fraud_2025_mm_dd (whistleblowers and financial Fraud)

Starting from Subsection 5.2 (line 116) change of notation: "C" compensation changed to "R" reward. => R (revenue) was changed to B (benefit)

#1: CaseMode ≔ Sensitive

#2: InputMode ≔ Word

Num employees per type

#3: $N :\in \mathrm{Real}\ (0, \infty)$

time discount factor (used in Subsection 5.1)

#4: $\tau :\in \mathrm{Real}\ (0, 1)$

probability of conviction based on WB info

#5: $\lambda :\in \mathrm{Real}\ (0, 1)$

initial fraud loss before recovery

#6: $L :\in \mathrm{Real}\ (0, \infty)$

fraction of recovered amount

#7: $\rho :\in \mathrm{Real}\ (0, 1)$

Parameter of WB discomfort from WB (disutility parameter)

#8: $\delta :\in \mathrm{Real}\ (0, \infty)$

concavility/convexity of WB utility w.r.t. type

#9: $\gamma :\in \mathrm{Real}\ (0, \infty)$

Total and fraction of recovered money that paid to WB

#10: $C :\in \text{Real } (0, \infty)$

#11: $c :\in \text{Real } (0, 1)$

WB probability (endogenous).

#12: $pw :\in \text{Real } (0, 1]$

fraud prob (endogeneous)

#13: $pf :\in \text{Real } (0, 1]$

Penalty on convicted fraudster

#14: $F :\in \text{Real } [0, \infty)$

eq (1): utility of a WB

#15: $u = \lambda \cdot C - \delta \cdot d^{\gamma}$

eq (2): dhat

#16: $0 = \lambda \cdot C - \delta \cdot d^{\gamma}$

#17: $\text{SOLVE}(0 = \lambda \cdot C - \delta \cdot d^{\gamma}, d)$

#18: $$pw = dhat = \left(\frac{C \cdot \lambda}{\delta}\right)^{1/\gamma}$$

< 1 if

#19: $$\left(\frac{C \cdot \lambda}{\delta}\right)^{1/\gamma} < 1$$

#20: $\text{SOLVE}\left(\left(\frac{C\cdot\lambda}{\delta}\right)^{1/\gamma} < 1,\ C\right)$

#21: $$0 < C < \frac{\delta}{\lambda}$$

eq (3) net revenue maximization

#22: $er = pw\cdot\lambda\cdot(\rho\cdot L - C)$

#23: $er = \left(\frac{C\cdot\lambda}{\delta}\right)^{1/\gamma}\cdot\lambda\cdot(\rho\cdot L - C)$

eq (4) Appendix B

#24: $\frac{d}{dC}\left(er = \left(\frac{C\cdot\lambda}{\delta}\right)^{1/\gamma}\cdot\lambda\cdot(\rho\cdot L - C)\right)$

eq (B.1)

#25: $$0 = \frac{C^{(1 - \gamma)/\gamma}\cdot\delta^{- 1/\gamma}\cdot\lambda^{(\gamma + 1)/\gamma}\cdot(L\cdot\rho - C\cdot(\gamma + 1))}{\gamma}$$

#26: $\frac{d}{dC}\frac{d}{dC}\left(er = \left(\frac{C\cdot\lambda}{\delta}\right)^{1/\gamma}\cdot\lambda\cdot(\rho\cdot L - C)\right)$

#27: $$-\frac{C^{(1 - 2\cdot\gamma)/\gamma}\cdot\delta^{- 1/\gamma}\cdot\lambda^{(\gamma + 1)/\gamma}\cdot(C\cdot(\gamma + 1) + L\cdot\rho\cdot(\gamma - 1))}{\gamma^2}$$

< 0 if

#28: $C\cdot(\gamma + 1) + L\cdot\rho\cdot(\gamma - 1) > 0$

#29: $\mathrm{SOLVE}(C\cdot(\gamma + 1) + L\cdot\rho\cdot(\gamma - 1) > 0, C)$

#30: $$C > \frac{L\cdot\rho\cdot(1 - \gamma)}{\gamma + 1}$$

#31: $\mathrm{SOLVE}(C^{(1 - \gamma)/\gamma}\cdot\delta^{- 1/\gamma}\cdot\lambda^{(\gamma + 1)/\gamma}\cdot(L\cdot\rho - C\cdot(\gamma + 1)), C)$

