random_tariff_2025_mm_dd.dfw: Random Tariff Wars

#1: CaseMode := Sensitive

#2: InputMode := Word

*** Section 3: The model

eq (1): Demand system

#3: $pxh = \alpha - \beta \cdot xh - \gamma \cdot yh$

#4: $pyh = \alpha - \gamma \cdot xh - \beta \cdot yh$

#5: $pxf = \alpha - \beta \cdot xf - \gamma \cdot yf$

#6: $pyf = \alpha - \gamma \cdot xf - \beta \cdot yf$

Let λ be prob t=T => 1−λ prob t=0

*** Section 4: Random reciprocal tariff wars

eq (2): expected profit of x

#7: $eprofitxh = (pxh - c) \cdot xh$

#8: $eprofitxf = \lambda \cdot (pxf - c - T) \cdot xf + (1 - \lambda) \cdot (pxf - c) \cdot xf$

#9: $eprofitx = (pxh - c) \cdot xh + \lambda \cdot (pxf - c - T) \cdot xf + (1 - \lambda) \cdot (pxf - c) \cdot xf$

eq (3): expected profit of y

#10: $eprofityf = (pyf - c) \cdot yf$

#11: $eprofityh = \lambda \cdot (pyh - c - T) \cdot yh + (1 - \lambda) \cdot (pyh - c) \cdot yh$

#12: $eprofity = (pyf - c) \cdot yf + \lambda \cdot (pyh - c - T) \cdot yh + (1 - \lambda) \cdot (pyh - c) \cdot yh$

Derivation of (4) and Appendix A

#13: $\mathrm{eprofitx} = ((\alpha - \beta\cdot xh - \gamma\cdot yh) - c)\cdot xh + \lambda\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c - T)\cdot xf + (1 - \lambda)\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c)\cdot xf$

#14: $\frac{d}{d\ xf}(\mathrm{eprofitx} = ((\alpha - \beta\cdot xh - \gamma\cdot yh) - c)\cdot xh + \lambda\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c - T)\cdot xf + (1 - \lambda)\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c)\cdot xf)$

#15: $$0 = - T\cdot\lambda - c - 2\cdot xf\cdot\beta - yf\cdot\gamma + \alpha$$

#16: $\frac{d}{d\ xh}(\mathrm{eprofitx} = ((\alpha - \beta\cdot xh - \gamma\cdot yh) - c)\cdot xh + \lambda\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c - T)\cdot xf + (1 - \lambda)\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c)\cdot xf)$

#17: $$0 = -c - 2\cdot xh\cdot\beta - yh\cdot\gamma + \alpha$$

#18: $\mathrm{eprofity} = ((\alpha - \gamma\cdot xf - \beta\cdot yf) - c)\cdot yf + \lambda\cdot((\alpha - \gamma\cdot xh - \beta\cdot yh) - c - T)\cdot yh + (1 - \lambda)\cdot((\alpha - \gamma\cdot xh - \beta\cdot yh) - c)\cdot yh$

#19: $\frac{d}{d\ yf}(\mathrm{eprofity} = ((\alpha - \gamma\cdot xf - \beta\cdot yf) - c)\cdot yf + \lambda\cdot((\alpha - \gamma\cdot xh - \beta\cdot yh) - c - T)\cdot yh + (1 - \lambda)\cdot((\alpha - \gamma\cdot xh - \beta\cdot yh) - c)\cdot yh)$

#20: $$0 = -c - xf\cdot\gamma - 2\cdot yf\cdot\beta + \alpha$$

#21: $$\frac{d}{d\ yh}\left(eprofity = ((\alpha - \gamma\cdot xf - \beta\cdot yf) - c)\cdot yf + \lambda\cdot((\alpha - \gamma\cdot xh - \beta\cdot yh) - c - T)\cdot yh + (1 - \lambda)\cdot((\alpha - \gamma\cdot xh - \beta\cdot yh) - c)\cdot yh\right)$$

#22: $$0 = - T\cdot\lambda - c - xh\cdot\gamma - 2\cdot yh\cdot\beta + \alpha$$

#23: $$\text{SOLVE}([0 = - T\cdot\lambda - c - 2\cdot xf\cdot\beta - yf\cdot\gamma + \alpha,\ 0 = -c - 2\cdot xh\cdot\beta - yh\cdot\gamma + \alpha,\ 0 = -c - xf\cdot\gamma - 2\cdot yf\cdot\beta + \alpha,\ 0 = - T\cdot\lambda - c - xh\cdot\gamma - 2\cdot yh\cdot\beta + \alpha], [xf, xh, yf, yh])$$

eq (4) sales levels

#24: $$\left[xf = \frac{2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma)}{\gamma^2 - 4\cdot\beta^2} \wedge xh = \frac{T\cdot\gamma\cdot\lambda + (c - \alpha)\cdot(\gamma - 2\cdot\beta)}{4\cdot\beta^2 - \gamma^2} \wedge yf = \frac{T\cdot\gamma\cdot\lambda + (c - \alpha)\cdot(\gamma - 2\cdot\beta)}{4\cdot\beta^2 - \gamma^2} \wedge yh = \frac{2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma)}{\gamma^2 - 4\cdot\beta^2}\right]$$

xf = yh > 0 if

#25: $$2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma) < 0$$

#26: $$\text{SOLVE}(2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma) < 0, T)$$

