bank_liq_2023_7_23c.dfw => go to line 65: new model where interest is also bailed out

#1: CaseMode := Sensitive

#2: InputMode := Word

prob of bank failure

#3: $\phi :\in \text{Real } (0, 1)$

transp cost

#4: $\tau :\in \text{Real } (0, \infty)$

endogenous deposit rates

#5: $r1 :\in \text{Real } (0, \infty)$

#6: $r2 :\in \text{Real } (0, \infty)$

Uniform depositor density

#7: $n :\in \text{Real } (0, \infty)$

FDIC premium (potentially also policy variable)

#8: $\mu :\in \text{Real } (0, 1)$

required liquidty ratio (main policy variable)

#9: $q1 :\in \text{Real } (0, 1)$

#10: $q2 :\in \text{Real } (0, 1)$

net return on bank investment

#11: $r :\in \text{Real } (0, \infty)$

lobbying cost tuning parameter

#12: $\lambda :\in \text{Real}\ (0, \infty)$

Weights on banks in banking association objective function

#13: $\alpha :\in \text{Real}\ (0, 1)$

*** Section 2: The model

** subsection 2.1: Depositors

eq (1) depositor expeted utility (fully bailed out, no uncertainty, see footnote) Now, v1 < v2 (bank 1 is smaller than bank 2)

#14: $u1 = v1 + r1 - \tau \cdot x$

#15: $u2 = v2 + r2 - \tau \cdot (1 - x)$

#16: $v1 + r1 - \tau \cdot x = v2 + r2 - \tau \cdot (1 - x)$

eq (2)

#17: $\text{SOLVE}(v1 + r1 - \tau \cdot x = v2 + r2 - \tau \cdot (1 - x), x)$

#18: $$xhat = \frac{r1 - r2 + v1 - v2 + \tau}{2 \cdot \tau}$$

eq (3)

#19: $d1 = n \cdot xhat$

#20: $d2 = n \cdot (1 - xhat)$

#21: $$d1 = n \cdot \frac{r1 - r2 + v1 - v2 + \tau}{2 \cdot \tau}$$

#22: $$d2 = n\cdot\left(1 - \frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)$$

#23: $$d2 = - \frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}$$

** Subsection 2.3: Banks' profit. Eq (4)

bank does not fail, prob (1−ϕ)

#24: $profit1 = (1 - q1)\cdot d1\cdot r - d1\cdot r1 - \mu\cdot d1$

#25: $profit2 = (1 - q2)\cdot d2\cdot r - d2\cdot r2 - \mu\cdot d2$

bank profit (loss) when fails prob ϕ

#26: $profit1 = - \mu\cdot d1$

#27: $profit2 = - \mu\cdot d2$

*** Section 3: deposit rate competition

eq (6)

#28: $eprofit1 = (1 - \phi)\cdot((1 - q1)\cdot d1\cdot r - d1\cdot r1 - \mu\cdot d1) + \phi\cdot(- \mu\cdot d1)$

#29: $eprofit2 = (1 - \phi)\cdot((1 - q2)\cdot d2\cdot r - d2\cdot r2 - \mu\cdot d2) + \phi\cdot(- \mu\cdot d2)$

Equation (7) and Appendix B

#30: $$eprofit1 = (1 - \phi)\cdot\left((1 - q1)\cdot\left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\cdot r - \left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\cdot r1 -\right.$$

$$\mu\cdot\left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\Bigg) + \phi\cdot\left(-\ \mu\cdot\left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\right)\Bigg)$$

#31: $$eprofit2 = (1 - \phi)\cdot\left((1 - q2)\cdot\left(-\ \frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\cdot r - \left(-\ \frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\cdot r2 - \mu\cdot\left(-\ \frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\right) + \phi\cdot\left(-\ \mu\cdot\left(-\ \frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\right)$$

#32: $$\frac{d}{d\ r1}\left(eprofit1 = (1 - \phi)\cdot\left((1 - q1)\cdot\left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\cdot r - \left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\cdot r1 - \mu\cdot\left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\right) + \phi\cdot\left(-\ \mu\cdot\left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\right)\right)$$

#33: $$0 = \frac{n\cdot(q1\cdot r\cdot(\phi - 1) + r\cdot(1 - \phi) + 2\cdot r1\cdot(\phi - 1) + r2\cdot(1 - \phi) + v1\cdot(\phi - 1) + v2\cdot(1 - \phi) - \mu + \tau\cdot(\phi - 1))}{2\cdot\tau}$$

#34: $$0 = \frac{n\cdot(r\cdot(q1 - 1)\cdot(\phi - 1) + 2\cdot r1\cdot(\phi - 1) + r2\cdot(1 - \phi) + v1\cdot(\phi - 1) + v2\cdot(1 - \phi) - \mu + \tau\cdot(\phi - 1))}{2\cdot\tau}$$

#35: $$\frac{d}{d\ r1}\frac{d}{d\ r1}\left(eprofit1 = (1 - \phi)\cdot\left((1 - q1)\cdot\left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\cdot r - \left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\cdot r1 - \mu\cdot\left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\right) + \phi\cdot\left(-\mu\cdot\left(n\cdot\frac{r1 - r2 + v1 - v2 + \tau}{2\cdot\tau}\right)\right)\right)$$

#36: $$0 > \frac{n\cdot(\phi - 1)}{\tau}$$

#37: $$\frac{d}{d\ r2}\left(eprofit2 = (1 - \phi)\cdot\left((1 - q2)\cdot\left(-\frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\cdot r - \left(-\frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\cdot r2 - \mu\cdot\left(-\frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\right) + \phi\cdot\left(-\mu\cdot\left(-\frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\right)\right)$$

#38: $$0 = \frac{n\cdot(q2\cdot r\cdot(\phi - 1) + r\cdot(1 - \phi) + r1\cdot(1 - \phi) + 2\cdot r2\cdot(\phi - 1) + v1\cdot(1 - \phi) + v2\cdot(\phi - 1) - \mu + \tau\cdot(\phi - \sim}{2\cdot\tau}\sim$$

1))

#39: $$0 = \frac{n\cdot(r\cdot(q2 - 1)\cdot(\phi - 1) + r1\cdot(1 - \phi) + 2\cdot r2\cdot(\phi - 1) + v1\cdot(1 - \phi) + v2\cdot(\phi - 1) - \mu + \tau\cdot(\phi - 1))}{2\cdot\tau}$$

