wb_fin_2025_mm_dd (whistleblowers and financial Fraud)

#1: CaseMode := Sensitive

#2: InputMode := Word

fraction recovered loss

#3: $\lambda :\in \text{Real } (0, 1)$

initial fraud loss before recovery

#4: $L :\in \text{Real } (0, \infty)$

Amount recovered less cost paid to WB (amount returned to the party that lost from the fraud):

#5: $R = \lambda \cdot L - c \cdot \lambda \cdot L$

Parameter in the prob function

#6: $\gamma :\in \text{Real } (0, \infty)$

fraction of recovered money that paid to WB

#7: $c :\in \text{Real } (0, 1)$

eq (1): Compensation to WB

#8: $C = c \cdot \lambda \cdot L$

eq (2): probability

#9: $$p = \frac{c \cdot \lambda \cdot L}{\gamma + c \cdot \lambda \cdot L}$$

#10: $$p = \frac{C}{\gamma + C}$$

#11: $$\frac{d}{dC}\left(p = \frac{C}{\gamma + C}\right)$$

#12: $$0 < \frac{\gamma}{(C + \gamma)^2}$$

#13: $$\frac{d}{dC}\frac{d}{dC}\left(p = \frac{C}{\gamma + C}\right)$$

#14: $$0 > - \frac{2\cdot\gamma}{(C + \gamma)^3}$$

#15: $$\lim_{C\to\infty}\left(p = \frac{C}{\gamma + C}\right)$$

#16: $$p = 1$$

eq (3) Authority max w.r.t c

#17: $$eR = p\cdot(\lambda\cdot L - c\cdot\lambda\cdot L)$$

#18: $$eR = \frac{c\cdot\lambda\cdot L}{\gamma + c\cdot\lambda\cdot L}\cdot(\lambda\cdot L - c\cdot\lambda\cdot L)$$

eq (4) and Appendix A

#19: $$\frac{d}{dc}\left(eR = \frac{c\cdot\lambda\cdot L}{\gamma + c\cdot\lambda\cdot L}\cdot(\lambda\cdot L - c\cdot\lambda\cdot L)\right)$$

#20: $$0 = -\frac{L^2\cdot\lambda^2\cdot(L\cdot c^2\cdot\lambda + \gamma\cdot(2\cdot c - 1))}{(L\cdot c\cdot\lambda + \gamma)^2}$$

#21: $$\frac{d}{dc}\frac{d}{dc}\left(eR = \frac{c\cdot\lambda\cdot L}{\gamma + c\cdot\lambda\cdot L}\cdot(\lambda\cdot L - c\cdot\lambda\cdot L)\right)$$

#22: $$0 > -\frac{2\cdot L^2\cdot\gamma\cdot\lambda^2\cdot(L\cdot\lambda + \gamma)}{(L\cdot c\cdot\lambda + \gamma)^3}$$

#23: $$\mathrm{SOLVE}\left(0 = -\frac{L^2\cdot\lambda^2\cdot(L\cdot c^2\cdot\lambda + \gamma\cdot(2\cdot c - 1))}{(L\cdot c\cdot\lambda + \gamma)^2}, c\right)$$

#24: $$c = \frac{\sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma})}{L\cdot\lambda} \vee c = -\frac{\sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} + \sqrt{\gamma})}{L\cdot\lambda}$$

#25: $$c\_star = \frac{\sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma})}{L\cdot\lambda}$$

#26: $$c\_star = \frac{\sqrt{\gamma}\cdot\sqrt{(\gamma + L\cdot\lambda)}}{L\cdot\lambda} - \frac{\gamma}{L\cdot\lambda}$$

Result 1: Define

#27: $x = \lambda\cdot L$

#28: $c\_star = \dfrac{\sqrt{\gamma}\cdot(\sqrt{(x + \gamma)} - \sqrt{\gamma})}{x}$

eq (A.3) check the limit for figure 2

#29: $\lim_{x\to 0+} \left(c\_star = \dfrac{\sqrt{\gamma}\cdot(\sqrt{(x + \gamma)} - \sqrt{\gamma})}{x}\right)$

#30: $c\_star = \dfrac{1}{2}$

#31: $c\_star = \dfrac{\sqrt{\gamma}\cdot(\sqrt{(0 + \gamma)} - \sqrt{\gamma})}{0}$

#32: c_star = ?

