balance_2023_mm_dd See GDrive balance for summary of all derivations

#1: CaseMode ≔ Sensitive

#2: InputMode ≔ Word

transportation parameter

#3: τ ∈ Real (0, ∞)

consumer index

#4: x ∈ Real [0, 1]

Return profit per dollar of deposits

#5: ρ ∈ Real (0, ∞)

basic utility derived from bank services

#6: μ ∈ Real (0, ∞)

value derived from bank 1 (if pays monthly fee)

#7: μ − τ·x − f1

value derived from bank 1 (no fee)

#8: μ − τ·x

value derived from bank 2 (if pays monthly fee)

#9: μ − τ·(1 − x) − f2

value derived from bank 2 (no fee)

#10: μ − τ·(1 − x)

Total number of depositors

#11: $\eta \in \text{Real } (0, \infty)$

fraction of depositors with high balance

#12: $\theta \in \text{Real } [0, 1]$

Dollar amount of low and high balances

#13: $\beta l \in \text{Real } (0, \infty)$

#14: $\beta h \in \text{Real } (0, \infty)$

fees (if levied) by bank 1 and bank 2

#15: $f1 \in \text{Real } [0, \infty)$

#16: $f2 \in \text{Real } [0, \infty)$

*** Section 2.1 in paper: Both banks set min bal b1=b2=βh (high balance)

#17: $\mu - \tau \cdot x - f1 = \mu - \tau \cdot (1 - x) - f2$

#18: $\text{SOLVE}(\mu - \tau \cdot x - f1 = \mu - \tau \cdot (1 - x) - f2, x)$

eq (2)

#19: $$xmml = -\frac{f1 - f2 - \tau}{2 \cdot \tau}$$

eq (3)

#20: $nl1 = xmml \cdot (1 - \theta) \cdot \eta$

#21: $nl2 = (1 - xmml) \cdot (1 - \theta) \cdot \eta$

#22: $dl1 = nl1 \cdot \beta l$

#23: $dl1 = (xmml \cdot (1 - \theta) \cdot \eta) \cdot \beta l$

#24: dl2 = nl2·βl

#25: dl2 = ((1 − xmml)·(1 − θ)·η)·βl

Deriving eq (4) (high balance)

#26: μ − τ·x = μ − τ·(1 − x)

#27: SOLVE(μ − τ·x = μ − τ·(1 − x), x)

#28: $$xmmh = \frac{1}{2}$$

eq (5)

#29: nh1 = xmmh·θ·η

#30: nh2 = (1 − xmmh)·θ·η

#31: dh1 = nh1·βh

#32: dh1 = (xmmh·θ·η)·βh

#33: dh2 = nh2·βh

#34: dh2 = ((1 − xmmh)·θ·η)·βh

eq (6)

#35: profit1 = ρ·(dl1 + dh1) + f1·nl1

#36: profit2 = ρ·(dl2 + dh2) + f2·nl2

Deriving (7) and Appendix A

#37: profit1 = ρ·((xmml·(1 − θ)·η)·βl + (xmmh·θ·η)·βh) + f1·(xmml·(1 − θ)·η)

#38: profit2 = ρ·(((1 − xmml)·(1 − θ)·η)·βl + ((1 − xmmh)·θ·η)·βh) + f2·((1 − xmml)·(1 − θ)·η)

#39: $$\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)$$

#40: $$\text{profit2} = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)$$

#41: $$\frac{d}{d\ f1}\left(\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)$$

#42: $$0 = \frac{\eta\cdot(\theta - 1)\cdot(2\cdot f1 - f2 + \beta l\cdot\rho - \tau)}{2\cdot\tau}$$

#43: $$\frac{d}{d\ f1}\frac{d}{d\ f1}\left(\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)$$

#44: $$0 > \frac{\eta\cdot(\theta - 1)}{\tau}$$

#45: $$\frac{d}{d\ f2}\left(\text{profit2} = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\right.\right.\right.$$

$$\left(\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\Bigg)$$

#46: $$0 = \frac{\eta\cdot(1 - \theta)\cdot(f1 - 2\cdot f2 - \beta l\cdot\rho + \tau)}{2\cdot\tau}$$

#47: $$\frac{d}{d\ f2}\frac{d}{d\ f2}\left(profit2 = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\right.\right.\right.$$

$$\left.\left.\left.\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)$$

#48: $$0 > \frac{\eta\cdot(\theta - 1)}{\tau}$$

#49: $$\text{SOLVE}\left(\left[0 = \frac{\eta\cdot(\theta - 1)\cdot(2\cdot f1 - f2 + \beta l\cdot\rho - \tau)}{2\cdot\tau}, 0 = \frac{\eta\cdot(1 - \theta)\cdot(f1 - 2\cdot f2 - \beta l\cdot\rho + \tau)}{2\cdot\tau}\right], [f1, f2]\right)$$

eq (7)

#50: $$[fmm1 = \tau - \beta l\cdot\rho \wedge fmm2 = \tau - \beta l\cdot\rho]$$

#51: $$profitmm1 = \frac{\eta\cdot(\beta h\cdot\theta\cdot\rho - \tau\cdot(\theta - 1))}{2}$$

#52: $$profitmm2 = \frac{\eta\cdot(\beta h\cdot\theta\cdot\rho - \tau\cdot(\theta - 1))}{2}$$

Result 1a

$$\#53:\quad \frac{d}{d\theta}\left(\text{profitmm1} = \frac{\eta\cdot(\beta h\cdot\theta\cdot\rho - \tau\cdot(\theta - 1))}{2}\right)$$

$$\#54:\quad \frac{\eta\cdot(\beta h\cdot\rho - \tau)}{2}$$

** Section 2.2 no min balance

$$\#55:\quad \mu - \tau\cdot x - f1 = \mu - \tau\cdot(1 - x) - f2$$

$$\#56:\quad \text{SOLVE}(\mu - \tau\cdot x - f1 = \mu - \tau\cdot(1 - x) - f2,\ x)$$

eq (8)

$$\#57:\quad \text{xff} = -\frac{f1 - f2 - \tau}{2\cdot\tau}$$

eq (9)

$$\#58:\quad n1 = \text{xff}\cdot\eta$$

$$\#59:\quad n2 = (1 - \text{xff})\cdot\eta$$

$$\#60:\quad d1 = n1\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)$$

$$\#61:\quad d2 = n2\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)$$

$$\#62:\quad d1 = (\text{xff}\cdot\eta)\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)$$

$$\#63:\quad d2 = ((1 - \text{xff})\cdot\eta)\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)$$

eq (10)

$$\#64:\quad \text{profit1} = \rho\cdot d1 + f1\cdot n1$$

$$\#65:\quad \text{profit2} = \rho\cdot d2 + f2\cdot n2$$

Derivation of (11) and Appendix B

#66: $\text{profit1} = \rho\cdot((\text{xff}\cdot\eta)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)) + f1\cdot(\text{xff}\cdot\eta)$

#67: $\text{profit2} = \rho\cdot(((1-\text{xff})\cdot\eta)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)) + f2\cdot((1-\text{xff})\cdot\eta)$

#68: $$\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f1\cdot\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)$$

#69: $$\text{profit2} = \rho\cdot\left(\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f2\cdot\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)$$