#40: $$\frac{d}{d\ r2}\frac{d}{d\ r2}\left(eprofit2 = (1 - \phi)\cdot\left((1 - q2)\cdot\left(- \frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\cdot r - \left(- \frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\cdot r2 - \mu\cdot\left(- \frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\right) + \phi\cdot\left(- \mu\cdot\left(- \frac{n\cdot(r1 - r2 + v1 - v2 - \tau)}{2\cdot\tau}\right)\right)\right)$$

#41: $$0 > \frac{n\cdot(\phi - 1)}{\tau}$$

#42: SOLVE$\Bigg(\Bigg[0 =$

$$\frac{n\cdot(r\cdot(q1 - 1)\cdot(\phi - 1) + 2\cdot r1\cdot(\phi - 1) + r2\cdot(1 - \phi) + v1\cdot(\phi - 1) + v2\cdot(1 - \phi) - \mu + \tau\cdot(\phi - 1))}{2\cdot\tau},$$

$0 =$

$$\left.\frac{n\cdot(r\cdot(q2-1)\cdot(\phi-1)+r1\cdot(1-\phi)+2\cdot r2\cdot(\phi-1)+v1\cdot(1-\phi)+v2\cdot(\phi-1)-\mu+\tau\cdot(\phi-1))}{2\cdot\tau}\right],$$

$$\left.[r1,\ r2]\right)$$

eq (7)

#43: $$\left[r1=\frac{2\cdot q1\cdot r\cdot(\phi-1)+q2\cdot r\cdot(\phi-1)+3\cdot r\cdot(1-\phi)+v1\cdot(\phi-1)+v2\cdot(1-\phi)-3\cdot(\mu+\tau\cdot(1-\phi))}{3\cdot(1-\phi)}\right.$$

$$\wedge\ r2=$$

$$\left.\frac{q1\cdot r\cdot(\phi-1)+2\cdot q2\cdot r\cdot(\phi-1)+3\cdot r\cdot(1-\phi)+v1\cdot(1-\phi)+v2\cdot(\phi-1)-3\cdot(\mu+\tau\cdot(1-\phi))}{3\cdot(1-\phi)}\right]$$

#44: $$\left[r1=\frac{r\cdot(\phi-1)\cdot(2\cdot q1+q2-3)+v1\cdot(\phi-1)+v2\cdot(1-\phi)-3\cdot(\mu+\tau\cdot(1-\phi))}{3\cdot(1-\phi)}\wedge r2=\right.$$

$$\left.\frac{r\cdot(q1+2\cdot q2-3)\cdot(\phi-1)+v1\cdot(1-\phi)+v2\cdot(\phi-1)-3\cdot(\mu+\tau\cdot(1-\phi))}{3\cdot(1-\phi)}\right]$$

eq (8) and assumption 4 to avoid corner xhat

#45: $$xhat=-\frac{q1\cdot r-q2\cdot r-v1+v2-3\cdot\tau}{6\cdot\tau}$$

#46: $$xhat=-\frac{-r\cdot\Delta q+\Delta v-3\cdot\tau}{6\cdot\tau}$$

#47: $$\text{xhat} = \frac{r\cdot\Delta q - \Delta v + 3\cdot\tau}{6\cdot\tau}$$

xhat > 0 if

#48: $- r\cdot\Delta q + \Delta v - 3\cdot\tau < 0$

#49: $\text{SOLVE}(- r\cdot\Delta q + \Delta v - 3\cdot\tau < 0, \Delta v)$

#50: $$\Delta v < r\cdot\Delta q + 3\cdot\tau$$

xhat < 0.5 if

#51: $$- \frac{- r\cdot\Delta q + \Delta v - 3\cdot\tau}{6\cdot\tau} < \frac{1}{2}$$

#52: $$\text{SOLVE}\left(- \frac{- r\cdot\Delta q + \Delta v - 3\cdot\tau}{6\cdot\tau} < \frac{1}{2}, \Delta v\right)$$

#53: $$\Delta v > r\cdot\Delta q$$

eq (9)

#54: $$r1 - r2 = \frac{r\cdot(\phi - 1)\cdot(2\cdot q1 + q2 - 3) + v1\cdot(\phi - 1) + v2\cdot(1 - \phi) - 3\cdot(\mu + \tau\cdot(1 - \phi))}{3\cdot(1 - \phi)} - \frac{r\cdot(q1 + 2\cdot q2 - 3)\cdot(\phi - 1) + v1\cdot(1 - \phi) + v2\cdot(\phi - 1) - 3\cdot(\mu + \tau\cdot(1 - \phi))}{3\cdot(1 - \phi)}$$

#55: $$r1 - r2 = - \frac{q1\cdot r - q2\cdot r + 2\cdot(v1 - v2)}{3}$$

#56: $$r1 - r2 = - \frac{- r\cdot\Delta q - 2\cdot\Delta v}{3}$$

#57: $$r1 - r2 = \frac{r\cdot\Delta q + 2\cdot\Delta v}{3}$$

eq (10) equilibrium expected profits

#58: eprofit1 =

$$\frac{n\cdot(1 - \phi)\cdot(q1^2\cdot r^2 - 2\cdot q1\cdot r\cdot(q2\cdot r + v1 - v2 + 3\cdot\tau) + q2^2\cdot r^2 + 2\cdot q2\cdot r\cdot(v1 - v2 + 3\cdot\tau) + v1^2 + 2\cdot v\sim}{18\cdot\tau}\sim$$

$$\frac{1\cdot(3\cdot\tau - v2) + (v2 - 3\cdot\tau)^2)}{}$$

#59: eprofit2 =

$$\frac{n\cdot(1 - \phi)\cdot(q1^2\cdot r^2 - 2\cdot q1\cdot r\cdot(q2\cdot r + v1 - v2 - 3\cdot\tau) + q2^2\cdot r^2 + 2\cdot q2\cdot r\cdot(v1 - v2 - 3\cdot\tau) + v1^2 - 2\cdot v\sim}{18\cdot\tau}\sim$$

$$\frac{1\cdot(v2 + 3\cdot\tau) + (v2 + 3\cdot\tau)^2)}{}$$

#60: $$\text{eprofit1} = \frac{n\cdot(1 - \phi)\cdot(q1\cdot r - q2\cdot r - v1 + v2 - 3\cdot\tau)^2}{18\cdot\tau}$$