#27: $$\text{IF}\left(\beta\cdot\lambda < 0, T > \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda}\right) \vee \text{IF}\left(\beta\cdot\lambda > 0, T < \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda}\right)$$

Assumption 3

#28: $T < \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda}$

soc in Appendix A

#29: $\frac{d}{d\ xf}\frac{d}{d\ xf}(eprofitx = ((\alpha - \beta\cdot xh - \gamma\cdot yh) - c)\cdot xh + \lambda\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c - T)\cdot xf + (1 - \lambda)\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c)\cdot xf)$

#30: $0 > - 2\cdot\beta$

#31: $\frac{d}{d\ xh}\frac{d}{d\ xh}(eprofitx = ((\alpha - \beta\cdot xh - \gamma\cdot yh) - c)\cdot xh + \lambda\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c - T)\cdot xf + (1 - \lambda)\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c)\cdot xf)$

#32: $0 > - 2\cdot\beta$

#33: $\frac{d}{d\ xh}\frac{d}{d\ xf}(eprofitx = ((\alpha - \beta\cdot xh - \gamma\cdot yh) - c)\cdot xh + \lambda\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c - T)\cdot xf + (1 - \lambda)\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c)\cdot xf)$

#34: $0 = 0$

#35: $\frac{d}{d\ yf}\frac{d}{d\ yf}(eprofity = ((\alpha - \gamma\cdot xf - \beta\cdot yf) - c)\cdot yf + \lambda\cdot((\alpha - \gamma\cdot xh - \beta\cdot yh) - c - T)\cdot yh + (1 - \lambda)\cdot((\alpha$

$- \gamma \cdot xh - \beta \cdot yh) - c) \cdot yh)$

#36: $$0 > -2 \cdot \beta$$

#37: $$\frac{d}{d\ yh} \frac{d}{d\ yh} (eprofity = ((\alpha - \gamma \cdot xf - \beta \cdot yf) - c) \cdot yf + \lambda \cdot ((\alpha - \gamma \cdot xh - \beta \cdot yh) - c - T) \cdot yh + (1 - \lambda) \cdot ((\alpha$$

$- \gamma \cdot xh - \beta \cdot yh) - c) \cdot yh)$

#38: $$0 > -2 \cdot \beta$$

#39: $$\frac{d}{d\ yf} \frac{d}{d\ yh} (eprofity = ((\alpha - \gamma \cdot xf - \beta \cdot yf) - c) \cdot yf + \lambda \cdot ((\alpha - \gamma \cdot xh - \beta \cdot yh) - c - T) \cdot yh + (1 - \lambda) \cdot ((\alpha$$

$- \gamma \cdot xh - \beta \cdot yh) - c) \cdot yh)$

#40: $$0 = 0$$

eq (5): equlibrium prices

#41: $$pxh = \frac{T \cdot \beta \cdot \gamma \cdot \lambda + (2 \cdot \beta - \gamma) \cdot (c \cdot (\beta + \gamma) + \alpha \cdot \beta)}{4 \cdot \beta^2 - \gamma^2}$$

#42: $$pyh = \frac{T \cdot \lambda \cdot (2 \cdot \beta^2 - \gamma^2) + (2 \cdot \beta - \gamma) \cdot (c \cdot (\beta + \gamma) + \alpha \cdot \beta)}{4 \cdot \beta^2 - \gamma^2}$$

#43:
$$pxf = \frac{T\cdot\lambda\cdot(2\cdot\beta^2 - \gamma^2) + (2\cdot\beta - \gamma)\cdot(c\cdot(\beta + \gamma) + \alpha\cdot\beta)}{4\cdot\beta^2 - \gamma^2}$$

#44:
$$pyf = \frac{T\cdot\beta\cdot\gamma\cdot\lambda + (2\cdot\beta - \gamma)\cdot(c\cdot(\beta + \gamma) + \alpha\cdot\beta)}{4\cdot\beta^2 - \gamma^2}$$

eq (7) profits in domestic markets

#45:
$$eprofitxh = \frac{\beta\cdot(T\cdot\gamma\cdot\lambda + (c - \alpha)\cdot(\gamma - 2\cdot\beta))^2}{(4\cdot\beta^2 - \gamma^2)^2}$$

#46:
$$eprofityf = \frac{\beta\cdot(T\cdot\gamma\cdot\lambda + (c - \alpha)\cdot(\gamma - 2\cdot\beta))^2}{(4\cdot\beta^2 - \gamma^2)^2}$$

eq (8) profits in export markets

#47:
$$eprofitxf = \frac{\beta\cdot(2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma))^2}{(4\cdot\beta^2 - \gamma^2)^2}$$

#48:
$$eprofityh = \frac{\beta\cdot(2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma))^2}{(4\cdot\beta^2 - \gamma^2)^2}$$

Result 2, Appendix B

eq (9): The condition of this result

#49: $$T > T\_bar = \frac{(\alpha - c)\cdot(2\cdot\beta - \gamma)^2}{4\cdot\beta^2 - \gamma^2}$$

Is this consistent with Assumption 3?

#50: $$\frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda} - \frac{(\alpha - c)\cdot(2\cdot\beta - \gamma)^2}{4\cdot\beta^2 - \gamma^2}$$

eq (B.1)

#51: $$\frac{(c - \alpha)\cdot(2\cdot\beta - \gamma)\cdot(2\cdot\beta\cdot(\lambda - 1) - \gamma)}{2\cdot\beta\cdot\lambda\cdot(2\cdot\beta + \gamma)}$$

> 0 if [always!]

