Optimal Liquidity Management and Bail-Out Policy in the Banking Industry

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Abstract

We characterize the profit-maximizing reserves held by a commercial bank, and the generated probability of a liquidity crisis, as a function of the penalty imposed by the central bank, the probability of depositors' liquidity needs, and the return on outside investment opportunities. We demonstrate that commercial banks do not fully internalize the social cost associated with the bail-out policy in the presence of correlation among the liquidity needs of individual depositors.

Key words: liquidity crisis, Optimal Reserve Ratio, Deposit Insurance

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1 Introduction

The literature on demand deposits has focused on liquidity crises generated by expectation-driven panics, but it has not offered any general method for calculating the probability of bank runs generated by a realization of liquidity needs by a large, but finite, number of depositors. Therefore, in this article we propose a method for calculating the probability of realizing a liquidity crisis and banks' optimal reserve ratio assuming that depositors face *real* liquidity needs as opposed to rumors or panics concerning a liquidity crisis. We associate liquidity crises with bank runs.

The existing banking literature views the depository institutions as "pools of liquidity" providing consumers with insurance against idiosyncratic liquidity shocks. In the influential model by Diamond and Dybvig (1983), banks provide liquidity to depositors who are, ex ante, uncertain about their inter-temporal preferences with respect to consumption sequences. They demonstrate how deposit contracts offer insurance to consumers and how such contracts can support a Pareto efficient allocation of risk. However, as they show, there exists a second, inefficient Nash equilibrium where the interaction between pessimistic depositor expectations generates a liquidity crisis. Such liquidity crisis confronting individual banks may trigger socially costly bank panics.

Against this background, most countries apply explicit or implicit deposit insurance policies as a mechanism for the elimination of inefficient Nash equilibria driven by pessimistic expectations. Despite the indisputable insurance benefits, empirical observations as well as theoretical research convincingly demonstrate how federal deposit insurance will encourage banks to engage in excessive risk taking and to keep lower levels of liquid reserves than what would be socially optimal (cf. Cooper and Ross, 1998). Consequently, researchers have systematically investigated mechanisms other than deposit insurance as instruments for reducing the instability of the banking system. Bhattacharya et al. (1998) categorize those regulatory measures. In addition, all policy commitments relative to distressed financial institutions face a severe time-consistency problem as governments and central banks seem to have an incentive of bailing out distressed financial institutions with the intention of eliminating potential contagion problems (e.g. Chen, 1999). Freixas (1999) investigates such bail-out policies.

A meaningful evaluation of all policy measures directed towards the banking industry rely on the knowledge of how ex-ante uncertain liquidity needs translate into probabilities of realizing a liquidity crisis and of how the characteristics of this transmission mechanism interacts with banks' optimal allocation of their portfolios between liquid low-yield assets and illiquid high-yield investments.

In this paper we delineate the bank's optimal management and characterize how the profit-maximizing reserves adjust to the interest rate applied by the central bank for liquidity provision to a bank facing a liquidity crisis. In particular, the optimal reserves are found to be an increasing function of the correlation between the liquidity shocks facing individual depositors. As Holmström and Tirole (1998) show, the private sector cannot satisfy its own liquidity needs when aggregate uncertainty dominates the liquidity shocks. In this paper we characterize the socially optimal penalty rate applied when public liquidity is provided. We find the socially optimal penalty rate to be increasing as a function of the correlation between the liquidity shocks facing depositors. Thus, the private banking industry fails to fully internalize the full social costs of public liquidity provision.

Our study is organized as follows. Section 2 presents our model and characterizes the bank's optimal liquidity management. In Section 3 we carry out a welfare analysis and delineate the socially optimal bail-out policy. Section 4 concludes.

2 The Model

Consider a three-period economy with one representative commercial bank and n depositors with known bounded liquidity needs distributions. Each depositor has d_i dollars to deposit. Therefore, the total amount of money deposited in the bank is $D = d_1 + \ldots + d_n$ dollars.

