wb_fin_2025_mm_dd (whistleblowers and financial Fraud)

#1: CaseMode ≔ Sensitive

#2: InputMode ≔ Word

probability of conviction based on WB info

#3: $\lambda :\in \text{Real } (0, 1)$

initial fraud loss before recovery

#4: $L :\in \text{Real } (0, \infty)$

fraction of recovered amount

#5: $\rho :\in \text{Real } (0, 1)$

Parameter of WB discomfort from WB (disutility parameter)

#6: $\delta :\in \text{Real } (0, \infty)$

concavility/convexity of WB utility w.r.t. type

#7: $\gamma :\in \text{Real } (0, \infty)$

Total and fraction of recovered money that paid to WB

#8: $C :\in \text{Real } (0, \infty)$

#9: $c :\in \text{Real } (0, 1)$

WB probability (endogenous).

#10: $pw :\in \text{Real } (0, 1]$

fraud prob (endogeneous)

#11: $pf :\in \text{Real } (0, 1]$

Penalty on convicted fraudster

#12: $F :\in \text{Real } [0, \infty)$

eq (1): utility of a WB

#13: $u = \lambda \cdot C - \delta \cdot d^{\gamma}$

eq (2): dhat

#14: $0 = \lambda \cdot C - \delta \cdot d^{\gamma}$

#15: $\text{SOLVE}(0 = \lambda \cdot C - \delta \cdot d^{\gamma}, d)$

#16: $$pw = dhat = \left(\frac{C \cdot \lambda}{\delta}\right)^{1/\gamma}$$

< 1 if

#17: $\left(\frac{C \cdot \lambda}{\delta}\right)^{1/\gamma} < 1$

#18: $\text{SOLVE}\left(\left(\frac{C \cdot \lambda}{\delta}\right)^{1/\gamma} < 1, C\right)$

#19: $$0 < C < \frac{\delta}{\lambda}$$

eq (3) net revenue maximization

#20: $er = pw \cdot \lambda \cdot (\rho \cdot L - C)$

#21: $er = \left(\frac{C\cdot\lambda}{\delta}\right)^{1/\gamma}\cdot\lambda\cdot(\rho\cdot L - C)$

eq (4) Appendix B

#22: $\frac{d}{dC}\left(er = \left(\frac{C\cdot\lambda}{\delta}\right)^{1/\gamma}\cdot\lambda\cdot(\rho\cdot L - C)\right)$

eq (B.1)

#23: $$0 = \frac{C^{(1 - \gamma)/\gamma}\cdot\delta^{- 1/\gamma}\cdot\lambda^{(\gamma + 1)/\gamma}\cdot(L\cdot\rho - C\cdot(\gamma + 1))}{\gamma}$$

#24: $\frac{d}{dC}\frac{d}{dC}\left(er = \left(\frac{C\cdot\lambda}{\delta}\right)^{1/\gamma}\cdot\lambda\cdot(\rho\cdot L - C)\right)$

#25: $$- \frac{C^{(1 - 2\cdot\gamma)/\gamma}\cdot\delta^{- 1/\gamma}\cdot\lambda^{(\gamma + 1)/\gamma}\cdot(C\cdot(\gamma + 1) + L\cdot\rho\cdot(\gamma - 1))}{\gamma^{2}}$$

< 0 if

#26: $C\cdot(\gamma + 1) + L\cdot\rho\cdot(\gamma - 1) > 0$

#27: $\mathrm{SOLVE}(C\cdot(\gamma + 1) + L\cdot\rho\cdot(\gamma - 1) > 0, C)$

#28: $$C > \frac{L\cdot\rho\cdot(1 - \gamma)}{\gamma + 1}$$

#29: $\mathrm{SOLVE}(C^{(1 - \gamma)/\gamma}\cdot\delta^{- 1/\gamma}\cdot\lambda^{(\gamma + 1)/\gamma}\cdot(L\cdot\rho - C\cdot(\gamma + 1)), C)$

#30: $$\mathrm{Cstar} = \frac{L \cdot \rho}{\gamma + 1}$$

explainng Figure 2 horizontal axis

Cstar = δ/λ when

#31: $$\frac{L \cdot \rho}{\gamma + 1} = \frac{\delta}{\lambda}$$

Result 2b

#32: $$\frac{d}{d\gamma} \left(\mathrm{Cstar} = \frac{L \cdot \rho}{\gamma + 1} \right)$$

#33: $$0 > - \frac{L \cdot \rho}{(\gamma + 1)^2}$$

erstar is not in the paper.

