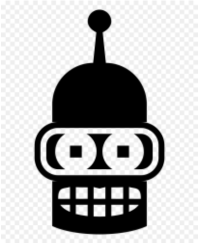
 

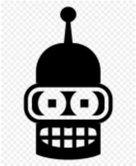
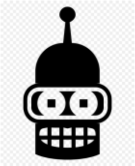
$k = 1$ $\qquad \frac{M_0}{2} \times \frac{1}{1+1} \qquad \frac{M_0}{2} \times \frac{1}{1+1}$

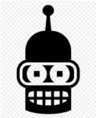
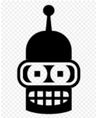
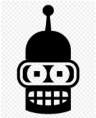
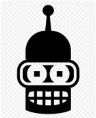
$k = 2$ $\qquad \frac{M_0}{4} \times \frac{1}{2+1} \qquad \frac{M_0}{4} \times \frac{1}{2+1} \qquad \frac{M_0}{4} \times \frac{1}{2+1} \qquad \frac{M_0}{4} \times \frac{1}{2+1}$



$k = 0$ $\qquad M_0$

$k = 1$ $\qquad \frac{M_0}{2} \times \frac{1}{1+1} \qquad \frac{M_0}{2} \times \frac{1}{1+1}$

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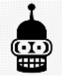 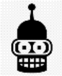 