#32: $$Cstar = \frac{L\cdot\rho}{\gamma + 1}$$

explainng Figure 2 horizontal axis

Cstar = δ/λ when

#33: $\frac{L\cdot\rho}{\gamma + 1} = \frac{\delta}{\lambda}$

Result 2b

#34: $\frac{d}{d\gamma}\left(Cstar = \frac{L\cdot\rho}{\gamma + 1}\right)$

#35: $$0 > - \frac{L\cdot\rho}{(\gamma + 1)^2}$$

erstar is not in the paper.

#36: $$\mathrm{erstar} = \left(\frac{\frac{L\cdot\rho}{\gamma + 1}\cdot\lambda}{\delta}\right)^{1/\gamma}\cdot\lambda\cdot\left(\rho\cdot L - \frac{L\cdot\rho}{\gamma + 1}\right)$$

#37: $$\mathrm{erstar} = \gamma\cdot\delta^{-1/\gamma}\cdot\left(\frac{L\cdot\lambda\cdot\rho}{\gamma + 1}\right)^{(\gamma + 1)/\gamma}$$

#38: $$\frac{d}{d\gamma}\left(\mathrm{erstar} = \gamma\cdot\delta^{-1/\gamma}\cdot\left(\frac{L\cdot\lambda\cdot\rho}{\gamma + 1}\right)^{(\gamma + 1)/\gamma}\right)$$

#39: $$0 > -\frac{\delta^{-1/\gamma}\cdot\left(\frac{L\cdot\lambda\cdot\rho}{\gamma + 1}\right)^{(\gamma + 1)/\gamma}\cdot\mathrm{LN}\left(\frac{L\cdot\lambda\cdot\rho}{\delta\cdot(\gamma + 1)}\right)}{\gamma}$$

eq (5): optimal probability

#40: $$\mathrm{pw} = \left(\frac{L\cdot\lambda\cdot\rho}{\delta\cdot(\gamma + 1)}\right)^{1/\gamma}$$

Result 3 and Figure 3

#41: $$\mathrm{pw} = \left(\frac{\lambda\cdot x}{\delta\cdot(\gamma + 1)}\right)^{1/\gamma}$$

#42: $$\frac{d}{dx}\left(\mathrm{pw} = \left(\frac{\lambda\cdot x}{\delta\cdot(\gamma + 1)}\right)^{1/\gamma}\right)$$

#43: $$0 < \frac{x^{(1 - \gamma)/\gamma}\cdot\left(\frac{\delta\cdot(\gamma + 1)}{\lambda}\right)^{- 1/\gamma}}{\gamma}$$

#44: $$\frac{d}{dx}\frac{d}{dx}\left(pw = \left(\frac{\lambda\cdot x}{\delta\cdot(\gamma + 1)}\right)^{1/\gamma}\right)$$

#45: $$\frac{x^{(1 - 2\cdot\gamma)/\gamma}\cdot(1 - \gamma)\cdot\left(\frac{\delta\cdot(\gamma + 1)}{\lambda}\right)^{- 1/\gamma}}{\gamma^2}$$

> 0 iff $\gamma < 1$.

#46: $$\frac{d}{d\delta}\left(pw = \left(\frac{L\cdot\lambda\cdot\rho}{\delta\cdot(\gamma + 1)}\right)^{1/\gamma}\right)$$

#47: $$0 > - \frac{\delta^{- (\gamma + 1)/\gamma}\cdot\left(\frac{L\cdot\lambda\cdot\rho}{\gamma + 1}\right)^{1/\gamma}}{\gamma}$$

