runfast_2024_mm_dd.dfw

#1: CaseMode := Sensitive

#2: InputMode := Word

time

#3: $t :\in \text{Real } [0, \infty)$

cost reduction parameter

#4: $\rho :\in \text{Real } (0, 1]$

interest bank pays to depositors

#5: $rd :\in \text{Real } (0, \infty)$

probability of a run = investment failure

#6: $\phi :\in \text{Real } (0, 1)$

speed of fund withdrawal during a run

#7: $\sigma :\in \text{Real } (0, \infty)$

Risky interest bank earns on investment

#8: $rk :\in \text{Real } (0, \infty)$

bank investment return concavity parameter

#9: $\alpha :\in \text{Real } (0, 1)$

*** Section 3: The model

equation (1): amount withdrawn at t during a run

#10: $dt = 1 + rd - \sigma \cdot t$

equation (2): bank liquidity at t during a run

#11: $qt = q - \sigma \cdot t$

time when bank becomesd insolvent

#12: $0 = q - \sigma \cdot tq$

#13: $\mathrm{SOLVE}(0 = q - \sigma \cdot ti,\ tq)$

eq (3)

#14: $$tq = \frac{q}{\sigma}$$

eq (4): Bank profit (bottom: no run, prob $1-\phi$). Otherwise: upper: profit=0 with prob ϕ

#15: $profit = rk \cdot (1 - q)^{\alpha} - rd$

eq (5) top: Depositor welfare under a run and slow bailout $tb \geq tq$

#16: $wds = rd - \lambda \cdot (tbs - tq) \cdot (1 + rd - q)$

eq (5) bottom: Depositor welfare under fast bailout and under no run

#17: $wdf = rd$

eq (6) top bailout cost $tb < \tau bar$

#18: $cb = 1 + rd - q$

eq (6) bottom: bailout cost $tb \geq \tau bar$ (reduced cost)

#19: $cb = \rho \cdot (1 + rd - q)$

*** Section 4: Bailout policy

eq (8): total welfare

#20: $w = wd + profit - cb$

** Subsection 4.1: Optimal bailout time (q is given)

Deriving Result 1 and eq (9)

eq (11) top: tb = τbar (welfare under slow bailout)

#21: $ws = (rd - \lambda\cdot(\tau bar - tq)\cdot(1 + rd - q)) + 0 - \rho\cdot(1 + rd - q)$

eq (11) bottom: tb < td (welfare under fast bailout)

#22: $wf = rd + 0 - (1 + rd - q)$

ws − wf =

#23: $(rd - \lambda\cdot(\tau bar - tq)\cdot(1 + rd - q)) + 0 - \rho\cdot(1 + rd - q) - (rd + 0 - (1 + rd - q))$

subs for tq

#24: $\left(rd - \lambda\cdot\left(\tau bar - \frac{q}{\sigma}\right)\cdot(1 + rd - q)\right) + 0 - \rho\cdot(1 + rd - q) - (rd + 0 - (1 + rd - q))$

#25: $$- \frac{q^2\cdot\lambda - q\cdot(rd\cdot\lambda + \lambda\cdot(\sigma\cdot\tau bar + 1) + \rho\cdot\sigma - \sigma) + \sigma\cdot(rd\cdot(\lambda\cdot\tau bar + \rho - 1) + \lambda\cdot\tau bar + \rho - 1)}{\sigma}$$

≥ 0 if

#26: $q^2\cdot\lambda - q\cdot(rd\cdot\lambda + \lambda\cdot(\sigma\cdot\tau bar + 1) + \rho\cdot\sigma - \sigma) + \sigma\cdot(rd\cdot(\lambda\cdot\tau bar + \rho - 1) + \lambda\cdot\tau bar + \rho - 1) \le 0$

#27: $\mathrm{SOLVE}(q^2\cdot\lambda - q\cdot(rd\cdot\lambda + \lambda\cdot(\sigma\cdot\tau bar + 1) + \rho\cdot\sigma - \sigma) + \sigma\cdot(rd\cdot(\lambda\cdot\tau bar + \rho - 1) + \lambda\cdot\tau bar + \rho - 1) \le 0, q)$

