balance_2023_mm_dd See GDrive balance for summary of all derivations

#1: CaseMode ≔ Sensitive

#2: InputMode ≔ Word

transportation parameter

#3: τ ∈̔ Real (0, ∞)

consumer index

#4: x ∈̔ Real [0, 1]

Return profit per dollar of deposits

#5: ρ ∈̔ Real (0, ∞)

basic utility derived from bank services

#6: μ ∈̔ Real (0, ∞)

value derived from bank 1 (if pays monthly fee)

#7: μ − τ·x − f1

value derived from bank 1 (no fee)

#8: μ − τ·x

value derived from bank 2 (if pays monthly fee)

#9: μ − τ·(1 − x) − f2

value derived from bank 2 (no fee)

#10: μ − τ·(1 − x)

Total number of depositors

#11: η ∈ Real (0, ∞)

fraction of depositors with high balance

#12: θ ∈ Real [0, 1]

Dollar amount of low and high balances

#13: βl ∈ Real (0, ∞)

#14: βh ∈ Real (0, ∞)

fees (if levied) by bank 1 and bank 2

#15: f1 ∈ Real [0, ∞)

#16: f2 ∈ Real [0, ∞)

*** Section 2.1 in paper: Both banks set min bal b1=b2=βh (high balance)

#17: μ − τ·x − f1 = μ − τ·(1 − x) − f2

#18: SOLVE(μ − τ·x − f1 = μ − τ·(1 − x) − f2, x)

eq (2)

#19: $$xmml = -\frac{f1 - f2 - \tau}{2 \cdot \tau}$$

eq (3)

#20: nl1 = xmml·(1 − θ)·η

#21: nl2 = (1 − xmml)·(1 − θ)·η

#22: dl1 = nl1·βl

#23: dl1 = (xmml·(1 − θ)·η)·βl

#24: $dl2 = nl2 \cdot \beta l$

#25: $dl2 = ((1 - xmml) \cdot (1 - \theta) \cdot \eta) \cdot \beta l$

Deriving eq (4) (high balance)

#26: $\mu - \tau \cdot x = \mu - \tau \cdot (1 - x)$

#27: $\text{SOLVE}(\mu - \tau \cdot x = \mu - \tau \cdot (1 - x), x)$

#28: $$xmmh = \frac{1}{2}$$

eq (5)

#29: $nh1 = xmmh \cdot \theta \cdot \eta$

#30: $nh2 = (1 - xmmh) \cdot \theta \cdot \eta$

#31: $dh1 = nh1 \cdot \beta h$

#32: $dh1 = (xmmh \cdot \theta \cdot \eta) \cdot \beta h$

#33: $dh2 = nh2 \cdot \beta h$

#34: $dh2 = ((1 - xmmh) \cdot \theta \cdot \eta) \cdot \beta h$

eq (6)

#35: $profit1 = \rho \cdot (dl1 + dh1) + f1 \cdot nl1$

#36: $profit2 = \rho \cdot (dl2 + dh2) + f2 \cdot nl2$

Deriving (7) and Appendix A

#37: $profit1 = \rho \cdot ((xmml \cdot (1 - \theta) \cdot \eta) \cdot \beta l + (xmmh \cdot \theta \cdot \eta) \cdot \beta h) + f1 \cdot (xmml \cdot (1 - \theta) \cdot \eta)$

#38: $profit2 = \rho \cdot (((1 - xmml) \cdot (1 - \theta) \cdot \eta) \cdot \beta l + ((1 - xmmh) \cdot \theta \cdot \eta) \cdot \beta h) + f2 \cdot ((1 - xmml) \cdot (1 - \theta) \cdot \eta)$

#39: $$\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)$$

#40: $$\text{profit2} = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)$$

#41: $$\frac{d}{d\ f1}\left(\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)$$

#42: $$0 = \frac{\eta\cdot(\theta - 1)\cdot(2\cdot f1 - f2 + \beta l\cdot\rho - \tau)}{2\cdot\tau}$$

#43: $$\frac{d}{d\ f1}\frac{d}{d\ f1}\left(\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{1}{2}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)$$

#44: $$0 > \frac{\eta\cdot(\theta - 1)}{\tau}$$

#45: $$\frac{d}{d\ f2}\left(\text{profit2} = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\right.\right.\right.$$

$$\left(\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\Bigg)$$

#46: $$0 = \frac{\eta\cdot(1 - \theta)\cdot(f1 - 2\cdot f2 - \beta l\cdot\rho + \tau)}{2\cdot\tau}$$