*** Section 4: Incentives to commit fraud

eq (6): Fraud expected payoff

#48: $epayoff = L - pw\cdot\lambda\cdot(\phi + \rho\cdot L)$

#49: $L - pw\cdot\lambda\cdot(\phi + \rho\cdot L) \ge 0$

#50: $SOLVE(L - pw\cdot\lambda\cdot(\phi + \rho\cdot L) \ge 0, \lambda)$

#51: $$\lambda \le \frac{L}{pw\cdot(L\cdot\rho + \phi)}$$

eq (7): λhat

#52: $$\lambda hat = \frac{L}{pw\cdot(L\cdot\rho + \phi)}$$

λhat < 1 if

#53: $$\frac{L}{pw\cdot(L\cdot\rho + \phi)} < 1$$

#54: $L < pw\cdot(L\cdot\rho + \phi)$

#55: SOLVE($L < pw\cdot(L\cdot\rho + \phi)$, pw)

#56: $$pw > \frac{L}{L\cdot\rho + \phi}$$

eq (8): expected conviction rate

#57: $$e\lambda = \int_0^{\lambda hat} \lambda \, d\lambda$$

#58: $$e\lambda = \frac{\lambda hat^2}{2}$$

#59: $$e\lambda = \frac{L^2}{2\cdot pw^2\cdot(L\cdot\rho + \phi)^2}$$

eq (9): whistleblowing probability pw (for ɣ=1)

#60: $$pw = \left(\frac{C\cdot\lambda}{\delta}\right)^{1/1}$$

#61: $$pw = \frac{C\cdot\lambda}{\delta}$$

#62: $$pw = \frac{C\cdot\frac{L^2}{2\cdot pw^2\cdot(L\cdot\rho + \phi)^2}}{\delta}$$

extracting pw

#63: $$\mathrm{SOLVE}\left(pw = \frac{C\cdot\frac{L^2}{2\cdot pw^2\cdot(L\cdot\rho + \phi)^2}}{\delta}, pw\right)$$

#64: $$pw = -\frac{2^{2/3}\cdot C^{1/3}\cdot L^{2/3}}{4\cdot\delta^{1/3}\cdot(L\cdot\rho + \phi)^{2/3}} - \frac{\sqrt{3}\cdot 2^{2/3}\cdot i\cdot C^{1/3}\cdot L^{2/3}}{4\cdot\delta^{1/3}\cdot(L\cdot\rho + \phi)^{2/3}} \vee pw = -\frac{2^{2/3}\cdot C^{1/3}\cdot L^{2/3}}{4\cdot\delta^{1/3}\cdot(L\cdot\rho + \phi)^{2/3}} +$$

$$\frac{\sqrt{3}\cdot 2^{2/3}\cdot i\cdot C^{1/3}\cdot L^{2/3}}{4\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi)^{2/3}} \vee pw = \frac{2^{2/3}\cdot C^{1/3}\cdot L^{2/3}}{2\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi)^{2/3}}$$

eq (9)

#65: $$pw = \frac{2^{2/3}\cdot C^{1/3}\cdot L^{2/3}}{2\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi)^{2/3}}$$

eq (10) pf

#66: $$pf = \lambda hat = \frac{2^{1/3}\cdot L^{1/3}\cdot \delta^{1/3}}{C^{1/3}\cdot (L\cdot \rho + \phi)^{1/3}}$$

eq (10) Eλ

#67: $$e\lambda = \frac{2^{2/3}\cdot L^{2/3}\cdot \delta^{2/3}}{2\cdot C^{2/3}\cdot (L\cdot \rho + \phi)^{2/3}}$$

eq (11) Boundaries on C: Cmin and Cmax

C in which pf ≤ 1 implies

#68: $$\frac{2^{1/3}\cdot L^{1/3}\cdot \delta^{1/3}}{C^{1/3}\cdot (L\cdot \rho + \phi)^{1/3}} \le 1$$

#69: $2^{1/3} \cdot L^{1/3} \cdot \delta^{1/3} \le C^{1/3} \cdot (L \cdot \rho + \phi)^{1/3}$

