(1) [10 points According to the four-largest firm concentration index, industry A is more concentrated than industry B since

$$I_4^A = 40 + 15 + 15 + 15 = 85 > 78 = 45 + 11 + 11 + 11 = I_4^B$$
.

According to the Hirschman-Herfindahl concentration index, industry B is more concentrated than industry A since

$$I_{HH}^A = 40^2 + 4 \cdot 15^2 = 2500 < 2630 = 45^2 + 5 \cdot 11^2 = I_{HH}^B$$

Country	Firms						Concentration Index	
	1	2	3	4	5	6	I_4	$I_{ m HH}$
Albania	40%	15%	15%	15%	15%	0%	85	2500
Bolivia	45%	11%	11%	11%	11%	11%	78	2630

(2) [5 points] Section 2 of the 1914 Clayton Act states that price discrimination is unlawful if its effect is "to lessen competition or tend to create a monopoly...or to injure destroy or prevent competition." In addition, price differentials are also allowed to account for "differences in the cost of manufactures, sale or delivery."

This, in part, implies that price discrimination that does not reduce competition should not be viewed as illegal.

(3a) [10 points]

$$p_G = BR_G(p_F) = \begin{cases} p^L & \text{if } p_F = p^L \\ p^H & \text{if } p_F = p^M \\ p^M & \text{if } p_F = p^H \end{cases} \quad \text{and} \quad p_F = BR_F(p_G) = \begin{cases} p^L & \text{if } p_G = p^L \\ p^H & \text{if } p_F = p^M \\ p^M & \text{if } p_F = p^H \end{cases}$$

Therefore, there are three Nash equilibria:

$$\langle p_G, p_F \rangle = \langle p^H, p^M \rangle, \ \langle p_G, p_F \rangle = \langle p^M, p^H \rangle, \ \text{and} \ \langle p_G, p_F \rangle = \langle p^L, p^L \rangle.$$

(3b) [5 points] No, because
$$\pi_G(p^H,p^M)=250<300=\pi_G(p^H,p^H)$$
, but $\pi_F(p^H,p^M)=350>300=\pi_F(p^H,p^H)$.

(3c) [10 points] The equilibrium strategies are:

$$p_F = p^M$$
 and $p_G = BR_G(p_F) = egin{cases} p^L & \text{if } p_F = p^L \\ p^H & \text{if } p_F = p^M \\ p^M & \text{if } p_F = p^H. \end{cases}$

In this equilibrium $p_G = p^H$ and hence $\pi_F(p^M, p^H) = 350$ and $\pi_G(p^M, p^H) = 250$.

To prove that the above is a SPE, note that GM's strategy is its best-response function. Next, if Ford sets different prices then if

$$p_F = p^L \Longrightarrow p_F = p^L \Longrightarrow \pi_F(p^L, p^L) = 100 < 350$$

and if

$$p_F = p^H \Longrightarrow p_F = p^M \Longrightarrow \pi_F(p^H, p^M) = 250 < 350.$$

(4) [10 points] The direct demand function facing this monopoly is:

$$Q(p) = \begin{cases} 0 & \text{if } p > 500 \\ 1000 & \text{if } 300 500 \\ (500 - 100)1000 & \text{if } p = 500 \\ (300 - 100)4000 & \text{if } p = 300 \\ (200 - 100)9000 & \text{if } p = 200. \end{cases}$$

Therefore, the monopoly's profit-maximizing price is p=200 yielding a profit of $\pi=(200-100)9000=\$900,000$.

(5a) [15 points] In the second stage of this game, firm B solves

$$\max_{p_B} \pi_B = \left(12 - \frac{q_A}{3} - \frac{q_B}{3}\right) q_B - 0 \cdot q_B,$$

yielding B's best-response function

$$q_B(q_A) = 18 - \frac{q_A}{2}.$$

In the first stage, firm A solves

$$\max_{p_A} \pi_A = \left[12 - \frac{1}{3} q_A - \frac{1}{3} \left(18 - \frac{q_A}{2} \right) \right]$$

yielding $q_A=18$. Therefore $q_B=18-18/2=9$, Q=18+9=27, p=12-27/3=\$3. Hence, $\pi_A=(3-0)18=\$54$ and $\pi_B=(3-0)9=\$27$.

