Advance Booking, Cancellations, and Partial Refunds

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12th January 2004

Abstract

We develop methods for calculating profit-maximizing and socially optimal rates of partial refunds on customers' no-shows and cancellations. We demonstrate how partial refunds can be used to screen consumers according to their different probabilities of cancellation and no-shows. Finally, we show that the socially-optimal rate of partial refund exceeds the equilibrium rate of partial refund.

Keywords: Partial Refund, Advance Booking, Reservations, Cancellations, No-show

JEL Classification Numbers: M2, L9

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1 Introduction

Partial refunds are widely used in advance booking and reservation systems. In this note we propose how to compute the profit-maximizing rates of partial refunds. Partial refunds are used to control for the selection of potential customers who make reservations but differ with respect to their cancellation probabilities. In addition, we explore the social welfare consequences associated with the profit-maximizing partial refund rates.

Consumers making reservations for products and services differ in their probability of showing up to collect the good or the service at the pre-agreed time of delivery. Agencies and dealers that sell these goods and services can save on unused capacity costs, generated by consumers' cancellations and no-shows, by varying the degree of partial refunds offered to those consumers who cancel or do not show. Partial refunds are widely observed in almost all privately-provided services. They are heavily used by airline companies in selling discounted tickets where cheaper tickets allow for a very small refund (if any) on cancellations, whereas full-fare tickets are either fully-refundable or subject to low penalty rates.

In the economics literature there are a few papers analyzing *post-delivery* contracts in the form of money-back-warranties, see Mann and Wissink (1988, 1990), Moorthy and Srinivasan (1995), and Shiou (1996). In their models, money-back-guarantees serve as a signaling device for products' quality which is observable only after the product is delivered. Here the quality of the product is exogenous.

Advance booking mechanisms received a limited attention in the economic literature. Gale and Holmes (1992, 1993) compare monopoly pricing with social optimum. In their analysis, the price mechanism is used to shift the demand from the peak period to the off-peak period. Gale (1993) analyzes consumers who learn their preferences after they are offered an advance purchase option. On this line, Miravete (1996) and more recently Courty and Li (2000) further investigate how consumers who learn their valuation over time can be screened via the introduction of refunds in their advance booking mechanism. Dana (1998) also investigates market segmentation under advance booking of price-taking firms. There also exist a literature dealing with supply-rationing under demand uncertainty in revenue management which is broadly related to this paper (see McGill and van Ryzin 1999 for a survey). Ringbom and Shy (2003) analyze the benefits of fully-refundable reservation systems without allowing for partial refunds. Xie and Shugan (2001) analyze advance sales in the presence of spot sales for exogenously-given participation levels. In this note, we endogenously solve for the participation rates, which can be adjusted by the rate of partial refunds. We also show how welfare statements can be drawn from our model.

2 Two consumer types

Let β denote consumers' basic valuation (maximal willingness to pay) for a certain service or a product. Let p denote the market price. We assume that p is exogenously given by the manufacturers and service providers. What we have in mind are products and services that are sold under resale-price maintenance agreements, such as airline and train tickets, car rental fees, books with a "recommended" price, and telecommunication services.

2.1 Consumer types and sellers

The are n potential consumers who vary according to their cancellation habits, as reflected by the probability of showing up of each customer. Suppose that there are only two types of potential customers, indexed by $i = H, L$. More precisely, there are $\alpha_H n$ consumers with a "high" probability of showing up given by σ_H, and $\alpha_L n$ consumers with a "low" probability of showing up given by σ_L, where $\alpha_H + \alpha_L = 1$. We assume that $0 < \sigma_L < \sigma_H < 1$.

Suppose that a representative seller purchases the service/good at the cost of $c \geq 0$. If the customer does not show up at the delivery time, the seller's salvage value of the product is $s \geq 0$. Given the service price p, the seller's only choice variable is the rate of partial refund offered to those consumers who do not show up or cancel their reservations. We denote this choice variable by r, where $0 \leq r \leq 1$. Thus, on each reservation, the seller earns a profit of $p - c$ if the customer shows up, and loses $c + rp - s$ if the customer cancels, or simply does not show up.

The expected utility of a type i consumer, $i = H, L$, is given by

$$\mathrm{E}U = \begin{cases} \sigma_i(\beta - p) - (1 - \sigma_i)(1 - r)p & \text{if he makes a reservation} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Thus, if the consumer shows up at the delivery time, the net benefit is $\beta - p$. However, if the consumer cancels, the consumer loses the non-refundable portion of the price, which amounts to $(1 - r)p$.