2.1 Timing

The economy operates in periods, $t = 0, 1, 2$. In period 0 consumer i, $i = 1, .., n$, makes a deposit of $\$d_i$, followed by the bank's decision of which reserve ratio to maintain and thereby which proportion of the accumulated deposits to allocate to an illiquid outside investment project. In period 1 depositors face uncertain liquidity needs, which may generate a liquidity crisis. In period 2 the bank collects the return on the outside investment project and pays a penalty to the central bank in the event that a liquidity crises (run on the bank) occurred in period 1.

2.2 The commercial bank

Let r, $0 \leq r \leq 1$, be the reserve ratio which is set by the commercial bank. The bank keeps rD as reserves. The remaining amount, $(1-r)D$, is invested into an outside investment project which bears a safe net return (gain) of $g > 0$. This investment project cannot be liquidated until period 2.[2] The depositors are paid a safe net return g^d, $(0 < g^d < g/2)$ per period. We assume that the banking industry is competitive and therefore the commercial bank considers itself unable to strategically affect g^d.

We consider a system in which the banks' liquidity management is centralized in the sense that the central bank acts as a counterpart and guarantees the finality of the banks' obligations (see Rochet and Tirole, 1996). More precisely, we assume that the central bank maintains a deposit insurance system where the central bank is committed to bail out the bank in case of a liquidity crisis. The central bank imposes a penalty of γ for every dollar it lends to the bank so as to make this survive the liquidity crisis. The bank has to compensate the central bank for this loan in period 2 after it collects the return from the illiquid high-return investment project. This penalty serves as a general policy instrument which can be given several interpretations. The most natural interpretation is that of an insurance premium which the bank is forced to accept in order to qualify to be covered by the bail-out policy. Another interpretation would be to think of γ as a partial deposit insurance with the intention of reducing the bank's incentives to exploit the option value incorporated in the deposit insurance system. Still, we could also view γ as capturing the probability of the bank losing its charter in case of a liquidity

[2] Thus, g is the net gain on a two-period investment project.

crisis. All of these interpretations represent policy measures to mitigate the agency costs generated through moral hazard.

Let X $(0 \leq X \leq 1)$ be the random withdrawal rate of the bank's deposits, with an associated strictly increasing and absolutely continuous distribution function F. The distribution function F is known by the bank and the withdrawal rate is further characterized in subsection 2.3. In the presence of a bail-out policy the expected profit of the bank, is given by

$$\begin{aligned} \mathrm{E}\Pi = & (1-r)gD - \gamma D \mathrm{E}\,(X-r)^{+} \\ & -g^{d} D \mathrm{E} X - \left(\left(1+g^{d}\right)^{2} - 1\right)(1-\mathrm{E}X)D, \end{aligned} \tag{1}$$

where

$$\gamma D \mathrm{E}(X-r)^{+} = \gamma D \int_{r}^{1} (y-r)\,dF(y) \tag{2}$$

The first term in (1) measures the bank's profit (net return) generated by investing $(1-r)D$ in the two-period illiquid investment project. The second term, $\gamma D\mathrm{E}(X-r)^{+}$, which is spelled out in (2), measures the expected penalty imposed on the bank. This term is the product of the penalty rate, γ, and the expected amount withdrawn in $t = 1$ beyond the reserves held by the bank. The second row in (1) expresses the total interest paid to depositors by the commercial bank, in periods $t = 1$ and $t = 2$ respectively. The interest payments are realized at the end of period 2. Obviously, the penalty will depend on the size of reserves. We assume that the central bank will bail out a bank facing a liquidity crisis at a penalty rate γ. Section 3 provides an analysis for determining the socially optimal penalty rate.