#70: $$\frac{d}{d\ f1}\left(\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f1\cdot\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\right)$$

#71: $$0 = -\frac{\eta\cdot(2\cdot f1 - f2 + \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(1-\theta) - \tau)}{2\cdot\tau}$$

#72: $$\frac{d}{d\ f1}\frac{d}{d\ f1}\left(\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f1\cdot\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\right)$$

#73: $$0 > -\frac{\eta}{\tau}$$

#74: $$\frac{d}{d\ f2}\left(\text{profit2} = \rho\cdot\left(\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f2\cdot\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\right)$$

#75: $$0 = \frac{\eta\cdot(f1 - 2\cdot f2 - \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(\theta-1) + \tau)}{2\cdot\tau}$$

$$\#76:\quad \frac{d}{d\ f2}\,\frac{d}{d\ f2}\left(\text{profit2} = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\right)$$

$$\#77:\quad 0 > -\frac{\eta}{\tau}$$

$$\#78:\quad \text{SOLVE}\left(\left[0 = -\frac{\eta\cdot(2\cdot f1 - f2 + \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(1-\theta) - \tau)}{2\cdot\tau},\ 0 = \frac{\eta\cdot(f1 - 2\cdot f2 - \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(\theta - 1) + \tau)}{2\cdot\tau}\right],\ [f1,\ f2]\right)$$

eq (11)

$$\#79:\quad [\text{fff1} = -\beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(\theta - 1) + \tau \wedge \text{fff2} = -\beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(\theta - 1) + \tau]$$

$$\#80:\quad \text{profitff1} = \frac{\eta\cdot\tau}{2}$$

Result 2a

$$\#81:\quad \frac{d}{d\theta}\,(f1 = -\beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(\theta - 1) + \tau)$$

$$\#82:\quad 0 > \rho\cdot(\beta l - \beta h)$$

*** Section 3: Do banks benefit from imposing minimum balance?

Recall equilibrium fees and profits under min bal (line 50 above)

#83: fmm1 = τ − βl·ρ

#84: $$\text{profitmm1} = \frac{\eta\cdot(\beta h\cdot\theta\cdot\rho - \tau\cdot(\theta - 1))}{2}$$

Recall equilibrium fees and profits with NO min bal (line 79 above)

#85: fff1 = − βh·θ·ρ + βl·ρ·(θ − 1) + τ

#86: $$\text{profitff1} = \frac{\eta\cdot\tau}{2}$$

comparing fees: (m,m) minus (f,f), eq (12): fmm1 − fff1

#87: τ − βl·ρ − (− βh·θ·ρ + βl·ρ·(θ − 1) + τ)

#88: $$\beta h\cdot\theta\cdot\rho - \beta l\cdot\theta\cdot\rho > 0$$

comparing profits: (m,m) minus (f,f), eq (13): profitmm1 − profitff1

#89: $$\frac{\eta\cdot(\beta h\cdot\theta\cdot\rho - \tau\cdot(\theta - 1))}{2} - \frac{\eta\cdot\tau}{2}$$

#90: $$\frac{\eta\cdot\theta\cdot(\beta h\cdot\rho - \tau)}{2}$$

*** Section 4: Asymmetric strategies

** Subsection 4a: Eql fees

Deriving eq (14): xmfl (low-balance, both charge fees)

#91: μ − τ·x − f1 = μ − τ·(1 − x) − f2

#92: SOLVE(μ − τ·x − f1 = μ − τ·(1 − x) − f2, x)

#93: $$xmfl = -\frac{f1 - f2 - \tau}{2\cdot\tau}$$

Also in eq (14): xmfh (high-balance, only bank 2 charges a fee f2)

#94: $\mu - \tau\cdot x = \mu - \tau\cdot(1 - x) - f2$

#95: SOLVE($\mu - \tau\cdot x = \mu - \tau\cdot(1 - x) - f2$, x)

#96: $$xmfh = \frac{f2 + \tau}{2\cdot\tau}$$

eq (15) num low-balance depositors

#97: $nl1 = xmfl\cdot(1 - \theta)\cdot\eta$

#98: $nl2 = (1 - xmfl)\cdot(1 - \theta)\cdot\eta$

#99: $dl1 = nl1\cdot\beta l$

#100: $dl2 = nl2\cdot\beta l$

#101: $dl1 = (xmfl\cdot(1 - \theta)\cdot\eta)\cdot\beta l$

#102: $dl2 = ((1 - xmfl)\cdot(1 - \theta)\cdot\eta)\cdot\beta l$

eq (16) num high-balance depositors

#103: $nh1 = xmfh\cdot\theta\cdot\eta$

#104: $nh2 = (1 - xmfh)\cdot\theta\cdot\eta$

#105: $dh1 = nh1\cdot\beta h$

#106: $dh2 = nh2\cdot\beta h$

#107: $dh1 = (xmfh\cdot\theta\cdot\eta)\cdot\beta h$

#108: dh2 = ((1 − xmfh)·θ·η)·βh

eq (17): Profit functions

#109: profitmf1 = ρ·(dl1 + dh1) + f1·nl1

#110: profitmf2 = ρ·(dl2 + dh2) + f2·(nl2 + nh2)

#111: profitmf1 = ρ·((xmfl·(1 − θ)·η)·βl + (xmfh·θ·η)·βh) + f1·(xmfl·(1 − θ)·η)

#112: profitmf2 = ρ·(((1 − xmfl)·(1 − θ)·η)·βl + ((1 − xmfh)·θ·η)·βh) + f2·((1 − xmfl)·(1 − θ)·η + (1 − xmfh)·θ·η)

Deriving eq (18) and Appendix C

$$\#113:\ \text{profitmf1} = \rho\cdot\left(\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\frac{f2+\tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)$$

$$\#114:\ \text{profitmf2} = \rho\cdot\left(\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\left(1-\frac{f2+\tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta + \left(1-\frac{f2+\tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)$$

$$\#115:\ \frac{d}{d\ f1}\left(\text{profitmf1} = \rho\cdot\left(\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\frac{f2+\tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\right.\right.\right.$$

$$\left(\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\Bigg)$$

#116: $$0 = \frac{\eta\cdot(\theta - 1)\cdot(2\cdot f1 - f2 + \beta l\cdot\rho - \tau)}{2\cdot\tau}$$

#117: $$\frac{d}{d\ f1}\ \frac{d}{d\ f1}\left(profitmf1 = \rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{f2 + \tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\right.\right.\right.$$

$$\left.\left.\left.\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)$$

#118: $$0 > \frac{\eta\cdot(\theta - 1)}{\tau}$$

#119: $$\frac{d}{d\ f2}\left(profitmf2 = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{f2 + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\right.\right.\right.$$

$$\left.\left.\left.\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta + \left(1 - \frac{f2 + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\right)$$

#120: $$0 = -\frac{\eta\cdot(f1\cdot(\theta - 1) + 2\cdot f2 + \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(1 - \theta) - \tau)}{2\cdot\tau}$$

#121: $$\frac{d}{d\ f2}\ \frac{d}{d\ f2}\left(profitmf2 = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{f2 + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1\right.\right.\right.$$

$$- - \frac{f1 - f2 - \tau}{2 \cdot \tau}\right) \cdot (1 - \theta) \cdot \eta + \left(1 - \frac{f2 + \tau}{2 \cdot \tau}\right) \cdot \theta \cdot \eta\right)\right)$$