#61: $$\text{eprofit2} = \frac{n\cdot(1 - \phi)\cdot(q1\cdot r - q2\cdot r - v1 + v2 + 3\cdot\tau)^2}{18\cdot\tau}$$

#62: $$\text{eprofit1} = \frac{n\cdot(1 - \phi)\cdot(- r\cdot\Delta q + \Delta v - 3\cdot\tau)^2}{18\cdot\tau}$$

#63: $$\text{eprofit2} = \frac{n\cdot(1 - \phi)\cdot(- r\cdot\Delta q + \Delta v + 3\cdot\tau)^2}{18\cdot\tau}$$

#64: $$\text{eprofit2} - \text{eprofit1} = \frac{n\cdot(1 - \phi)\cdot(- r\cdot\Delta q + \Delta v + 3\cdot\tau)^2}{18\cdot\tau} - \frac{n\cdot(1 - \phi)\cdot(- r\cdot\Delta q + \Delta v - 3\cdot\tau)^2}{18\cdot\tau}$$

#65: $$\text{eprofit2} - \text{eprofit1} = \frac{2\cdot n\cdot(\phi - 1)\cdot(r\cdot\Delta q - \Delta v)}{3}$$

Result 2 and Appendix C

#66: $$\frac{d}{d\ q1}\left(r1 = \frac{r\cdot(\phi - 1)\cdot(2\cdot q1 + q2 - 3) + v1\cdot(\phi - 1) + v2\cdot(1 - \phi) - 3\cdot(\mu + \tau\cdot(1 - \phi))}{3\cdot(1 - \phi)}\right)$$

#67: $$0 > - \frac{2 \cdot r}{3}$$

#68: $$\frac{d}{d\ q2} \left(r1 = \frac{r \cdot (\phi - 1) \cdot (2 \cdot q1 + q2 - 3) + v1 \cdot (\phi - 1) + v2 \cdot (1 - \phi) - 3 \cdot (\mu + \tau \cdot (1 - \phi))}{3 \cdot (1 - \phi)} \right)$$

#69: $$0 > - \frac{r}{3}$$

#70: $$\frac{d}{d\ q1} \left(eprofit1 = \frac{n \cdot (1 - \phi) \cdot (q1 \cdot r - q2 \cdot r - v1 + v2 - 3 \cdot \tau)^2}{18 \cdot \tau} \right)$$

#71: $$\frac{n \cdot r \cdot (1 - \phi) \cdot (q1 \cdot r - q2 \cdot r - v1 + v2 - 3 \cdot \tau)}{9 \cdot \tau}$$

#72: $$\frac{n \cdot r \cdot (1 - \phi) \cdot (- r \cdot \Delta q + \Delta v - 3 \cdot \tau)}{9 \cdot \tau} < 0$$

#73: $$\frac{d}{d\ q2} \left(eprofit1 = \frac{n \cdot (1 - \phi) \cdot (q1 \cdot r - q2 \cdot r - v1 + v2 - 3 \cdot \tau)^2}{18 \cdot \tau} \right)$$

#74: $$\frac{n \cdot r \cdot (\phi - 1) \cdot (q1 \cdot r - q2 \cdot r - v1 + v2 - 3 \cdot \tau)}{9 \cdot \tau}$$

#75: $$\frac{n \cdot r \cdot (\phi - 1) \cdot (- r \cdot \Delta q + \Delta v - 3 \cdot \tau)}{9 \cdot \tau} > 0$$

Result 3

#76: $$\frac{d}{d\phi}\left(r1 = \frac{r\cdot(\phi - 1)\cdot(2\cdot q1 + q2 - 3) + v1\cdot(\phi - 1) + v2\cdot(1 - \phi) - 3\cdot(\mu + \tau\cdot(1 - \phi))}{3\cdot(1 - \phi)}\right)$$

#77: $$0 > - \frac{\mu}{(\phi - 1)^2}$$

#78: $$\frac{d}{d\phi}\left(eprofit1 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot\Delta q + \Delta v - 3\cdot\tau)^2}{18\cdot\tau}\right)$$

#79: $$0 > - \frac{n\cdot(r\cdot\Delta q - \Delta v + 3\cdot\tau)^2}{18\cdot\tau}$$

#80: $$\frac{d}{dr}\left(r1 = \frac{r\cdot(\phi - 1)\cdot(2\cdot q1 + q2 - 3) + v1\cdot(\phi - 1) + v2\cdot(1 - \phi) - 3\cdot(\mu + \tau\cdot(1 - \phi))}{3\cdot(1 - \phi)}\right)$$

#81: $$0 < - \frac{2\cdot q1 + q2 - 3}{3}$$

#82: $$\frac{d}{dr}\left(eprofit1 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot\Delta q + \Delta v - 3\cdot\tau)^2}{18\cdot\tau}\right)$$

#83: $$0 < \frac{n\cdot\Delta q\cdot(1 - \phi)\cdot(r\cdot\Delta q - \Delta v + 3\cdot\tau)}{9\cdot\tau}$$

#84: $$\frac{d}{d\tau}\left(r1 = \frac{r\cdot(\phi - 1)\cdot(2\cdot q1 + q2 - 3) + v1\cdot(\phi - 1) + v2\cdot(1 - \phi) - 3\cdot(\mu + \tau\cdot(1 - \phi))}{3\cdot(1 - \phi)}\right)$$

#85: $0 > -1$

#86: $$\frac{d}{d\tau}\left(eprofit1 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot\Delta q + \Delta v - 3\cdot\tau)^2}{18\cdot\tau}\right)$$

#87: $$0 < \frac{n\cdot(\phi - 1)\cdot(r\cdot\Delta q - \Delta v + 3\cdot\tau)\cdot(r\cdot\Delta q - \Delta v - 3\cdot\tau)}{18\cdot\tau^2}$$

*** Section 4: Lobbying to reduce liquidity

eq (11)

#88: $l1 = \lambda\cdot qq1^2$

#89: $l2 = \lambda\cdot qq2^2$

from #61 above, (recall eq (10) in the paper)

#90: $$eprofit1 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot\Delta q + \Delta v - 3\cdot\tau)^2}{18\cdot\tau}$$

#91: $$eprofit2 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot\Delta q + \Delta v + 3\cdot\tau)^2}{18\cdot\tau}$$

eq (13) and (14)