#52: $$(c - \alpha)\cdot(2\cdot\beta - \gamma)\cdot(2\cdot\beta\cdot(\lambda - 1) - \gamma) > 0$$

#53: $$\mathrm{SOLVE}((c - \alpha)\cdot(2\cdot\beta - \gamma)\cdot(2\cdot\beta\cdot(\lambda - 1) - \gamma) > 0, \lambda)$$

#54: $$\mathrm{IF}\left(\beta\cdot(c - \alpha)\cdot(2\cdot\beta - \gamma) < 0, \lambda < \frac{2\cdot\beta + \gamma}{2\cdot\beta}\right) \vee \mathrm{IF}\left(\beta\cdot(c - \alpha)\cdot(2\cdot\beta - \gamma) > 0, \lambda > \frac{2\cdot\beta + \gamma}{2\cdot\beta}\right)$$

#55: $$\lambda < \frac{2\cdot\beta + \gamma}{2\cdot\beta}$$

#56: $$\text{eprofitx} = \frac{\beta\cdot(T\cdot\gamma\cdot\lambda + (c - \alpha)\cdot(\gamma - 2\cdot\beta))^2}{(4\cdot\beta^2 - \gamma^2)^2} + \frac{\beta\cdot(2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma))^2}{(4\cdot\beta^2 - \gamma^2)^2}$$

#57: $$\text{eprofitx} = \frac{\beta\cdot(T^2\cdot\lambda^2\cdot(4\cdot\beta^2 + \gamma^2) + 2\cdot T\cdot\lambda\cdot(c - \alpha)\cdot(2\cdot\beta - \gamma)^2 + 2\cdot(c - \alpha)^2\cdot(2\cdot\beta - \gamma)^2)}{(4\cdot\beta^2 - \gamma^2)^2}$$

#58: $$\frac{d}{d\lambda}\left(\text{eprofitx} = \frac{\beta\cdot(T^2\cdot\lambda^2\cdot(4\cdot\beta^2 + \gamma^2) + 2\cdot T\cdot\lambda\cdot(c - \alpha)\cdot(2\cdot\beta - \gamma)^2 + 2\cdot(c - \alpha)^2\cdot(2\cdot\beta - \gamma)^2)}{(4\cdot\beta^2 - \gamma^2)^2}\right)$$

eq (B.2) FOC for a minimum

#59: $$0 = \frac{2\cdot T\cdot\beta\cdot(T\cdot\lambda\cdot(4\cdot\beta^2 + \gamma^2) + (c - \alpha)\cdot(2\cdot\beta - \gamma)^2)}{(4\cdot\beta^2 - \gamma^2)^2}$$

#60: $$\frac{d}{d\lambda}\frac{d}{d\lambda}\left(\text{eprofitx} = \frac{\beta\cdot(T^2\cdot\lambda^2\cdot(4\cdot\beta^2 + \gamma^2) + 2\cdot T\cdot\lambda\cdot(c - \alpha)\cdot(2\cdot\beta - \gamma)^2 + 2\cdot(c - \alpha)^2\cdot(2\cdot\beta - \gamma)^2)}{(4\cdot\beta^2 - \gamma^2)^2}\right)$$

eq (B.3) SOC for minimum

#61: $$0 < \frac{2\cdot T^2\cdot\beta\cdot(4\cdot\beta^2 + \gamma^2)}{(4\cdot\beta^2 - \gamma^2)^2}$$

#62: $$\mathrm{SOLVE}\left(0 = \frac{2\cdot T\cdot \beta\cdot (T\cdot \lambda\cdot (4\cdot \beta^2 + \gamma^2) + (c - \alpha)\cdot (2\cdot \beta - \gamma)^2)}{(4\cdot \beta^2 - \gamma^2)^2}, \lambda\right)$$

eq (10)

#63: $$\lambda hat = \frac{(\alpha - c)\cdot (2\cdot \beta - \gamma)^2}{T\cdot (4\cdot \beta^2 + \gamma^2)} \vee T = 0 \vee \beta = 0$$

Result 3 and Appendix C:

#64: $$\frac{d}{d\lambda}\left(eprofitxh = \frac{\beta\cdot (T\cdot \gamma\cdot \lambda + (c - \alpha)\cdot (\gamma - 2\cdot \beta))^2}{(4\cdot \beta^2 - \gamma^2)^2}\right)$$

eq (c.1)

#65: $$0 < \frac{2\cdot T\cdot \beta\cdot \gamma\cdot (T\cdot \gamma\cdot \lambda + (c - \alpha)\cdot (\gamma - 2\cdot \beta))}{(4\cdot \beta^2 - \gamma^2)^2}$$

#66: $$\frac{d}{d\lambda}\left(eprofitxf = \frac{\beta\cdot (2\cdot T\cdot \beta\cdot \lambda + (c - \alpha)\cdot (2\cdot \beta - \gamma))^2}{(4\cdot \beta^2 - \gamma^2)^2}\right)$$

eq (C.2)

#67: $$\frac{4\cdot T\cdot \beta^2\cdot(2\cdot T\cdot \beta\cdot \lambda + (c - \alpha)\cdot(2\cdot \beta - \gamma))}{(4\cdot \beta^2 - \gamma^2)^2}$$