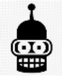 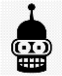 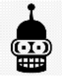 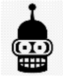 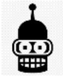 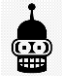

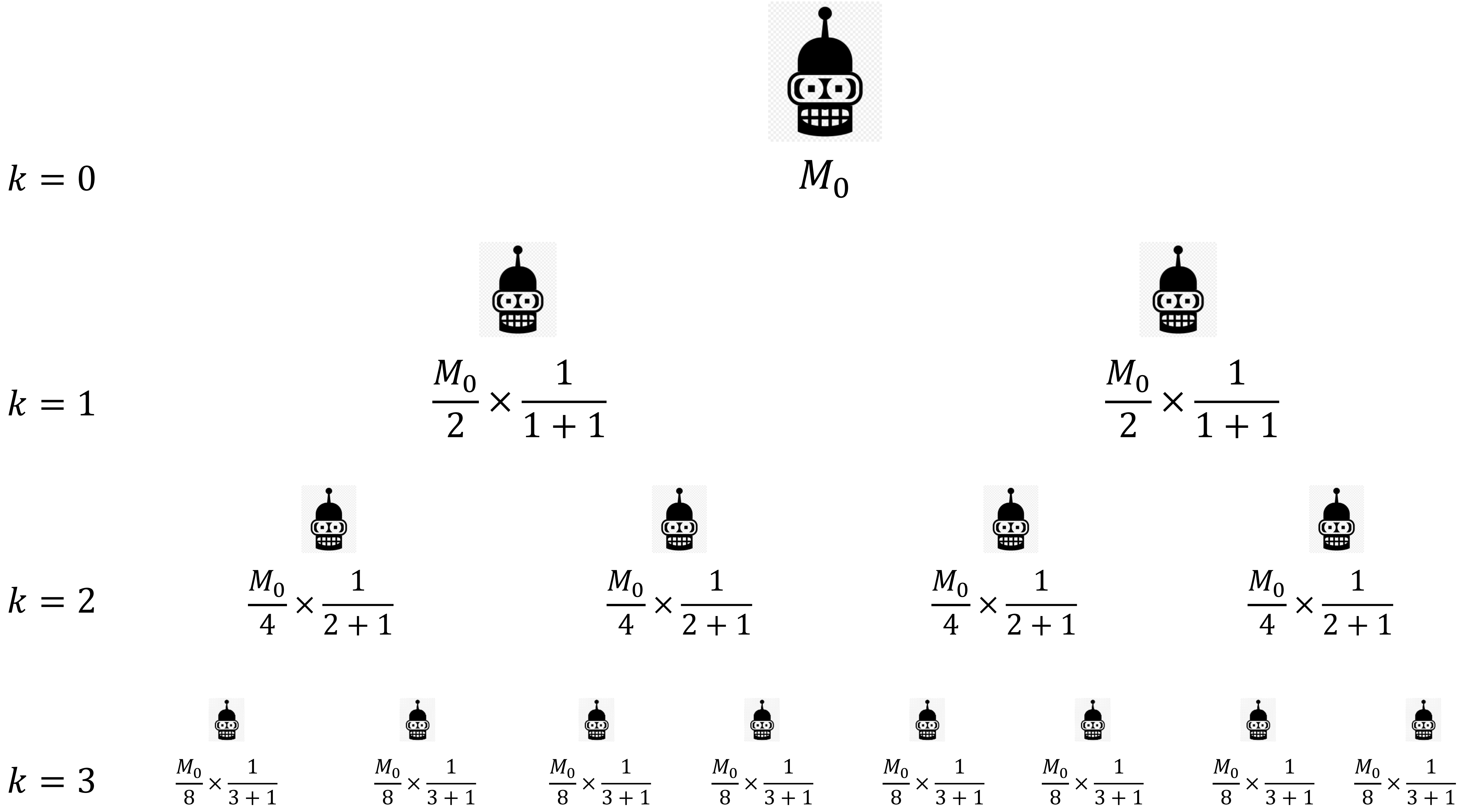
$k = 0$
M_0
$k = 1$
$\frac{M_0}{2} \times \frac{1}{1+1}$
$\frac{M_0}{2} \times \frac{1}{1+1}$
$k = 2$
$\frac{M_0}{4} \times \frac{1}{2+1}$
$\frac{M_0}{4} \times \frac{1}{2+1}$
$\frac{M_0}{4} \times \frac{1}{2+1}$
$\frac{M_0}{4} \times \frac{1}{2+1}$
$k = 3$
$\frac{M_0}{8} \times \frac{1}{3+1}$
$\frac{M_0}{8} \times \frac{1}{3+1}$
$\frac{M_0}{8} \times \frac{1}{3+1}$
$\frac{M_0}{8} \times \frac{1}{3+1}$
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Fórmula geral para a massa de cada Bender no k-esimo processo de duplicação