#47: $$\frac{d}{d\ f2}\frac{d}{d\ f2}\left(profit2 = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{1}{2}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\right.\right.\right.$$

$$\left.\left.\left.\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)$$

#48: $$0 > \frac{\eta\cdot(\theta - 1)}{\tau}$$

#49: $$\text{SOLVE}\left(\left[0 = \frac{\eta\cdot(\theta - 1)\cdot(2\cdot f1 - f2 + \beta l\cdot\rho - \tau)}{2\cdot\tau}, 0 = \frac{\eta\cdot(1 - \theta)\cdot(f1 - 2\cdot f2 - \beta l\cdot\rho + \tau)}{2\cdot\tau}\right], [f1, f2]\right)$$

eq (7)

#50: $$[fmm1 = \tau - \beta l\cdot\rho \wedge fmm2 = \tau - \beta l\cdot\rho]$$

#51: $$profitmm1 = \frac{\eta\cdot(\beta h\cdot\theta\cdot\rho - \tau\cdot(\theta - 1))}{2}$$

#52: $$profitmm2 = \frac{\eta\cdot(\beta h\cdot\theta\cdot\rho - \tau\cdot(\theta - 1))}{2}$$

Result 1a

$$\#53:\quad \frac{d}{d\theta}\left(\text{profitmm1} = \frac{\eta\cdot(\beta h\cdot\theta\cdot\rho - \tau\cdot(\theta - 1))}{2}\right)$$

$$\#54:\quad \frac{\eta\cdot(\beta h\cdot\rho - \tau)}{2}$$

** Section 2.2 no min balance

$$\#55:\quad \mu - \tau\cdot x - f1 = \mu - \tau\cdot(1 - x) - f2$$

$$\#56:\quad \text{SOLVE}(\mu - \tau\cdot x - f1 = \mu - \tau\cdot(1 - x) - f2,\ x)$$

eq (8)

$$\#57:\quad \text{xff} = -\frac{f1 - f2 - \tau}{2\cdot\tau}$$

eq (9)

$$\#58:\quad n1 = \text{xff}\cdot\eta$$

$$\#59:\quad n2 = (1 - \text{xff})\cdot\eta$$

$$\#60:\quad d1 = n1\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)$$

$$\#61:\quad d2 = n2\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)$$

$$\#62:\quad d1 = (\text{xff}\cdot\eta)\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)$$

$$\#63:\quad d2 = ((1 - \text{xff})\cdot\eta)\cdot((1 - \theta)\cdot\beta l + \theta\cdot\beta h)$$

eq (10)

$$\#64:\quad \text{profit1} = \rho\cdot d1 + f1\cdot n1$$

$$\#65:\quad \text{profit2} = \rho\cdot d2 + f2\cdot n2$$

Derivation of (11) and Appendix B

#66: $\text{profit1} = \rho\cdot((\text{xff}\cdot\eta)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)) + f1\cdot(\text{xff}\cdot\eta)$

#67: $\text{profit2} = \rho\cdot(((1-\text{xff})\cdot\eta)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)) + f2\cdot((1-\text{xff})\cdot\eta)$

#68: $\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f1\cdot\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)$

#69: $\text{profit2} = \rho\cdot\left(\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f2\cdot\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)$

#70: $\frac{d}{d\ f1}\left(\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f1\cdot\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\right)$

#71: $$0 = -\frac{\eta\cdot(2\cdot f1 - f2 + \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(1-\theta) - \tau)}{2\cdot\tau}$$

#72: $\frac{d}{d\ f1}\frac{d}{d\ f1}\left(\text{profit1} = \rho\cdot\left(\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f1\cdot\left(\left(-\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\right)$

#73: $$0 > -\frac{\eta}{\tau}$$

#74: $\frac{d}{d\ f2}\left(\text{profit2} = \rho\cdot\left(\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f2\cdot\left(\left(1--\frac{f1-f2-\tau}{2\cdot\tau}\right)\cdot\eta\right)\right)$

#75: $$0 = \frac{\eta\cdot(f1 - 2\cdot f2 - \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(\theta-1) + \tau)}{2\cdot\tau}$$