< 0 if [Assumption 3]

#68: $4\cdot T\cdot \beta^2\cdot(2\cdot T\cdot \beta\cdot \lambda + (c - \alpha)\cdot(2\cdot \beta - \gamma)) < 0$

#69: $\mathrm{SOLVE}(4\cdot T\cdot \beta^2\cdot(2\cdot T\cdot \beta\cdot \lambda + (c - \alpha)\cdot(2\cdot \beta - \gamma)) < 0, T)$

#70: $$\left(\beta \ne 0 \wedge T < 0 \wedge \mathrm{IF}\left(\beta\cdot\lambda < 0, T < \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda}\right)\right) \vee \left(\beta \ne 0 \wedge T < 0 \wedge \mathrm{IF}\left(\beta\cdot\lambda > 0, T > \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda}\right)\right) \vee \left(\beta \ne 0 \wedge T > 0 \wedge \mathrm{IF}\left(\beta\cdot\lambda < 0, T > \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda}\right)\right) \vee \left(\beta \ne 0 \wedge T > 0 \wedge \mathrm{IF}\left(\beta\cdot\lambda > 0, T < \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda}\right)\right)$$

which is Assumption 3

#71: $$T < \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda}$$

#72: $$\frac{d}{d\lambda}\frac{d}{d\lambda}\left(\mathrm{eprofitxh} = \frac{\beta\cdot(T\cdot\gamma\cdot\lambda + (c - \alpha)\cdot(\gamma - 2\cdot\beta))^2}{(4\cdot\beta^2 - \gamma^2)^2}\right)$$

#73: $$0 < \frac{2\cdot T^2\cdot\beta\cdot\gamma^2}{(4\cdot\beta^2 - \gamma^2)^2}$$

#74: $$\frac{d}{d\lambda}\frac{d}{d\lambda}\left(eprofitxf = \frac{\beta\cdot(2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma))^2}{(4\cdot\beta^2 - \gamma^2)^2}\right)$$

#75: $$0 < \frac{8\cdot T^2\cdot\beta^3}{(4\cdot\beta^2 - \gamma^2)^2}$$

proving Result 3c, eq (C.3)

#76: $$\frac{2\cdot T\cdot\beta\cdot\gamma\cdot(T\cdot\gamma\cdot\lambda + (c - \alpha)\cdot(\gamma - 2\cdot\beta))}{(4\cdot\beta^2 - \gamma^2)^2} = - \frac{4\cdot T\cdot\beta^2\cdot(2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma))}{(4\cdot\beta^2 - \gamma^2)^2}$$

#77: $$\text{SOLVE}\left(\frac{2\cdot T\cdot\beta\cdot\gamma\cdot(T\cdot\gamma\cdot\lambda + (c - \alpha)\cdot(\gamma - 2\cdot\beta))}{(4\cdot\beta^2 - \gamma^2)^2} = - \frac{4\cdot T\cdot\beta^2\cdot(2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma))}{(4\cdot\beta^2 - \gamma^2)^2}, \lambda\right)$$

#78: $$\lambda = \frac{(\alpha - c)\cdot(2\cdot\beta - \gamma)^2}{T\cdot(4\cdot\beta^2 + \gamma^2)} \vee T = 0 \vee \beta = 0$$

Proving Result 3d, eq (C.4)

#79: $$\frac{8\cdot T^2\cdot\beta^3}{(4\cdot\beta^2-\gamma^2)^2}-\frac{2\cdot T^2\cdot\beta\cdot\gamma^2}{(4\cdot\beta^2-\gamma^2)^2}$$

#80: $$0<\frac{2\cdot T^2\cdot\beta}{4\cdot\beta^2-\gamma^2}$$

*** Section 5: Ad-valorem tariff wars

eq (11) profit X

#81: $eprofitxh = (pxh - c)\cdot xh$

#82: $eprofitxf = \lambda\cdot((1 - \tau)\cdot pxf - c)\cdot xf + (1 - \lambda)\cdot(pxf - c)\cdot xf$

#83: $eprofitx = (pxh - c)\cdot xh + \lambda\cdot((1 - \tau)\cdot pxf - c)\cdot xf + (1 - \lambda)\cdot(pxf - c)\cdot xf$

eq (12) profit Y

#84: $eprofityf = (pyf - c)\cdot yf$

#85: $eprofityh = \lambda\cdot((1 - \tau)\cdot pyh - c)\cdot yh + (1 - \lambda)\cdot(pyh - c)\cdot yh$

#86: $eprofity = (pyf - c)\cdot yf + \lambda\cdot((1 - \tau)\cdot pyh - c)\cdot yh + (1 - \lambda)\cdot(pyh - c)\cdot yh$

Derivations of (13) and (14) equlibrium sales and Appendix D

#87: $eprofitx = ((\alpha - \beta\cdot xh - \gamma\cdot yh) - c)\cdot xh + \lambda\cdot((1 - \tau)\cdot(\alpha - \beta\cdot xf - \gamma\cdot yf) - c)\cdot xf + (1 - \lambda)\cdot((\alpha - \beta\cdot xf - \gamma\cdot yf) - c)\cdot xf$

#88: $eprofity = ((\alpha - \gamma\cdot xf - \beta\cdot yf) - c)\cdot yf + \lambda\cdot((1 - \tau)\cdot(\alpha - \gamma\cdot xh - \beta\cdot yh) - c)\cdot yh + (1 - \lambda)\cdot((\alpha - \gamma\cdot xh$

