hotel_2022_1_24.dfw Redo all calculations based on hotel_4.tex. switching to L=1 and separation of global from marginal deviations

#1: CaseMode :≔ Sensitive

#2: InputMode :≔ Word

#3: $\tau :\in \text{Real } (0, \infty)$

#4: $L :\in \text{Real } [1, \infty)$

#5: $pa :\in \text{Real } (0, \infty)$

#6: $pb :\in \text{Real } (0, \infty)$

#7: $xhat :\in \text{Real } [0, L]$

#8: $a :\in \text{Real } [0, 1]$

#9: $b :\in \text{Real } [0, 1]$

Eq (1)

#10: $pa + \tau\cdot(xhat - a)$

#11: $pb + \tau\cdot(1 - b - xhat)$

Eq (2)

#12: $pb + \tau\cdot(1 - b - xhat) = pa + \tau\cdot(xhat - a)$

#13: $\text{SOLVE}(pb + \tau\cdot(1 - b - xhat) = pa + \tau\cdot(xhat - a), xhat)$

#14: $$xhat = \frac{a\cdot\tau - b\cdot\tau - pa + pb + \tau}{2\cdot\tau}$$

#15: $$xhat = \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}$$

Equations (3) and (4), and Appendix A

I will derive equation (4) first disregarding condition (3) which is global deviation

#16: $$profita\_n = pa\cdot\frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}$$

#17: $$profitb\_n = pb\cdot\left(1 - \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)$$

#18: $$\frac{d}{d\ pa}\left(profita\_n = pa\cdot\frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)$$

#19: $$0 = \frac{a\cdot\tau - b\cdot\tau - 2\cdot pa + pb + \tau}{2\cdot\tau}$$

#20: $$\frac{d}{d\ pb}\left(profitb\_n = pb\cdot\left(1 - \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)\right)$$

#21: $$0 = - \frac{a\cdot\tau - b\cdot\tau - pa + 2\cdot pb - \tau}{2\cdot\tau}$$

#22: $$SOLVE\left(\left[0 = \frac{a\cdot\tau - b\cdot\tau - 2\cdot pa + pb + \tau}{2\cdot\tau},\ 0 = - \frac{a\cdot\tau - b\cdot\tau - pa + 2\cdot pb - \tau}{2\cdot\tau}\right], [pa, pb]\right)$$

#23: $$\left[pa = \frac{\tau\cdot(a - b + 3)}{3} \wedge pb = - \frac{\tau\cdot(a - b - 3)}{3}\right]$$

#24: $$\text{xhat} = \frac{a - b + 3}{6}$$

#25: $$\text{profita\_n} = \frac{\tau\cdot(a - b + 3)^2}{18}$$

#26: $$\text{profitb\_n} = \frac{\tau\cdot(a - b - 3)\cdot(a - b - 3)}{18}$$

#27: $$\text{profitb\_n} = \frac{\tau\cdot(a - b - 3)^2}{18}$$

back to Appendix A: Deriving (3)
eq (A.1)

#28: $$\frac{\tau\cdot(a - b + 3)^2}{18} \ge - \frac{\tau\cdot(a - b - 3)}{3} - \tau\cdot(1 - b - a)$$

#29: $$\tau\cdot(a - b + 3)^2 \ge 12\cdot\tau\cdot(a + 2\cdot b)$$

eq (A.2)

#30: $$\frac{\tau\cdot(a - b - 3)^2}{18} \ge \frac{\tau\cdot(a - b + 3)}{3} - \tau\cdot(1 - b - a)$$

#31: $$\tau\cdot(a - b - 3)^2 \ge 12\cdot\tau\cdot(2\cdot a + b)$$

Deriving (5a)

#32: $\tau\cdot(a - b + 3)^2 = 12\cdot\tau\cdot(a + 2\cdot b)$

#33: $\text{SOLVE}(\tau\cdot(a - b + 3)^2 = 12\cdot\tau\cdot(a + 2\cdot b), b)$

#34: $b = - 6\cdot\sqrt{(a + 6)} + a + 15 \vee b = 6\cdot\sqrt{(a + 6)} + a + 15$

Deriving (5b)

#35: $\tau\cdot(a - b - 3)^2 = 12\cdot\tau\cdot(2\cdot a + b)$

#36: $\text{SOLVE}(\tau\cdot(a - b - 3)^2 = 12\cdot\tau\cdot(2\cdot a + b), b)$

#37: $b = a - 6\cdot\sqrt{a} + 3 \vee b = a + 6\cdot\sqrt{a} + 3$

Explaining Figure 2
(5a), what is b when a=0?

#38: $b = - 6\cdot\sqrt{(0 + 6)} + 0 + 15$

#39: $b = 15 - 6\cdot\sqrt{6}$

#40: $b = 0.3030615433$

5(b): what is a when b=0?