#70: $\mathrm{SOLVE}(2^{1/3} \cdot L^{1/3} \cdot \delta^{1/3} \le C^{1/3} \cdot (L \cdot \rho + \phi)^{1/3}, C)$

c_min below

#71: $$C \ge \frac{2 \cdot L \cdot \delta}{L \cdot \rho + \phi}$$

Range of C in which pw ≤ 1

#72: $$\frac{2^{2/3} \cdot C^{1/3} \cdot L^{2/3}}{2 \cdot \delta^{1/3} \cdot (L \cdot \rho + \phi)^{2/3}} \le 1$$

#73: $2^{2/3} \cdot C^{1/3} \cdot L^{2/3} \le 2 \cdot \delta^{1/3} \cdot (L \cdot \rho + \phi)^{2/3}$

#74: $\mathrm{SOLVE}(2^{2/3} \cdot C^{1/3} \cdot L^{2/3} \le 2 \cdot \delta^{1/3} \cdot (L \cdot \rho + \phi)^{2/3}, C)$

c_max below

#75: $$0 \le C \le \frac{2 \cdot \delta \cdot (L \cdot \rho + \phi)^2}{L^2}$$

c_max − c_min =

#76: $\dfrac{2\cdot\delta\cdot(L\cdot\rho + \phi)^2}{L^2} - \dfrac{2\cdot L\cdot\delta}{L\cdot\rho + \phi}$

#77: $\dfrac{2\cdot\delta\cdot(L^3\cdot(\rho^3 - 1) + 3\cdot L^2\cdot\rho^2\cdot\phi + 3\cdot L\cdot\rho\cdot\phi^2 + \phi^3)}{L^2\cdot(L\cdot\rho + \phi)}$

>0 if

#78: $2\cdot\delta\cdot(L^3\cdot(\rho^3 - 1) + 3\cdot L^2\cdot\rho^2\cdot\phi + 3\cdot L\cdot\rho\cdot\phi^2 + \phi^3) > 0$

#79: $\mathrm{SOLVE}(2\cdot\delta\cdot(L^3\cdot(\rho^3 - 1) + 3\cdot L^2\cdot\rho^2\cdot\phi + 3\cdot L\cdot\rho\cdot\phi^2 + \phi^3) > 0, \phi)$

#80: $(\phi^2 + L\cdot\phi\cdot(2\cdot\rho + 1) < - L^2\cdot(\rho^2 + \rho + 1) \wedge \phi < L\cdot(1 - \rho)) \vee (\phi^2 + L\cdot\phi\cdot(2\cdot\rho + 1) > - L^2\cdot(\rho^2 + \rho + 1) \wedge \phi > L\cdot(1 - \rho))$

#81: $\phi > L\cdot(1 - \rho)$

eq (12) Expected net fraud loss

#82: $\mathrm{eloss} = \mathrm{pf}\cdot(L - \mathrm{pw}\cdot e\lambda\cdot(\phi + \rho\cdot L - C))$

deriving FOC and SOC

#83: $\mathrm{eloss} = \dfrac{2^{1/3}\cdot L^{1/3}\cdot\delta^{1/3}}{C^{1/3}\cdot(L\cdot\rho + \phi)^{1/3}}\cdot\left(L - \dfrac{2^{2/3}\cdot C^{1/3}\cdot L^{2/3}}{2\cdot\delta^{1/3}\cdot(L\cdot\rho + \phi)^{2/3}}\cdot\dfrac{2^{2/3}\cdot L^{2/3}\cdot\delta^{2/3}}{2\cdot C^{2/3}\cdot(L\cdot\rho + \phi)^{2/3}}\cdot(\phi + \rho\cdot L - C)\right)$

#84: $$\mathrm{eloss} = \frac{2^{2/3}\cdot L^{4/3}\cdot \delta^{1/3}\cdot (C\cdot L^{1/3}\cdot \delta^{1/3} + 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi))^{4/3} - L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi))}{2\cdot C^{2/3}\cdot (L\cdot \rho + \phi)^{5/3}}$$

