wb_fin_2025_mm_dd (whistleblowers and financial Fraud)

#1: CaseMode ≔ Sensitive

#2: InputMode ≔ Word

fraction recovered loss

#3: $\lambda :\in \text{Real}\ (0, 1)$

initial fraud loss before recovery

#4: $L :\in \text{Real}\ (0, \infty)$

Amount recovered less cost paid to WB (amount returned to the party that lost from the fraud):

#5: $R = \lambda \cdot L - c \cdot \lambda \cdot L$

Parameter in the prob function

#6: $\gamma :\in \text{Real}\ (0, \infty)$

fraction of recovered money that paid to WB

#7: $c :\in \text{Real}\ (0, 1)$

eq (1): probability

#8: $$p = \frac{C}{\gamma + C}$$

#9: $$\frac{d}{dC}\left(p = \frac{C}{\gamma + C}\right)$$

#10: $$0 < \frac{\gamma}{(C + \gamma)^2}$$

#11: $$\frac{d}{dC} \frac{d}{dC} \left(p = \frac{C}{\gamma + C}\right)$$

#12: $$0 > - \frac{2 \cdot \gamma}{(C + \gamma)^3}$$

#13: $$\lim_{C \to \infty} \left(p = \frac{C}{\gamma + C}\right)$$

#14: $$p = 1$$

eq (2) Authority max w.r.t C and Appendix A, eq (A.1)

#15: $eR = p \cdot (\lambda \cdot L - C)$

#16: $$eR = \frac{C}{\gamma + C} \cdot (\lambda \cdot L - C)$$

#17: $$\frac{d}{dC} \left(eR = \frac{C}{\gamma + C} \cdot (\lambda \cdot L - C)\right)$$

#18: $$0 = - \frac{C^2 + 2 \cdot C \cdot \gamma - L \cdot \gamma \cdot \lambda}{(C + \gamma)^2}$$

#19: $\frac{d}{dC}\frac{d}{dC}\left(eR = \frac{C}{\gamma + C}\cdot(\lambda\cdot L - C)\right)$

#20: $0 > -\frac{2\cdot\gamma\cdot(L\cdot\lambda + \gamma)}{(C + \gamma)^3}$

#21: $\text{SOLVE}\left(0 = -\frac{C^2 + 2\cdot C\cdot\gamma - L\cdot\gamma\cdot\lambda}{(C + \gamma)^2}, C\right)$

#22: $C = \sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma}) \vee C = -\sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} + \sqrt{\gamma})$

eq (3) optimal compensation and Appendix A

#23: $C = \sqrt{\gamma}\cdot(\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma})$

#24: $C = \sqrt{\gamma}\cdot\sqrt{(\gamma + L\cdot\lambda)} - \gamma$

#25: $C = \sqrt{\gamma}\cdot(\sqrt{(x + \gamma)} - \sqrt{\gamma})$

Result 1 & Appendix A eq (A.2)

#26: $\frac{d}{dx}(C = \sqrt{\gamma}\cdot(\sqrt{(x + \gamma)} - \sqrt{\gamma}))$

#27: $0 < \frac{\sqrt{\gamma}}{2\cdot\sqrt{(x + \gamma)}}$

#28: $\frac{d}{dx}\frac{d}{dx}(C = \sqrt{\gamma}\cdot(\sqrt{(x + \gamma)} - \sqrt{\gamma}))$

#29: $$0 > - \frac{\sqrt{\gamma}}{4\cdot(x + \gamma)^{3/2}}$$

eq (4): p*

#30: $$p\_star = \frac{\sqrt{(L\cdot\lambda + \gamma)} - \sqrt{\gamma}}{\sqrt{(L\cdot\lambda + \gamma)}}$$

#31: $$p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(L\cdot\lambda + \gamma)}}$$

#32: $$p\_star = \frac{\sqrt{\gamma}\cdot\sqrt{(\gamma + L\cdot\lambda)} - \gamma}{\gamma + (\sqrt{\gamma}\cdot\sqrt{(\gamma + L\cdot\lambda)} - \gamma)}$$

Result 2 and Appendix A eq (A.3)

#33: $$p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(x + \gamma)}}$$

#34: $$\frac{d}{dx}\left(p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(x + \gamma)}}\right)$$

#35: $$0 < \frac{\sqrt{\gamma}}{2\cdot(x + \gamma)^{3/2}}$$

#36: $$\frac{d}{dx}\,\frac{d}{dx}\left(p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(x + \gamma)}}\right)$$

#37: $$0 > -\frac{3\cdot\sqrt{\gamma}}{4\cdot(x+\gamma)^{5/2}}$$

eq (5): c*

#38: $$c\_star = \frac{\sqrt{\gamma}\cdot\sqrt{(\gamma + L\cdot\lambda)} - \gamma}{\lambda\cdot L}$$

Result 3b and Appendix A, eq (A.4)