#28: $$\frac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda} \le q \le rd + 1 \vee rd + 1 \le q \le \frac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda}$$

eq (9) and Result 1

#29: $$\frac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda} \le q$$

Deriving Assumption 2 and eq (7) showing that qtitle is between 0 and 1

#30: $$\frac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda} < 1$$

#31: $$\mathrm{SOLVE}\left(\frac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda} < 1, \tau bar\right)$$

#32: $$\tau bar < \frac{\lambda - \sigma\cdot(\rho - 1)}{\lambda\cdot\sigma}$$

#33: $$\frac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda} > 0$$

#34: $$\mathrm{SOLVE}\left(\frac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda} > 0, \tau bar\right)$$

#35: $$\tau bar > \frac{1 - \rho}{\lambda}$$

difference between upper bound and lower bound (nonempty interval) because

#36: $\dfrac{\lambda - \sigma\cdot(\rho - 1)}{\lambda\cdot\sigma} - \dfrac{1 - \rho}{\lambda}$

#37: $\dfrac{1}{\sigma} > 0$

** Subsection 4.2: Effect of faster runs on optimal bailout time
go back to eq (9) q_tilde to prove Result 2

#38: $\dfrac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda} = q\_tilde$

#39: $\dfrac{d}{d\sigma}\,\dfrac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda}$

#40: $\dfrac{\lambda\cdot\tau bar + \rho - 1}{\lambda}$

> 0 if [Yes, by Assumption 2]

#41: $\lambda\cdot\tau bar + \rho - 1 > 0$

#42: SOLVE($\lambda\cdot\tau bar + \rho - 1 > 0$, τbar)

#43: $\tau bar > \dfrac{1 - \rho}{\lambda}$

*** Section 5: Optimal liquidity requirement

eq (12): Expected total welfare

#44: $ew = \phi\cdot w\_run + (1 - \phi)\cdot w\_no\_run$

eq (13) computing qmax

#45: $$\frac{qmax}{\sigma} = \tau bar$$

#46: $$\text{SOLVE}\left(\frac{qmax}{\sigma} = \tau bar,\ qmax\right)$$

#47: $$qmax = \sigma \cdot \tau bar$$

eq (14)

#48: $$w\_no\_run = rd + rk \cdot (1 - q)^{\alpha} - rd + 0$$

#49: $$w\_no\_run = rk \cdot (1 - q)^{\alpha}$$

w_run for high q: q > q_tilde. top of eq (15)

#50: $$w\_run\_high\_q = (rd - \lambda \cdot (\tau bar - tq) \cdot (1 + rd - q)) + 0 - \rho \cdot (1 + rd - q)$$

#51: $$w\_run\_high\_q = \left(rd - \lambda \cdot \left(\tau bar - \frac{q}{\sigma}\right) \cdot (1 + rd - q)\right) + 0 - \rho \cdot (1 + rd - q)$$

w_run for low q: q < q_tilde. bottom of eq (15)

#52: $$w\_run\_low\_q = rd + 0 - (1 + rd - q)$$

#53: $$w\_run\_low\_q = q - 1$$

* Proof of Result 3:

max ew for q ≥ q_tilde by adding 14 to top part of (15)

#54: $$ew = \phi \cdot \left(\left(rd - \lambda \cdot \left(\tau bar - \frac{q}{\sigma} \right) \cdot (1 + rd - q) \right) + 0 - \rho \cdot (1 + rd - q) \right) + (1 - \phi) \cdot (rk \cdot (1 - q)^{\alpha})$$

#55: $$\frac{d}{dq} \left(ew = \phi \cdot \left(\left(rd - \lambda \cdot \left(\tau bar - \frac{q}{\sigma} \right) \cdot (1 + rd - q) \right) + 0 - \rho \cdot (1 + rd - q) \right) + (1 - \phi) \cdot (rk \cdot (1 - q)^{\alpha}) \right)$$