Appendix C eq (C.1): FOC

#85: $$\frac{d}{dC}\left(\mathrm{eloss} = \frac{2^{2/3}\cdot L^{4/3}\cdot \delta^{1/3}\cdot (C\cdot L^{1/3}\cdot \delta^{1/3} + 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi))^{4/3} - L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi))}{2\cdot C^{2/3}\cdot (L\cdot \rho + \phi)^{5/3}}\right)$$

#86: $$0 = \frac{2^{2/3}\cdot L^{4/3}\cdot \delta^{1/3}\cdot (C\cdot L^{1/3}\cdot \delta^{1/3} - 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi))^{4/3} + 2\cdot L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi))}{6\cdot C^{5/3}\cdot (L\cdot \rho + \phi)^{5/3}}$$

eq (C.2): SOC

#87: $$\frac{d}{dC}\frac{d}{dC}\left(\mathrm{eloss} = \frac{2^{2/3}\cdot L^{4/3}\cdot \delta^{1/3}\cdot (C\cdot L^{1/3}\cdot \delta^{1/3} + 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi))^{4/3} - L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi))}{2\cdot C^{2/3}\cdot (L\cdot \rho + \phi)^{5/3}}\right)$$

#88: $$-\frac{2^{2/3}\cdot L^{4/3}\cdot \delta^{1/3}\cdot (C\cdot L^{1/3}\cdot \delta^{1/3} - 2\cdot 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi))^{4/3} + 5\cdot L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi))}{9\cdot C^{8/3}\cdot (L\cdot \rho + \phi)^{5/3}}$$

#89: $$C\cdot L^{1/3}\cdot \delta^{1/3} - 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi)^{4/3} + 2\cdot L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi) = 0$$

#90: $$\mathrm{SOLVE}(C\cdot L^{1/3}\cdot \delta^{1/3} - 2^{2/3}\cdot C^{1/3}\cdot (L\cdot \rho + \phi)^{4/3} + 2\cdot L^{1/3}\cdot \delta^{1/3}\cdot (L\cdot \rho + \phi) = 0, C)$$

=> not solvable => requires numerical simulations.

Derivation of Result 4(b)
Recall c_min and c_max

#91: $c\_min = \frac{2 \cdot L \cdot \delta}{L \cdot \rho + \phi}$

#92: $c\_max = \frac{2 \cdot \delta \cdot (L \cdot \rho + \phi)^2}{L^2}$

eq (C.3): SOC evaluated at c_min

#93: $- \frac{L^2 \cdot \rho^2 + 2 \cdot L \cdot (\delta + \rho \cdot \phi) + \phi^2}{36 \cdot L \cdot \delta^2} < 0$

eq (C.4): Evaluating SOC at c_max

#94: $\frac{L^5 \cdot (L^2 \cdot (4 \cdot \rho - 5) + 2 \cdot L \cdot (2 \cdot \phi - \delta \cdot \rho) - 2 \cdot \delta \cdot \phi)}{36 \cdot \delta^2 \cdot (L \cdot \rho + \phi)^6}$

< 0 if [not in paper]

#95: $L^2 \cdot (4 \cdot \rho - 5) + 2 \cdot L \cdot (2 \cdot \phi - \delta \cdot \rho) - 2 \cdot \delta \cdot \phi < 0$

#96: $\mathrm{SOLVE}(L^2 \cdot (4 \cdot \rho - 5) + 2 \cdot L \cdot (2 \cdot \phi - \delta \cdot \rho) - 2 \cdot \delta \cdot \phi < 0, \phi)$

#97: $$\mathrm{IF}\left(2\cdot L - \delta < 0,\ \phi > \frac{L\cdot(L\cdot(4\cdot\rho - 5) - 2\cdot\delta\cdot\rho)}{2\cdot(\delta - 2\cdot L)}\right) \vee \mathrm{IF}\left(2\cdot L - \delta > 0,\ \phi < \frac{L\cdot(L\cdot(4\cdot\rho - 5) - 2\cdot\delta\cdot\rho)}{2\cdot(\delta - 2\cdot L)}\right)$$

