hotel_2022_1_24.dfw Redo all calculations based on hotel_4.tex. switching to L=1 and separation of global from marginal deviations

#1: CaseMode := Sensitive

#2: InputMode := Word

#3: $\tau :\in \text{Real } (0, \infty)$

#4: $L :\in \text{Real } [1, \infty)$

#5: $pa :\in \text{Real } (0, \infty)$

#6: $pb :\in \text{Real } (0, \infty)$

#7: $xhat :\in \text{Real } [0, L]$

#8: $a :\in \text{Real } [0, 1]$

#9: $b :\in \text{Real } [0, 1]$

Eq (1)

#10: $pa + \tau\cdot(xhat - a)$

#11: $pb + \tau\cdot(1 - b - xhat)$

Eq (2)

#12: $pb + \tau\cdot(1 - b - xhat) = pa + \tau\cdot(xhat - a)$

#13: $\text{SOLVE}(pb + \tau\cdot(1 - b - xhat) = pa + \tau\cdot(xhat - a), xhat)$

#14:
$$xhat = \frac{a\cdot\tau - b\cdot\tau - pa + pb + \tau}{2\cdot\tau}$$

#15: $$xhat = \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}$$

Equations (3) and (4), and Appendix A

I will derive equation (4) first disregarding condition (3) which is global deviation

#16: $$profita\_n = pa\cdot\frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}$$

#17: $$profitb\_n = pb\cdot\left(1 - \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)$$

#18: $$\frac{d}{d\ pa}\left(profita\_n = pa\cdot\frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)$$

#19: $$0 = \frac{a\cdot\tau - b\cdot\tau - 2\cdot pa + pb + \tau}{2\cdot\tau}$$

#20: $$\frac{d}{d\ pb}\left(profitb\_n = pb\cdot\left(1 - \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)\right)$$

#21: $$0 = -\frac{a\cdot\tau - b\cdot\tau - pa + 2\cdot pb - \tau}{2\cdot\tau}$$

#22: $$SOLVE\left(\left[0 = \frac{a\cdot\tau - b\cdot\tau - 2\cdot pa + pb + \tau}{2\cdot\tau}, 0 = -\frac{a\cdot\tau - b\cdot\tau - pa + 2\cdot pb - \tau}{2\cdot\tau}\right], [pa, pb]\right)$$

#23: $$\left[pa = \frac{\tau\cdot(a - b + 3)}{3} \wedge pb = -\frac{\tau\cdot(a - b - 3)}{3}\right]$$

#24: $$xhat = \frac{a - b + 3}{6}$$

#25: $$profita\_n = \frac{\tau \cdot (a - b + 3)^2}{18}$$

#26: $$profitb\_n = \frac{\tau \cdot (a - b - 3) \cdot (a - b - 3)}{18}$$

#27: $$profitb\_n = \frac{\tau \cdot (a - b - 3)^2}{18}$$

back to Appendix A: Deriving (3)
eq (A.1)

#28: $$\frac{\tau \cdot (a - b + 3)^2}{18} \ge - \frac{\tau \cdot (a - b - 3)}{3} - \tau \cdot (1 - b - a)$$

#29: $$\tau \cdot (a - b + 3)^2 \ge 12 \cdot \tau \cdot (a + 2 \cdot b)$$

eq (A.2)

#30: $$\frac{\tau \cdot (a - b - 3)^2}{18} \ge \frac{\tau \cdot (a - b + 3)}{3} - \tau \cdot (1 - b - a)$$

#31: $$\tau \cdot (a - b - 3)^2 \ge 12 \cdot \tau \cdot (2 \cdot a + b)$$

Deriving (5a)

#32: $\tau\cdot(a - b + 3)^2 = 12\cdot\tau\cdot(a + 2\cdot b)$

#33: $\text{SOLVE}(\tau\cdot(a - b + 3)^2 = 12\cdot\tau\cdot(a + 2\cdot b), b)$

#34: $b = - 6\cdot\sqrt{(a + 6)} + a + 15 \vee b = 6\cdot\sqrt{(a + 6)} + a + 15$

Deriving (5b)

#35: $\tau\cdot(a - b - 3)^2 = 12\cdot\tau\cdot(2\cdot a + b)$

#36: $\text{SOLVE}(\tau\cdot(a - b - 3)^2 = 12\cdot\tau\cdot(2\cdot a + b), b)$

#37: $b = a - 6\cdot\sqrt{a} + 3 \vee b = a + 6\cdot\sqrt{a} + 3$

Explaining Figure 2
(5a), what is b when a=0?