#34: $$\mathrm{erstar} = \left(\frac{\frac{L \cdot \rho}{\gamma + 1} \cdot \lambda}{\delta} \right)^{1/\gamma} \cdot \lambda \cdot \left(\rho \cdot L - \frac{L \cdot \rho}{\gamma + 1} \right)$$

#35: $$\mathrm{erstar} = \gamma \cdot \delta^{-1/\gamma} \cdot \left(\frac{L \cdot \lambda \cdot \rho}{\gamma + 1} \right)^{(\gamma + 1)/\gamma}$$

#36: $$\frac{d}{d\gamma} \left(\mathrm{erstar} = \gamma \cdot \delta^{-1/\gamma} \cdot \left(\frac{L \cdot \lambda \cdot \rho}{\gamma + 1} \right)^{(\gamma + 1)/\gamma} \right)$$

#37: $$0 > - \frac{\delta^{-1/\gamma}\cdot\left(\frac{L\cdot\lambda\cdot\rho}{\gamma + 1}\right)^{(\gamma + 1)/\gamma}\cdot LN\left(\frac{L\cdot\lambda\cdot\rho}{\delta\cdot(\gamma + 1)}\right)}{\gamma}$$

eq (5): optimal probability

#38: $$pw = \left(\frac{L\cdot\lambda\cdot\rho}{\delta\cdot(\gamma + 1)}\right)^{1/\gamma}$$

Result 3 and Figure 3

#39: $$pw = \left(\frac{\lambda\cdot x}{\delta\cdot(\gamma + 1)}\right)^{1/\gamma}$$

#40: $$\frac{d}{dx}\left(pw = \left(\frac{\lambda\cdot x}{\delta\cdot(\gamma + 1)}\right)^{1/\gamma}\right)$$

#41: $$0 < \frac{x^{(1 - \gamma)/\gamma}\cdot\left(\frac{\delta\cdot(\gamma + 1)}{\lambda}\right)^{- 1/\gamma}}{\gamma}$$

#42: $$\frac{d}{dx}\frac{d}{dx}\left(pw = \left(\frac{\lambda\cdot x}{\delta\cdot(\gamma + 1)}\right)^{1/\gamma}\right)$$

#43: $$\frac{x^{(1 - 2\cdot\gamma)/\gamma}\cdot(1 - \gamma)\cdot\left(\frac{\delta\cdot(\gamma + 1)}{\lambda}\right)^{- 1/\gamma}}{\gamma^{2}}$$

> 0 iff $\gamma < 1$.

#44: $$\frac{d}{d\delta}\left(pw = \left(\frac{L\cdot\lambda\cdot\rho}{\delta\cdot(\gamma + 1)}\right)^{1/\gamma}\right)$$

#45: $$0 > -\frac{\delta^{-(\gamma + 1)/\gamma}\cdot\left(\frac{L\cdot\lambda\cdot\rho}{\gamma + 1}\right)^{1/\gamma}}{\gamma}$$

*** Section 4: Incentives to commit fraud

eq (6): Fraud expected payoff

#46: $epayoff = L - pw\cdot\lambda\cdot(\phi + \rho\cdot L)$

#47: $L - pw\cdot\lambda\cdot(\phi + \rho\cdot L) \ge 0$

#48: $\mathrm{SOLVE}(L - pw\cdot\lambda\cdot(\phi + \rho\cdot L) \ge 0, \lambda)$