Proving Result 4b:

eq (C.5): Loss evaluated at c_min

#98: $$\text{eloss\_cmin} = \frac{L\cdot(L^2\cdot\rho^2 + 2\cdot L\cdot(\delta + \rho\cdot\phi) + \phi^2)}{2\cdot(L\cdot\rho + \phi)^2}$$

eq (C.6): Loss evaluated at c_max

#99: $$\text{eloss\_cmax} = \frac{L\cdot(L^2\cdot(2\cdot\rho - 1) + 2\cdot L\cdot(\delta\cdot\rho + \phi) + 2\cdot\delta\cdot\phi)}{2\cdot(L\cdot\rho + \phi)^2}$$

eq (C.7) eloss_cmin - eloss_cmax =

#100: $$\frac{L\cdot(L^2\cdot\rho^2 + 2\cdot L\cdot(\delta + \rho\cdot\phi) + \phi^2)}{2\cdot(L\cdot\rho + \phi)^2} - \frac{L\cdot(L^2\cdot(2\cdot\rho - 1) + 2\cdot L\cdot(\delta\cdot\rho + \phi) + 2\cdot\delta\cdot\phi)}{2\cdot(L\cdot\rho + \phi)^2}$$

#101: $$\frac{L\cdot(L^2\cdot(\rho^2 - 2\cdot\rho + 1) + 2\cdot L\cdot(1 - \rho)\cdot(\delta - \phi) - \phi\cdot(2\cdot\delta - \phi))}{2\cdot(L\cdot\rho + \phi)^2}$$

> 0 if

#102: $L^2\cdot(\rho^2 - 2\cdot\rho + 1) + 2\cdot L\cdot(1 - \rho)\cdot(\delta - \phi) - \phi\cdot(2\cdot\delta - \phi) > 0$

#103: $\mathrm{SOLVE}(L^2\cdot(\rho^2 - 2\cdot\rho + 1) + 2\cdot L\cdot(1 - \rho)\cdot(\delta - \phi) - \phi\cdot(2\cdot\delta - \phi) > 0, \phi)$

#104: $(\phi < L\cdot(1 - \rho) + 2\cdot\delta \wedge \phi < L\cdot(1 - \rho)) \vee (\phi > L\cdot(1 - \rho) + 2\cdot\delta \wedge \phi > L\cdot(1 - \rho))$

#105: $\phi > L\cdot(1 - \rho) + 2\cdot\delta \wedge \phi > L\cdot(1 - \rho)$

which is the condition specified in Result 4b.

** Subsection 5.1: Wait to inflate

eq (13) Delay yields higher utility if

#106: $\tau\cdot\left(\frac{\lambda\cdot(\rho\cdot 2\cdot L)}{1 + \gamma} - \delta\cdot d^{\gamma}\right) > \frac{\lambda\cdot(\rho\cdot L)}{1 + \gamma} - \delta\cdot d^{\gamma}$

#107: $\mathrm{SOLVE}\left(\tau\cdot\left(\frac{\lambda\cdot(\rho\cdot 2\cdot L)}{1 + \gamma} - \delta\cdot d^{\gamma}\right) > \frac{\lambda\cdot(\rho\cdot L)}{1 + \gamma} - \delta\cdot d^{\gamma}, \tau\right)$

#108: $\mathrm{IF}\left(\delta\cdot d^{\gamma}\cdot(\gamma + 1) - 2\cdot L\cdot\lambda\cdot\rho < 0, \tau > \frac{\delta\cdot d^{\gamma}\cdot(\gamma + 1) - L\cdot\lambda\cdot\rho}{\delta\cdot d^{\gamma}\cdot(\gamma + 1) - 2\cdot L\cdot\lambda\cdot\rho}\right) \vee \mathrm{IF}\left(\delta\cdot d^{\gamma}\cdot(\gamma + 1) - 2\cdot L\cdot\lambda\cdot\rho > 0, \tau < \frac{\delta\cdot d^{\gamma}\cdot(\gamma + 1) - L\cdot\lambda\cdot\rho}{\delta\cdot d^{\gamma}\cdot(\gamma + 1) - 2\cdot L\cdot\lambda\cdot\rho}\right)$