2.2 Profit-maximizing refund rate

Inspecting the utility functions (1) reveals that the seller can adjust the participation of each consumer group by the decision variable r. A "high" refund rate would induce both consumer types to make a reservation, whereas a low refund rate would make the reservation non-beneficial (in expected terms) to at least type-L consumers (i.e., those consumers who are more likely to cancel).

Suppose that the seller sets up a "high" refund rate, so that both consumer types make reservations. Formally, let r_2 be the refund rate that makes type-L consumers indifferent between making a reservation and not consuming. From (1) we have

$$r_2 = \frac{p - \beta\sigma_L}{p(1-\sigma_L)}. \tag{2}$$

Similarly, let r_1 be the refund rate that would make type H consumers indifferent between making reservations and not consuming. Then,

$$r_1 = \frac{p - \beta\sigma_H}{p(1-\sigma_H)}. \tag{3}$$

Clearly, $r_2 > r_1$ reflecting the fact that the refund rate must be higher if the seller would like consumers with high cancellation rates (type L) to make reservations.

The seller's expected profit levels at any refund level $r \in [0,1]$ is then given by

$$\mathrm{E}\pi(r) = \begin{cases} n(p-c) - [\alpha_L n(1-\sigma_L) + \alpha_H n(1-\sigma_H)]\,(r_2 p - s) & r \geq r_2 \\ \alpha_H n\,[p - c - (1-\sigma_H)(r_1 p - s)] & r_1 \leq r < r_2 \\ 0 & r < r_1. \end{cases} \tag{4}$$

We observe from (4) that a high refund rate is more profitable than the low refund rate which excludes type L consumers, that is $\mathrm{E}\pi(r_2) \geq \mathrm{E}\pi(r_1)$, if and only if

$$(p-c) - [\alpha_L(1-\sigma_L) + \alpha_H(1-\sigma_H)]\,(r_2 p - s) \geq \alpha_H\,[p - c - (1-\sigma_H)(r_1 p - s)]\,. \tag{5}$$

By substituting (2) and (3) into (5), with some manipulation we can state the following proposition

Proposition 1. *The profit-maximizing partial refund rate is r_2 if and only if*

$$\alpha_L[\sigma_L(\beta - s) - c + s] \geq \alpha_H(\sigma_H - \sigma_L)(\beta - p). \tag{6}$$

Otherwise, the profit maximizing rate is r_1.

The insights to draw from Proposition 1 are summarized in the following corollary.

Corollary 1. *Both consumer types are served if p is sufficiently high, α_H or σ_H are sufficiently low, or σ_L is sufficiently high.*

3 General Distribution of Consumer Types

In this section we generalize the two-point distribution of consumers by assuming that there is a continuum of consumers indexed by $\sigma \in (0,1]$, who differ according to their probability of showing up. The distribution function of the consumers' showing up probability σ is F_σ, which is absolutely continuous with respect to σ, $F_\sigma(0) = 0$ and $F_\sigma(1) = 1$, and $f_\sigma = dF_\sigma/d\sigma$. Formally, similar to (1) the expected utility of a type σ consumer is

$$\mathrm{E}U(\sigma) \;=\; \begin{cases} \sigma\,(\beta - p) - (1-\sigma)(1-r)p \geq 0 & \text{if makes reservation} \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

3.1 Refundability and profits

Let $\hat{\sigma}$ denote a consumer who is indifferent between making reservations or not. From (7), $\hat{\sigma}$ is implicitly defined by $\hat{\sigma}\,(\beta - p) - (1-\hat{\sigma})(1-r)p = 0$. Hence,

$$\hat{\sigma}(r) = \frac{p - rp}{\beta - rp}. \tag{8}$$

$\hat{\sigma}$ is the threshold showing up probability, which is controllable through the refund rate, r. Therefore, only consumers indexed by $\sigma \geq \hat{\sigma}$ make reservations. Next, observe that the expected unit gross profit from a type σ consumer who makes a reservation, is given by

$$\begin{aligned} g_\sigma \;&=\; p - c - \underbrace{(1-\sigma)(rp - s)}_{\text{no}-\text{show cost}} \\ &=\; p - rp + \sigma(rp - s) + s - c. \end{aligned} \tag{9}$$