The bank chooses a reserve ratio, r, to maximize its profit given in (1). By applying Leibniz' rule, the necessary and sufficient conditions for the profit-maximizing reserve ratio are implicitly given by [3]

$$\frac{1}{D} \cdot \frac{\partial \mathrm{E}\Pi}{\partial r} = -g + \gamma(1 - F(r)) = 0, \tag{3}$$

meaning that the probability of a liquidity crisis is given by

$$1 - F(r^*) \ = \ \frac{g}{\gamma}. \tag{4}$$

Reformulating (4) we find the profit-maximizing reserve ratio to satisfy

$$r^* = F^{-1}\left(\frac{\gamma - g}{\gamma}\right), \quad \text{for}\, \gamma > g, \tag{5}$$

where F^{-1} is the inverse of the distribution function . Equation (4) implies that the commercial bank sets its reserve ratio so that the probability of a liquidity crisis, $1 - F(r^*)$, equals g/γ, which is the ratio between the return on the external illiquid investment project and the penalty rate. In order to induce the commercial bank to hold any reserves at all, the penalty rate γ must exceed the rate of return on the bank's investment project, g. We summarize the results in the following proposition.

Proposition 1 *The bank adjusts its profit-maximizing reserve ratio so that, the probability of a liquidity crises is invariant to the need by a representative depositor. In addition, the bank's optimal reserve ratio is a decreasing (increasing) function of* g (γ)*.*

The probability of a liquidity of a liquidity crises depends only on the ratio between the return on the investment project, g and the penalty rate, γ. The

[3] Sufficiency follows from the fact that F is increasing in r.

commercial bank always adjusts its optimal reserve ratio so as to keep the probability of a liquidity crisis at the level g/γ.

2.3 The aggregate distribution of withdrawals.

In this section we analyze how the liquidity of a large number of representative depositors translates to an aggregate withdrawal rate. Assume that depositor i faces an uncertain liquidity need, $d_i X_i$, in period 1, where, $X_i \in [0, 1]$ is the fraction of the deposit d_i that depositor i would like to withdraw in period 1. In the general case the distribution F of the random aggregate withdrawal rate facing the bank, X, is a convolution which has to be identified by the bank. It can be seen that

$$D\mathrm{E}X = \sum_{i=1}^{n} d_i \mathrm{E}X_i \quad \text{and} \quad D^2\sigma_X^2 = \sum_{i,j=1}^{n} \rho_{i,j} d_i d_j \sigma_{X_i}\sigma_{X_j}, \tag{6}$$

where σ_X^2 denotes the variance of X and ρ_{ij} denotes the correlation between the liquidity needs of depositors i and j. Under fairly mild conditions, the distribution of the aggregate withdrawal rate X can be approximated by a normally distributed random variable [4]

$$X \sim_{approx} \mathbf{N}\left(\mathrm{E}X, \sigma_X^2\right). \tag{7}$$

With this approximation, the probability of a liquidity crisis (4) and the profit maximizing reserve ratio (5) then simplify to

[4] A normal approximation is justified if, for example, n is large and if the $d_i X_i$'s are identically distributed and $0 < \rho_{ij} < 1$, when $i \neq j$, or if $X_i \sim \mathbf{N}(\mathrm{E}X_i, \mathrm{Var}X_i)$. This approximation also applies to an aggregation over a heterogeneous pool of depositors as long as there are sufficiently many depositors within each pool.

$$1 - \Phi\left(\frac{r^* - EX}{\sigma_X}\right) = \frac{g}{\gamma}, \tag{8}$$

and

$$r^* = \mathrm{E}X + \sigma_X \cdot \Phi^{-1}\left(\frac{\gamma - g}{\gamma}\right), \tag{9}$$

where Φ is the standard normal distribution function. The implications of equation (9) are summarized in the following Proposition.

Proposition 2 *For a normally distributed X , the optimal reserve ratio r^* is*

(a) linearly increasing in the depositors' expected liquidity need EX,

(b) linearly increasing (decreasing) in the standard deviation of the depositors' aggregated liquidity need σ_X, if $\gamma \geq 2g$ ($\gamma \leq 2g$).