$$\#109:\ \tau > \tau bar = \frac{\delta \cdot d^{\gamma} \cdot (\gamma + 1) - L \cdot \lambda \cdot \rho}{\delta \cdot d^{\gamma} \cdot (\gamma + 1) - 2 \cdot L \cdot \lambda \cdot \rho}$$

eq (14)

$$\#110:\ \tau \cdot (\lambda \cdot c\_bar - \delta \cdot d^{\gamma}) \le \frac{\lambda \cdot (\rho \cdot L)}{1 + \gamma} - \delta \cdot d^{\gamma}$$

$$\#111:\ \mathrm{SOLVE}\left(\tau \cdot (\lambda \cdot c\_bar - \delta \cdot d^{\gamma}) \le \frac{\lambda \cdot (\rho \cdot L)}{1 + \gamma} - \delta \cdot d^{\gamma},\ c\_bar\right)$$

$$\#112:\ c\_bar \le \frac{\delta \cdot d^{\gamma} \cdot (\gamma + 1) \cdot (\tau - 1) + L \cdot \lambda \cdot \rho}{\lambda \cdot \tau \cdot (\gamma + 1)}$$

$$\#113:\ c\_bar \le \frac{\delta \cdot d^{\gamma} \cdot (\tau - 1)}{\lambda \cdot \tau} + \frac{L \cdot \rho}{\tau \cdot (\gamma + 1)}$$

as $\tau \to 1$,

$$\#114:\ \lim_{\tau \to 1-} \left(c\_bar \le \frac{\delta \cdot d^{\gamma} \cdot (\gamma + 1) \cdot (\tau - 1) + L \cdot \lambda \cdot \rho}{\lambda \cdot \tau \cdot (\gamma + 1)}\right)$$

$$\#115:\ c\_bar \le \frac{L \cdot \rho}{\gamma + 1}$$

** Subsection 5.2: Multiple WBs

Modfied utility level of d

#116: $u = \lambda \cdot \frac{R}{dhat} - \delta \cdot d^{\gamma}$

#117: $0 = \lambda \cdot \frac{R}{dhat} - \delta \cdot dhat^{\gamma}$

eq (15)

#118: $\mathrm{SOLVE}\left(0 = \lambda \cdot \frac{R}{dhat} - \delta \cdot dhat^{\gamma}, dhat\right)$

#119: $dhat = \left(\frac{R \cdot \lambda}{\delta}\right)^{1/(\gamma + 1)}$

dhat < 1 if

#120: $\left(\frac{R \cdot \lambda}{\delta}\right)^{1/(\gamma + 1)} < 1$

#121: $\mathrm{SOLVE}\left(\left(\frac{R \cdot \lambda}{\delta}\right)^{1/(\gamma + 1)} < 1, R\right)$

#122: $0 < R < \frac{\delta}{\lambda}$

deriving eq (16) dhat_mult - dhat_single

#123: $\left(\frac{R \cdot \lambda}{\delta}\right)^{1/(\gamma + 1)} - \left(\frac{R \cdot \lambda}{\delta}\right)^{1/\gamma}$

#124: $$\left(\frac{R\cdot\lambda}{\delta}\right)^{1/(\gamma + 1)} - \left(\frac{R\cdot\lambda}{\delta}\right)^{1/\gamma}$$

must be < 0

eq (17): Reward choice

#125: $$eb = \left(\frac{R\cdot\lambda}{\delta}\right)^{1/(1 + \gamma)}\cdot\lambda\cdot(\rho\cdot L - R)$$

Appendix D and derivation of (18)

#126: $$\frac{d}{dR}\left(eb = \left(\frac{R\cdot\lambda}{\delta}\right)^{1/(1 + \gamma)}\cdot\lambda\cdot(\rho\cdot L - R)\right)$$

#127: $$0 = \frac{R^{-\gamma/(\gamma + 1)}\cdot\delta^{-1/(\gamma + 1)}\cdot\lambda^{(\gamma + 2)/(\gamma + 1)}\cdot(L\cdot\rho - R\cdot(\gamma + 2))}{\gamma + 1}$$