#92: $$eb1 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot(q2 - qq2 - (q1 - qq1)) + \Delta v - 3\cdot\tau)^2}{18\cdot\tau} - \lambda\cdot qq1^2$$

#93: $$eb2 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot((q2 - qq2) - (q1 - qq1)) + \Delta v + 3\cdot\tau)^2}{18\cdot\tau} - \lambda\cdot qq2^2$$

Appendix E

#94: $$\frac{d}{d\ qq1}\left(eb1 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot(q2 - qq2 - (q1 - qq1)) + \Delta v - 3\cdot\tau)^2}{18\cdot\tau} - \lambda\cdot qq1^2\right)$$

eq (E.1)

#95: $$0 = \frac{n\cdot r\cdot(\phi - 1)\cdot(q1\cdot r - q2\cdot r - qq1\cdot r + qq2\cdot r + \Delta v - 3\cdot\tau) - 18\cdot qq1\cdot\lambda\cdot\tau}{9\cdot\tau}$$

#96: $$\frac{d}{d\ qq1}\frac{d}{d\ qq1}\left(eb1 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot(q2 - qq2 - (q1 - qq1)) + \Delta v - 3\cdot\tau)^2}{18\cdot\tau} - \lambda\cdot qq1^2\right)$$

#97: $$0 > - \frac{n\cdot r^2\cdot(\phi - 1) + 18\cdot\lambda\cdot\tau}{9\cdot\tau}$$

if [implied by Assumption 5 (which is stronger because of eq (17) and (18) and (21) below)]

#98: $$\lambda > \frac{n\cdot r^2\cdot(1 - \phi)}{18\cdot\tau}$$

#99: $$\frac{d}{d\ qq2}\left(eb2 = \frac{n\cdot(1-\phi)\cdot(-r\cdot((q2-qq2)-(q1-qq1))+\Delta v+3\cdot\tau)^2}{18\cdot\tau}-\lambda\cdot qq2^2\right)$$

eq (E.2)

#100: $$0 = -\frac{n\cdot r\cdot(\phi-1)\cdot(q1\cdot r-q2\cdot r-qq1\cdot r+qq2\cdot r+\Delta v+3\cdot\tau)+18\cdot qq2\cdot\lambda\cdot\tau}{9\cdot\tau}$$

#101: $$\frac{d}{d\ qq2}\frac{d}{d\ qq2}\left(eb2 = \frac{n\cdot(1-\phi)\cdot(-r\cdot((q2-qq2)-(q1-qq1))+\Delta v+3\cdot\tau)^2}{18\cdot\tau}-\lambda\cdot qq2^2\right)$$

#102: $$0 > -\frac{n\cdot r^2\cdot(\phi-1)+18\cdot\lambda\cdot\tau}{9\cdot\tau}$$

eq (15) BR function

#103: $$\mathrm{SOLVE}\left(0 = \frac{n\cdot r\cdot(\phi-1)\cdot(q1\cdot r-q2\cdot r-qq1\cdot r+qq2\cdot r+\Delta v-3\cdot\tau)-18\cdot qq1\cdot\lambda\cdot\tau}{9\cdot\tau}, qq1\right)$$

#104: $$qq1 = \frac{n\cdot r\cdot(\phi-1)\cdot(q1\cdot r-q2\cdot r+qq2\cdot r+\Delta v-3\cdot\tau)}{n\cdot r^2\cdot(\phi-1)+18\cdot\lambda\cdot\tau}$$

eq (16) BR function

#105: $$\mathrm{SOLVE}\left(0 = -\frac{n\cdot r\cdot(\phi-1)\cdot(q1\cdot r-q2\cdot r-qq1\cdot r+qq2\cdot r+\Delta v+3\cdot\tau)+18\cdot qq2\cdot\lambda\cdot\tau}{9\cdot\tau}, qq2\right)$$

#106: $$qq2 = \frac{n\cdot r\cdot(1 - \phi)\cdot(q1\cdot r - q2\cdot r - qq1\cdot r + \Delta v + 3\cdot\tau)}{n\cdot r^2\cdot(\phi - 1) + 18\cdot\lambda\cdot\tau}$$

finding slopes of the BR functions

#107: $$\frac{d}{d\ qq2}\left(qq1 = \frac{n\cdot r\cdot(\phi - 1)\cdot(q1\cdot r - q2\cdot r + qq2\cdot r + \Delta v - 3\cdot\tau)}{n\cdot r^2\cdot(\phi - 1) + 18\cdot\lambda\cdot\tau}\right)$$

#108: $$0 > \frac{n\cdot r^2\cdot(\phi - 1)}{n\cdot r^2\cdot(\phi - 1) + 18\cdot\lambda\cdot\tau}$$

denominator > 0 if [implied by Assumption 5]

#109: $n\cdot r^2\cdot(\phi - 1) + 18\cdot\lambda\cdot\tau > 0$

#110: $\mathrm{SOLVE}(n\cdot r^2\cdot(\phi - 1) + 18\cdot\lambda\cdot\tau > 0, \lambda)$

#111: $$\lambda > \frac{n\cdot r^2\cdot(1 - \phi)}{18\cdot\tau}$$

eq (17) and (18)

#112: $$\mathrm{SOLVE}\left(\left[0 = \frac{n\cdot r\cdot(\phi - 1)\cdot(q1\cdot r - q2\cdot r - qq1\cdot r + qq2\cdot r + \Delta v - 3\cdot\tau) - 18\cdot qq1\cdot\lambda\cdot\tau}{9\cdot\tau}, 0 = -\right.\right.$$

$$\left.\frac{n\cdot r\cdot(\phi - 1)\cdot(q1\cdot r - q2\cdot r - qq1\cdot r + qq2\cdot r + \Delta v + 3\cdot\tau) + 18\cdot qq2\cdot\lambda\cdot\tau}{9\cdot\tau}\right], [qq1, qq2]\Big)$$

#113: $$\left[qq1 = \frac{n\cdot r\cdot(1 - \phi)\cdot(n\cdot r^2\cdot(\phi - 1) - 3\cdot\lambda\cdot(q1\cdot r - q2\cdot r + \Delta v - 3\cdot\tau))}{6\cdot\lambda\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)} \wedge qq2 = \frac{n\cdot r\cdot(1 - \phi)\cdot(n\cdot r^2\cdot(\phi - 1) + 3\cdot\lambda\cdot(q1\cdot r - q2\cdot r + \Delta v + 3\cdot\tau))}{6\cdot\lambda\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)}\right]$$