Proposition 2 can intuitively be explained in the following way. The illiquid investment can be regarded as a call option for the commercial bank and the reserves can be regarded as a put option. The value $\gamma = 2g$, which is the median of X, represents a threshold such that these option values are invariant to an increase in the variance σ_X^2. If the penalty rate γ is large enough ($\gamma > 2g$), the option value associated with the illiquid investment is lower than the option value associated with holding reserves. For that reason an increase in the underlying variance σ_X^2, will increase the bank's optimal reserve ratio. Conversely, if the penalty rate is low enough ($\gamma < 2g$), an increased variance will lower the commercial bank's incentives to hold reserves, promoting allocations towards the illiquid high-return investment. As shown in Appendix A.1 it is seen that the an analogous result to Proposition 2, holds for an arbitrary withdrawal distribution.

Under the assumption of mutually identical, possibly correlated depositors, it can be seen that the variance of total withdrawals can be written as

$$D^2\sigma_X^2 = n^2d^2\sigma_X^2 = d^2\sum_{i,j=1}^{n}\rho_{i,j}\sigma_{X_i}\sigma_{X_j} = nd^2(1+(n-1)\rho)\sigma_{X_i}^2, \tag{10}$$

where $\rho = \rho_{ij},\ 0 < \rho < 1$ when $i \neq j$. Therefore, it holds that

$$\sigma_X^2 = (\rho + (1-\rho)/n)\,\sigma_{X_i}^2. \tag{11}$$

This variance is decreasing in the number of customers and it approaches

$$\lim_{n\to\infty}\sigma_X^2 = \rho\sigma_{X_i}^2. \tag{12}$$

A substitution of (11) into (9) yields the following finding.

Proposition 3 *When the depositors face mutually identical and correlated liquidity shocks, the profit-maximizing reserve ratio r^* is*

(a) increasing (decreasing) in the correlation of liquidity shocks, ρ, if $\gamma \geq 2g$ $(\gamma \leq 2g)$

(b) asymptotically decreasing (increasing) in the number of depositors, n, if $\gamma \geq 2g$ $(\gamma \leq 2g)$.

It is seen from equation (12) that, for the determination of the optimal reserves the systematic component $\rho\sigma_{X_i}^2$ becomes more significant when there are more depositors. As Proposition 3 (a) demonstrates, the bank adjusts to an increased correlation among depositors by allocating more funds to the reserves. However, as will be seen in Section 3, the bank has socially insufficient incentives to increase the reserves in response to an increase in the correlation among the depositors' the liquidity shocks.

3 The socially optimal bail-out policy

The central bank is assumed to have commitment power relative to the banking industry. As a rational policymaker it anticipates the optimal reserve response r^* (equation (5)) set by the commercial bank. The central bank determines the optimal penalty rate γ on lending intended to bail-out the banking industry during a liquidity crisis.

Assume that society has to pay the interest rate, δ, for the liquidity channeled to maintain the banking system operative. This interest rate is an increasing and convex function of the expected amount of the liquidity needed to bail out the bank. Hence, the social cost of liquidity is given by a function $\delta = \delta\left(D\mathrm{E}(X - r^*)^+\right)$, where $\delta', \delta'' > 0$.

We define the economy's welfare function as the sum of the commercial bank's expected profit and the depositors' expected yield minus the cost for the central bank of providing the banking system with the required liquidity. Formally, the benevolent central bank determines the penalty rate in order to maximize social welfare, determined by

$$\begin{aligned} \mathrm{E}W(\gamma,\cdot) &= (1-r^*)gD - \gamma D\mathrm{E}(X-r^*)^+ - \delta\left(D\mathrm{E}(X-r^*)^+\right) + \gamma D\mathrm{E}(X-r^*)^+ \\ . \quad & (1-r^*)gD - \delta\left(D\mathrm{E}(X-r^*)^+\right) \end{aligned} \tag{13}$$