#76: $$\frac{d}{d\ f2}\ \frac{d}{d\ f2}\left(\text{profit2} = \rho\cdot\left(\left(\left(1 - -\ \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\cdot((1-\theta)\cdot\beta l + \theta\cdot\beta h)\right) + f2\cdot\left(\left(1 - -\ \frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot\eta\right)\right)$$

#77: $$0 > -\ \frac{\eta}{\tau}$$

#78: $$\text{SOLVE}\left(\left[0 = -\ \frac{\eta\cdot(2\cdot f1 - f2 + \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(1-\theta) - \tau)}{2\cdot\tau},\ 0 = \frac{\eta\cdot(f1 - 2\cdot f2 - \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(\theta - 1) + \tau)}{2\cdot\tau}\right],\ [f1,\ f2]\right)$$

eq (11)

#79: $$[\text{fff1} = -\ \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(\theta - 1) + \tau \wedge \text{fff2} = -\ \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(\theta - 1) + \tau]$$

#80: $$\text{profitff1} = \frac{\eta\cdot\tau}{2}$$

Result 2a

#81: $$\frac{d}{d\theta}\ (f1 = -\ \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(\theta - 1) + \tau)$$

#82: $$0 > \rho\cdot(\beta l - \beta h)$$

*** Section 3: Do banks benefit from imposing minimum balance?

Recall equilibrium fees and profits under min bal (line 50 above)

#83: fmm1 = τ − βl·ρ

#84: $$\text{profitmm1} = \frac{\eta\cdot(\beta h\cdot\theta\cdot\rho - \tau\cdot(\theta - 1))}{2}$$

Recall equilibrium fees and profits with NO min bal (line 79 above)

#85: fff1 = − βh·θ·ρ + βl·ρ·(θ − 1) + τ

#86: $$\text{profitff1} = \frac{\eta\cdot\tau}{2}$$

comparing fees: (m,m) minus (f,f), eq (12): fmm1 − fff1

#87: τ − βl·ρ − (− βh·θ·ρ + βl·ρ·(θ − 1) + τ)

#88: $$\beta h\cdot\theta\cdot\rho - \beta l\cdot\theta\cdot\rho > 0$$

comparing profits: (m,m) minus (f,f), eq (13): profitmm1 − profitff1

#89: $$\frac{\eta\cdot(\beta h\cdot\theta\cdot\rho - \tau\cdot(\theta - 1))}{2} - \frac{\eta\cdot\tau}{2}$$

#90: $$\frac{\eta\cdot\theta\cdot(\beta h\cdot\rho - \tau)}{2}$$

*** Section 4: Asymmetric strategies

** Subsection 4a: Eql fees

Deriving eq (14): xmfl (low-balance, both charge fees)

#91: μ − τ·x − f1 = μ − τ·(1 − x) − f2

#92: SOLVE(μ − τ·x − f1 = μ − τ·(1 − x) − f2, x)

#93: $$xmfl = -\frac{f1 - f2 - \tau}{2\cdot\tau}$$

Also in eq (14): xmfh (high-balance, only bank 2 charges a fee f2)

#94: $\mu - \tau\cdot x = \mu - \tau\cdot(1 - x) - f2$

#95: SOLVE($\mu - \tau\cdot x = \mu - \tau\cdot(1 - x) - f2$, x)

#96: $$xmfh = \frac{f2 + \tau}{2\cdot\tau}$$

eq (15) num low-balance depositors

#97: $nl1 = xmfl\cdot(1 - \theta)\cdot\eta$

#98: $nl2 = (1 - xmfl)\cdot(1 - \theta)\cdot\eta$

#99: $dl1 = nl1\cdot\beta l$

#100: $dl2 = nl2\cdot\beta l$

#101: $dl1 = (xmfl\cdot(1 - \theta)\cdot\eta)\cdot\beta l$

#102: $dl2 = ((1 - xmfl)\cdot(1 - \theta)\cdot\eta)\cdot\beta l$

eq (16) num high-balance depositors

#103: $nh1 = xmfh\cdot\theta\cdot\eta$

#104: $nh2 = (1 - xmfh)\cdot\theta\cdot\eta$

#105: $dh1 = nh1\cdot\beta h$

#106: $dh2 = nh2\cdot\beta h$

#107: $dh1 = (xmfh\cdot\theta\cdot\eta)\cdot\beta h$

#108: $dh2 = ((1 - xmfh)\cdot\theta\cdot\eta)\cdot\beta h$

eq (17): Profit functions

#109: $profitmf1 = \rho\cdot(dl1 + dh1) + f1\cdot nl1$

#110: $profitmf2 = \rho\cdot(dl2 + dh2) + f2\cdot(nl2 + nh2)$

#111: $profitmf1 = \rho\cdot((xmfl\cdot(1 - \theta)\cdot\eta)\cdot\beta l + (xmfh\cdot\theta\cdot\eta)\cdot\beta h) + f1\cdot(xmfl\cdot(1 - \theta)\cdot\eta)$