#33: $\lim_{x\to\infty} \left(c\_star = \dfrac{\sqrt{\gamma}\cdot(\sqrt{(x + \gamma)} - \sqrt{\gamma})}{x}\right)$

#34: $c\_star = 0$

#35: $\dfrac{d}{dx} \left(c\_star = \dfrac{\sqrt{\gamma}\cdot(\sqrt{(x + \gamma)} - \sqrt{\gamma})}{x}\right)$

#36: $\dfrac{\sqrt{\gamma}\cdot(2\cdot\sqrt{\gamma}\cdot\sqrt{(x + \gamma)} - x - 2\cdot\gamma)}{2\cdot x^2\cdot\sqrt{(x + \gamma)}}$

eq (A.2)

#37: $$\frac{\sqrt{\gamma}\cdot(2\cdot\sqrt{\gamma}\cdot\sqrt{(\lambda\cdot L + \gamma)} - \lambda\cdot L - 2\cdot\gamma)}{2\cdot(\lambda\cdot L)^2\cdot\sqrt{(\lambda\cdot L + \gamma)}}$$

> 0 if [Yes]

#38: $\sqrt{\gamma}\cdot(2\cdot\sqrt{\gamma}\cdot\sqrt{(\lambda\cdot L + \gamma)} - \lambda\cdot L - 2\cdot\gamma) > 0$

#39: $\mathrm{SOLVE}(\sqrt{\gamma}\cdot(2\cdot\sqrt{\gamma}\cdot\sqrt{(\lambda\cdot L + \gamma)} - \lambda\cdot L - 2\cdot\gamma) > 0, \gamma)$

#40: $$\gamma < -\frac{L\cdot\lambda}{2} \wedge \gamma > 0 \wedge \gamma \geq -L\cdot\lambda$$

Result 2a and Appendix A

#41: $$\frac{\sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma})}{L\cdot\lambda} > 0$$

if

#42: $\sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma}) > 0$

#43: $\mathrm{SOLVE}(\sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma}) > 0, \gamma)$

#44: $$\gamma > 0 \wedge \gamma \geq -L\cdot\lambda$$

#45: $$\lim_{x\to 0+}\left(c\_star = \frac{\sqrt{\gamma}\cdot(\sqrt{(x + \gamma)} - \sqrt{\gamma})}{x}\right)$$

#46: $$c\_star = \frac{1}{2}$$

#47: $$\frac{\sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma})}{L\cdot\lambda} < 1$$

#48: $$\mathrm{SOLVE}\left(\frac{\sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma})}{L\cdot\lambda} < 1,\ \gamma\right)$$

#49: $$\gamma > -L\cdot\lambda \wedge \gamma \ge 0$$

Result 2b and Appendix A

#50: $$\frac{d}{d\gamma}\left(c\_star = \frac{\sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma})}{L\cdot\lambda}\right)$$

eq (A.4)

#51: $$\frac{(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma})^2}{2\cdot L\cdot\sqrt{\gamma}\cdot\lambda\cdot\sqrt{(L\cdot\lambda + \gamma)}}$$

> 0 if

#52: $$(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma})^2 > 0$$

#53: $$\mathrm{SOLVE}((\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma})^2 > 0,\ \gamma)$$

#54: true

eq (5) equilibrium total compensation and probability

#55: $$C\_star = \frac{\sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma})}{L\cdot\lambda}\cdot\lambda\cdot L$$

#56: $$C\_star = \sqrt{\gamma}\cdot(\sqrt{L\cdot\lambda + \gamma} - \sqrt{\gamma})$$

#57: $$C\_star = \sqrt{\gamma}\cdot\sqrt{\gamma + L\cdot\lambda} - \gamma$$

#58: $$p\_star = \frac{\sqrt{\gamma}\cdot\sqrt{\gamma + L\cdot\lambda} - \gamma}{\gamma + (\sqrt{\gamma}\cdot\sqrt{\gamma + L\cdot\lambda} - \gamma)}$$

#59: $$p\_star = \frac{\sqrt{L\cdot\lambda + \gamma} - \sqrt{\gamma}}{\sqrt{L\cdot\lambda + \gamma}}$$

#60: $$p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{\gamma + L\cdot\lambda}}$$

Result 3 and Appendix A. Define x

#61: $x = \lambda\cdot L$

#62: $C\_star = \sqrt{\gamma}\cdot\sqrt{\gamma + x} - \gamma$

#63: $$\frac{d}{dx}\left(C\_star = \sqrt{\gamma}\cdot\sqrt{\gamma + x} - \gamma\right)$$