$- \beta \cdot yh) - c) \cdot yh$

eqs (D.1)

#89: $\frac{d}{d\ xf}$ (eprofitx = $((\alpha - \beta \cdot xh - \gamma \cdot yh) - c) \cdot xh + \lambda \cdot ((1 - \tau) \cdot (\alpha - \beta \cdot xf - \gamma \cdot yf) - c) \cdot xf + (1 - \lambda) \cdot ((\alpha -$

$\beta \cdot xf - \gamma \cdot yf) - c) \cdot xf)$

#90: $$0 = (2 \cdot xf \cdot \beta + yf \cdot \gamma - \alpha) \cdot (\lambda \cdot \tau - 1) - c$$

#91: $\frac{d}{d\ xh}$ (eprofitx = $((\alpha - \beta \cdot xh - \gamma \cdot yh) - c) \cdot xh + \lambda \cdot ((1 - \tau) \cdot (\alpha - \beta \cdot xf - \gamma \cdot yf) - c) \cdot xf + (1 - \lambda) \cdot ((\alpha -$

$\beta \cdot xf - \gamma \cdot yf) - c) \cdot xf)$

#92: $$0 = -c - 2 \cdot xh \cdot \beta - yh \cdot \gamma + \alpha$$

eqs (D.2)

#93: $\frac{d}{d\ yf}$ (eprofity = $((\alpha - \gamma \cdot xf - \beta \cdot yf) - c) \cdot yf + \lambda \cdot ((1 - \tau) \cdot (\alpha - \gamma \cdot xh - \beta \cdot yh) - c) \cdot yh + (1 - \lambda) \cdot ((\alpha -$

$\gamma \cdot xh - \beta \cdot yh) - c) \cdot yh)$

#94: $$0 = -c - xf \cdot \gamma - 2 \cdot yf \cdot \beta + \alpha$$

#95: $\frac{d}{d\ yh}$ (eprofity = $((\alpha - \gamma \cdot xf - \beta \cdot yf) - c) \cdot yf + \lambda \cdot ((1 - \tau) \cdot (\alpha - \gamma \cdot xh - \beta \cdot yh) - c) \cdot yh + (1 - \lambda) \cdot ((\alpha -$

$\gamma \cdot xh - \beta \cdot yh) - c) \cdot yh)$

#96: $$0 = (xh \cdot \gamma + 2 \cdot yh \cdot \beta - \alpha) \cdot (\lambda \cdot \tau - 1) - c$$

Second-order conditions

#97: $\frac{d}{d\ xf} \frac{d}{d\ xf}$ (eprofitx = $((\alpha - \beta \cdot xh - \gamma \cdot yh) - c) \cdot xh + \lambda \cdot ((1 - \tau) \cdot (\alpha - \beta \cdot xf - \gamma \cdot yf) - c) \cdot xf + (1 -$

$\lambda) \cdot ((\alpha - \beta \cdot xf - \gamma \cdot yf) - c) \cdot xf)$

#98: $$0 > 2 \cdot \beta \cdot (\lambda \cdot \tau - 1)$$

#99: $\frac{d}{d\ xh} \frac{d}{d\ xh}$ (eprofitx = $((\alpha - \beta \cdot xh - \gamma \cdot yh) - c) \cdot xh + \lambda \cdot ((1 - \tau) \cdot (\alpha - \beta \cdot xf - \gamma \cdot yf) - c) \cdot xf + (1 -$

$\lambda) \cdot ((\alpha - \beta \cdot xf - \gamma \cdot yf) - c) \cdot xf)$

#100: $$0 > - 2 \cdot \beta$$

#101: $\frac{d}{d\ xh} \frac{d}{d\ xf}$ (eprofitx = $((\alpha - \beta \cdot xh - \gamma \cdot yh) - c) \cdot xh + \lambda \cdot ((1 - \tau) \cdot (\alpha - \beta \cdot xf - \gamma \cdot yf) - c) \cdot xf + (1 -$

$\lambda) \cdot ((\alpha - \beta \cdot xf - \gamma \cdot yf) - c) \cdot xf)$

#102: $$0 = 0$$

Solving the 4 FOC for sales levels:

#103: SOLVE([0 = (2·xf·β + yf·γ − α)·(λ·τ − 1) − c, 0 = −c − 2·xh·β − yh·γ + α, 0 = −c − xf·γ − 2·yf·β + α, 0 = (xh·γ + 2·yh·β − α)·(λ·τ − 1) − c], [xf, xh, yf, yh])

eqs (13) and (14)