#128: $$\frac{d}{dR}\frac{d}{dR}\left(eb = \left(\frac{R\cdot\lambda}{\delta}\right)^{1/(1 + \gamma)}\cdot\lambda\cdot(\rho\cdot L - R)\right)$$

#129: $$0 > -\frac{R^{-(2\cdot\gamma + 1)/(\gamma + 1)}\cdot\delta^{-1/(\gamma + 1)}\cdot\lambda^{(\gamma + 2)/(\gamma + 1)}\cdot(L\cdot\gamma\cdot\rho + R\cdot(\gamma + 2))}{(\gamma + 1)^2}$$

#130: $$\text{SOLVE}(R^{-\gamma/(\gamma + 1)}\cdot\delta^{-1/(\gamma + 1)}\cdot\lambda^{(\gamma + 2)/(\gamma + 1)}\cdot(L\cdot\rho - R\cdot(\gamma + 2)), R)$$

eq (18)

#131: $$R = \frac{L\cdot\rho}{\gamma + 2}$$

#132: $$dhat = \left(\frac{L\cdot\lambda\cdot\rho}{\delta\cdot(\gamma + 2)}\right)^{1/(\gamma + 1)}$$

dhat < 1 if

#133: $$\left(\frac{L\cdot\lambda\cdot\rho}{\delta\cdot(\gamma + 2)}\right)^{1/(\gamma + 1)} < 1$$

#134: $$\mathrm{SOLVE}\left(\left(\frac{L\cdot\lambda\cdot\rho}{\delta\cdot(\gamma + 2)}\right)^{1/(\gamma + 1)} < 1,\ L\right)$$

#135: $$0 < L < \frac{\delta\cdot(\gamma + 2)}{\lambda\cdot\rho}$$

#136: $$\mathrm{SOLVE}\left(\left(\frac{L\cdot\lambda\cdot\rho}{\delta\cdot(\gamma + 2)}\right)^{1/(\gamma + 1)} < 1,\ \delta\right)$$

#137: $$\frac{1}{\delta} < \frac{\gamma + 2}{L\cdot\lambda\cdot\rho} \wedge \delta > 0$$

Result 6: comparing R multiple with R single. R_mult - R_single =

#138: $$\frac{L\cdot\rho}{\gamma + 2} - \frac{L\cdot\rho}{\gamma + 1}$$

#139: $$-\frac{L\cdot\rho}{(\gamma + 1)\cdot(\gamma + 2)} < 0$$

Discussions about shares after Result 6:

#140: $1 - \frac{1}{2 + \gamma}$

#141: $$\frac{\gamma + 1}{\gamma + 2}$$

** Subsection 5.3: Calibrations

#142: $R\_star\_single = \frac{L \cdot \rho}{\gamma + 1}$

#143: $SOLVE\left(R\_star\_single = \frac{L \cdot \rho}{\gamma + 1}, \gamma\right)$

#144: $$\gamma\_single = \frac{L \cdot \rho}{R\_star\_single} - 1$$

#145: $R\_star\_multiple = \frac{L \cdot \rho}{\gamma + 2}$

#146: $SOLVE\left(R\_star\_multiple = \frac{L \cdot \rho}{\gamma + 2}, \gamma\right)$

#147: $$\gamma\_multiple = \frac{L \cdot \rho}{R\_star\_multiple} - 2$$

let r define rate of reward

#148: $r = \frac{R}{\rho \cdot L}$

#149: $\gamma\_single = \frac{1}{r\_single} - 1$

Then,

#150: $\gamma\_multiple = \frac{1}{r\_multiple} - 2$

inverting for the discussion after Table 3

#151: $\mathrm{SOLVE}\left(\gamma\_multiple = \frac{1}{r\_multiple} - 2,\ r\_multiple\right)$

#152: $r\_multiple = \frac{1}{\gamma\_multiple + 2}$

#153: $r\_multiple = \frac{1}{1 + 2}$

#154: $r\_multiple = \frac{1}{3}$