#122: $$0 > - \frac{\eta}{\tau}$$

#123: $$\text{SOLVE}\left(\left[0 = \frac{\eta \cdot (\theta - 1) \cdot (2 \cdot f1 - f2 + \beta l \cdot \rho - \tau)}{2 \cdot \tau}, 0 = - \frac{\eta \cdot (f1 \cdot (\theta - 1) + 2 \cdot f2 + \beta h \cdot \theta \cdot \rho + \beta l \cdot \rho \cdot (1 - \theta) - \tau)}{2 \cdot \tau}\right], [f1, f2]\right)$$

eq (18)

#124: $$\left[f1 = - \frac{\beta h \cdot \theta \cdot \rho + \beta l \cdot \rho \cdot (3 - \theta) - 3 \cdot \tau}{\theta + 3} \wedge f2 = - \frac{2 \cdot \beta h \cdot \theta \cdot \rho + 3 \cdot \beta l \cdot \rho \cdot (1 - \theta) + \tau \cdot (\theta - 3)}{\theta + 3}\right]$$

*** Section 5: Common Ownership: Investor A, B, and passive investors

Share of investor A in bank 1 and share of investor B in bank 2

#125: $\sigma \in \text{Real}\ (0, 1)$

eq (19) in paper

Investor A chooses f1 to max

#126: profita = $\sigma \cdot$profitmm1 + $(1 - \sigma) \cdot$profitmm2

where profitmm1 and profitmm2 are defined in (6) and spelled out in (B.1).

Investor B chooses f2 to max

#127: profitb = $(1 - \sigma) \cdot$profitmm1 + $\sigma \cdot$profitmm2

Substituting from (B.1) in paper, line #39 above: Derving (20) and Appendix E in paper

$$\#128:\ \text{profita} = \sigma\cdot\left(\rho\cdot\left(\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l+\left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right)+f1\cdot\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)+(1-\sigma)\cdot\left(\rho\cdot\left(\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l+\left(\left(1-\frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right)+f2\cdot\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)$$

$$\#129:\ \text{profitb} = (1-\sigma)\cdot\left(\rho\cdot\left(\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l+\left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right)+f1\cdot\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)+\sigma\cdot\left(\rho\cdot\left(\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l+\left(\left(1-\frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right)+f2\cdot\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)$$

$$\#130:\ \frac{d}{d\ f1}\left(\text{profita} = \sigma\cdot\left(\rho\cdot\left(\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l+\left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right)+f1\cdot\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)+(1-\sigma)\cdot\left(\rho\cdot\left(\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l+\left(\left(1-\frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right)+f2\cdot\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)\right)$$

#131: $$0 = \frac{\eta\cdot(\theta - 1)\cdot(2\cdot f1\cdot\sigma - f2 + \beta l\cdot\rho\cdot(2\cdot\sigma - 1) - \sigma\cdot\tau)}{2\cdot\tau}$$

#132: $$\frac{d}{d\ f1}\frac{d}{d\ f1}\left(profita = \sigma\cdot\left(\rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right) + (1 - \sigma)\cdot\left(\rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)\right)$$

#133: $$0 > \frac{\eta\cdot\sigma\cdot(\theta - 1)}{\tau}$$

#134: $$\frac{d}{d\ f2}\left(profitb = (1 - \sigma)\cdot\left(\rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right) + \sigma\cdot\left(\rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)\right)$$

#135: $$0 = \frac{\eta\cdot(1 - \theta)\cdot(f1 - 2\cdot f2\cdot\sigma + \beta l\cdot\rho\cdot(1 - 2\cdot\sigma) + \sigma\cdot\tau)}{2\cdot\tau}$$

#136: $$\frac{d}{d\ f2}\frac{d}{d\ f2}\left(profitb = (1 - \sigma)\cdot\left(\rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\right.\right.\right.\right.$$

$$\left(\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\Bigg) + \sigma\cdot\left(\rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)\right)$$

#137: $$0 > \frac{\eta\cdot\sigma\cdot(\theta - 1)}{\tau}$$

#138: $$\text{SOLVE}\left(\left[0 = \frac{\eta\cdot(\theta - 1)\cdot(2\cdot f1\cdot\sigma - f2 + \beta l\cdot\rho\cdot(2\cdot\sigma - 1) - \sigma\cdot\tau)}{2\cdot\tau}, 0 = \frac{\eta\cdot(1 - \theta)\cdot(f1 - 2\cdot f2\cdot\sigma + \beta l\cdot\rho\cdot(1 - 2\cdot\sigma) + \sigma\cdot\tau)}{2\cdot\tau}\right], [f1, f2]\right)$$

eq (20)

#139: $$\left[fmm1 = \frac{\beta l\cdot\rho\cdot(2\cdot\sigma - 1) - \sigma\cdot\tau}{1 - 2\cdot\sigma} \wedge fmm2 = \frac{\beta l\cdot\rho\cdot(2\cdot\sigma - 1) - \sigma\cdot\tau}{1 - 2\cdot\sigma}\right]$$

#140: $$profitmma = \frac{\eta\cdot(\beta h\cdot\theta\cdot\rho\cdot(2\cdot\sigma - 1) - \sigma\cdot\tau\cdot(\theta - 1))}{2\cdot(2\cdot\sigma - 1)}$$

#141: $$profitmmb = \frac{\eta\cdot(\beta h\cdot\theta\cdot\rho\cdot(2\cdot\sigma - 1) - \sigma\cdot\tau\cdot(\theta - 1))}{2\cdot(2\cdot\sigma - 1)}$$

Show that (20)→(7) as σ→1

#142: $$\lim_{\sigma\to 1-}\left(f1 = \frac{\beta l\cdot\rho\cdot(2\cdot\sigma - 1) - \sigma\cdot\tau}{1 - 2\cdot\sigma}\right)$$

#143: $$f1 = \tau - \beta l \cdot \rho$$

#144: $$\lim_{\sigma \to 1-} \left(profita = \frac{\eta \cdot (\beta h \cdot \theta \cdot \rho \cdot (2 \cdot \sigma - 1) - \sigma \cdot \tau \cdot (\theta - 1))}{2 \cdot (2 \cdot \sigma - 1)} \right)$$

#145: $$profita = \frac{\eta \cdot (\beta h \cdot \theta \cdot \rho - \tau \cdot (\theta - 1))}{2}$$

Subsection 5.2: Common ownership no min bal
Deriving eq (21) and Appendix F

#146: $profita = \sigma \cdot profitff1 + (1 - \sigma) \cdot profitff2$

#147: $profitb = (1 - \sigma) \cdot profitff1 + \sigma \cdot profitff2$

substituting (C.1) line #68 above

#148: $$profita = \sigma \cdot \left(\rho \cdot \left(\left(\left(- \frac{f1 - f2 - \tau}{2 \cdot \tau} \right) \cdot \eta \right) \cdot ((1 - \theta) \cdot \beta l + \theta \cdot \beta h) \right) + f1 \cdot \left(\left(- \frac{f1 - f2 - \tau}{2 \cdot \tau} \right) \cdot \eta \right) \right) + (1 - \sigma) \cdot \left(\rho \cdot \left(\left(\left(1 - - \frac{f1 - f2 - \tau}{2 \cdot \tau} \right) \cdot \eta \right) \cdot ((1 - \theta) \cdot \beta l + \theta \cdot \beta h) \right) + f2 \cdot \left(\left(1 - - \frac{f1 - f2 - \tau}{2 \cdot \tau} \right) \cdot \eta \right) \right)$$