#41: $0 = a - 6\cdot\sqrt{a} + 3$

#42: $\text{SOLVE}(0 = a - 6\cdot\sqrt{a} + 3, a)$

#43: $a = 15 - 6\cdot\sqrt{6} \vee a = 6\cdot\sqrt{6} + 15$

#44: $a = 0.3030615433$

verify instersection at 1/4

#45: SOLVE([b = a − 6·√a + 3, b = − 6·√(a + 6) + a + 15], [a, b])

#46: $\left[a = \frac{1}{4} \wedge b = \frac{1}{4}\right]$

eq (6) and Result 2

#47: $\int_0^{0.25} (-6\cdot\sqrt{a+6} + a + 15)\, da$

#48: $\int_{0.25}^{15-6\cdot\sqrt{6}} (a - 6\cdot\sqrt{a} + 3)\, da$

#49: $F = \frac{\int_0^{0.25} (-6\cdot\sqrt{a+6} + a + 15)\, da + \int_{0.25}^{15-6\cdot\sqrt{6}} (a - 6\cdot\sqrt{a} + 3)\, da}{0.5}$

#50: $F = 96\cdot\sqrt{6} - 235$

#51: $F = 0.151015307$

*** section 5 UPE begins

Definition 3 (global property), eq (7)

#52: pb_u·(1 − xhat_u) ≥ pa_u − τ·(1 − b − a)

eq (8)

#53: pa_u·xhat_u ≥ pb_u − τ·(1 − b − a)

recall xhat

#54: $$\text{xhat} = \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}$$

Definition 4 (marginal property): eq (9)-(10), should be ≥0 to satisfy the marginal property

#55: $$\frac{d}{d\ pa}\left(\text{profita\_u} = pa\cdot\frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)$$

#56: $$\frac{a\cdot\tau - b\cdot\tau - 2\cdot pa + pb + \tau}{2\cdot\tau} \ge 0$$

#57: $$\frac{d}{d\ pb}\left(\text{profitb\_u} = pb\cdot\left(1 - \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)\right)$$

#58: $$-\frac{a\cdot\tau - b\cdot\tau - pa + 2\cdot pb - \tau}{2\cdot\tau} \ge 0$$

*** Section 4: UPE under symmetry a=b=d

eq (11)

#59: $$\text{xhat\_u} = -\frac{\text{pa\_u} - \text{pb\_u} - \tau}{2\cdot\tau}$$

eq (12) From #52 #53,

#60: $$\text{pb\_u}\cdot\left(1 - -\frac{\text{pa\_u} - \text{pb\_u} - \tau}{2\cdot\tau}\right) = \text{pa\_u} - \tau\cdot(1 - d - d)$$

eq (13)

$$\#61: \quad pa\_u\cdot\left(-\frac{pa\_u - pb\_u - \tau}{2\cdot\tau}\right) = pb\_u - \tau\cdot(1 - d - d)$$

deriving eq (14)

$$\#62: \quad \text{SOLVE}\left(\left[pb\_u\cdot\left(1 - -\frac{pa\_u - pb\_u - \tau}{2\cdot\tau}\right) = pa\_u - \tau\cdot(1 - d - d),\ pa\_u\cdot\left(-\frac{pa\_u - pb\_u - \tau}{2\cdot\tau}\right) = pb\_u - \tau\cdot(1 - d - d)\right], [pa\_u, pb\_u]\right)$$

$$\#63: \quad \left[pa\_u = 2\cdot\tau\cdot(1 - 2\cdot d) \wedge pb\_u = 2\cdot\tau\cdot(1 - 2\cdot d),\ pa\_u = \frac{\tau\cdot(\sqrt{(1 - 8\cdot d)} + 3)}{2} \wedge pb\_u = \frac{\tau\cdot(3 - \sqrt{(1 - 8\cdot d)})}{2},\ pa\_u = \frac{\tau\cdot(3 - \sqrt{(1 - 8\cdot d)})}{2} \wedge pb\_u = \frac{\tau\cdot(\sqrt{(1 - 8\cdot d)} + 3)}{2}\right]$$

$$\#64: \quad pa\_u = 2\cdot\tau\cdot(1 - 2\cdot d) \wedge pb\_u = 2\cdot\tau\cdot(1 - 2\cdot d)$$

$$\#65: \quad xhat\_u = \frac{1}{2}$$

$$\#66: \quad profita\_u = profitb\_u = (2\cdot\tau\cdot(1 - 2\cdot d))\cdot\frac{1}{2}$$

$$\#67: \quad profita\_u = profitb\_u = \tau\cdot(1 - 2\cdot d)$$

eq (15) marginal conditions from #55 and #56

$$\#68: \quad \frac{d}{d\ pa}\left(profita\_u = pa\cdot\frac{\tau\cdot(d - d + 1) - pa + 2\cdot\tau\cdot(1 - 2\cdot d)}{2\cdot\tau}\right)$$