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$$M_k = \frac{1}{2^k}\frac{M_0}{k+1} \tag{1}$$

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Massa total dos 2^k Benders no k-ésimo processo de duplicação

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$$M_k = \frac{1}{2^k}\frac{M_0}{k+1} \tag{1}$$

Massa total dos 2^k Benders no k-ésimo processo de duplicação

$$M = M_0 + 2M_1 + 4M_2 + 8M_3 + \cdots + 2^k M_k \tag{2}$$

Fórmula geral para a massa de cada Bender no k-esimo processo de duplicação

$$M_k = \frac{1}{2^k}\frac{M_0}{k+1} \tag{1}$$

Massa total dos 2^k Benders no k-ésimo processo de duplicação

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Em notação de somatório

Fórmula geral para a massa de cada Bender no k-esimo processo de duplicação

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$$M = M_0 + 2M_1 + 4M_2 + 8M_3 + \cdots + 2^k M_k \tag{2}$$

Em notação de somatório

$$M = \sum_{k=0}^{n} 2^k \frac{M_0}{2^k(k+1)} \tag{3}$$

Fórmula geral para a massa de cada Bender no k-esimo processo de duplicação

$$M_k = \frac{1}{2^k}\frac{M_0}{k+1} \tag{1}$$

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Em notação de somatório

$$M = \sum_{k=0}^{n} 2^k \frac{M_0}{2^k(k+1)} \tag{3}$$

Simplificando, obtemos a famosa série Harmônica

Fórmula geral para a massa de cada Bender no k-esimo processo de duplicação

$$M_k = \frac{1}{2^k}\frac{M_0}{k+1} \tag{1}$$

Massa total dos 2^k Benders no k-ésimo processo de duplicação

$$M = M_0 + 2M_1 + 4M_2 + 8M_3 + \cdots + 2^k M_k \tag{2}$$

Em notação de somatório

$$M = \sum_{k=0}^{n} 2^k \frac{M_0}{2^k(k+1)} \tag{3}$$

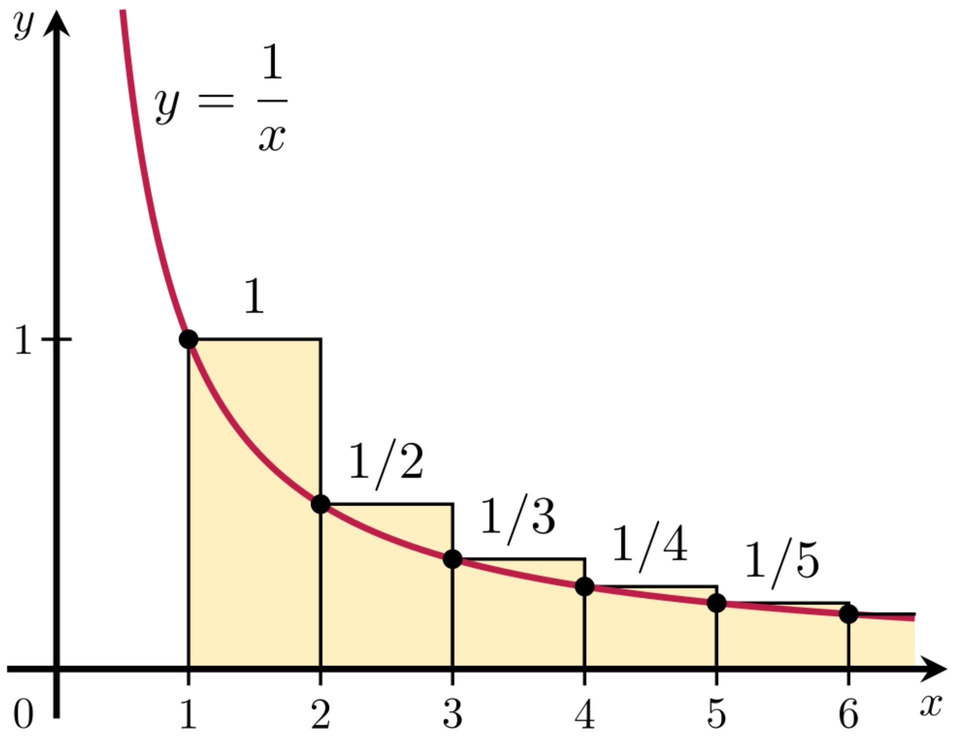
Simplificando, obtemos a famosa série Harmônica

$$M = \sum_{k=0}^{n} \frac{M_0}{k+1} \tag{4}$$

A série harmônica de fato é tida quando $n \to \infty$ e trocando de variável $s = k + 1$