#149: $$profitb = (1 - \sigma) \cdot \left(\rho \cdot \left(\left(\left(- \frac{f1 - f2 - \tau}{2 \cdot \tau} \right) \cdot \eta \right) \cdot ((1 - \theta) \cdot \beta l + \theta \cdot \beta h) \right) + f1 \cdot \left(\left(- \frac{f1 - f2 - \tau}{2 \cdot \tau} \right) \cdot \eta \right) \right) + \sigma \cdot \left(\rho \cdot \left(\left(\left(1 - - \frac{f1 - f2 - \tau}{2 \cdot \tau} \right) \cdot \eta \right) \cdot ((1 - \theta) \cdot \beta l + \theta \cdot \beta h) \right) + f2 \cdot \left(\left(1 - - \frac{f1 - f2 - \tau}{2 \cdot \tau} \right) \cdot \eta \right) \right)$$

#150: $$\frac{d}{d\ f1} \left(profita = \sigma \cdot \left(\rho \cdot \left(\left(\left(- \frac{f1 - f2 - \tau}{2 \cdot \tau} \right) \cdot \eta \right) \cdot ((1 - \theta) \cdot \beta l + \theta \cdot \beta h) \right) + f1 \cdot \left(\left(- \frac{f1 - f2 - \tau}{2 \cdot \tau} \right) \cdot \eta \right) \right) + (1$$

$$- \sigma)\cdot\left(\rho\cdot\left(\left(\left(1 - - \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)\right) + f2\cdot\left(\left(1 - - \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\right)\right)$$

#151: $$0 = - \frac{\eta\cdot(2\cdot f1\cdot\sigma - f2 + \beta h\cdot\theta\cdot\rho\cdot(2\cdot\sigma - 1) + \beta l\cdot\rho\cdot(1 - 2\cdot\sigma)\cdot(\theta - 1) - \sigma\cdot\tau)}{2\cdot\tau}$$

#152: $$\frac{d}{d\ f1}\ \frac{d}{d\ f1}\left(\text{profita} = \sigma\cdot\left(\rho\cdot\left(\left(\left(- \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)\right) + f1\cdot\left(\left(- \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\right)\right.$$
$$\left. + (1 - \sigma)\cdot\left(\rho\cdot\left(\left(\left(1 - - \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)\right) + f2\cdot\left(\left(1 - - \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\right)\right)$$

#153: $$0 > - \frac{\eta\cdot\sigma}{\tau}$$

#154: $$\frac{d}{d\ f2}\left(\text{profitb} = (1 - \sigma)\cdot\left(\rho\cdot\left(\left(\left(- \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)\right) + f1\cdot\left(\left(- \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\right)\right.$$
$$\left. + \sigma\cdot\left(\rho\cdot\left(\left(\left(1 - - \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)\right) + f2\cdot\left(\left(1 - - \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\right)\right)$$

#155: $$0 = \frac{\eta\cdot(f1 - 2\cdot f2\cdot\sigma + \beta h\cdot\theta\cdot\rho\cdot(1 - 2\cdot\sigma) + \beta l\cdot\rho\cdot(\theta - 1)\cdot(2\cdot\sigma - 1) + \sigma\cdot\tau)}{2\cdot\tau}$$

#156: $$\frac{d}{d\ f2}\ \frac{d}{d\ f2}\left(\text{profitb} = (1 - \sigma)\cdot\left(\rho\cdot\left(\left(\left(- \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)\right) + f1\cdot\left(\left(-\right.\right.\right.\right.$$

$$\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\right) + \sigma\cdot\left(\rho\cdot\left(\left(\left(1 - - \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)\right) + f2\cdot\left(\left(1 - -$$

$$\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\right)\right)$$

#157: $$0 > - \frac{\eta\cdot\sigma}{\tau}$$

#158: $$\text{SOLVE}\left(\left[0 = - \frac{\eta\cdot(2\cdot f1\cdot\sigma - f2 + \beta h\cdot\theta\cdot\rho\cdot(2\cdot\sigma - 1) + \beta l\cdot\rho\cdot(1 - 2\cdot\sigma)\cdot(\theta - 1) - \sigma\cdot\tau)}{2\cdot\tau}, 0 = \frac{\eta\cdot(f1 - 2\cdot f2\cdot\sigma + \beta h\cdot\theta\cdot\rho\cdot(1 - 2\cdot\sigma) + \beta l\cdot\rho\cdot(\theta - 1)\cdot(2\cdot\sigma - 1) + \sigma\cdot\tau)}{2\cdot\tau}\right], [f1, f2]\right)$$

eq (21)

#159: $$\left[fff1 = \frac{\beta h\cdot\theta\cdot\rho\cdot(2\cdot\sigma - 1) + \beta l\cdot\rho\cdot(1 - 2\cdot\sigma)\cdot(\theta - 1) - \sigma\cdot\tau}{1 - 2\cdot\sigma} \wedge fff2 = \frac{\beta h\cdot\theta\cdot\rho\cdot(2\cdot\sigma - 1) + \beta l\cdot\rho\cdot(1 - 2\cdot\sigma)\cdot(\theta - 1) - \sigma\cdot\tau}{1 - 2\cdot\sigma}\right]$$

#160: $$profitffa = \frac{\eta\cdot\sigma\cdot\tau}{2\cdot(2\cdot\sigma - 1)}$$

#161: $$profitffb = \frac{\eta\cdot\sigma\cdot\tau}{2\cdot(2\cdot\sigma - 1)}$$

Verify convergence of (21) → (11) as σ→1

$$\#162: \lim_{\sigma \to 1-} \frac{\beta h \cdot \theta \cdot \rho \cdot (2 \cdot \sigma - 1) + \beta l \cdot \rho \cdot (1 - 2 \cdot \sigma) \cdot (\theta - 1) - \sigma \cdot \tau}{1 - 2 \cdot \sigma}$$

$$\#163: -\beta h \cdot \theta \cdot \rho + \beta l \cdot \rho \cdot (\theta - 1) + \tau$$

$$\#164: \lim_{\sigma \to 1-} \left(\text{profitffa} = \frac{\eta \cdot \sigma \cdot \tau}{2 \cdot (2 \cdot \sigma - 1)} \right)$$

$$\#165: \text{profitffa} = \frac{\eta \cdot \tau}{2}$$

** subsection 5.3: Common ownership benefit from min bal?

$$\#166: \text{profitmm1} - \text{profitff1} = \frac{\eta \cdot (\beta h \cdot \theta \cdot \rho \cdot (2 \cdot \sigma - 1) - \sigma \cdot \tau \cdot (\theta - 1))}{2 \cdot (2 \cdot \sigma - 1)} - \frac{\eta \cdot \sigma \cdot \tau}{2 \cdot (2 \cdot \sigma - 1)}$$

$$\#167: \text{profitmm1} - \text{profitff1} = \frac{\eta \cdot \theta \cdot (\beta h \cdot \rho \cdot (2 \cdot \sigma - 1) - \sigma \cdot \tau)}{2 \cdot (2 \cdot \sigma - 1)}$$