#38: $b = - 6\cdot\sqrt{(0 + 6)} + 0 + 15$

#39: $b = 15 - 6\cdot\sqrt{6}$

#40: $b = 0.3030615433$

5(b): what is a when b=0?

#41: $0 = a - 6\cdot\sqrt{a} + 3$

#42: $\text{SOLVE}(0 = a - 6\cdot\sqrt{a} + 3, a)$

#43: $a = 15 - 6\cdot\sqrt{6} \vee a = 6\cdot\sqrt{6} + 15$

#44: $a = 0.3030615433$

verify instersection at 1/4

#45: SOLVE([b = a − 6·√a + 3, b = − 6·√(a + 6) + a + 15], [a, b])

#46: $$\left[a = \frac{1}{4} \wedge b = \frac{1}{4}\right]$$

eq (6) and Result 2

#47: $$\int_{0}^{0.25} (-6\cdot\sqrt{a + 6} + a + 15)\, da$$

#48: $$\int_{0.25}^{15 - 6\cdot\sqrt{6}} (a - 6\cdot\sqrt{a} + 3)\, da$$

#49: $$F = \frac{\int_{0}^{0.25} (-6\cdot\sqrt{a + 6} + a + 15)\, da + \int_{0.25}^{15 - 6\cdot\sqrt{6}} (a - 6\cdot\sqrt{a} + 3)\, da}{0.5}$$

#50: $$F = 96\cdot\sqrt{6} - 235$$

#51: $$F = 0.151015307$$

*** section 5 UPE begins

Definition 3 (global property), eq (7)

#52: pb_u·(1 − xhat_u) ≥ pa_u − τ·(1 − b − a)

eq (8)

#53: pa_u·xhat_u ≥ pb_u − τ·(1 − b − a)

recall xhat

#54: $$\mathrm{xhat} = \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}$$

Definition 4 (marginal property): eq (9)-(10), should be ≥0 to satisfy the marginal property

#55: $$\frac{d}{d\ pa}\left(\mathrm{profita\_u} = pa\cdot\frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)$$

#56: $$\frac{a\cdot\tau - b\cdot\tau - 2\cdot pa + pb + \tau}{2\cdot\tau} \ge 0$$

#57: $$\frac{d}{d\ pb}\left(\mathrm{profitb\_u} = pb\cdot\left(1 - \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)\right)$$

#58: $$- \frac{a\cdot\tau - b\cdot\tau - pa + 2\cdot pb - \tau}{2\cdot\tau} \ge 0$$

*** Section 4: UPE under symmetry a=b=d

eq (11)

#59: $$\mathrm{xhat\_u} = - \frac{pa\_u - pb\_u - \tau}{2\cdot\tau}$$

eq (12) From #52 #53,

#60: $$pb\_u\cdot\left(1 - - \frac{pa\_u - pb\_u - \tau}{2\cdot\tau}\right) = pa\_u - \tau\cdot(1 - d - d)$$

eq (13)

#61: $$pa\_u\cdot\left(-\frac{pa\_u - pb\_u - \tau}{2\cdot\tau}\right) = pb\_u - \tau\cdot(1 - d - d)$$

deriving eq (14)

#62: $$\text{SOLVE}\left(\left[pb\_u\cdot\left(1 - -\frac{pa\_u - pb\_u - \tau}{2\cdot\tau}\right) = pa\_u - \tau\cdot(1 - d - d),\ pa\_u\cdot\left(-\frac{pa\_u - pb\_u - \tau}{2\cdot\tau}\right) = pb\_u - \tau\cdot(1 - d - d)\right], [pa\_u, pb\_u]\right)$$

#63: $$\left[pa\_u = 2\cdot\tau\cdot(1 - 2\cdot d) \wedge pb\_u = 2\cdot\tau\cdot(1 - 2\cdot d),\ pa\_u = \frac{\tau\cdot(\sqrt{(1 - 8\cdot d)} + 3)}{2} \wedge pb\_u = \frac{\tau\cdot(3 - \sqrt{(1 - 8\cdot d)})}{2},\ pa\_u = \frac{\tau\cdot(3 - \sqrt{(1 - 8\cdot d)})}{2} \wedge pb\_u = \frac{\tau\cdot(\sqrt{(1 - 8\cdot d)} + 3)}{2}\right]$$

#64: $$pa\_u = 2\cdot\tau\cdot(1 - 2\cdot d) \wedge pb\_u = 2\cdot\tau\cdot(1 - 2\cdot d)$$