#49: $$\lambda \le \frac{L}{pw\cdot(L\cdot\rho + \phi)}$$

eq (7): λhat

#50: $$\lambda hat = \frac{L}{pw\cdot(L\cdot\rho + \phi)}$$

λhat < 1 if

#51: $$\frac{L}{pw\cdot(L\cdot\rho + \phi)} < 1$$

#52: $L < pw\cdot(L\cdot\rho + \phi)$

#53: SOLVE(L < pw·(L·ρ + φ), pw)

#54: $$pw > \frac{L}{L\cdot\rho + \phi}$$

eq (8): expected conviction rate

#55: $$e\lambda = \int_0^{\lambda hat} \lambda \, d\lambda$$

#56: $$e\lambda = \frac{\lambda hat^2}{2}$$

#57: $$e\lambda = \frac{L^2}{2\cdot pw^2\cdot(L\cdot\rho + \phi)^2}$$

eq (9): whistleblowing probability pw (for γ=1)

#58: $$pw = \left(\frac{C\cdot\lambda}{\delta}\right)^{1/1}$$

#59: $$pw = \frac{C\cdot\lambda}{\delta}$$

#60:
$$pw = \frac{2 \cdot pw^2 \cdot (L \cdot \rho + \phi)^2}{\delta} \cdot C \cdot \frac{L^2}{\,}$$

extracting pw

#61:
$$\mathrm{SOLVE}\left(pw = \frac{C \cdot \dfrac{L^2}{2 \cdot pw^2 \cdot (L \cdot \rho + \phi)^2}}{\delta},\ pw\right)$$

#62:
$$pw = -\frac{2^{2/3} \cdot C^{1/3} \cdot L^{2/3}}{4 \cdot \delta^{1/3} \cdot (L \cdot \rho + \phi)^{2/3}} - \frac{\sqrt{3} \cdot 2^{2/3} \cdot i \cdot C^{1/3} \cdot L^{2/3}}{4 \cdot \delta^{1/3} \cdot (L \cdot \rho + \phi)^{2/3}} \lor pw = -\frac{2^{2/3} \cdot C^{1/3} \cdot L^{2/3}}{4 \cdot \delta^{1/3} \cdot (L \cdot \rho + \phi)^{2/3}} + \frac{\sqrt{3} \cdot 2^{2/3} \cdot i \cdot C^{1/3} \cdot L^{2/3}}{4 \cdot \delta^{1/3} \cdot (L \cdot \rho + \phi)^{2/3}} \lor pw = \frac{2^{2/3} \cdot C^{1/3} \cdot L^{2/3}}{2 \cdot \delta^{1/3} \cdot (L \cdot \rho + \phi)^{2/3}}$$

eq (9)

#63:
$$pw = \frac{2^{2/3} \cdot C^{1/3} \cdot L^{2/3}}{2 \cdot \delta^{1/3} \cdot (L \cdot \rho + \phi)^{2/3}}$$

eq (10) pf

#64: $$pf = \lambda hat = \frac{2^{1/3}\cdot L^{1/3}\cdot \delta^{1/3}}{C^{1/3}\cdot (L\cdot \rho + \phi)^{1/3}}$$

eq (10) Eλ

#65: $$e\lambda = \frac{2^{2/3}\cdot L^{2/3}\cdot \delta^{2/3}}{2\cdot C^{2/3}\cdot (L\cdot \rho + \phi)^{2/3}}$$

eq (11) Boundaries on C: Cmin and Cmax

C in which pf $\le$ 1 implies

#66: $$\frac{2^{1/3}\cdot L^{1/3}\cdot \delta^{1/3}}{C^{1/3}\cdot (L\cdot \rho + \phi)^{1/3}} \le 1$$

#67: $$2^{1/3}\cdot L^{1/3}\cdot \delta^{1/3} \le C^{1/3}\cdot (L\cdot \rho + \phi)^{1/3}$$

#68: $$\mathrm{SOLVE}(2^{1/3}\cdot L^{1/3}\cdot \delta^{1/3} \le C^{1/3}\cdot (L\cdot \rho + \phi)^{1/3}, C)$$