> 0 if

$$\#168: \beta h \cdot \rho \cdot (2 \cdot \sigma - 1) - \sigma \cdot \tau > 0$$

#169: SOLVE(βh·ρ·(2·σ − 1) − σ·τ > 0, τ)

eq (22) and τ_tilde

$$\#170: \tau < \frac{\beta h \cdot \rho \cdot (2 \cdot \sigma - 1)}{\sigma}$$

try to give intuition for Result 6 by subtracting the fees ==> fee difference is NOT affected by σ ! same as eq (12).

$$\#171:\ fmm1 - fff1 = \frac{\beta l\cdot\rho\cdot(2\cdot\sigma - 1) - \sigma\cdot\tau}{1 - 2\cdot\sigma} - \frac{\beta h\cdot\theta\cdot\rho\cdot(2\cdot\sigma - 1) + \beta l\cdot\rho\cdot(1 - 2\cdot\sigma)\cdot(\theta - 1) - \sigma\cdot\tau}{1 - 2\cdot\sigma}$$

$$\#172:\quad fmm1 - fff1 = \theta\cdot\rho\cdot(\beta h - \beta l) > 0$$

try another way: how the profit difference is affected by σ ==> diff in profit rises with σ
eq (23)

$$\#173:\ \frac{d}{d\sigma}\left(profitmm1 - profitff1 = \frac{\eta\cdot\theta\cdot(\beta h\cdot\rho\cdot(2\cdot\sigma - 1) - \sigma\cdot\tau)}{2\cdot(2\cdot\sigma - 1)}\right)$$

$$\#174:\quad 0 < \frac{\eta\cdot\theta\cdot\tau}{2\cdot(2\cdot\sigma - 1)^2}$$

look at the behavior of fees w.r.t σ

$$\#175:\ \frac{d}{d\sigma}\left(fmm1 = \frac{\beta l\cdot\rho\cdot(2\cdot\sigma - 1) - \sigma\cdot\tau}{1 - 2\cdot\sigma}\right)$$

$$\#176:\quad 0 > -\frac{\tau}{2\cdot(2\cdot\sigma - 1)^2}$$

$$\#177:\ \frac{d}{d\sigma}\left(fff1 = \frac{\beta h\cdot\theta\cdot\rho\cdot(2\cdot\sigma - 1) + \beta l\cdot\rho\cdot(1 - 2\cdot\sigma)\cdot(\theta - 1) - \sigma\cdot\tau}{1 - 2\cdot\sigma}\right)$$

$$\#178:\quad 0 > -\frac{\tau}{2\cdot(2\cdot\sigma - 1)^2}$$

*** Section 6: Interest rate competition

Modified utility function (not displayed equation in the paper):

#179: $\mu + r1 \cdot \beta h - \tau \cdot x - f1$

#180: $\mu + r1 \cdot \beta l - \tau \cdot x - f1$

#181: $\mu + r2 \cdot \beta h - \tau \cdot (1 - x) - f2$

#182: $\mu + r2 \cdot \beta l - \tau \cdot (1 - x) - f2$

#183: $\mu + r1 \cdot \beta h - \tau \cdot x$

#184: $\mu + r1 \cdot \beta l - \tau \cdot x$

#185: $\mu + r2 \cdot \beta h - \tau \cdot (1 - x)$

#186: $\mu + r2 \cdot \beta l - \tau \cdot (1 - x)$

** Subsection 6.1: Min balance βh to avoid a fee

#187: $\mu + r1 \cdot \beta l - \tau \cdot x - f1 = \mu + r2 \cdot \beta l - \tau \cdot (1 - x) - f2$

eq (24)

#188: $\text{SOLVE}(\mu + r1 \cdot \beta l - \tau \cdot x - f1 = \mu + r2 \cdot \beta l - \tau \cdot (1 - x) - f2, x)$

#189: $$xmml = -\frac{f1 - f2 - r1 \cdot \beta l + r2 \cdot \beta l - \tau}{2 \cdot \tau}$$

#190: $\mu + r1 \cdot \beta h - \tau \cdot x = \mu + r2 \cdot \beta h - \tau \cdot (1 - x)$

#191: $\text{SOLVE}(\mu + r1 \cdot \beta h - \tau \cdot x = \mu + r2 \cdot \beta h - \tau \cdot (1 - x), x)$

#192: $$xmmh = \frac{r1 \cdot \beta h - r2 \cdot \beta h + \tau}{2 \cdot \tau}$$

#193: $nl1 = xmml \cdot (1 - \theta) \cdot \eta$

#194: nl2 = (1 − xmml)·(1 − θ)·η

#195: dl1 = nl1·βl

#196: dl2 = nl2·βl

#197: dl1 = (xmml·(1 − θ)·η)·βl

#198: dl2 = ((1 − xmml)·(1 − θ)·η)·βl

#199: nh1 = xmmh·θ·η

#200: nh2 = (1 − xmmh)·θ·η

#201: dh1 = nh1·βh

#202: dh2 = nh2·βh

#203: dh1 = (xmmh·θ·η)·βh

#204: dh2 = ((1 − xmmh)·θ·η)·βh

eq (25)

#205: profitmm1 = (ρ − r1)·(dl1 + dh1) + f1·nl1

#206: profitmm2 = (ρ − r2)·(dl2 + dh2) + f2·nl2

Deriving eq (26) Appendix G

#207: profitmm1 = (ρ − r1)·((xmml·(1 − θ)·η)·βl + (xmmh·θ·η)·βh) + f1·(xmml·(1 − θ)·η)

#208: profitmm2 = (ρ − r2)·(((1 − xmml)·(1 − θ)·η)·βl + ((1 − xmmh)·θ·η)·βh) + f2·((1 − xmml)·(1 − θ)·η)

eq (G.1)

$$\#209:\ profitmm1 = (\rho - r1)\cdot\left(\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l +\right.$$

$$\left(\frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)$$

#210: $profitmm2 = (\rho - r2)\cdot\left(\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)$

eq (G.2) FOCs

#211: $\frac{d}{d\ f1}\left(profitmm1 = (\rho - r1)\cdot\left(\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\left(\frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)\right)$

#212: $$0 = \frac{\eta\cdot(\theta - 1)\cdot(2\cdot f1 - f2 - 2\cdot r1\cdot\beta l + r2\cdot\beta l + \beta l\cdot\rho - \tau)}{2\cdot\tau}$$

#213: $\frac{d}{d\ f1}\frac{d}{d\ f1}\left(profitmm1 = (\rho - r1)\cdot\left(\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\left(\frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)\right)$