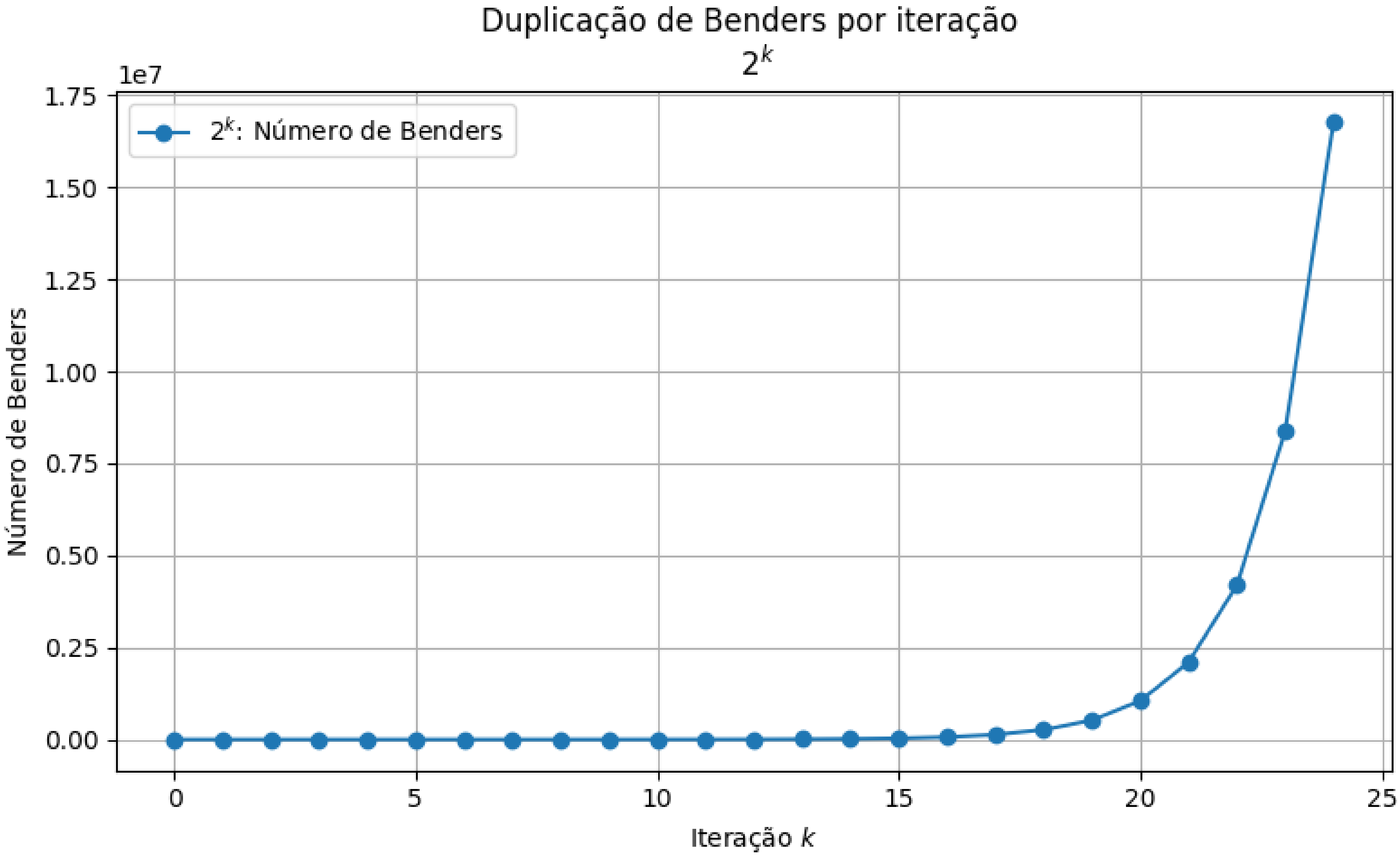
$$M = M_0 \sum_{s=1}^{\infty} \frac{1}{s} \tag{5}$$