The social cost of emergency funding from the central bank will in general depend on external parameters as well as institutional distortions in the economy in (13). As we show below, a higher penalty rate γ will increase the reserves and reduce the expected amount of liquidity needed to bail out the bank. The expected social welfare, $\mathrm{E}\,W$, is the difference between the expected profit of the commercial bank (1) and the social cost of raising funds as to support the bail out policy. The additional cost imposed on society for the bailout policy is $\delta\,(D\mathrm{E}(X-r)^+)$ - $D\gamma\mathrm{E}(X-r)^+$. The expected net interest paid to the depositors does not enter into the welfare function (13), since it represents only a transfer from the bank to consumers.

In order to transform the social welfare function into a more tractable form we use the following lemma.

Lemma 4 *For $X \sim \boldsymbol{N}(EX, \sigma_X^2)$ it holds that*

$$E(X-r^*)^+ = \int_{r^*}^{1} (y-r^*)dF(y) = \sigma_X(\phi(\underline{z}) - \underline{z}g/\gamma), \tag{14}$$

where $\underline{z}(\gamma) = \Phi^{-1}\left(\frac{\gamma-g}{\gamma}\right)$ and ϕ is the standard normal density function. Furthermore,

$$\frac{\partial E(X-r^*)^+}{\partial\gamma} = \sigma_X\frac{\partial}{\partial\gamma}(\phi(\underline{z}) - \underline{z}g/\gamma) = -\sigma_X\underline{z}'g/\gamma < 0. \tag{15}$$

PROOF. In Appendix A.2.

Observe that the optimal reserves (9) can be rewritten as $r^* = EX + \sigma_X\underline{z}$. Therefore, equation (15) in Lemma 4, implies that the necessary and sufficient optimality condition is transformed into

$$\begin{aligned}\frac{\partial \mathrm{E}W}{\partial\gamma} &= -D\sigma_X\underline{z}'g - \delta'\left(D\sigma_X\left(\phi\left(\underline{z}(\gamma)\right) - \underline{z}(\gamma)g/\gamma\right)\right)\cdot\left(-D\sigma_X\underline{z}'g/\gamma\right)\\ &= D\sigma_X\underline{z}'g\left(\delta'/\,\gamma - 1\right) = 0. \end{aligned}\tag{16}$$

Sufficiency follows from the fact that $\partial\delta'/\,\partial\gamma = \delta''(\cdot)\cdot(-\underline{z}'g/\gamma)\cdot D\sigma_X < 0 \Rightarrow (\partial\delta'/\,\gamma)/\,\partial\gamma < 0$. Now we are ready to state our final proposition. Let γ^* be the solution to the optimality condition (16).

Proposition 5 *The socially optimal penalty rate, γ^*, imposed on the commercial bank is an increasing function of the correlation of the depositors' withdrawals. Formally, $\partial\gamma^*/\partial\rho > 0$.*

PROOF. From the optimality condition (16) it is observed that $\delta'(\cdot) - \gamma = 0$. The implicit function theorem implies that

$$\delta''(\cdot)\,D\cdot\left(\mathrm{E}\,(X-r^*)^+\big/\,\sigma_X\right)\frac{\partial\sigma_X}{\partial\rho}d\rho = \delta''(\cdot)\cdot D\sigma_X\underline{z}'g{\gamma^*}^{-1}d\gamma^* + d\gamma^*. \quad (17)$$

From equation (11) $\sigma_X^2 = (\rho + (1-\rho)/n)\,\sigma_{X_i}^2$ it follows that

$$\frac{\partial\sigma_X}{\partial\rho} = \frac{\partial\sigma_X^2}{2\sigma_X\partial\rho} = \frac{(n-1)\sigma_{X_i}^2}{2n\sigma_X}. \quad (18)$$