#214: $$0 > \frac{\eta\cdot(\theta - 1)}{\tau}$$

$$\#215:\ \frac{d}{d\ r1}\left(profitmm1 = (\rho - r1)\cdot\left(\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)$$

$$\#216:\ 0 = -\frac{\eta\cdot(2\cdot f1\cdot\beta l\cdot(\theta-1) + f2\cdot\beta l\cdot(1-\theta) + 2\cdot r1\cdot(\beta h^2\cdot\theta + \beta l^2\cdot(1-\theta)) - r2\cdot(\beta h^2\cdot\theta + \beta l^2\cdot(1-\theta)) - \beta h\sim}{2\cdot\tau}$$

$$\frac{2\cdot\theta\cdot\rho + \beta h\cdot\theta\cdot\tau + \beta l^2\cdot\rho\cdot(\theta-1) + \beta l\cdot\tau\cdot(1-\theta))}{}$$

$$\#217:\ \frac{d}{d\ r1}\frac{d}{d\ r1}\left(profitmm1 = (\rho - r1)\cdot\left(\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)$$

$$\#218:\quad 0 > -\frac{\eta\cdot(\beta h^2\cdot\theta + \beta l^2\cdot(1-\theta))}{\tau}$$

Hessian for bank 1: Cross derivative

$$\#219:\ \frac{d}{d\ f1}\left(\frac{d}{d\ r1}\left(profitmm1 = (\rho - r1)\cdot\left(\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)\right)$$

$$\#220:\ \frac{\beta l\cdot\eta\cdot(1 - \theta)}{\tau}$$

$$\#221:\ H = \frac{\eta\cdot(\theta - 1)}{\tau}\cdot\left(-\frac{\eta\cdot(\beta h^{2}\cdot\theta + \beta l^{2}\cdot(1 - \theta))}{\tau}\right) - \frac{\beta l\cdot\eta\cdot(1 - \theta)}{\tau}\cdot\frac{\beta l\cdot\eta\cdot(1 - \theta)}{\tau}$$

$$\#222:\ H = \frac{\beta h^{2}\cdot\eta^{2}\cdot\theta\cdot(1 - \theta)}{\tau^{2}} > 0$$

$$\#223:\ \frac{d}{d\ f2}\left(profitmm2 = (\rho - r2)\cdot\left(\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)$$

$$\#224:\ 0 = \frac{\eta\cdot(1 - \theta)\cdot(f1 - 2\cdot f2 - r1\cdot\beta l + 2\cdot r2\cdot\beta l - \beta l\cdot\rho + \tau)}{2\cdot\tau}$$

$$\#225:\ \frac{d}{d\ f2}\left(\frac{d}{d\ f2}\left(profitmm2 = (\rho - r2)\cdot\left(\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 -\right.\right.\right.\right.$$

$$\left(\frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\Bigg)\cdot\beta h\Bigg) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)$$

#226: $$0 > \frac{\eta\cdot(\theta - 1)}{\tau}$$

#227: $$\frac{d}{d\ r2}\left(\text{profitmm2} = (\rho - r2)\cdot\left(\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \left(\frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)\right)$$

#228: $$0 = \frac{\eta\cdot(f1\cdot\beta l\cdot(\theta - 1) + 2\cdot f2\cdot\beta l\cdot(1-\theta) + r1\cdot(\beta h^2\cdot\theta + \beta l^2\cdot(1-\theta)) - 2\cdot r2\cdot(\beta h^2\cdot\theta + \beta l^2\cdot(1-\theta)) + \beta h\sim}{2\cdot\tau}$$

$$\frac{\beta h^2\cdot\theta\cdot\rho - \beta h\cdot\theta\cdot\tau + \beta l^2\cdot\rho\cdot(1-\theta) + \beta l\cdot\tau\cdot(\theta - 1))}{}$$

#229: $$\frac{d}{d\ r2}\frac{d}{d\ r2}\left(\text{profitmm2} = (\rho - r2)\cdot\left(\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \left(\frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)\right)$$

#230: $$0 > -\frac{\eta\cdot(\beta h^{2}\cdot\theta + \beta l^{2}\cdot(1-\theta))}{\tau}$$

#231: $$\frac{d}{d\ r2}\frac{d}{d\ f2}\left(\text{profitmm2} = (\rho - r2)\cdot\left(\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{r1\cdot\beta h - r2\cdot\beta h + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\right)$$

#232: $$\frac{\beta l\cdot\eta\cdot(1-\theta)}{\tau}$$

#233: $$H = \frac{\eta\cdot(\theta - 1)}{\tau}\cdot\left(-\frac{\eta\cdot(\beta h^{2}\cdot\theta + \beta l^{2}\cdot(1-\theta))}{\tau}\right) - \frac{\beta l\cdot\eta\cdot(1-\theta)}{\tau}\cdot\frac{\beta l\cdot\eta\cdot(1-\theta)}{\tau}$$

#234: $$H = \frac{\beta h^{2}\cdot\eta^{2}\cdot\theta\cdot(1-\theta)}{\tau^{2}} > 0$$

eq (26) equlibrium under mm

#235: $$\text{SOLVE}\left(\left[0 = \frac{\eta\cdot(\theta - 1)\cdot(2\cdot f1 - f2 - 2\cdot r1\cdot\beta l + r2\cdot\beta l + \beta l\cdot\rho - \tau)}{2\cdot\tau}, 0 = -\right.\right.$$

$$\frac{\eta\cdot(2\cdot f1\cdot\beta l\cdot(\theta-1)+f2\cdot\beta l\cdot(1-\theta)+2\cdot r1\cdot(\beta h^2\cdot\theta+\beta l^2\cdot(1-\theta))-r2\cdot(\beta h^2\cdot\theta+\beta l^2\cdot(1-\theta))-\beta h}{2\cdot\tau}$$

$$\frac{2\cdot\theta\cdot\rho+\beta h\cdot\theta\cdot\tau+\beta l^2\cdot\rho\cdot(\theta-1)+\beta l\cdot\tau\cdot(1-\theta))}{}, 0 =$$

$$\frac{\eta\cdot(1-\theta)\cdot(f1-2\cdot f2-r1\cdot\beta l+2\cdot r2\cdot\beta l-\beta l\cdot\rho+\tau)}{2\cdot\tau}, 0 =$$

$$\left[\frac{\eta\cdot(f1\cdot\beta l\cdot(\theta-1)+2\cdot f2\cdot\beta l\cdot(1-\theta)+r1\cdot(\beta h^2\cdot\theta+\beta l^2\cdot(1-\theta))-2\cdot r2\cdot(\beta h^2\cdot\theta+\beta l^2\cdot(1-\theta))+\beta h}{2\cdot\tau}\right.$$

$$\left.\frac{2\cdot\theta\cdot\rho-\beta h\cdot\theta\cdot\tau+\beta l^2\cdot\rho\cdot(1-\theta)+\beta l\cdot\tau\cdot(\theta-1))}{}\right], [f1, f2, r1, r2]\Bigg)$$

eq (26) equilibrium under mm with interest

#236: $$\left[f1=\frac{\tau\cdot(\beta h-\beta l)}{\beta h}\wedge f2=\frac{\tau\cdot(\beta h-\beta l)}{\beta h}\wedge r1=\frac{\beta h\cdot\rho-\tau}{\beta h}\wedge r2=\frac{\beta h\cdot\rho-\tau}{\beta h}\right]$$