#114: $$qq1 = \frac{n\cdot r\cdot(1 - \phi)\cdot(n\cdot r^2\cdot(\phi - 1) - 3\cdot\lambda\cdot(- r\cdot\Delta q + \Delta v - 3\cdot\tau))}{6\cdot\lambda\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)}$$

#115: $$qq2 = \frac{n\cdot r\cdot(1 - \phi)\cdot(n\cdot r^2\cdot(\phi - 1) + 3\cdot\lambda\cdot(- r\cdot\Delta q + \Delta v + 3\cdot\tau))}{6\cdot\lambda\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)}$$

denominator > 0 if [Assumption 5]

#116: $n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau > 0$

#117: $\mathrm{SOLVE}(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau > 0, \lambda)$

#118: $$\lambda > \frac{n\cdot r^2\cdot(1 - \phi)}{9\cdot\tau}$$

eq (19)

#119: $$qq2 - qq1 = \frac{n\cdot r\cdot(1 - \phi)\cdot(n\cdot r^2\cdot(\phi - 1) + 3\cdot\lambda\cdot(- r\cdot\Delta q + \Delta v + 3\cdot\tau))}{6\cdot\lambda\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)} - \frac{n\cdot r\cdot(1 - \phi)\cdot(n\cdot r^2\cdot(\phi - 1) - 3\cdot\lambda\cdot(- r\cdot\Delta q + \Delta v - 3\cdot\tau))}{6\cdot\lambda\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)}$$

#120: $$qq2 - qq1 = \frac{n\cdot r\cdot(\phi - 1)\cdot(r\cdot\Delta q - \Delta v)}{n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau}$$

*** Section 5: Lobbying by bank assocations
new version with weights and unqual membership fees

eq (20)

#121: $ea = \alpha\cdot eprofit1 + (1 - \alpha)\cdot eprofit2 - f1 - f2$

where

#122: $f1 + f2 = \lambda\cdot qq1^2 + \lambda\cdot qq2^2$

$$\#123:\ \text{eprofit1} = \frac{n\cdot(1 - \phi)\cdot(q1\cdot r - q2\cdot r - v1 + v2 - 3\cdot\tau)^2}{18\cdot\tau}$$

$$\#124:\ \text{eprofit2} = \frac{n\cdot(1 - \phi)\cdot(q1\cdot r - q2\cdot r - v1 + v2 + 3\cdot\tau)^2}{18\cdot\tau}$$

$$\#125:\ \text{eprofit1} = \frac{n\cdot(1 - \phi)\cdot((q1 - qq1)\cdot r - (q2 - qq2)\cdot r + \Delta v - 3\cdot\tau)^2}{18\cdot\tau}$$

$$\#126:\ \text{eprofit2} = \frac{n\cdot(1 - \phi)\cdot((q1 - qq1)\cdot r - (q2 - qq2)\cdot r + \Delta v + 3\cdot\tau)^2}{18\cdot\tau}$$

eq (20)

$$\#127:\ ea = \alpha\cdot\frac{n\cdot(1 - \phi)\cdot((q1 - qq1)\cdot r - (q2 - qq2)\cdot r + \Delta v - 3\cdot\tau)^2}{18\cdot\tau} + (1 - \alpha)\cdot\frac{n\cdot(1 - \phi)\cdot((q1 - qq1)\cdot r - (q2 - qq2)\cdot r + \Delta v + 3\cdot\tau)^2}{18\cdot\tau} - \lambda\cdot qq1^2 - \lambda\cdot qq2^2$$

Appendix F

$$\#128:\ \frac{d}{d\ qq1}\left(ea = \alpha\cdot\frac{n\cdot(1 - \phi)\cdot((q1 - qq1)\cdot r - (q2 - qq2)\cdot r + \Delta v - 3\cdot\tau)^2}{18\cdot\tau} + (1 - \right.$$

$$\alpha)\cdot\frac{n\cdot(1-\phi)\cdot((q1-qq1)\cdot r-(q2-qq2)\cdot r+\Delta v+3\cdot\tau)^2}{18\cdot\tau}-\lambda\cdot qq1^2-\lambda\cdot qq2^2\Bigg)$$

#129: $$0=\frac{n\cdot r\cdot(\phi-1)\cdot(q1\cdot r-q2\cdot r-qq1\cdot r+qq2\cdot r+\Delta v-3\cdot\tau\cdot(2\cdot\alpha-1))-18\cdot qq1\cdot\lambda\cdot\tau}{9\cdot\tau}$$

#130: $$\frac{d}{d\ qq1}\frac{d}{d\ qq1}\Bigg(ea=\alpha\cdot\frac{n\cdot(1-\phi)\cdot((q1-qq1)\cdot r-(q2-qq2)\cdot r+\Delta v-3\cdot\tau)^2}{18\cdot\tau}+(1-$$

$$\alpha)\cdot\frac{n\cdot(1-\phi)\cdot((q1-qq1)\cdot r-(q2-qq2)\cdot r+\Delta v+3\cdot\tau)^2}{18\cdot\tau}-\lambda\cdot qq1^2-\lambda\cdot qq2^2\Bigg)$$

#131: $$0>-\frac{n\cdot r^2\cdot(\phi-1)+18\cdot\lambda\cdot\tau}{9\cdot\tau}$$

if [implied by Assumption 5]

#132: $n\cdot r^2\cdot(\phi-1)+18\cdot\lambda\cdot\tau>0$

#133: SOLVE($n\cdot r^2\cdot(\phi-1)+18\cdot\lambda\cdot\tau>0$, λ)

#134: $$\lambda>\frac{n\cdot r^2\cdot(1-\phi)}{18\cdot\tau}$$

#135: $$\frac{d}{d\ qq2}\left(ea = \alpha\cdot\frac{n\cdot(1-\phi)\cdot((q1-qq1)\cdot r-(q2-qq2)\cdot r+\Delta v-3\cdot\tau)^2}{18\cdot\tau} + (1-\alpha)\cdot\frac{n\cdot(1-\phi)\cdot((q1-qq1)\cdot r-(q2-qq2)\cdot r+\Delta v+3\cdot\tau)^2}{18\cdot\tau} - \lambda\cdot qq1^2 - \lambda\cdot qq2^2\right)$$