Consequently, the gross profit of the seller is

$$\pi(r) = n \int_{(\hat{\sigma},1]} g_\sigma dF_\sigma. \tag{10}$$

We observe by implementing Leibnitz' rule, that the first order condition for an internal profit maximum, $0 < r < 1$, becomes

$$\frac{1}{n}\,\frac{d\pi(r)}{dr} = \int_{(\hat{\sigma},1]} \frac{\partial g_\sigma}{\partial r} dF_\sigma - \frac{d\hat{\sigma}}{dr} f(\hat{\sigma})\; g_\sigma|_{\sigma=\hat{\sigma}} = 0. \tag{11}$$

From (8) we conclude that $d\hat{\sigma}/dr = -p(\beta - p)/(\beta - rp)^2 = -p(1-\hat{\sigma})/(\beta - rp) < 0$, and from (9) that $dg_\sigma/dr = p(\sigma - 1)$. Therefore, (11) simplifies to

$$\frac{1}{np}\frac{d\pi(r)}{dr} = \int_{(\hat{\sigma},1]} (\sigma - 1) f(\sigma) d\sigma + (\beta - p) f(\hat{\sigma}) \frac{g_\sigma|_{\sigma=\hat{\sigma}}}{(\beta - rp)^2} = 0. \tag{12}$$

If an interior solution to (12) does not exist, $(0 < r < 1)$, then profit is maximized by selling non-refundable tickets, that is, the corner solution $r = 0$ maximizes (10). In order to be able to interpret our results we now simplify the distribution of σ.

3.2 Partial refunds with uniformly distributed showing up probability

Suppose that σ is uniformly distributed over the unit interval, $\sigma \sim U[0,1]$. From (8) we conclude that $1 - \hat{\sigma} = (\beta - p)/(\beta - rp)$. In this case the first order condition for internal profit maximum (12) can be rewritten as

$$\begin{aligned}\frac{1}{np}\frac{d\pi(r)}{dr} &= -(1-\hat{\sigma}) + \frac{(1-\hat{\sigma}^2)}{2} + \frac{(\beta - p)}{(\beta - rp)^2}\, g_\sigma|_{\sigma=\hat{\sigma}} = \\ &= -\frac{(\beta - p)^2}{2(\beta - rp)^2} + \frac{(\beta - p)}{(\beta - rp)^2}\, g_\sigma|_{\sigma=\hat{\sigma}} = 0.\end{aligned} \tag{13}$$

Therefore, $(\beta - p)/2 = g_\sigma|_{\sigma=\hat{\sigma}} > 0$. We now solve for the unique profit-maximizing refund rate $\bar{r} \geq 0$. From (8), (9) and (13) we obtain

$$\bar{r} = \max\left\{0, \frac{\beta(3p - 2(c-s)) - 2ps - \beta^2}{p(\beta - 2c + p)}\right\}, \tag{14}$$

which is strictly positive for a sufficiently high price satisfying

$$p > \tilde{p} \stackrel{\text{def}}{=} \beta\left(1 - 2\frac{\beta - c}{3\beta - 2s}\right). \tag{15}$$

By combining (8) and (14) we observe that

$$\hat{\sigma}(\bar{r}) = \min\left\{\frac{p}{\beta}, \frac{(\beta - p) + 2(c-s)}{2(\beta - s)}\right\}. \tag{16}$$

We conclude that:

Proposition 2. *If $p > \tilde{p}$, the profit maximizing refund rate $\bar{r}$ is an increasing function of the price p and the salvage value s, and a decreasing function of the marginal cost c. Otherwise*

$\bar{r} = 0$.

Numerical examples of profit maximizing refund rates are given in Table 1 below.

4 Welfare analysis

We define the social welfare as the sum the consumers' utilities and seller's profit. Formally,

$$\begin{aligned} W(r) &= CS + \mathrm{E}\pi = n \int_{\hat{\sigma}(r)}^{1} (\sigma(\beta - p) - (1-\sigma)(1-r)p) dF_\sigma + n \int_{\hat{\sigma}(r)}^{1} g_\sigma dF_\sigma \\ &= n(\beta - s) \int_{\hat{\sigma}(r)}^{1} \sigma dF_\sigma - n(c-s) \int_{\hat{\sigma}(r)}^{1} dF_\sigma. \end{aligned} \tag{17}$$

Lemma 1. *Let r^* be the socially optimal refund rate that maximizes (17). Then $r^* = 1 - (c-s)(\beta - p)/[p(\beta - c)]$. In addition, $g_\sigma|_{\hat{\sigma}(r^*)} = 0$, where g_σ is defined in (9).*

Proof. By applying Leibnitz' rule on (17) we observe that the critical showing up probability $\hat{\sigma}(r^*)$ which maximizes welfare is $\hat{\sigma} = (c-s)/(\beta - s)$. Therefore, $c - s = \hat{\sigma}(\beta - s)$, and by (8), $p - rp = (\beta - rp)\hat{\sigma}$. Substituting these values into (9) yields $g_\sigma = \hat{\sigma} \cdot 0 = 0$. $\hat{\sigma}(r^*) = (c-s)/(\beta - s) = (p - r^*p)/(\beta - r^*p)$ determines r^*. □

Intuitively, Lemma 1 implies that the social planner increases the refund rate until the unit gross profit goes to zero. Our welfare conclusion is given in the following proposition.