Hence,

$$\frac{\partial\gamma^*}{\partial\rho} = \frac{\mathrm{E}\,(X-r^*)^+\,(n-1)\,\sigma_{X_i}^2}{2n\sigma_X\left[\underline{z}'g\sigma_X/\gamma^* + 1/\left(D\delta''\right)\right]} > 0. \quad (19)$$

A higher correlation in the liquidity shocks facing depositors will increase the probability of a liquidity crisis for the representative bank. As Proposition 5 shows, the socially optimal penalty rate is an increasing function of the correlation between the liquidity shocks facing the depositors. This is an intuitive result in light of the fact that an increased probability of a liquidity crisis makes it increasingly costly for the policymaker to raise the funds necessary to support the bail-out program required to avoid a liquidity crisis. Furthermore, as Proposition 3 (a) establishes a private banking industry does adjust the reserve ratio to an increase in the correlation, but this adjustment is insufficient from the social point of view, unless the central bank raisis the penalty rate, γ. Namely, the private banking industry does not fully internalize the increasing costs of additional liquidity to support the bail-out program.[5]

In qualitative terms Proposition 5 means that the correlation of depositor withdrawals should be an essential element in the design of the regulatory framework for the banking industry. In fact, Proposition 5 can be viewed as a formalization of the policy conclusion that an increased systemic risk, measured as an increased degree of correlation of liquidity shocks among depositors, calls for stronger intervention in the sense of higher penalty rates. This conclusion can be seen to complement the analysis in Rochet and Tirole (1996) and Freixas et al. (2000), which analytically explore how the design of interbank markets will impact on the vulnerability of the banking industry to systemic liquidity risks.

4 Concluding Comments

This paper focuses on a competitive banking industry facing liquidity risks caused by depositors facing premature liquidity needs. We develop a method for calculating the profit-maximizing amount of reserves of a representative bank, and characterize the associated probability of a liquidity crisis. We show that the only information needed to predict the probability of a liquidity crisis is the cost of maintaining reserves and the penalty rate charged to a bank facing a run.

[5] This result is generally in line with Holmström and Tirole (1998), however, these authors study a completely different environment and they do not explicitly focus on the correlation between the depositors liquidity needs.

Within the framework of our welfare analysis we delineated a characterization of the socially optimal penalty rate, which by taking the optimal response of the banking industry into account, will determine the socially optimal fraction of reserves. Importantly, this socially optimal penalty rate was found to be an increasing function of correlation between the liquidity shocks facing depositors. Indeed, as was established in our analysis, the private banking industry will have an incentive to adjust the reserves upwards when facing an increased correlation, but this incentive will for structural reasons be too weak from a social point of view. Namely, the banking industry does not fully internalize the increasing social costs associated with a need to raise additional liquidity in order to support a more extensive bail-out program.

The expected profit of the bank (1) could be formulated in alternative ways. An example of a plausible reformulation of (1) would be: $\mathrm{E}\Pi = (1-r)gD$ - $\gamma D \int_{r/\left(1+g^d\right)}^{1} \left(y\left(1+g^d\right)-r\right) dF(y)$ - $\mathrm{E}Xg^dD$ - $\left(\left(1+g^d\right)^2-1\right)(1-\mathrm{E}X)D$, where the interest on the principal withdrawn in period 1 would be paid at the end of period one. In such an alternative setting the probability of a bank run would be unchanged, but the optimal reserves must be scaled up with the coefficient $\left(1+g^d\right)$. However, the qualitative findings of our analysis are easily adapted to such an alteration.

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A Appendix

A.1 Extension of Proposition 2

Fact 6 *The optimal reserve rate r^* is linearly increasing in the expected value and the median of X, EX and MdX such that $\Delta r^* = \Delta EX = \Delta MdX$.*

PROOF. A shift in a centering parameter, shifts the inverse distribution function F^{-1} with an equal amount.