#104: $$\left[xf = \frac{c\cdot(2\cdot\beta + \gamma\cdot(\lambda\cdot\tau - 1)) + \alpha\cdot(2\cdot\beta - \gamma)\cdot(\lambda\cdot\tau - 1)}{(4\cdot\beta^2 - \gamma^2)\cdot(\lambda\cdot\tau - 1)} \wedge xh = \frac{c\cdot(2\cdot\beta\cdot(\lambda\cdot\tau - 1) + \gamma) + \alpha\cdot(1 - \lambda\cdot\tau)\cdot(2\cdot\beta - \gamma)}{(1 - \lambda\cdot\tau)\cdot(4\cdot\beta^2 - \gamma^2)} \wedge yf = \frac{c\cdot(2\cdot\beta\cdot(\lambda\cdot\tau - 1) + \gamma) + \alpha\cdot(1 - \lambda\cdot\tau)\cdot(2\cdot\beta - \gamma)}{(1 - \lambda\cdot\tau)\cdot(4\cdot\beta^2 - \gamma^2)} \wedge yh = \frac{c\cdot(2\cdot\beta + \gamma\cdot(\lambda\cdot\tau - 1)) + \alpha\cdot(2\cdot\beta - \gamma)\cdot(\lambda\cdot\tau - 1)}{(4\cdot\beta^2 - \gamma^2)\cdot(\lambda\cdot\tau - 1)}\right]$$

xh = yf > 0 if

#105: c·(2·β·(λ·τ − 1) + γ) + α·(1 − λ·τ)·(2·β − γ) > 0

#106: SOLVE(c·(2·β·(λ·τ − 1) + γ) + α·(1 − λ·τ)·(2·β − γ) > 0, τ)

#107: $$\text{IF}\left(2\cdot c\cdot\beta\cdot\lambda + \alpha\cdot\lambda\cdot(\gamma - 2\cdot\beta) < 0,\ \tau < \frac{(c - \alpha)\cdot(2\cdot\beta - \gamma)}{\lambda\cdot(2\cdot c\cdot\beta + \alpha\cdot(\gamma - 2\cdot\beta))}\right) \vee \text{IF}\left(2\cdot c\cdot\beta\cdot\lambda + \alpha\cdot\lambda\cdot(\gamma - 2\cdot\beta) > 0,\ \tau\right.$$

$$> \frac{(c - \alpha)\cdot(2\cdot\beta - \gamma)}{\lambda\cdot(2\cdot c\cdot\beta + \alpha\cdot(\gamma - 2\cdot\beta))}\Bigg)$$

xf = yh > 0 if

#108: $c\cdot(2\cdot\beta + \gamma\cdot(\lambda\cdot\tau - 1)) + \alpha\cdot(2\cdot\beta - \gamma)\cdot(\lambda\cdot\tau - 1) < 0$

#109: $\mathrm{SOLVE}(c\cdot(2\cdot\beta + \gamma\cdot(\lambda\cdot\tau - 1)) + \alpha\cdot(2\cdot\beta - \gamma)\cdot(\lambda\cdot\tau - 1) < 0,\ \tau)$

#110: $\mathrm{IF}\left(c\cdot\gamma\cdot\lambda + \alpha\cdot\lambda\cdot(2\cdot\beta - \gamma) < 0,\ \tau > \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{\lambda\cdot(c\cdot\gamma + \alpha\cdot(2\cdot\beta - \gamma))}\right) \vee \mathrm{IF}\left(c\cdot\gamma\cdot\lambda + \alpha\cdot\lambda\cdot(2\cdot\beta - \gamma) > 0,\ \tau < \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{\lambda\cdot(c\cdot\gamma + \alpha\cdot(2\cdot\beta - \gamma))}\right)$

Assumption 4

#111: $\tau < \tau\_bar = \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{\lambda\cdot(c\cdot\gamma + \alpha\cdot(2\cdot\beta - \gamma))}$

which one is higher?

#112: $\frac{(c - \alpha)\cdot(2\cdot\beta - \gamma)}{\lambda\cdot(2\cdot c\cdot\beta + \alpha\cdot(\gamma - 2\cdot\beta))} - \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{\lambda\cdot(c\cdot\gamma + \alpha\cdot(2\cdot\beta - \gamma))}$

#113: $\frac{c\cdot(c - \alpha)\cdot(2\cdot\beta + \gamma)\cdot(2\cdot\beta - \gamma)}{\lambda\cdot(2\cdot c\cdot\beta + \alpha\cdot(\gamma - 2\cdot\beta))\cdot(c\cdot\gamma + \alpha\cdot(2\cdot\beta - \gamma))}$

should be > 0 implies that Assumption 4 is sufficient.

eqs (15) and (16) equilbrium prices

#114: $$pxh = \frac{c\cdot(2\cdot\beta^2\cdot(\lambda\cdot\tau - 1) - \beta\cdot\gamma + \gamma^2\cdot(1 - \lambda\cdot\tau)) + \alpha\cdot\beta\cdot(2\cdot\beta - \gamma)\cdot(\lambda\cdot\tau - 1)}{(4\cdot\beta^2 - \gamma^2)\cdot(\lambda\cdot\tau - 1)}$$

#115: $$pyh = \frac{c\cdot(2\cdot\beta^2 + \beta\cdot\gamma\cdot(1 - \lambda\cdot\tau) - \gamma^2) + \alpha\cdot\beta\cdot(1 - \lambda\cdot\tau)\cdot(2\cdot\beta - \gamma)}{(1 - \lambda\cdot\tau)\cdot(4\cdot\beta^2 - \gamma^2)}$$

#116: $$pxf = \frac{c\cdot(2\cdot\beta^2 + \beta\cdot\gamma\cdot(1 - \lambda\cdot\tau) - \gamma^2) + \alpha\cdot\beta\cdot(1 - \lambda\cdot\tau)\cdot(2\cdot\beta - \gamma)}{(1 - \lambda\cdot\tau)\cdot(4\cdot\beta^2 - \gamma^2)}$$