#64: $$\frac{\sqrt{\gamma}}{2\cdot\sqrt{x + \gamma}}$$

eqs (A.5)

#65: $$\frac{\sqrt{\gamma}}{2\cdot\sqrt{\lambda\cdot L + \gamma}} > 0$$

#66: $$\frac{d}{dx}\frac{d}{dx}\left(C\_star = \sqrt{\gamma}\cdot\sqrt{\gamma + x} - \gamma\right)$$

#67: $$0 > -\frac{\sqrt{\gamma}}{4\cdot(x+\gamma)^{3/2}}$$

#68: $$0 > -\frac{\sqrt{\gamma}}{4\cdot(\lambda\cdot L+\gamma)^{3/2}}$$

#69: $$p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(\gamma + x)}}$$

#70: $$\frac{d}{dx}\left(p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(\gamma + x)}}\right)$$

#71: $$0 < \frac{\sqrt{\gamma}}{2\cdot(x+\gamma)^{3/2}}$$

eqs (A.6)

#72: $$0 < \frac{\sqrt{\gamma}}{2\cdot(\lambda\cdot L+\gamma)^{3/2}}$$

#73: $$\frac{d}{dx}\frac{d}{dx}\left(p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(\gamma + x)}}\right)$$

#74: $$0 > -\frac{3\cdot\sqrt{\gamma}}{4\cdot(x+\gamma)^{5/2}}$$

#75: $$0 > - \frac{3\cdot\sqrt{\gamma}}{4\cdot(\lambda\cdot L + \gamma)^{5/2}}$$

*** Section 4: WB and incentives to commit fraud

eq (6)

#76: $eprofit = L - p\cdot(F + \lambda\cdot L)$

> 0 => fraud is profitable if

#77: $L - p\cdot(F + \lambda\cdot L) > 0$

#78: SOLVE(L - p·(F + λ·L) > 0, p)

#79: $$\mathrm{IF}\left(F + L\cdot\lambda < 0,\ p > \frac{L}{F + L\cdot\lambda}\right) \vee \mathrm{IF}\left(F + L\cdot\lambda > 0,\ p < \frac{L}{F + L\cdot\lambda}\right)$$

eq (7)

#80: $$p < \frac{L}{F + L\cdot\lambda}$$

#81: $$p\_bar = \frac{L}{F + L\cdot\lambda}$$

eq (8)

#82: $$\lim_{F\to 0+}\left(p\_bar = \frac{L}{F + L\cdot\lambda}\right)$$

#83: $$p\_bar = \frac{1}{\lambda}$$

Result 5 and Appendix B

Recall p_bar and p_star

#84: $$p\_bar = \frac{L}{F + L\cdot\lambda}$$

#85: $$p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(\gamma + L\cdot\lambda)}}$$

eq (B.1)

#86: $$\frac{d}{dL}\left(p\_bar = \frac{L}{F + L\cdot\lambda}\right)$$

#87: $$0 < \frac{F}{(F + L\cdot\lambda)^2}$$

#88: $$\frac{d}{dL}\frac{d}{dL}\left(p\_bar = \frac{L}{F + L\cdot\lambda}\right)$$

#89: $$0 > -\frac{2\cdot F\cdot\lambda}{(F + L\cdot\lambda)^3}$$

eq (B.3)

#90: $$\frac{d}{dL}\left(p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(\gamma + L\cdot\lambda)}}\right)$$

#91: $$0 < \frac{\sqrt{\gamma}\cdot\lambda}{2\cdot(L\cdot\lambda + \gamma)^{3/2}}$$

#92: $$\frac{d}{dL}\frac{d}{dL}\left(p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(\gamma + L\cdot\lambda)}}\right)$$

#93: $$0 > - \frac{3\cdot\sqrt{\gamma}\cdot\lambda^{2}}{4\cdot(L\cdot\lambda + \gamma)^{5/2}}$$

eq (B.3)

#94: $$\lim_{L\to\infty}\left(p\_bar = \frac{L}{F + L\cdot\lambda}\right)$$

#95: $$p\_bar = \frac{1}{\lambda}$$

#96: $$\lim_{L\to\infty}\left(p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(\gamma + L\cdot\lambda)}}\right)$$

#97: $$p\_star = 1$$

eq (B.4)

#98: $\dfrac{F}{(F + 0\cdot\lambda)^2}$

#99: $\dfrac{1}{F}$

#100: $\dfrac{\sqrt{\gamma}\cdot\lambda}{2\cdot(0\cdot\lambda + \gamma)^{3/2}}$

#101: $\dfrac{\lambda}{2\cdot\gamma}$

#102: $\dfrac{1}{F} < \dfrac{\lambda}{2\cdot\gamma}$

proving Result 5c, eq (B.5): at L = L_bar, 0=p_bar - p_star = ***Cannot prove it w/o additional condition. Part c deleted!