Let $\eta > 0$ be a (linear) median preserving stretch-parameter for the withdrawal distribution F, the stretched distribution be F_η, and let r_η^* be the optimal reserve rate for the stretched distribution. Consequently, the withdrawals become more (less) "volatile" when η grows above (below) one. The median preserving spread has the property $F_\eta^{-1}(x) = F^{-1}(x) + \eta\left[F^{-1}(x) - F^{-1}(0.5)\right]$, for all withdrawal probabilities.

Fact 7 *r_η^* is linearly increasing (decreasing) in the spread parameter η when $\gamma > 2g$ $(\gamma < 2g)$.*

PROOF. $r_\eta^* = F_\eta^{-1}\left(\frac{\gamma-g}{\gamma}\right) = F^{-1}\left(\frac{\gamma-g}{\gamma}\right) + \eta\left[F^{-1}\left(\frac{\gamma-g}{\gamma}\right) - F^{-1}(0.5)\right]$. As F^{-1} is an increasing function, the factor in the last term $F^{-1}\left(\frac{\gamma-g}{\gamma}\right)$ - $F^{-1}(0.5)$ is strictly positive (negative) when $\gamma > 2g$ $(\gamma < 2g)$.

A.2 Proofs of the formulae (14) and (15) in Lemma 4.

Here we prove the equation (14) $\mathrm{E}(X - r^*)^+ = \int_{r^*}^1 (y - r^*)dF(y) = \sigma_X(\phi(\underline{z}) - \underline{z}g/\gamma)$ and the equation (15) $\partial \mathrm{E}(X - r^*)^+ / \partial\gamma = \partial(\phi(\underline{z}) - \underline{z}g/\gamma)/\partial\gamma = -\underline{z}'g/\gamma < 0$, used in the main text.

PROOF. Substitution for $r^* = EX + \sigma_X \underline{z}$ into the integral $\int_{r^*}^1 (y - r^*)dF(y)$ yields the equation (14), as

$$\int_{r^*}^{1} (y - \mathrm{E}X + \mathrm{E}X - r^*)dF(y) = \int_{r^*}^{1} (y - \mathrm{E}X)dF(y) - \int_{r^*}^{1} (r^* - \mathrm{E}X)dF(y)$$

$$= \sigma_X \int_{\underline{z}}^{\infty} z\phi(z)d(z) - (r^* - \mathrm{E}X) \int_{r^*}^{1} dF(y) = \sigma_X(\phi(\underline{z}) - \underline{z}g/\gamma). \tag{A.1}$$

By applying the well known rule for the derivative of an inverse function it is seen that

$$\underline{z}' = \frac{\partial \underline{z}}{\partial\gamma} = \frac{\partial}{\partial\gamma}\Phi^{-1}\left(\frac{\gamma - g}{\gamma}\right) = \frac{g}{\gamma^2\phi\left(\underline{z}\right)} > 0 \quad \forall\gamma > 0. \tag{A.2}$$

Now, (A.2) in combination with $\phi(z) = (2\pi)^{-0.5}\exp\left(-z^2/2\right)$ imply that

$$\frac{\partial}{\partial\gamma}\phi\left(\underline{z}(\gamma)\right) = -\phi\left(\underline{z}(\gamma)\right)\underline{z}(\gamma)\underline{z}'(\gamma) = -\underline{z}(\gamma)\phi(\underline{z})\frac{g}{\phi(\underline{z})\gamma^2} < 0,\ \forall\gamma > g. \tag{A.3}$$

Consequently,

$$\begin{aligned}\frac{\partial}{\partial\gamma}(\phi(\underline{z}(\gamma)) - \underline{z}(\gamma)g/\gamma) &= \underline{z}(\gamma)g/\gamma^2 - \underline{z}(\gamma)g/\gamma^2 - \underline{z}'(\gamma)g/\gamma \\ &= -\underline{z}'(\gamma)g/\gamma.\end{aligned} \tag{A.4}$$