#136: $$0 = -\frac{n\cdot r\cdot(\phi-1)\cdot(q1\cdot r-q2\cdot r-qq1\cdot r+qq2\cdot r+\Delta v-3\cdot\tau\cdot(2\cdot\alpha-1))+18\cdot qq2\cdot\lambda\cdot\tau}{9\cdot\tau}$$

#137: $$\frac{d}{d\ qq2}\frac{d}{d\ qq2}\left(ea = \alpha\cdot\frac{n\cdot(1-\phi)\cdot((q1-qq1)\cdot r-(q2-qq2)\cdot r+\Delta v-3\cdot\tau)^2}{18\cdot\tau} + (1-\alpha)\cdot\frac{n\cdot(1-\phi)\cdot((q1-qq1)\cdot r-(q2-qq2)\cdot r+\Delta v+3\cdot\tau)^2}{18\cdot\tau} - \lambda\cdot qq1^2 - \lambda\cdot qq2^2\right)$$

#138: $$0 > -\frac{n\cdot r^2\cdot(\phi-1)+18\cdot\lambda\cdot\tau}{9\cdot\tau}$$

#139: $$\mathrm{SOLVE}\left(\left[0 = \frac{n\cdot r\cdot(\phi-1)\cdot(q1\cdot r-q2\cdot r-qq1\cdot r+qq2\cdot r+\Delta v-3\cdot\tau\cdot(2\cdot\alpha-1))-18\cdot qq1\cdot\lambda\cdot\tau}{9\cdot\tau},\ 0 = -\frac{n\cdot r\cdot(\phi-1)\cdot(q1\cdot r-q2\cdot r-qq1\cdot r+qq2\cdot r+\Delta v-3\cdot\tau\cdot(2\cdot\alpha-1))+18\cdot qq2\cdot\lambda\cdot\tau}{9\cdot\tau}\right], [qq1, qq2]\right)$$

eq (21)

#140: $$\left[qq1 = \frac{n\cdot r\cdot(\phi - 1)\cdot(q1\cdot r - q2\cdot r + \Delta v - 3\cdot\tau\cdot(2\cdot\alpha - 1))}{2\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)} \wedge qq2 = \frac{n\cdot r\cdot(1 - \phi)\cdot(q1\cdot r - q2\cdot r + \Delta v - 3\cdot\tau\cdot(2\cdot\alpha - 1))}{2\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)}\right]$$

The denominator > 0 if [Assumption 5]

#141: $n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau > 0$

#142: $\text{SOLVE}(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau > 0, \lambda)$

#143: $$\lambda > \frac{n\cdot r^2\cdot(1 - \phi)}{9\cdot\tau}$$

The difference below is not used in the paper.

#144: $$qq2 - qq1 = \frac{n\cdot r\cdot(1 - \phi)\cdot(q1\cdot r - q2\cdot r + \Delta v - 3\cdot\tau\cdot(2\cdot\alpha - 1))}{2\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)} - \frac{n\cdot r\cdot(\phi - 1)\cdot(q1\cdot r - q2\cdot r + \Delta v - 3\cdot\tau\cdot(2\cdot\alpha - 1))}{2\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)}$$

#145: $$qq2 - qq1 = \frac{n\cdot r\cdot(1 - \phi)\cdot(q1\cdot r - q2\cdot r + \Delta v - 3\cdot\tau\cdot(2\cdot\alpha - 1))}{n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau}$$

= 0 if

#146: $q1\cdot r - q2\cdot r + \Delta v - 3\cdot\tau\cdot(2\cdot\alpha - 1) = 0$

#147: SOLVE($q1\cdot r - q2\cdot r + \Delta v - 3\cdot\tau\cdot(2\cdot\alpha - 1) = 0, \alpha$)

#148: $$\alpha = \frac{q1\cdot r - q2\cdot r + \Delta v + 3\cdot\tau}{6\cdot\tau}$$

Problem! Sum is zero

#149: $$qq1 + qq2 = \frac{n\cdot r\cdot(\phi - 1)\cdot(q1\cdot r - q2\cdot r + \Delta v - 3\cdot\tau\cdot(2\cdot\alpha - 1))}{2\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)} + \frac{n\cdot r\cdot(1 - \phi)\cdot(q1\cdot r - q2\cdot r + \Delta v - 3\cdot\tau\cdot(2\cdot\alpha - 1))}{2\cdot(n\cdot r^2\cdot(\phi - 1) + 9\cdot\lambda\cdot\tau)}$$

#150: $$qq1 + qq2 = 0$$

** Subsection 6.2: Multiple banks (Salop's model) [note: subection 6.1 is derived after 6.2 on this file]

eq (25)

#151: $u1 = v + r1 - \tau\cdot x$

#152: $u2 = v + r2 - \tau\cdot\left(\frac{1}{b} - x\right)$

#153: $ub = v + rb - \tau\cdot\left(\frac{1}{b} - x\right)$

#154: $v + r1 - \tau\cdot x = v + r2 - \tau\cdot\left(\frac{1}{b} - x\right)$

#155: $\mathrm{SOLVE}\left(v + r1 - \tau\cdot x = v + r2 - \tau\cdot\left(\frac{1}{b} - x\right), x\right)$

eq (26)

#156: $$x12 = \frac{b\cdot(r1 - r2) + \tau}{2\cdot b\cdot\tau}$$

#157: $v + r1 - \tau\cdot\left(\frac{1}{b} - x\right) = v + rb - \tau\cdot x$

#158: $\mathrm{SOLVE}\left(v + r1 - \tau\cdot\left(\frac{1}{b} - x\right) = v + rb - \tau\cdot x, x\right)$

#159: $$xb1 = \frac{\tau - b\cdot(r1 - rb)}{2\cdot b\cdot\tau}$$

eq (27)

#160: $d1 = n\cdot\left(x12 + \frac{1}{b} - xb1\right)$

#161: $d1 = n\cdot\left(\frac{b\cdot(r1 - r2) + \tau}{2\cdot b\cdot\tau} + \frac{1}{b} - \frac{\tau - b\cdot(r1 - rb)}{2\cdot b\cdot\tau}\right)$

#162: $$d1 = \frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}$$

eq (29)

#163: $\text{profit1} = (1 - \phi)\cdot((1 - (q1 - qq1))\cdot d1\cdot R - d1\cdot r1) - \mu\cdot d1 - \lambda\cdot qq1^{2}$

Appenix G

#164: $$\text{profit1} = (1 - \phi)\cdot\left((1 - (q1 - qq1))\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot R - \frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot r1\right) - \mu\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau} - \lambda\cdot qq1^{2}$$