#69: $$C \ge \frac{2\cdot L\cdot \delta}{L\cdot \rho + \phi}$$

Range of C in which pw $\le$ 1

#70: $$\frac{2^{2/3}\cdot C^{1/3}\cdot L^{2/3}}{2\cdot\delta^{1/3}\cdot(L\cdot\rho+\phi)^{2/3}} \le 1$$

#71: $$2^{2/3}\cdot C^{1/3}\cdot L^{2/3} \le 2\cdot\delta^{1/3}\cdot(L\cdot\rho+\phi)^{2/3}$$

#72: $$\mathrm{SOLVE}(2^{2/3}\cdot C^{1/3}\cdot L^{2/3} \le 2\cdot\delta^{1/3}\cdot(L\cdot\rho+\phi)^{2/3},\ C)$$

#73: $$0 \le C \le \frac{2\cdot\delta\cdot(L\cdot\rho+\phi)^{2}}{L^{2}}$$

eq (12) Expected net fraud loss

#74: $$\mathrm{eloss} = \mathrm{pf}\cdot(L - \mathrm{pw}\cdot e\lambda\cdot(\phi+\rho\cdot L - C))$$

deriving FOC and SOC

#75: $$\mathrm{eloss} = \frac{2^{1/3}\cdot L^{1/3}\cdot\delta^{1/3}}{C^{1/3}\cdot(L\cdot\rho+\phi)^{1/3}}\cdot\left(L - \frac{2^{2/3}\cdot C^{1/3}\cdot L^{2/3}}{2\cdot\delta^{1/3}\cdot(L\cdot\rho+\phi)^{2/3}}\cdot\frac{2^{2/3}\cdot L^{2/3}\cdot\delta^{2/3}}{2\cdot C^{2/3}\cdot(L\cdot\rho+\phi)^{2/3}}\cdot(\phi+\rho\cdot L - C)\right)$$

#76: $$\mathrm{eloss} = \frac{2^{2/3}\cdot L^{4/3}\cdot\delta^{1/3}\cdot(C\cdot L^{1/3}\cdot\delta^{1/3} + 2^{2/3}\cdot C^{1/3}\cdot(L\cdot\rho+\phi)^{4/3} - L^{1/3}\cdot\delta^{1/3}\cdot(L\cdot\rho+\phi))}{2\cdot C^{2/3}\cdot(L\cdot\rho+\phi)^{5/3}}$$

#77: $$\frac{d}{dC}\left(eloss = \frac{2^{2/3}\cdot L^{4/3}\cdot \delta^{1/3}\cdot (C\cdot L^{1/3}\cdot \delta^{1/3} + 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi)^{4/3} - L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi))}{2\cdot C^{2/3}\cdot (L\cdot \rho + \phi)^{5/3}}\right)$$

#78: $$0 = \frac{2^{2/3}\cdot L^{4/3}\cdot \delta^{1/3}\cdot (C\cdot L^{1/3}\cdot \delta^{1/3} - 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi)^{4/3} + 2\cdot L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi))}{6\cdot C^{5/3}\cdot (L\cdot \rho + \phi)^{5/3}}$$

#79: $$\frac{d}{dC}\frac{d}{dC}\left(eloss = \frac{2^{2/3}\cdot L^{4/3}\cdot \delta^{1/3}\cdot (C\cdot L^{1/3}\cdot \delta^{1/3} + 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi)^{4/3} - L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi))}{2\cdot C^{2/3}\cdot (L\cdot \rho + \phi)^{5/3}}\right)$$

#80: $$- \frac{2^{2/3}\cdot L^{4/3}\cdot \delta^{1/3}\cdot (C\cdot L^{1/3}\cdot \delta^{1/3} - 2\cdot 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi)^{4/3} + 5\cdot L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi))}{9\cdot C^{8/3}\cdot (L\cdot \rho + \phi)^{5/3}}$$

#81: $$C\cdot L^{1/3}\cdot \delta^{1/3} - 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi)^{4/3} + 2\cdot L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi) = 0$$

#82: $$\mathrm{SOLVE}(C\cdot L^{1/3}\cdot \delta^{1/3} - 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi)^{4/3} + 2\cdot L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi) = 0, C)$$

=> not solvable => requires numerical simulations.