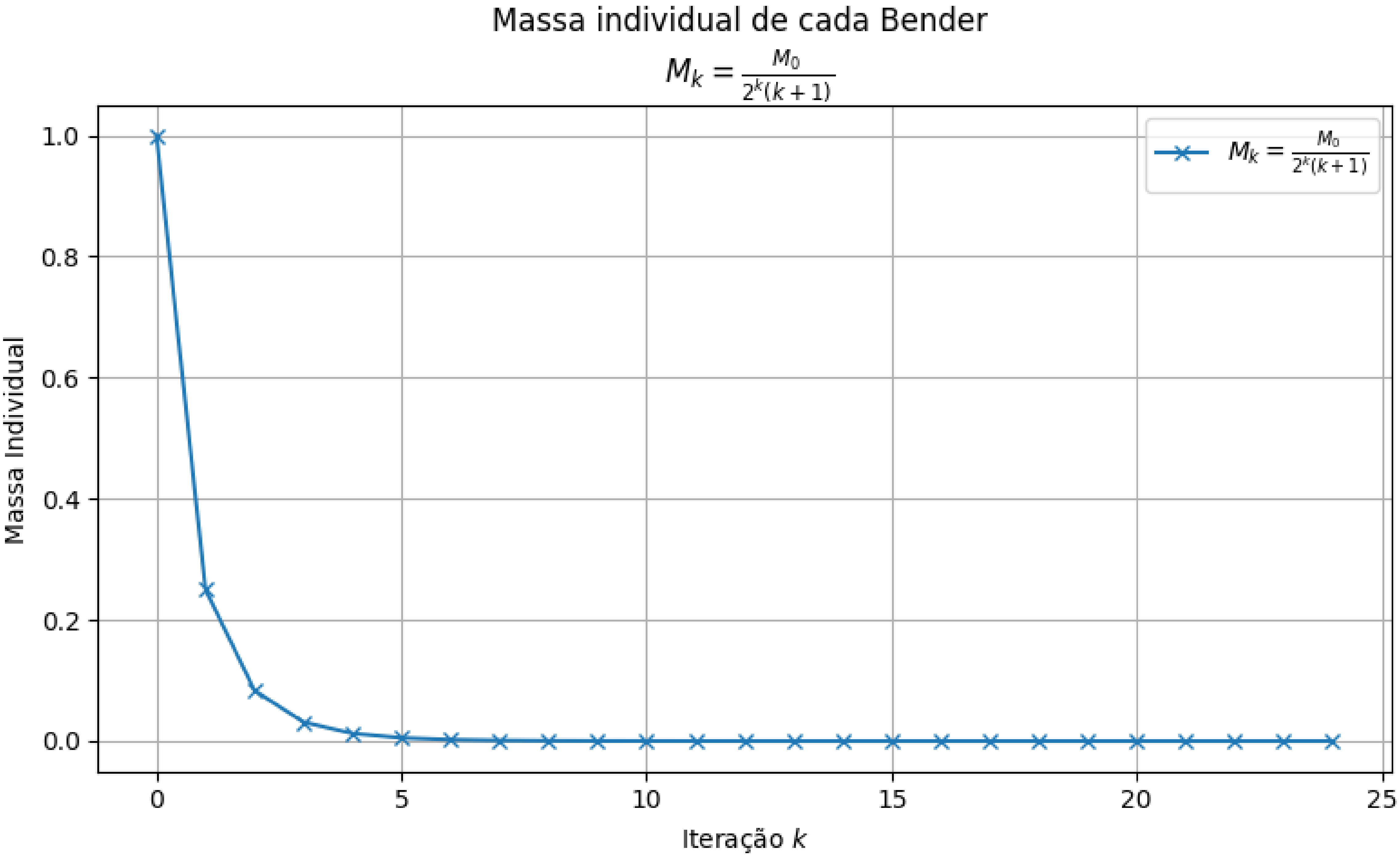
A série harmônica de fato é tida quando $n \to \infty$