#117: $$pyf = \frac{c\cdot(2\cdot\beta^2\cdot(\lambda\cdot\tau - 1) - \beta\cdot\gamma + \gamma^2\cdot(1 - \lambda\cdot\tau)) + \alpha\cdot\beta\cdot(2\cdot\beta - \gamma)\cdot(\lambda\cdot\tau - 1)}{(4\cdot\beta^2 - \gamma^2)\cdot(\lambda\cdot\tau - 1)}$$

eqs (17): equilibrium expected domestic profits

#118: $$eprofitxh = \frac{\beta\cdot(c\cdot(2\cdot\beta\cdot(\lambda\cdot\tau - 1) + \gamma) + \alpha\cdot(1 - \lambda\cdot\tau)\cdot(2\cdot\beta - \gamma))^2}{(4\cdot\beta^2 - \gamma^2)^2\cdot(\lambda\cdot\tau - 1)^2}$$

#119: $$eprofityf = \frac{\beta\cdot(c\cdot(2\cdot\beta\cdot(\lambda\cdot\tau - 1) + \gamma) + \alpha\cdot(1 - \lambda\cdot\tau)\cdot(2\cdot\beta - \gamma))^2}{(4\cdot\beta^2 - \gamma^2)^2\cdot(\lambda\cdot\tau - 1)^2}$$

eqs (18): equilibrium expected export profits

#120: $$eprofitxf = \frac{\beta\cdot(c\cdot(2\cdot\beta + \gamma\cdot(\lambda\cdot\tau - 1)) + \alpha\cdot(2\cdot\beta - \gamma)\cdot(\lambda\cdot\tau - 1))^2}{(1 - \lambda\cdot\tau)\cdot(4\cdot\beta^2 - \gamma^2)^2}$$

#121: $$eprofityh = \frac{\beta\cdot(c\cdot(2\cdot\beta + \gamma\cdot(\lambda\cdot\tau - 1)) + \alpha\cdot(2\cdot\beta - \gamma)\cdot(\lambda\cdot\tau - 1))^2}{(1 - \lambda\cdot\tau)\cdot(4\cdot\beta^2 - \gamma^2)^2}$$

*** Section 6 (deleted starting v11): Unilateral tariff with no retaliation
Ignore equation numbers as section 6 was removed: last version was v10

eq (18) profit of X

#122: $eprofitx = (pxh - c)\cdot xh + (pxf - c)\cdot xf$

eq (19) profit of Y

#123: $eprofity = (pyf - c)\cdot yf + \lambda\cdot(pyh - c - T)\cdot yh + (1 - \lambda)\cdot(pyh - c)\cdot yh$

Appendix E

#124: $eprofitx = ((\alpha - \beta\cdot xh - \gamma\cdot yh) - c)\cdot xh + ((\alpha - \beta\cdot xf - \gamma\cdot yf) - c)\cdot xf$

#125: $eprofity = ((\alpha - \gamma\cdot xf - \beta\cdot yf) - c)\cdot yf + \lambda\cdot((\alpha - \gamma\cdot xh - \beta\cdot yh) - c - T)\cdot yh + (1 - \lambda)\cdot((\alpha - \gamma\cdot xh - \beta\cdot yh) - c)\cdot yh$

eqs (E.1)

#126: $$\frac{d}{d\ xf}(eprofitx = ((\alpha - \beta\cdot xh - \gamma\cdot yh) - c)\cdot xh + ((\alpha - \beta\cdot xf - \gamma\cdot yf) - c)\cdot xf)$$

#127: $$0 = -c - 2\cdot xf\cdot\beta - yf\cdot\gamma + \alpha$$

#128: $\frac{d}{d\ xh}$ (eprofitx = ((α − β·xh − γ·yh) − c)·xh + ((α − β·xf − γ·yf) − c)·xf)

#129: $0 = -c - 2\cdot xh\cdot\beta - yh\cdot\gamma + \alpha$

eqs (E.2)

#130: $\frac{d}{d\ yf}$ (eprofity = ((α − γ·xf − β·yf) − c)·yf + λ·((α − γ·xh − β·yh) − c − T)·yh + (1 − λ)·((α − γ·xh − β·yh) − c)·yh)

#131: $0 = -c - xf\cdot\gamma - 2\cdot yf\cdot\beta + \alpha$

#132: $\frac{d}{d\ yh}$ (eprofity = ((α − γ·xf − β·yf) − c)·yf + λ·((α − γ·xh − β·yh) − c − T)·yh + (1 − λ)·((α − γ·xh − β·yh) − c)·yh)

#133: $0 = -\ T\cdot\lambda - c - xh\cdot\gamma - 2\cdot yh\cdot\beta + \alpha$

eq (20) eql sales under no retaliation

#134: SOLVE([0 = −c − 2·xf·β − yf·γ + α, 0 = −c − 2·xh·β − yh·γ + α, 0 = −c − xf·γ − 2·yf·β + α, 0 = − T·λ − c − xh·γ − 2·yh·β + α], [xf, xh, yf, yh])

#135: $\left[xf = \frac{\alpha - c}{2\cdot\beta + \gamma} \wedge xh = \frac{T\cdot\gamma\cdot\lambda + (c - \alpha)\cdot(\gamma - 2\cdot\beta)}{4\cdot\beta^2 - \gamma^2} \wedge yf = \frac{\alpha - c}{2\cdot\beta + \gamma} \wedge yh = \right.$

$$\left.\frac{2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma)}{\gamma^2 - 4\cdot\beta^2}\right]$$

y > 0 if [Assumption 3]