#69: $$- \frac{4\cdot d\cdot \tau + 2\cdot pa - 3\cdot \tau}{2\cdot \tau}$$

substitute UPE pa yields

#70: $$- \frac{4\cdot d\cdot \tau + 2\cdot (2\cdot \tau\cdot (1 - 2\cdot d)) - 3\cdot \tau}{2\cdot \tau}$$

#71: $$\frac{4\cdot d - 1}{2}$$

#72: $$\frac{d}{d\ pb}\left(profitb\_u = - \frac{pb\cdot (4\cdot d\cdot \tau + pb - 3\cdot \tau)}{2\cdot \tau}\right)$$

#73: $$\frac{4\cdot d - 1}{2}$$

$\geq$ 0 for d>1/4

#74: $$\tau\cdot (1 - 2\cdot d)$$

#75: $$\frac{d}{d\ pb}\left(profitb\_u = pb\cdot \left(1 - \frac{\tau\cdot (d - d + 1) - 2\cdot \tau\cdot (1 - 2\cdot d) + pb}{2\cdot \tau}\right)\right)$$

#76: $$- \frac{4\cdot d\cdot \tau + 2\cdot pb - 3\cdot \tau}{2\cdot \tau}$$

#77: $$\frac{4\cdot d - 1}{2}$$

$\geq$ 0 for d>1/4

*** section 6: UPE under asymmetry a ≠ b

This section is computed using R

*** section proving a=0.9 and b=0 is not an UPEG. eq (17) and (18) in paper

#78: $pb\cdot\left(1 - \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right) = pa - \tau\cdot(1 - b - a)$

#79: $pa\cdot\frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau} = pb - \tau\cdot(1 - b - a)$

#80: $pb\cdot\left(1 - \frac{\tau\cdot(0.9 - 0 + 1) - pa + pb}{2\cdot\tau}\right) = pa - \tau\cdot(1 - 0 - 0.9)$

#81: $pa\cdot\frac{\tau\cdot(0.9 - 0 + 1) - pa + pb}{2\cdot\tau} = pb - \tau\cdot(1 - 0 - 0.9)$

#82: $\mathrm{SOLVE}\left(\left[pb\cdot\left(1 - \frac{\tau\cdot(0.9 - 0 + 1) - pa + pb}{2\cdot\tau}\right) = pa - \tau\cdot(1 - 0 - 0.9),\ pa\cdot\frac{\tau\cdot(0.9 - 0 + 1) - pa + pb}{2\cdot\tau} = pb - \tau\cdot(1 - 0 - 0.9)\right], [pa, pb]\right)$

#83: $\left[pa = 2\cdot\tau \wedge pb = \frac{21\cdot\tau}{20} + \frac{\sqrt{1079}\cdot i\cdot\tau}{20},\ pa = 2\cdot\tau \wedge pb = \frac{21\cdot\tau}{20} - \frac{\sqrt{1079}\cdot i\cdot\tau}{20},\ pa = \frac{\tau}{10} \wedge pb = \frac{\tau}{5}\right]$

#84: $[\mathrm{false}, \mathrm{false}, pa = 0.1\cdot\tau \wedge pb = 0.2\cdot\tau]$

#85: $\frac{d}{d\ pb}\left(profitb\_u = pb\cdot\left(1 - \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)\right)$

#86: $$- \frac{a\cdot\tau - b\cdot\tau - pa + 2\cdot pb - \tau}{2\cdot\tau}$$

#87: $$- \frac{0.9\cdot\tau - 0\cdot\tau - 0.1\cdot\tau + 2\cdot(0.2\cdot\tau) - \tau}{2\cdot\tau}$$

#88: $$- \frac{1}{10} < 0$$

which means that firm B can increase its profit by marginally lowering its price.

*** Checking why case I agam (both are bidning for A) yields constant pa
These confirm! the agam case, which alway yields a constant pa.