#237: $$profitmm1=\frac{\eta\cdot\tau}{2}$$

#238: $$profitmm2=\frac{\eta\cdot\tau}{2}$$

** Subsection 6.2: Interest with NO min balance ==> all pay a fee

utlity from line 179,

#239: μ + r1·βl − τ·x − f1 = μ + r2·βl − τ·(1 − x) − f2

#240: SOLVE(μ + r1·βl − τ·x − f1 = μ + r2·βl − τ·(1 − x) − f2, x)

eq (27)

$$\#241: \quad xffl = -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}$$

#242: μ + r1·βh − τ·x − f1 = μ + r2·βh − τ·(1 − x) − f2

#243: SOLVE(μ + r1·βh − τ·x − f1 = μ + r2·βh − τ·(1 − x) − f2, x)

$$\#244: \quad xffh = -\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}$$

#245: nl1 = xffl·(1 − θ)·η

#246: nl2 = (1 − xffl)·(1 − θ)·η

#247: dl1 = nl1·βl

#248: dl2 = nl2·βl

#249: dl1 = (xffl·(1 − θ)·η)·βl

#250: dl2 = ((1 − xffl)·(1 − θ)·η)·βl

#251: nh1 = xffh·θ·η

#252: nh2 = (1 − xffh)·θ·η

#253: dh1 = nh1·βh

#254: dh2 = nh2·βh

#255: dh1 = (xffh·θ·η)·βh

#256: dh2 = ((1 − xffh)·θ·η)·βh

eq (28) profit functions

#257: profitff1 = (ρ − r1)·(dl1 + dh1) + f1·(nl1 + nh1)

#258: profitff2 = (ρ − r2)·(dl2 + dh2) + f2·(nl2 + nh2)

Derivation of eq (29) and Appendix H

#259: profitff1 = (ρ − r1)·((xffl·(1 − θ)·η)·βl + (xffh·θ·η)·βh) + f1·(xffl·(1 − θ)·η + xffh·θ·η)

#260: profitff2 = (ρ − r2)·(((1 − xffl)·(1 − θ)·η)·βl + ((1 − xffh)·θ·η)·βh) + f2·((1 − xffl)·(1 − θ)·η
+ (1 − xffh)·θ·η)

eq (H.1) in paper Appendix

$$\#261:\ \text{profitff1} = (\rho - r1)\cdot\left(\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\left(-\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta + \left(-\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)$$

$$\#262:\ \text{profitff2} = (\rho - r2)\cdot\left(\left(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - -\right.\right.\right.$$

$$\left(\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\Bigg)\cdot\beta h\Bigg) + f2\cdot\Bigg(\Bigg(1 - -\left(\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta + \left(1 - -\left(\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)$$

eq (H.2) FOC

$$\#263:\ \frac{d}{d\ f1}\left(profitff1 = (\rho - r1)\cdot\Bigg(\Bigg(\Bigg(-\left(\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\Bigg)\cdot\beta l + \Bigg(\Bigg(-\left(\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\Bigg)\cdot\beta h\Bigg) + f1\cdot\Bigg(\Bigg(-\left(\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta + \left(-\left(\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\Bigg)\right)$$

$$\#264:\ 0 = -\frac{\eta\cdot(2\cdot f1 - f2 - 2\cdot r1\cdot(\beta h\cdot\theta + \beta l\cdot(1 - \theta)) + r2\cdot(\beta h\cdot\theta + \beta l\cdot(1 - \theta)) + \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(1 - \theta) - \tau)}{2\cdot\tau}$$

$$\#265:\ \frac{d}{d\ f1}\frac{d}{d\ f1}\left(profitff1 = (\rho - r1)\cdot\Bigg(\Bigg(\Bigg(-\left(\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\Bigg)\cdot\beta l + \Bigg(\Bigg(-\left(\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\Bigg)\cdot\beta h\Bigg) + f1\cdot\Bigg(\Bigg(-\left(\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta + \left(-\right.\right.\right.$$

$$\left(\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\Bigg)$$

#266: $$0 > -\frac{\eta}{\tau}$$

#267: $$\frac{d}{d\ r1}\left(\text{profitff1} = (\rho - r1)\cdot\left(\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\left(-\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta + \left(-\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\right)$$

#268: $$0 = \frac{\eta\cdot(2\cdot f1\cdot(\beta h\cdot\theta + \beta l\cdot(1-\theta)) - f2\cdot(\beta h\cdot\theta + \beta l\cdot(1-\theta)) - 2\cdot r1\cdot(\beta h^2\cdot\theta + \beta l^2\cdot(1-\theta)) + r2\cdot(\beta h^2\cdot\theta + \sim}{2\cdot\tau}$$
$$\frac{\beta l^2\cdot(1-\theta)) + \beta h^2\cdot\theta\cdot\rho - \beta h\cdot\theta\cdot\tau + \beta l^2\cdot\rho\cdot(1-\theta) + \beta l\cdot\tau\cdot(\theta - 1))}{}$$

#269: $$\frac{d}{d\ r1}\frac{d}{d\ r1}\left(\text{profitff1} = (\rho - r1)\cdot\left(\left(\left(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta\right)\cdot\beta l + \left(\left(-\right.\right.\right.\right.$$

$$\left(\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\Bigg)\cdot\beta h\Bigg) + f1\cdot\Bigg(\Bigg(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\Bigg)\cdot(1 - \theta)\cdot\eta + \Bigg(-\left(\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\Bigg)\Bigg)$$

#270: $$0 > -\frac{\eta\cdot(\beta h^2\cdot\theta + \beta l^2\cdot(1 - \theta))}{\tau}$$

cross derivative bank 1

#271: $$\frac{d}{d\,f1}\frac{d}{d\,r1}\Bigg(\text{profitff1} = (\rho - r1)\cdot\Bigg(\Bigg(\Bigg(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\Bigg)\cdot(1 - \theta)\cdot\eta\Bigg)\cdot\beta l + \Bigg(\Bigg(-\left(\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\Bigg)\cdot\beta h\Bigg) + f1\cdot\Bigg(\Bigg(-\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\Bigg)\cdot(1 - \theta)\cdot\eta + \Bigg(-\left(\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\Bigg)\Bigg)$$

#272: $$\frac{\eta\cdot(\beta h\cdot\theta + \beta l\cdot(1 - \theta))}{\tau}$$

#273: $$H = \left(-\frac{\eta}{\tau}\right)\cdot\left(-\frac{\eta\cdot(\beta h^2\cdot\theta + \beta l^2\cdot(1 - \theta))}{\tau}\right) - \frac{\eta\cdot(\beta h\cdot\theta + \beta l\cdot(1 - \theta))}{\tau}\cdot\frac{\eta\cdot(\beta h\cdot\theta + \beta l\cdot(1 - \theta))}{\tau}$$

#274: $$H = \frac{\eta^2 \cdot \theta \cdot (1 - \theta) \cdot (\beta h^2 - 2 \cdot \beta h \cdot \beta l + \beta l^2)}{\tau^2} > 0$$

#275: $$H = \frac{\eta^2 \cdot \theta \cdot (1 - \theta) \cdot (\beta h - \beta l)^2}{\tau^2} > 0$$