#65: $$xhat\_u = \frac{1}{2}$$

#66: $$profita\_u = profitb\_u = (2\cdot\tau\cdot(1 - 2\cdot d))\cdot\frac{1}{2}$$

#67: $$profita\_u = profitb\_u = \tau\cdot(1 - 2\cdot d)$$

eq (15) marginal conditions from #55 and #56

#68: $$\frac{d}{d\ pa}\left(profita\_u = pa\cdot\frac{\tau\cdot(d - d + 1) - pa + 2\cdot\tau\cdot(1 - 2\cdot d)}{2\cdot\tau}\right)$$

#69: $$- \frac{4\cdot d\cdot \tau + 2\cdot pa - 3\cdot \tau}{2\cdot \tau}$$

substitute UPE pa yields

#70: $$- \frac{4\cdot d\cdot \tau + 2\cdot (2\cdot \tau\cdot (1 - 2\cdot d)) - 3\cdot \tau}{2\cdot \tau}$$

#71: $$\frac{4\cdot d - 1}{2}$$

#72: $$\frac{d}{d\ pb}\left(profitb\_u = - \frac{pb\cdot (4\cdot d\cdot \tau + pb - 3\cdot \tau)}{2\cdot \tau}\right)$$

#73: $$\frac{4\cdot d - 1}{2}$$

$\geq$ 0 for d>1/4

#74: $$\tau\cdot (1 - 2\cdot d)$$

#75: $$\frac{d}{d\ pb}\left(profitb\_u = pb\cdot \left(1 - \frac{\tau\cdot (d - d + 1) - 2\cdot \tau\cdot (1 - 2\cdot d) + pb}{2\cdot \tau}\right)\right)$$

#76: $$- \frac{4\cdot d\cdot \tau + 2\cdot pb - 3\cdot \tau}{2\cdot \tau}$$

#77: $$\frac{4\cdot d - 1}{2}$$

$\geq$ 0 for d>1/4

*** section 6: UPE under asymmetry a ≠ b

This section is computed using R

*** section proving a=0.9 and b=0 is not an UPEG. eq (17) and (18) in paper

#78: $$pb\cdot\left(1 - \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right) = pa - \tau\cdot(1 - b - a)$$

#79: $$pa\cdot\frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau} = pb - \tau\cdot(1 - b - a)$$

#80: $$pb\cdot\left(1 - \frac{\tau\cdot(0.9 - 0 + 1) - pa + pb}{2\cdot\tau}\right) = pa - \tau\cdot(1 - 0 - 0.9)$$

#81: $$pa\cdot\frac{\tau\cdot(0.9 - 0 + 1) - pa + pb}{2\cdot\tau} = pb - \tau\cdot(1 - 0 - 0.9)$$

#82: $$\text{SOLVE}\left(\left[pb\cdot\left(1 - \frac{\tau\cdot(0.9 - 0 + 1) - pa + pb}{2\cdot\tau}\right) = pa - \tau\cdot(1 - 0 - 0.9),\ pa\cdot\frac{\tau\cdot(0.9 - 0 + 1) - pa + pb}{2\cdot\tau} = pb - \tau\cdot(1 - 0 - 0.9)\right], [pa, pb]\right)$$

#83: $$\left[pa = 2\cdot\tau \wedge pb = \frac{21\cdot\tau}{20} + \frac{\sqrt{1079}\cdot i\cdot\tau}{20},\ pa = 2\cdot\tau \wedge pb = \frac{21\cdot\tau}{20} - \frac{\sqrt{1079}\cdot i\cdot\tau}{20},\ pa = \frac{\tau}{10} \wedge pb = \frac{\tau}{5}\right]$$

#84: $$[\text{false}, \text{false}, pa = 0.1\cdot\tau \wedge pb = 0.2\cdot\tau]$$

#85: $$\frac{d}{d\ pb}\left(profitb\_u = pb\cdot\left(1 - \frac{\tau\cdot(a - b + 1) - pa + pb}{2\cdot\tau}\right)\right)$$

#86: $$- \frac{a\cdot\tau - b\cdot\tau - pa + 2\cdot pb - \tau}{2\cdot\tau}$$

#87: $$- \frac{0.9\cdot\tau - 0\cdot\tau - 0.1\cdot\tau + 2\cdot(0.2\cdot\tau) - \tau}{2\cdot\tau}$$

#88: $$- \frac{1}{10} < 0$$

which means that firm B can increase its profit by marginally lowering its price.