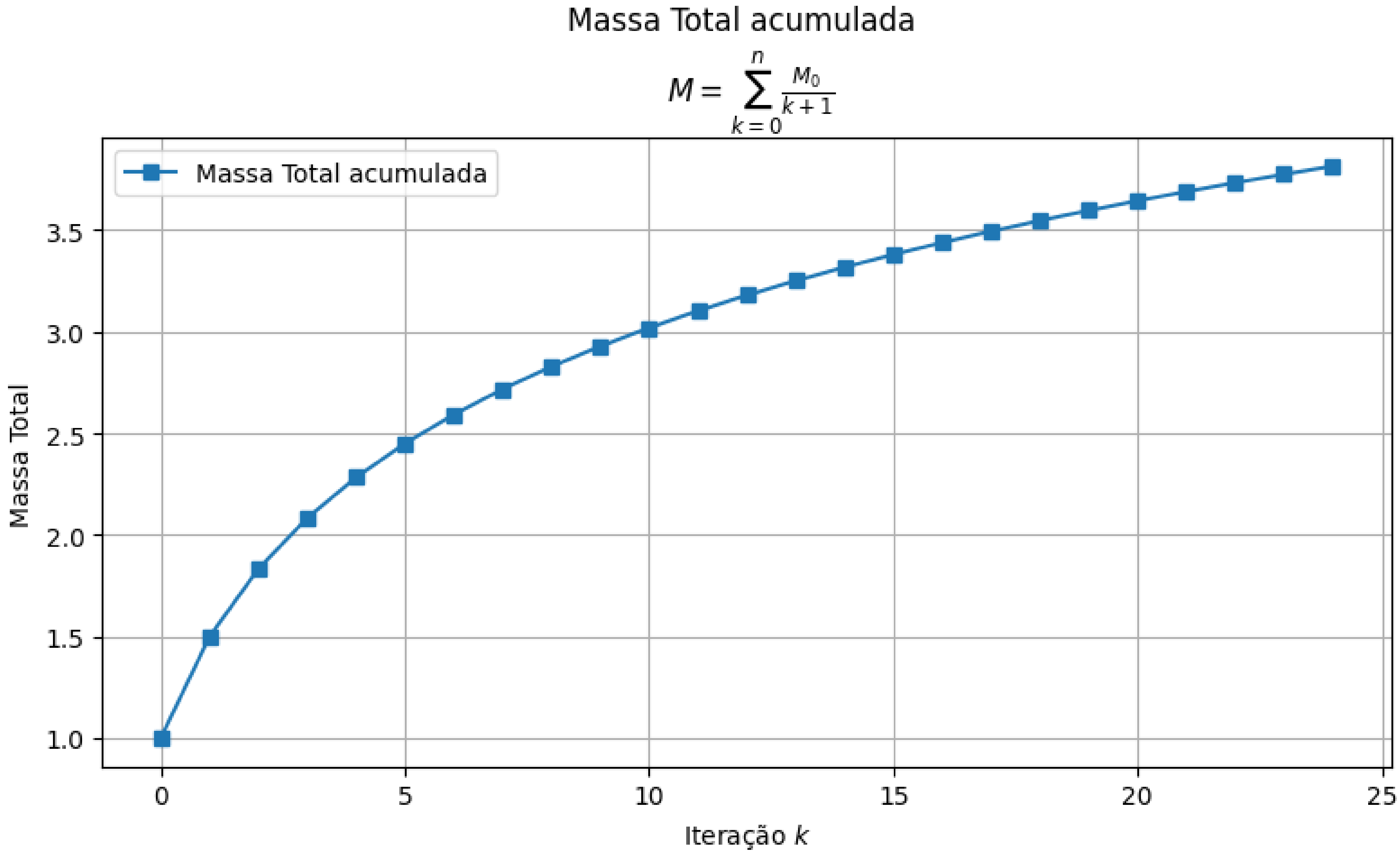


$$\int_1^{\infty} \frac{1}{x}\, dx = \infty. \tag{6}$$

Como a integral da equação (6) não converge, isso demonstra que soma dada pela série harmônica também não pode convergir.


Duplicação de Benders por iteração
2^k
1e7
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
2^k: Número de Benders
Número de Benders
0
5
10
15
20
25
Iteração k


Massa individual de cada Bender
$M_k = \frac{M_0}{2^k(k+1)}$
Massa Individual
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
Iteração k
$M_k = \frac{M_0}{2^k(k+1)}$


Massa Total acumulada
$M = \sum_{k=0}^{n} \frac{M_0}{k+1}$
Massa Total acumulada
Massa Total
Iteração k
1.0
1.5
2.0
2.5
3.0
3.5
0
5
10
15
20
25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