#165: $$\frac{d}{d\ r1}\left(\text{profit1} = (1 - \phi)\cdot\left((1 - (q1 - qq1))\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot R - \frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot r1\right) - \mu\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau} - \lambda\cdot qq1^{2}\right)$$

eq (G.1)

#166: $$0 = \frac{n\cdot(2\cdot R\cdot b\cdot(q1 - qq1 - 1)\cdot(\phi - 1) + b\cdot(4\cdot r1\cdot(\phi - 1) + r2\cdot(1 - \phi) + rb\cdot(1 - \phi) - 2\cdot\mu) + 2\cdot\tau\cdot(\phi - 1\sim}{2\cdot b\cdot\tau}\sim$$

$$\underline{))}$$

#167: $$\frac{d}{d\ r1}\frac{d}{d\ r1}\left(profit1 = (1 - \phi)\cdot\left((1 - (q1 - qq1))\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot R - \frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot r1\right) - \mu\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau} - \lambda\cdot qq1^2\right)$$

eq (G.3)

#168: $$0 > \frac{2\cdot n\cdot(\phi - 1)}{\tau}$$

#169: $$\frac{d}{d\ qq1}\left(profit1 = (1 - \phi)\cdot\left((1 - (q1 - qq1))\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot R - \frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot r1\right) - \mu\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau} - \lambda\cdot qq1^2\right)$$

eq (G.2)

#170: $$0 = - \frac{R\cdot n\cdot(\phi - 1)\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau) + 4\cdot b\cdot qq1\cdot\lambda\cdot\tau}{2\cdot b\cdot\tau}$$

#171: $$\frac{d}{d\ qq1}\frac{d}{d\ qq1}\left(profit1 = (1 - \phi)\cdot\left((1 - (q1 - qq1))\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot R - \frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot r1\right) - \mu\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau} - \lambda\cdot qq1^2\right)$$

eq (G.3) 2nd part

#172: $$0 > - 2\cdot\lambda$$

cross derivative

$$\#173: \frac{d}{d\ r1}\frac{d}{d\ qq1}\left(profit1 = (1 - \phi)\cdot\left((1 - (q1 - qq1))\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot R - \frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau}\cdot r1\right) - \mu\cdot\frac{n\cdot(b\cdot(2\cdot r1 - r2 - rb) + 2\cdot\tau)}{2\cdot b\cdot\tau} - \lambda\cdot qq1^2\right)$$

$$\#174: \frac{R\cdot n\cdot(1 - \phi)}{\tau}$$

eq (G.4) Hessian

$$\#175: H = \frac{2\cdot n\cdot(\phi - 1)}{\tau}\cdot(- 2\cdot\lambda) - \left(\frac{R\cdot n\cdot(1 - \phi)}{\tau}\right)^2$$

$$\#176: H = \frac{4\cdot n\cdot\lambda\cdot(1 - \phi)}{\tau} - \frac{R^2\cdot n^2\cdot(\phi - 1)^2}{\tau^2}$$

> 0 if

$$\#177: \frac{4\cdot n\cdot\lambda\cdot(1 - \phi)}{\tau} - \frac{R^2\cdot n^2\cdot(\phi - 1)^2}{\tau^2} > 0$$

$$\#178: \mathrm{SOLVE}\left(\frac{4\cdot n\cdot\lambda\cdot(1 - \phi)}{\tau} - \frac{R^2\cdot n^2\cdot(\phi - 1)^2}{\tau^2} > 0, \lambda\right)$$

#179:
$$\lambda > \frac{R^2 \cdot n \cdot (1 - \phi)}{4 \cdot \tau}$$

Which is Assumption 9.

#180:
$$\mathrm{SOLVE}\left(\left[0 = \frac{n \cdot (2 \cdot R \cdot b \cdot (q1 - qq1 - 1) \cdot (\phi - 1) + b \cdot (4 \cdot r1 \cdot (\phi - 1) + r2 \cdot (1 - \phi) + rb \cdot (1 - \phi) - 2 \cdot \mu) + 2 \cdot \tau \cdot (\phi - 1))}{2 \cdot b \cdot \tau}, 0 = - \frac{R \cdot n \cdot (\phi - 1) \cdot (b \cdot (2 \cdot r1 - r2 - rb) + 2 \cdot \tau) + 4 \cdot b \cdot qq1 \cdot \lambda \cdot \tau}{2 \cdot b \cdot \tau}\right], [qq1, r1]\right)$$

eq (30)

#181:
$$\left[qq1 = \frac{R \cdot n \cdot (2 \cdot R \cdot b \cdot (q1 - 1) \cdot (\phi - 1) + b \cdot (r2 \cdot (\phi - 1) + rb \cdot (\phi - 1) - 2 \cdot \mu) + 2 \cdot \tau \cdot (1 - \phi))}{2 \cdot b \cdot (R^2 \cdot n \cdot (\phi - 1) + 4 \cdot \lambda \cdot \tau)} \wedge r1 = \frac{R^2 \cdot n \cdot (\phi - 1)^2 \cdot (b \cdot (r2 + rb) - 2 \cdot \tau) + 4 \cdot R \cdot b \cdot \lambda \cdot \tau \cdot (1 - \phi) \cdot (q1 - 1) + 2 \cdot \lambda \cdot \tau \cdot (b \cdot (r2 \cdot (\phi - 1) + rb \cdot (\phi - 1) + 2 \cdot \mu) + 2 \cdot \tau \cdot (1 - \phi))}{2 \cdot b \cdot (\phi - 1) \cdot (R^2 \cdot n \cdot (\phi - 1) + 4 \cdot \lambda \cdot \tau)}\right]$$

eq (31) Result 7

#182: $$\frac{d}{db}\left(qq1 = \frac{R\cdot n\cdot(2\cdot R\cdot b\cdot(q1 - 1)\cdot(\phi - 1) + b\cdot(r2\cdot(\phi - 1) + rb\cdot(\phi - 1) - 2\cdot\mu) + 2\cdot\tau\cdot(1 - \phi))}{2\cdot b\cdot(R^{2}\cdot n\cdot(\phi - 1) + 4\cdot\lambda\cdot\tau)}\right)$$

#183: $$0 > \frac{R\cdot n\cdot\tau\cdot(\phi - 1)}{b^{2}\cdot(R^{2}\cdot n\cdot(\phi - 1) + 4\cdot\lambda\cdot\tau)}$$