Proposition 3. *The socially optimal refund rate exceeds the equilibrium refund rate. Formally, $r^* > \bar{r}$. If the salvage value s equals marginal cost ($s = c$), a full refund is socially optimal, otherwise, ($s < c$), the socially optimal refund rate is partial, $0 < r^* < 1$.*

Proof. $p > c \geq s \Longrightarrow r^* > 0$. Substituting $g_\sigma|_{\hat{\sigma}(r^*)} = 0$ into (12) yields $d\mathrm{E}\pi/dr\,|_{r^*} < 0$. □

Proposition 3 implies that the seller excludes "too many" customers from a social point of view. The profit maximizing and socially optimal refund rates are compared in Table 1.

$\bar{r} < r^*$	$c = 0.2,\ s = 0.1$	$c = s = 0.1$	$c = 0.1,\ s = 0$	$c = s = 0$
$p = 0.8$	$0.929 < 0.969$	$0.969 < 1$	$0.9375 < 0.972$	$0.972 < 1$
$p = 0.6$	$0.667 < 0.917$	$0.810 < 1$	$0.714 < 0.926$	$0.833 < 1$
$p = 0.5$	$0.364 < 0.875$	$0.250 < 1$	$0.462 < 0.889$	$0.357 < 1$
$p = 0.4$	$0.000 < 0.8125$	$0.250 < 1$	$0.000 < 0.833$	$0.357 < 1$

Table 1: Simulations of profit maximizing refunds $\bar{r}$ and socially optimal refunds r^* assuming that $\sigma \sim U[0,1]$, $\beta = 1$.

Using (17) we measure the welfare loss by

$$\Delta W = W(r^*) - W(\bar{r}) = n \int_{(c-s)/(\beta-s)}^{\hat{\sigma}(\bar{r})} [(\beta - s)\,\sigma - (c-s)] dF_\sigma > 0, \tag{18}$$

Assuming a uniform distribution of σ, (18) becomes

$$\begin{aligned}
\Delta W &= n \cdot \left\{ \frac{\hat{\sigma}^2(\bar{r})}{2}(\beta - s) - \hat{\sigma}(\bar{r})(c-s) + \frac{(c-s)^2}{2(\beta - s)} \right\} \\
&= \frac{n}{2(\beta - s)} \left[\underbrace{\min\left\{ \frac{p}{\beta}, \frac{(\beta - p) + 2(c-s)}{2(\beta - s)} \right\}}_{\hat{\sigma}(\bar{r})} (\beta - s) - (c-s) \right]^2 \\
&= \frac{n}{8} \frac{(\beta - p)^2}{(\beta - s)} \cdot \mathbf{1}_{\{p > \tilde{p}\}} + \frac{n}{2} \frac{[p(\beta - s) - \beta(c-s)]^2}{\beta^2(\beta - s)} \cdot \mathbf{1}_{\{p \le \tilde{p}\}},
\end{aligned} \tag{19}$$

where $\mathbf{1}_{\{.\}}$ is the indicator function taking value 1 if the subscript condition is true and zero otherwise, and $\tilde{p}$ was given in (15). Therefore,

Proposition 4. *If $\sigma \sim U[0,1]$ and $p > \tilde{p}$, the welfare loss ΔW is independent of the marginal cost, a decreasing function of the price p, and an increasing function of the salvage value s.*

Table 2 provides some numerical calculations of the welfare loss associated with an insufficiently high refund rate.

ΔW	$c = 0.2,\ s = 0.1$	$c = s = 0.1$	$c = 0.1,\ s = 0$	$c = s = 0$
$p = 0.8$	5.556	5.556	5.000	5.000
$p = 0.6$	22.22	22.22	20.00	20.00
$p = 0.5$	34.72	34.72	31.25	31.25
$p = 0.4$	37.56	50.00	45.00	45.00

Table 2: Simulations of welfare losses ΔW, $\sigma \sim U[0,1],\ \beta = 1,\ n = 1000$.

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