#56: $$0 = rk \cdot \alpha \cdot (1 - q)^{\alpha - 1} \cdot (\phi - 1) - \frac{\phi \cdot (2 \cdot q \cdot \lambda - rd \cdot \lambda - \lambda \cdot (\sigma \cdot \tau bar + 1) - \rho \cdot \sigma)}{\sigma}$$

Above FOC cannot be explicitly solved for q

#57: $$\frac{d}{dq} \frac{d}{dq} \left(ew = \phi \cdot \left(\left(rd - \lambda \cdot \left(\tau bar - \frac{q}{\sigma} \right) \cdot (1 + rd - q) \right) + 0 - \rho \cdot (1 + rd - q) \right) + (1 - \phi) \cdot (rk \cdot (1 - q)^{\alpha}) \right)$$

#58: $$0 > rk \cdot \alpha \cdot (1 - \phi) \cdot (1 - q)^{\alpha - 2} \cdot (\alpha - 1) - \frac{2 \cdot \lambda \cdot \phi}{\sigma}$$

Use IFT to evaluate dq/dσ = − dF/dσ div dF/dq

#59: $$\frac{d}{d\sigma} \left(rk \cdot \alpha \cdot (1 - q)^{\alpha - 1} \cdot (\phi - 1) - \frac{\phi \cdot (2 \cdot q \cdot \lambda - rd \cdot \lambda - \lambda \cdot (\sigma \cdot \tau bar + 1) - \rho \cdot \sigma)}{\sigma} \right)$$

#60: $$\frac{\lambda \cdot \phi \cdot (2 \cdot q - rd - 1)}{\sigma^2}$$

#61: $$\frac{d}{dq} \left(rk \cdot \alpha \cdot (1 - q)^{\alpha - 1} \cdot (\phi - 1) - \frac{\phi \cdot (2 \cdot q \cdot \lambda - rd \cdot \lambda - \lambda \cdot (\sigma \cdot \tau bar + 1) - \rho \cdot \sigma)}{\sigma} \right)$$

#62: $$rk\cdot\alpha\cdot(1-\phi)\cdot(1-q)^{\alpha-2}\cdot(\alpha-1)-\frac{2\cdot\lambda\cdot\phi}{\sigma}$$

Hence dq/dσ =

#63: $$-\frac{\dfrac{\lambda\cdot\phi\cdot(2\cdot q-rd-1)}{\sigma^2}}{rk\cdot\alpha\cdot(1-\phi)\cdot(1-q)^{\alpha-2}\cdot(\alpha-1)-\dfrac{2\cdot\lambda\cdot\phi}{\sigma}}$$

denominator < 0, Hence, dq/dσ < 0 if

#64: $$\frac{\lambda\cdot\phi\cdot(2\cdot q-rd-1)}{\sigma^2}<0$$

#65: $$\mathrm{SOLVE}\left(\frac{\lambda\cdot\phi\cdot(2\cdot q-rd-1)}{\sigma^2}<0,\ q\right)$$

#66: $$q<\frac{rd+1}{2}$$

#67: $$\mathrm{SOLVE}\left(q<\frac{rd+1}{2},\ rd\right)$$

#68: $$rd>2\cdot q-1$$

max ew for q < q_tilde by adding (14) to top part of (15). Proof of Result 3, Appendix A

#69: $ew = \phi\cdot(q - 1) + (1 - \phi)\cdot(rk\cdot(1 - q)^{\alpha})$

#70: $\frac{d}{dq}(ew = \phi\cdot(q - 1) + (1 - \phi)\cdot(rk\cdot(1 - q)^{\alpha}))$

eq (A.1)

#71: $$0 = rk\cdot\alpha\cdot(1 - q)^{\alpha - 1}\cdot(\phi - 1) + \phi$$

#72: $\frac{d}{dq}\frac{d}{dq}(ew = \phi\cdot(q - 1) + (1 - \phi)\cdot(rk\cdot(1 - q)^{\alpha}))$

#73: $$0 > rk\cdot\alpha\cdot(1 - \phi)\cdot(1 - q)^{\alpha - 2}\cdot(\alpha - 1)$$