** Section 6.1: Unequal lobbying cost [no appendix, just stating results]

#184: $$eb1 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot(q2 - qq2 - (q1 - qq1)) + \Delta v - 3\cdot\tau)^{2}}{18\cdot\tau} - \lambda1\cdot qq1^{2}$$

#185: $$eb2 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot((q2 - qq2) - (q1 - qq1)) + \Delta v + 3\cdot\tau)^{2}}{18\cdot\tau} - \lambda2\cdot qq2^{2}$$

#186: $$\frac{d}{d\ qq1}\left(eb1 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot(q2 - qq2 - (q1 - qq1)) + \Delta v - 3\cdot\tau)^{2}}{18\cdot\tau} - \lambda1\cdot qq1^{2}\right)$$

#187: $$0 = \frac{n\cdot r\cdot(\phi - 1)\cdot(q1\cdot r - q2\cdot r - qq1\cdot r + qq2\cdot r + \Delta v - 3\cdot\tau) - 18\cdot qq1\cdot\lambda1\cdot\tau}{9\cdot\tau}$$

#188: $$\frac{d}{d\ qq1}\frac{d}{d\ qq1}\left(eb1 = \frac{n\cdot(1 - \phi)\cdot(- r\cdot(q2 - qq2 - (q1 - qq1)) + \Delta v - 3\cdot\tau)^{2}}{18\cdot\tau} - \lambda1\cdot qq1^{2}\right)$$

#189: $$0 > -\frac{n\cdot r^2\cdot(\phi - 1) + 18\cdot\lambda 1\cdot\tau}{9\cdot\tau}$$

if

#190: $n\cdot r^2\cdot(\phi - 1) + 18\cdot\lambda 1\cdot\tau > 0$

#191: $\mathrm{SOLVE}(n\cdot r^2\cdot(\phi - 1) + 18\cdot\lambda 1\cdot\tau > 0, \lambda 1)$

#192: $$\lambda 1 > \frac{n\cdot r^2\cdot(1 - \phi)}{18\cdot\tau}$$

by Assumption 7, eq (22)

#193: $$\frac{d}{d\ qq2}\left(eb2 = \frac{n\cdot(1 - \phi)\cdot(-\ r\cdot((q2 - qq2) - (q1 - qq1)) + \Delta v + 3\cdot\tau)^2}{18\cdot\tau} - \lambda 2\cdot qq2^2\right)$$

#194: $$0 = -\frac{n\cdot r\cdot(\phi - 1)\cdot(q1\cdot r - q2\cdot r - qq1\cdot r + qq2\cdot r + \Delta v + 3\cdot\tau) + 18\cdot qq2\cdot\lambda 2\cdot\tau}{9\cdot\tau}$$

#195: $$\frac{d}{d\ qq2}\frac{d}{d\ qq2}\left(eb2 = \frac{n\cdot(1 - \phi)\cdot(-\ r\cdot((q2 - qq2) - (q1 - qq1)) + \Delta v + 3\cdot\tau)^2}{18\cdot\tau} - \lambda 2\cdot qq2^2\right)$$

#196: $$0 > -\frac{n\cdot r^2\cdot(\phi - 1) + 18\cdot\lambda 2\cdot\tau}{9\cdot\tau}$$

if

#197: $n \cdot r^2 \cdot (\phi - 1) + 18 \cdot \lambda 2 \cdot \tau > 0$

#198: $\mathrm{SOLVE}(n \cdot r^2 \cdot (\phi - 1) + 18 \cdot \lambda 2 \cdot \tau > 0, \lambda 2)$

#199: $$\lambda 2 > \frac{n \cdot r^2 \cdot (1 - \phi)}{18 \cdot \tau}$$

by Assumption 7, eq (22)

#200: $$\mathrm{SOLVE}\left(\left[0 = \frac{n \cdot r \cdot (\phi - 1) \cdot (q1 \cdot r - q2 \cdot r - qq1 \cdot r + qq2 \cdot r + \Delta v - 3 \cdot \tau) - 18 \cdot qq1 \cdot \lambda 1 \cdot \tau}{9 \cdot \tau}, 0 = - \frac{n \cdot r \cdot (\phi - 1) \cdot (q1 \cdot r - q2 \cdot r - qq1 \cdot r + qq2 \cdot r + \Delta v + 3 \cdot \tau) + 18 \cdot qq2 \cdot \lambda 2 \cdot \tau}{9 \cdot \tau}\right], [qq1, qq2]\right)$$

#201: $$\left[qq1 = \frac{n \cdot r \cdot (1 - \phi) \cdot (n \cdot r^2 \cdot (\phi - 1) - 3 \cdot \lambda 2 \cdot (q1 \cdot r - q2 \cdot r + \Delta v - 3 \cdot \tau))}{3 \cdot (n \cdot r^2 \cdot (\lambda 1 + \lambda 2) \cdot (\phi - 1) + 18 \cdot \lambda 1 \cdot \lambda 2 \cdot \tau)} \wedge qq2 = \frac{n \cdot r \cdot (1 - \phi) \cdot (n \cdot r^2 \cdot (\phi - 1) + 3 \cdot \lambda 1 \cdot (q1 \cdot r - q2 \cdot r + \Delta v + 3 \cdot \tau))}{3 \cdot (n \cdot r^2 \cdot (\lambda 1 + \lambda 2) \cdot (\phi - 1) + 18 \cdot \lambda 1 \cdot \lambda 2 \cdot \tau)}\right]$$

The denominators are > 0 by the 2nd part of Assumption 7, eq (22) because

#202: $n \cdot r^2 \cdot (\lambda 1 + \lambda 2) \cdot (\phi - 1) + 18 \cdot \lambda 1 \cdot \lambda 2 \cdot \tau > 0$

$$\#203:\ 18\cdot\lambda1\cdot\lambda2\cdot\tau > n\cdot r^2\cdot(\lambda1 + \lambda2)\cdot(1 - \phi)$$

$$\#204:\ \frac{18\cdot\lambda1\cdot\lambda2\cdot\tau}{\lambda1 + \lambda2} > n\cdot r^2\cdot(1 - \phi)$$

$$\#205:\ \frac{\lambda1\cdot\lambda2}{\lambda1 + \lambda2} > \frac{n\cdot r^2\cdot(1 - \phi)}{18\cdot\tau}$$