(5b) [5 points] We first compute the monopoly price from MR = 12 - 2Q/3 = c = 0 yielding $Q^m = 18$ hence $p^m = 12 - 18/3 = \$6$. Therefore, B's best response function (second stage) is

$$p_B(p_A) = BR_B(p_A) = \begin{cases} 6 & \text{if } p_A > 6\\ p_A - \epsilon & \text{if } 0 < p^A \le 6\\ 0 & \text{if } p_A = 0. \end{cases}$$

Notice that firm A is indifferent among all prices $p_A \ge 0$ since it makes zero profit regardless of which price it sets. Therefore, there are many equilibria consisting of the above B's best-response function and $p_A \ge 0$ (including $p_A = 0$, in which case $p_B = 0$.

To summarize the above analysis, any SPE takes the form of $p_A \ge 0$ and $p_B = BR_B(p_A)$ where the best-response function $BR_B(p_A)$ is defined above.

(5c) [5 points] We have already shown that the monopoly price (for firm A) is $p_A^m = \$3$. The monopoly price for firm B is computed from $MR = 12 - 2Q/3 = c_B = 4$ yielding $q_B^m = 12$ and hence $p_B^m = 12 - 12/3 = \$8$. Hence, B's best-response function is now given by

$$p_B(p_A) = \begin{cases} 8 & \text{if } p_A > 8 \\ p_A - \epsilon & \text{if } 4 < p^A \le 8 \\ 4 & \text{if } p_A \le 4. \end{cases}$$

Firm A sets its monopoly price $p_A = 4 - \epsilon$ and grabs the entire market.

To summarize the above analysis, the SPE strategies are: $p_A = 4 - \epsilon$ and $p_B = BR_B(p_A)$ where the best-response function $BR_B(p_A)$ is defined above.

(6) [10 points] In the market for nonstudents,

$$MR_N = p_N = \left[1 + \frac{1}{-3}\right] = c = 6 \Longrightarrow p_N = \$9.$$

$$MR_S = p_S = \left[1 + \frac{1}{-4}\right] = c = 6 \Longrightarrow p_N = \$8.$$

To find the amount of tickets sold to each group, solve

$$q_N = 7290 \cdot 9^{-3} = 10$$
 and $q_S = 40960 \cdot 8^{-4} = 10$ hence $Q = q_N + q_S = 20$.

(7a) [5 points] The monopoly price is $p^m = 10$. Hence, the best response function of the firms are

$$p_A(p_B) = \begin{cases} 10 & \text{if } p_B > 10 \\ p_B - \epsilon & \text{if } 2 < p_B \le 10 \\ 2 & \text{if } p_B \le 2 \end{cases} \quad \text{and} \quad p_B(p_A) = \begin{cases} 10 & \text{if } p_A > 10 \\ p_A - \epsilon & \text{if } 2 < p_A \le 10 \\ 2 & \text{if } p_A \le 2. \end{cases}$$

Hence, the unique Nash equilibrium is $p_A = p_B = \$2$.

(b) [10 points] Trigger price strategy of A is at each period τ

$$p_A^\tau = \begin{cases} 10 & \text{if } p_A^t = p_B^t = 10 \text{ for all } t = 1, 2, \dots, \tau - 1 \\ 2 & \text{otherwise.} \end{cases}$$

Trigger price strategy of B is at each period au

$$p_B^\tau = \begin{cases} 10 & \text{if } p_A^t = p_B^t = 10 \text{ for all } t = 1, 2, \dots, \tau - 1 \\ 2 & \text{otherwise}. \end{cases}$$

If firm A does not deviate, its discounted stream of profit is

$$\pi_A = \sum_{t=0}^{\infty} \rho^t (10 - 2) \frac{N}{2} = \frac{4N}{1 - \rho}.$$

Industrial Organization (Econ 431): Midterm exam (2008/05/20) Abbreviated Solution Page 4 of 4

If firm A deviates in period t=0, its discounted stream of profit is

$$\pi'_{A} = (10 - 2 - \epsilon)N + \rho \frac{0}{1 - \rho} \approx 8N.$$

Deviation is not profitable for firm A if $\pi_A \geq \pi_A'$ or

$$\frac{4N}{1-\rho} \ge 8N \quad \text{hence} \quad \rho \ge \frac{1}{2}.$$

Because firm B is identical to firm A, $\rho \geq 1/2$ is also sufficient for having firm B not deviating from the collusive price.