#112: $profitmf2 = \rho\cdot(((1 - xmfl)\cdot(1 - \theta)\cdot\eta)\cdot\beta l + ((1 - xmfh)\cdot\theta\cdot\eta)\cdot\beta h) + f2\cdot((1 - xmfl)\cdot(1 - \theta)\cdot\eta + (1 - xmfh)\cdot\theta\cdot\eta)$

Deriving eq (18) and Appendix C

#113: $$profitmf1 = \rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{f2 + \tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)$$

#114: $$profitmf2 = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{f2 + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta + \left(1 - \frac{f2 + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)$$

#115: $$\frac{d}{d\ f1}\left(profitmf1 = \rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{f2 + \tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\right.\right.\right.$$

$$\left(\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\Bigg)$$

#116: $$0 = \frac{\eta\cdot(\theta - 1)\cdot(2\cdot f1 - f2 + \beta l\cdot\rho - \tau)}{2\cdot\tau}$$

#117: $$\frac{d}{d\ f1}\ \frac{d}{d\ f1}\left(profitmf1 = \rho\cdot\left(\left(\left(-\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\frac{f2 + \tau}{2\cdot\tau}\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f1\cdot\left(\left(-\right.\right.\right.$$

$$\left.\left.\left.\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\right)$$

#118: $$0 > \frac{\eta\cdot(\theta - 1)}{\tau}$$

#119: $$\frac{d}{d\ f2}\left(profitmf2 = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{f2 + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1 - -\right.\right.\right.$$

$$\left.\left.\left.\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta + \left(1 - \frac{f2 + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\right)$$

#120: $$0 = -\frac{\eta\cdot(f1\cdot(\theta - 1) + 2\cdot f2 + \beta h\cdot\theta\cdot\rho + \beta l\cdot\rho\cdot(1 - \theta) - \tau)}{2\cdot\tau}$$

#121: $$\frac{d}{d\ f2}\ \frac{d}{d\ f2}\left(profitmf2 = \rho\cdot\left(\left(\left(1 - -\frac{f1 - f2 - \tau}{2\cdot\tau}\right)\cdot(1 - \theta)\cdot\eta\right)\cdot\beta l + \left(\left(1 - \frac{f2 + \tau}{2\cdot\tau}\right)\cdot\theta\cdot\eta\right)\cdot\beta h\right) + f2\cdot\left(\left(1\right.\right.\right.$$

$$- - \frac{f1 - f2 - \tau}{2 \cdot \tau}\right) \cdot (1 - \theta) \cdot \eta + \left(1 - \frac{f2 + \tau}{2 \cdot \tau}\right) \cdot \theta \cdot \eta\right)\right)$$

#122: $$0 > - \frac{\eta}{\tau}$$

#123: $$\text{SOLVE}\left(\left[0 = \frac{\eta \cdot (\theta - 1) \cdot (2 \cdot f1 - f2 + \beta l \cdot \rho - \tau)}{2 \cdot \tau}, \; 0 = - \frac{\eta \cdot (f1 \cdot (\theta - 1) + 2 \cdot f2 + \beta h \cdot \theta \cdot \rho + \beta l \cdot \rho \cdot (1 - \theta) - \tau)}{2 \cdot \tau}\right], [f1, f2]\right)$$

eq (18)

#124: $$\left[f1 = - \frac{\beta h \cdot \theta \cdot \rho + \beta l \cdot \rho \cdot (3 - \theta) - 3 \cdot \tau}{\theta + 3} \wedge f2 = - \frac{2 \cdot \beta h \cdot \theta \cdot \rho + 3 \cdot \beta l \cdot \rho \cdot (1 - \theta) + \tau \cdot (\theta - 3)}{\theta + 3}\right]$$