#39: $$c\_star = \frac{\sqrt{\gamma}\cdot\sqrt{(\gamma + x)} - \gamma}{x}$$

#40: $$\frac{d}{dx}\left(c\_star = \frac{\sqrt{\gamma}\cdot\sqrt{(\gamma + x)} - \gamma}{x}\right)$$

#41: $$\frac{\sqrt{\gamma}\cdot(2\cdot\sqrt{\gamma}\cdot\sqrt{(x+\gamma)} - x - 2\cdot\gamma)}{2\cdot x^2\cdot\sqrt{(x+\gamma)}}$$

< 0 if

#42: $2\cdot\sqrt{\gamma}\cdot\sqrt{(\lambda\cdot L + \gamma)} - \lambda\cdot L - 2\cdot\gamma < 0$

#43: $2\cdot\sqrt{\gamma}\cdot\sqrt{(x + \gamma)} - x - 2\cdot\gamma < 0$

#44: $\mathrm{SOLVE}(2\cdot\sqrt{\gamma}\cdot\sqrt{(x + \gamma)} - x - 2\cdot\gamma < 0, x)$

#45: $$x \neq 0 \wedge x > -2\cdot\gamma \wedge x \geq -\gamma$$

Result 3c and Appendix A, eq (A.5)

#46: $\lim_{x \to 0+} \left(c\_star = \frac{\sqrt{\gamma} \cdot \sqrt{(\gamma + x)} - \gamma}{x} \right)$

#47: $$c\_star = \frac{1}{2}$$

Result 3d and Appendix A, eq (A.6)

#48: $\frac{d}{d\gamma} \left(c\_star = \frac{\sqrt{\gamma} \cdot \sqrt{(\gamma + L \cdot \lambda)} - \gamma}{\lambda \cdot L} \right)$

#49: $$0 < \frac{(\sqrt{(L \cdot \lambda + \gamma)} - \sqrt{\gamma})^2}{2 \cdot L \cdot \sqrt{\gamma} \cdot \lambda \cdot \sqrt{(L \cdot \lambda + \gamma)}}$$

*** Section 4: Incentives to commit fraud

eq (6): fraudster's payoff

#50: $epayoff = L - p \cdot (F + \lambda \cdot L)$

eq (7): p_bar prob below which fraud is profitable epayoff > 0

#51: $L - p \cdot (F + \lambda \cdot L) > 0$

#52: SOLVE($L - p \cdot (F + \lambda \cdot L) > 0$, p)

#53: $$\mathrm{IF}\left(F + L \cdot \lambda < 0,\ p > \frac{L}{F + L \cdot \lambda} \right) \vee \mathrm{IF}\left(F + L \cdot \lambda > 0,\ p < \frac{L}{F + L \cdot \lambda} \right)$$

#54: $p < \frac{L}{F + L \cdot \lambda}$

#55: $p\_bar = \frac{L}{F + L\cdot\lambda}$

Result 4 and eq (7)

#56: $\lim_{F\to 0+} \left(p\_bar = \frac{L}{F + L\cdot\lambda}\right)$

#57: $p\_bar = \frac{1}{\lambda}$

Result 5, Appenedix B

eq (B.1)

#58: $\frac{d}{dL}\left(p\_bar = \frac{L}{F + L\cdot\lambda}\right)$

#59: $0 < \frac{F}{(F + L\cdot\lambda)^2}$

#60: $\frac{d}{dL}\frac{d}{dL}\left(p\_bar = \frac{L}{F + L\cdot\lambda}\right)$

#61: $0 > - \frac{2\cdot F\cdot\lambda}{(F + L\cdot\lambda)^3}$

eq (B.2)

#62: $$\frac{d}{dL}\left(p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(L\cdot\lambda + \gamma)}}\right)$$

#63: $$0 < \frac{\sqrt{\gamma}\cdot\lambda}{2\cdot(L\cdot\lambda + \gamma)^{3/2}}$$

#64: $$\frac{d}{dL}\frac{d}{dL}\left(p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(L\cdot\lambda + \gamma)}}\right)$$

#65: $$0 > - \frac{3\cdot\sqrt{\gamma}\cdot\lambda^{2}}{4\cdot(L\cdot\lambda + \gamma)^{5/2}}$$

eq (B.3)

#66: $$\lim_{L\to\infty}\left(p\_bar = \frac{L}{F + L\cdot\lambda}\right)$$

#67: $$p\_bar = \frac{1}{\lambda}$$

#68: $$\lim_{L\to\infty}\left(p\_star = 1 - \frac{\sqrt{\gamma}}{\sqrt{(L\cdot\lambda + \gamma)}}\right)$$

#69: $$p\_star = 1$$

eq (B.4)

#70: $$\frac{F}{(F + 0\cdot\lambda)^2}$$

#71: $$\frac{1}{F}$$

#72: $$\frac{\sqrt{\gamma}\cdot\lambda}{2\cdot(0\cdot\lambda + \gamma)^{3/2}}$$

#73: $$\frac{\lambda}{2\cdot\gamma}$$

need to assume

#74: $$\frac{1}{F} < \frac{\lambda}{2\cdot\gamma}$$