#276: $$\frac{d}{d\ f2}\left(profitff2 = (\rho - r2) \cdot \left(\left(\left(1 - -\frac{f1 - f2 - r1 \cdot \beta l + r2 \cdot \beta l - \tau}{2 \cdot \tau}\right) \cdot (1 - \theta) \cdot \eta\right) \cdot \beta l + \left(\left(1 - -\frac{f1 - f2 - r1 \cdot \beta h + r2 \cdot \beta h - \tau}{2 \cdot \tau}\right) \cdot \theta \cdot \eta\right) \cdot \beta h\right) + f2 \cdot \left(\left(1 - -\frac{f1 - f2 - r1 \cdot \beta l + r2 \cdot \beta l - \tau}{2 \cdot \tau}\right) \cdot (1 - \theta) \cdot \eta + \left(1 - -\frac{f1 - f2 - r1 \cdot \beta h + r2 \cdot \beta h - \tau}{2 \cdot \tau}\right) \cdot \theta \cdot \eta\right)\right)$$

#277: $$0 = \frac{\eta \cdot (f1 - 2 \cdot f2 - r1 \cdot (\beta h \cdot \theta + \beta l \cdot (1 - \theta)) + 2 \cdot r2 \cdot (\beta h \cdot \theta + \beta l \cdot (1 - \theta)) - \beta h \cdot \theta \cdot \rho + \beta l \cdot \rho \cdot (\theta - 1) + \tau)}{2 \cdot \tau}$$

#278: $$\frac{d}{d\ f2}\frac{d}{d\ f2}\left(profitff2 = (\rho - r2) \cdot \left(\left(\left(1 - -\frac{f1 - f2 - r1 \cdot \beta l + r2 \cdot \beta l - \tau}{2 \cdot \tau}\right) \cdot (1 - \theta) \cdot \eta\right) \cdot \beta l + \left(\left(1 - -\frac{f1 - f2 - r1 \cdot \beta h + r2 \cdot \beta h - \tau}{2 \cdot \tau}\right) \cdot \theta \cdot \eta\right) \cdot \beta h\right) + f2 \cdot \left(\left(1 - -\frac{f1 - f2 - r1 \cdot \beta l + r2 \cdot \beta l - \tau}{2 \cdot \tau}\right) \cdot (1 - \theta) \cdot \eta +\right.$$

$$\left(1 - - \frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\right)$$

#279: $$0 > -\frac{\eta}{\tau}$$

#280: $$\frac{d}{d\ r2}\left(profitff2 = (\rho - r2)\cdot\left(\left(\left(\left(1 - - \frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - - \frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - - \frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta + \left(1 - - \frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\right)$$

#281: $$0 = -\frac{\eta\cdot(f1\cdot(\beta h\cdot\theta + \beta l\cdot(1 - \theta)) - 2\cdot f2\cdot(\beta h\cdot\theta + \beta l\cdot(1 - \theta)) - r1\cdot(\beta h^2\cdot\theta + \beta l^2\cdot(1 - \theta)) + 2\cdot r2\cdot(\beta h^2\cdot\theta + \sim \\ \sim \beta l^2\cdot(1 - \theta)) - \beta h^2\cdot\theta\cdot\rho + \beta h\cdot\theta\cdot\tau + \beta l^2\cdot\rho\cdot(\theta - 1) + \beta l\cdot\tau\cdot(1 - \theta))}{2\cdot\tau}$$

#282: $$\frac{d}{d\ r2}\ \frac{d}{d\ r2}\left(profitff2 = (\rho - r2)\cdot\left(\left(\left(\left(1 - - \frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - -\right.\right.\right.\right.$$

$$\left(\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\Bigg)\cdot\beta h\Bigg) + f2\cdot\Bigg(\Bigg(\left(1 - -\frac{f1 - f2 - r1\cdot\beta l + r2\cdot\beta l - \tau}{2\cdot\tau}\right)\cdot(1-\theta)\cdot\eta + \left(1 - -\frac{f1 - f2 - r1\cdot\beta h + r2\cdot\beta h - \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\Bigg)\Bigg)$$

#283: $$0 > -\frac{\eta\cdot(\beta h^2\cdot\theta + \beta l^2\cdot(1-\theta))}{\tau}$$

eq (29) in paper

#284: SOLVE$$\left(\left[0 = -\frac{\eta\cdot(2\cdot f1 - f2 - 2\cdot r1\cdot(\beta h\cdot\theta + \beta l\cdot(1-\theta)) + r2\cdot(\beta h\cdot\theta + \beta l\cdot(1-\theta)) + \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(1-\theta) - \tau)}{2\cdot\tau},\right.\right.$$

$$\left.\left.0 = \frac{\eta\cdot(2\cdot f1\cdot(\beta h\cdot\theta + \beta l\cdot(1-\theta)) - f2\cdot(\beta h\cdot\theta + \beta l\cdot(1-\theta)) - 2\cdot r1\cdot(\beta h^2\cdot\theta + \beta l^2\cdot(1-\theta)) + r2\cdot(\beta h^2\cdot\theta + \sim}{2\cdot\tau}\right.\right.$$

$$\frac{\beta l^2\cdot(1-\theta))+\beta h^2\cdot\theta\cdot\rho-\beta h\cdot\theta\cdot\tau+\beta l^2\cdot\rho\cdot(1-\theta)+\beta l\cdot\tau\cdot(\theta-1))}{\text{——}},\ 0=$$

$$\frac{\eta\cdot(f1-2\cdot f2-r1\cdot(\beta h\cdot\theta+\beta l\cdot(1-\theta))+2\cdot r2\cdot(\beta h\cdot\theta+\beta l\cdot(1-\theta))-\beta h\cdot\theta\cdot\rho+\beta l\cdot\rho\cdot(\theta-1)+\tau)}{2\cdot\tau},$$

$$0=-\left(\frac{\eta\cdot(f1\cdot(\beta h\cdot\theta+\beta l\cdot(1-\theta))-2\cdot f2\cdot(\beta h\cdot\theta+\beta l\cdot(1-\theta))-r1\cdot(\beta h^2\cdot\theta+\beta l^2\cdot(1-\theta))+2\cdot r2\cdot(\beta h^2\cdot\theta+\beta l^2\cdot(1-\theta))}{2\cdot\tau}\right.$$

$$\left.\frac{\beta l^2\cdot(1-\theta))-\beta h^2\cdot\theta\cdot\rho+\beta h\cdot\theta\cdot\tau+\beta l^2\cdot\rho\cdot(\theta-1)+\beta l\cdot\tau\cdot(1-\theta))}{\text{——}}\right],\ [f1,\ f2,\ r1,\ r2]\Bigg)$$

eq (29)

#285: $[f1=\tau\wedge f2=\tau\wedge r1=\rho\wedge r2=\rho]$

#286: $\text{profitff1}=\frac{\eta\cdot\tau}{2}$

#287: $\text{profitff2}=\frac{\eta\cdot\tau}{2}$

** Section 6.3 Interest, comparisons

eq (30) differences

$$\#288:\ fmm1 - fff1 = \frac{\tau\cdot(\beta h - \beta l)}{\beta h} - \tau$$

$$\#289:\ fmm1 - fff1 = -\frac{\beta l\cdot\tau}{\beta h}$$

$$\#290:\ rmm1 - rff1 = \frac{\beta h\cdot\rho - \tau}{\beta h} - \rho$$

$$\#291:\ rmm1 - rff1 = -\frac{\tau}{\beta h}$$