#136: $2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma) < 0$

#137: SOLVE$(2\cdot T\cdot\beta\cdot\lambda + (c - \alpha)\cdot(2\cdot\beta - \gamma) < 0, T)$

#138: $$\mathrm{IF}\left(\beta\cdot\lambda < 0, T > \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda}\right) \vee \mathrm{IF}\left(\beta\cdot\lambda > 0, T < \frac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda}\right)$$

#139: $T < \dfrac{(c - \alpha)\cdot(\gamma - 2\cdot\beta)}{2\cdot\beta\cdot\lambda}$

eq (21) equlibrium prices without retaliation

#140: $$pxh = \frac{T\cdot\beta\cdot\gamma\cdot\lambda + (2\cdot\beta - \gamma)\cdot(c\cdot(\beta + \gamma) + \alpha\cdot\beta)}{4\cdot\beta^2 - \gamma^2}$$

#141: $$pyh = \frac{T\cdot\lambda\cdot(2\cdot\beta^2 - \gamma^2) + (2\cdot\beta - \gamma)\cdot(c\cdot(\beta + \gamma) + \alpha\cdot\beta)}{4\cdot\beta^2 - \gamma^2}$$

#142: $$pxf = \frac{c\cdot(\beta + \gamma) + \alpha\cdot\beta}{2\cdot\beta + \gamma}$$

#143: $$pyf = \frac{c\cdot(\beta + \gamma) + \alpha\cdot\beta}{2\cdot\beta + \gamma}$$

eq (22) equilibrium profit of X without retaliation

#144: $$\mathrm{eprofitx} = \frac{\beta\cdot(T^2\cdot\gamma^2\cdot\lambda^2 + 2\cdot T\cdot\gamma\cdot\lambda\cdot(c - \alpha)\cdot(\gamma - 2\cdot\beta) + 2\cdot(c - \alpha)^2\cdot(2\cdot\beta - \gamma)^2)}{(2\cdot\beta + \gamma)^2\cdot(2\cdot\beta - \gamma)^2}$$

eq (20) equilibrium profit of Y without retaliation

#145: $$\mathrm{eprofity} = \frac{2\cdot\beta\cdot(2\cdot T^2\cdot\beta^2\cdot\lambda^2 + 2\cdot T\cdot\beta\cdot\lambda\cdot(c - \alpha)\cdot(2\cdot\beta - \gamma) + (c - \alpha)^2\cdot(2\cdot\beta - \gamma)^2)}{(2\cdot\beta + \gamma)^2\cdot(2\cdot\beta - \gamma)^2}$$

Result 4 and Appendix F

eq (F.1)

#146: $$\frac{d}{d\lambda}\left(\mathrm{eprofitx} = \frac{\beta\cdot(T^2\cdot\gamma^2\cdot\lambda^2 + 2\cdot T\cdot\gamma\cdot\lambda\cdot(c - \alpha)\cdot(\gamma - 2\cdot\beta) + 2\cdot(c - \alpha)^2\cdot(2\cdot\beta - \gamma)^2)}{(2\cdot\beta + \gamma)^2\cdot(2\cdot\beta - \gamma)^2}\right)$$

#147: $$0 > \frac{2\cdot T\cdot\beta\cdot\gamma\cdot(T\cdot\gamma\cdot\lambda + (c - \alpha)\cdot(\gamma - 2\cdot\beta))}{(2\cdot\beta + \gamma)^2\cdot(2\cdot\beta - \gamma)^2}$$

eq (F.2)

#148: $$\frac{d}{d\lambda}\left(\mathrm{eprofity} = \frac{2\cdot\beta\cdot(2\cdot T^2\cdot\beta^2\cdot\lambda^2 + 2\cdot T\cdot\beta\cdot\lambda\cdot(c - \alpha)\cdot(2\cdot\beta - \gamma) + (c - \alpha)^2\cdot(2\cdot\beta - \gamma)^2)}{(2\cdot\beta + \gamma)^2\cdot(2\cdot\beta - \gamma)^2}\right)$$

#149: $$\frac{4\cdot T\cdot \beta^2\cdot (2\cdot T\cdot \beta\cdot \lambda + (c - \alpha)\cdot (2\cdot \beta - \gamma))}{(2\cdot \beta + \gamma)^2\cdot (2\cdot \beta - \gamma)^2}$$

< 0 if [Assumption 3]

#150: $2\cdot T\cdot \beta\cdot \lambda + (c - \alpha)\cdot (2\cdot \beta - \gamma) < 0$

#151: SOLVE$(2\cdot T\cdot \beta\cdot \lambda + (c - \alpha)\cdot (2\cdot \beta - \gamma) < 0, T)$

#152: $$\mathrm{IF}\left(\beta\cdot \lambda < 0,\ T > \frac{(c - \alpha)\cdot (\gamma - 2\cdot \beta)}{2\cdot \beta\cdot \lambda}\right) \vee \mathrm{IF}\left(\beta\cdot \lambda > 0,\ T < \frac{(c - \alpha)\cdot (\gamma - 2\cdot \beta)}{2\cdot \beta\cdot \lambda}\right)$$

#153: $$T < \frac{(c - \alpha)\cdot (\gamma - 2\cdot \beta)}{2\cdot \beta\cdot \lambda}$$

End of Section 6 (removed from v11 and on)