#103: $0 = \dfrac{L}{F + L\cdot\lambda} - \left(1 - \dfrac{\sqrt{\gamma}}{\sqrt{(\gamma + L\cdot\lambda)}}\right)$

#104: $\dfrac{d}{dF}\left(\dfrac{L}{F + L\cdot\lambda} - \left(1 - \dfrac{\sqrt{\gamma}}{\sqrt{(\gamma + L\cdot\lambda)}}\right)\right)$

#105: $-\dfrac{L}{(F + L\cdot\lambda)^2}$

#106: $$\frac{d}{dL}\left(\frac{L}{F + L\cdot\lambda} - \left(1 - \frac{\sqrt{\gamma}}{\sqrt{(\gamma + L\cdot\lambda)}}\right)\right)$$

#107: $$-\frac{F^2\cdot\sqrt{\gamma}\cdot\lambda + 2\cdot F\cdot(L\cdot\sqrt{\gamma}\cdot\lambda^2 - (L\cdot\lambda + \gamma)^{3/2}) + L^2\cdot\sqrt{\gamma}\cdot\lambda^3}{2\cdot(F + L\cdot\lambda)^2\cdot(L\cdot\lambda + \gamma)^{3/2}}$$

#108: $$-\frac{-\dfrac{L}{(F + L\cdot\lambda)^2}}{-\dfrac{F^2\cdot\sqrt{\gamma}\cdot\lambda + 2\cdot F\cdot(L\cdot\sqrt{\gamma}\cdot\lambda^2 - (L\cdot\lambda + \gamma)^{3/2}) + L^2\cdot\sqrt{\gamma}\cdot\lambda^3}{2\cdot(F + L\cdot\lambda)^2\cdot(L\cdot\lambda + \gamma)^{3/2}}}$$

#109: $$-\frac{2\cdot L\cdot(L\cdot\lambda + \gamma)^{3/2}}{F^2\cdot\sqrt{\gamma}\cdot\lambda + 2\cdot F\cdot(L\cdot\sqrt{\gamma}\cdot\lambda^2 - (L\cdot\lambda + \gamma)^{3/2}) + L^2\cdot\sqrt{\gamma}\cdot\lambda^3}$$

> 0 if

#110: $$F^2\cdot\sqrt{\gamma}\cdot\lambda + 2\cdot F\cdot(L\cdot\sqrt{\gamma}\cdot\lambda^2 - (L\cdot\lambda + \gamma)^{3/2}) + L^2\cdot\sqrt{\gamma}\cdot\lambda^3 < 0$$

#111: $$\mathrm{SOLVE}(F^2\cdot\sqrt{\gamma}\cdot\lambda + 2\cdot F\cdot(L\cdot\sqrt{\gamma}\cdot\lambda^2 - (L\cdot\lambda + \gamma)^{3/2}) + L^2\cdot\sqrt{\gamma}\cdot\lambda^3 < 0, L)$$

#112: $$2\cdot F\cdot(L\cdot\lambda + \gamma)^{3/2} - L^2\cdot\sqrt{\gamma}\cdot\lambda^3 - 2\cdot F\cdot L\cdot\sqrt{\gamma}\cdot\lambda^2 > F^2\cdot\sqrt{\gamma}\cdot\lambda$$

#113: $2\cdot F\cdot(L\cdot\lambda + \gamma)^{3/2} > L^2\cdot\sqrt{\gamma}\cdot\lambda^3 + 2\cdot F\cdot L\cdot\sqrt{\gamma}\cdot\lambda^2 + F^2\cdot\sqrt{\gamma}\cdot\lambda$

#114: $L^2\cdot\sqrt{\gamma}\cdot\lambda^3 + 2\cdot F\cdot L\cdot\sqrt{\gamma}\cdot\lambda^2 + F^2\cdot\sqrt{\gamma}\cdot\lambda$

#115: $\sqrt{\gamma}\cdot\lambda\cdot(F + L\cdot\lambda)^2$

hence if,

#116: $2\cdot F\cdot(L\cdot\lambda + \gamma)^{3/2} > \sqrt{\gamma}\cdot\lambda\cdot(F + L\cdot\lambda)^2$