A global is binding, eq (8)

#89: $pa\cdot xhat - (pb - \tau\cdot(1 - b - a)) = 0$

#90: $$pa\cdot\frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau} - (pb - \tau\cdot(1 - b - a)) = 0$$

A marginal is binding, eq (9)

#91: $\tau\cdot(1 + a - b) - 2\cdot pa + pb = 0$

set b=0.25 and a=0 and τ=1

#92: $$pa\cdot\frac{1\cdot(0 - 0.25 + 1) - pa + pb}{2\cdot 1} - (pb - 1\cdot(1 - 0.25 - 0)) = 0$$

#93: $1\cdot(1 + 0 - 0.25) - 2\cdot pa + pb = 0$

#94: $\text{SOLVE}\left(\left[pa\cdot\frac{1\cdot(0-0.25+1)-pa+pb}{2\cdot 1}-(pb-1\cdot(1-0.25-0))=0,\ 1\cdot(1+0-0.25)-2\cdot pa+pb=0\right],\ [pa,\ pb]\right)$

#95: $\left[pa=1\wedge pb=\frac{5}{4},\ pa=3\wedge pb=\frac{21}{4}\right]$

#96: $[pa=1\wedge pb=1.25,\ pa=3\wedge pb=5.25]$

set b=0.5 and a=0 and τ=1

#97: $pa\cdot\frac{1\cdot(0-0.5+1)-pa+pb}{2\cdot 1}-(pb-1\cdot(1-0.5-0))=0$

#98: $1\cdot(1+0-0.5)-2\cdot pa+pb=0$

#99: $\text{SOLVE}\left(\left[pa\cdot\frac{1\cdot(0-0.5+1)-pa+pb}{2\cdot 1}-(pb-1\cdot(1-0.5-0))=0,\ 1\cdot(1+0-0.5)-2\cdot pa+pb=0\right],\ [pa,\ pb]\right)$

#100: $\left[pa=\sqrt{2}+2\wedge pb=2\cdot\sqrt{2}+\frac{7}{2},\ pa=2-\sqrt{2}\wedge pb=\frac{7}{2}-2\cdot\sqrt{2}\right]$

#101: $[pa=3.414213562\wedge pb=6.328427124,\ pa=0.5857864376\wedge pb=0.6715728752]$

set b=0.5 and a=0.2 and τ=1

#102: $pa\cdot\frac{1\cdot(0.2 - 0.5 + 1) - pa + pb}{2\cdot1} - (pb - 1\cdot(1 - 0.5 - 0.2)) = 0$

#103: $1\cdot(1 + 0.2 - 0.5) - 2\cdot pa + pb = 0$

#104: $\text{SOLVE}\left(\left[pa\cdot\frac{1\cdot(0.2 - 0.5 + 1) - pa + pb}{2\cdot1} - (pb - 1\cdot(1 - 0.5 - 0.2)) = 0,\ 1\cdot(1 + 0.2 - 0.5) - 2\cdot pa + pb = 0\right], [pa, pb]\right)$

#105: $\left[pa = \sqrt{2} + 2 \wedge pb = 2\cdot\sqrt{2} + \frac{33}{10},\ pa = 2 - \sqrt{2} \wedge pb = \frac{33}{10} - 2\cdot\sqrt{2}\right]$

#106: $[pa = 3.414213562 \wedge pb = 6.128427124,\ pa = 0.5857864376 \wedge pb = 0.4715728752]$

*** proof of Result 4

Since the global property is binding: (7) and (8) hold with equality

#107: $pb\_ug\cdot(1 - xhat\_ug) = pa\_ug - \tau\cdot(1 - b - a)$

#108: $pa\_ug\cdot xhat\_ug = pb\_ug - \tau\cdot(1 - b - a)$

solving for the UPEG prices:

#109: $\text{SOLVE}([pb\_ug\cdot(1 - xhat\_ug) = pa\_ug - \tau\cdot(1 - b - a),\ pa\_ug\cdot xhat\_ug = pb\_ug - \tau\cdot(1 - b - a)], [pa\_ug, pb\_ug])$

#110: $\left[pa\_ug = \frac{\tau\cdot(a + b - 1)\cdot(xhat\_ug - 2)}{xhat\_ug^2 - xhat\_ug + 1} \wedge pb\_ug = -\frac{\tau\cdot(a + b - 1)\cdot(xhat\_ug + 1)}{xhat\_ug^2 - xhat\_ug + 1}\right]$

hence, pa_ug - pb_ug = n

#111: $$\frac{\tau\cdot(a + b - 1)\cdot(xhat\_ug - 2)}{xhat\_ug^2 - xhat\_ug + 1} - - \frac{\tau\cdot(a + b - 1)\cdot(xhat\_ug + 1)}{xhat\_ug^2 - xhat\_ug + 1}$$

#112: $$\frac{\tau\cdot(a + b - 1)\cdot(2\cdot xhat\_ug - 1)}{xhat\_ug^2 - xhat\_ug + 1}$$

$\le$ 0 iff

#113: $2\cdot xhat\_ug - 1 \ge 0$

#114: SOLVE($2\cdot xhat\_ug - 1 \ge 0$, xhat_ug)

#115: $$xhat\_ug \ge \frac{1}{2}$$