#74: $\mathrm{SOLVE}(0 = rk\cdot\alpha\cdot(1 - q)^{\alpha - 1}\cdot(\phi - 1) + \phi, q)$

eq (16)

#75: $$q = 1 - \left(\frac{rk\cdot\alpha\cdot(1 - \phi)}{\phi}\right)^{1/(1 - \alpha)}$$

=> no effect of σ !!! [this is confirmed in Figure 6]

Result 4 and Appendix : measuring the effect σ on EW(high q) < or > EW(low q)

analyzing for which q: EW(high q) > EW(low q) eq (A.2)

#76: $\phi\cdot\left(\left(rd - \lambda\cdot\left(\tau bar - \frac{q}{\sigma}\right)\cdot(1 + rd - q)\right) + 0 - \rho\cdot(1 + rd - q)\right) + (1 - \phi)\cdot(rk\cdot(1 - q)^{\alpha}) \ge \phi\cdot(q - 1) +$

$$(1 - \phi)\cdot(rk\cdot(1 - q)^{\alpha})$$

#77: $$\phi\cdot\left(\left(rd - \lambda\cdot\left(\tau bar - \frac{q}{\sigma}\right)\cdot(1 + rd - q)\right) + 0 - \rho\cdot(1 + rd - q)\right) + (1 - \phi)\cdot(rk\cdot(1 - q)^{\alpha}) = \phi\cdot(q - 1) + (1 - \phi)\cdot(rk\cdot(1 - q)^{\alpha})$$

#78: $$\mathrm{SOLVE}\left(\phi\cdot\left(\left(rd - \lambda\cdot\left(\tau bar - \frac{q}{\sigma}\right)\cdot(1 + rd - q)\right) + 0 - \rho\cdot(1 + rd - q)\right) + (1 - \phi)\cdot(rk\cdot(1 - q)^{\alpha}) = \phi\cdot(q - 1) + (1 - \phi)\cdot(rk\cdot(1 - q)^{\alpha}),\ q\right)$$

#79: $$q = rd + 1 \lor q = \frac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda}$$

do it again in a different way:

#80: $$\phi\cdot\left(\left(rd - \lambda\cdot\left(\tau bar - \frac{q}{\sigma}\right)\cdot(1 + rd - q)\right) + 0 - \rho\cdot(1 + rd - q)\right) + (1 - \phi)\cdot(rk\cdot(1 - q)^{\alpha}) \ge \phi\cdot(q - 1) + (1 - \phi)\cdot(rk\cdot(1 - q)^{\alpha})$$

delete the right hand terms on both sides:

#81: $$\phi\cdot\left(\left(rd - \lambda\cdot\left(\tau bar - \frac{q}{\sigma}\right)\cdot(1 + rd - q)\right) + 0 - \rho\cdot(1 + rd - q)\right) \ge \phi\cdot(q - 1)$$

delete φ both sides

#82: $$\left(rd - \lambda\cdot\left(\tau bar - \frac{q}{\sigma}\right)\cdot(1 + rd - q)\right) + 0 - \rho\cdot(1 + rd - q) \ge q - 1$$

#83: $$rd - \lambda\cdot\left(\tau bar - \frac{q}{\sigma}\right)\cdot(1 + rd - q) + 0 - \rho\cdot(1 + rd - q) \ge q - 1$$

#84: $$\left(rd - \lambda\cdot\left(\tau bar - \frac{q}{\sigma}\right)\cdot(1 + rd - q) + 0 - \rho\cdot(1 + rd - q)\right) - (q - 1) \ge 0$$

#85: $$\mathrm{SOLVE}\left(\left(rd - \lambda\cdot\left(\tau bar - \frac{q}{\sigma}\right)\cdot(1 + rd - q) + 0 - \rho\cdot(1 + rd - q)\right) - (q - 1) \ge 0,\ q\right)$$

#86: $$\frac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda} \le q \le rd + 1 \vee rd + 1 \le q \le \frac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda}$$

#87: $$\frac{\sigma\cdot(\lambda\cdot\tau bar + \rho - 1)}{\lambda} \le q$$