runfast_2023_10_26.dfw redo all calc for runfast_v13.tex

#1: CaseMode ≔ Sensitive

#2: InputMode ≔ Word

time

#3: t ∈ Real [0, ∞)

power of t in cb

#4: τ ∈ Real [1, ∞)

interest bank pays to depositors

#5: rd ∈ Real (0, ∞)

probability of a run = investment failure

#6: ϕ ∈ Real (0, 1)

speed of fund withdrawal during a run

#7: σ ∈ Real (0, ∞)

Risky interest bank earns on investment

#8: rk ∈ Real (0, ∞)

bailout cost parameters

#9: δ ∈ Real (1, ∞)

equation (1): amount withdrawn at t during a run

#10: dt = 1 + rd − σ·t

equation (2): bank liquidity at t during a run

#11: qt = q − σ·t

time when bank becomesd insolvent

#12: 0 = q − σ·tq

#13: SOLVE(0 = q − σ·ti, tq)

eq (3)

#14: $$tq = \frac{q}{\sigma}$$

eq (4): Liquidity remaining at the time of a fast bailout

#15: qtbf = q − σ·tbf

eq (5): expected Bank profit

#16: profit = (1 − ϕ)·rk·(1 − q) − (1 − ϕ)·rd

Eq (5) depositors' cost of delay

eq (6) top: Depositor welfare under slow bailout

#17: $ewds = rd - \phi \cdot \lambda \cdot tbs \cdot (1 + rd - q)^{\delta}$

eq (6) bottom: Depositor welfare under fast bailout

#18: $ewdf = rd - \phi \cdot \lambda \cdot tbf \cdot (1 + rd - qtbf)^{\delta}$

eq (7) top bailout cost under slow bailout

#19: ecbs = ϕ·(1 + rd − q)

eq (7) bottom to bailout cost under fast bailout

#20: $ecbf = \phi \cdot (1 + rd - qtbf)$

*** Section 3, eq (8)

#21: $ew = ewd + profit - ecb$

** Subsection 3.1: Optimal q under slow bailout

#22: $ews = (rd - \phi \cdot \lambda \cdot tbs \cdot (1 + rd - q)^{\delta}) + ((1 - \phi) \cdot rk \cdot (1 - q) - (1 - \phi) \cdot rd) - \phi \cdot (1 + rd - q)$

Appendix A eq (A.1)

#23: $\frac{d}{dq}(ews = (rd - \phi \cdot \lambda \cdot tbs \cdot (1 + rd - q)^{\delta}) + ((1 - \phi) \cdot rk \cdot (1 - q) - (1 - \phi) \cdot rd) - \phi \cdot (1 + rd - q))$

#24: $0 = tbs \cdot \delta \cdot \lambda \cdot \phi \cdot (-q + rd + 1)^{\delta - 1} + rk \cdot (\phi - 1) + \phi$

#25: $\frac{d}{dq}\frac{d}{dq}(ews = (rd - \phi \cdot \lambda \cdot tbs \cdot (1 + rd - q)^{\delta}) + ((1 - \phi) \cdot rk \cdot (1 - q) - (1 - \phi) \cdot rd) - \phi \cdot (1 + rd - q))$

#26: $0 > tbs \cdot \delta \cdot \lambda \cdot \phi \cdot (1 - \delta) \cdot (-q + rd + 1)^{\delta - 2}$

eq (10) optimal q under slow bailout

#27: $SOLVE(0 = tbs \cdot \delta \cdot \lambda \cdot \phi \cdot (-q + rd + 1)^{\delta - 1} + rk \cdot (\phi - 1) + \phi, q)$

#28: $q = IF\left(\frac{rk \cdot (\phi - 1) + \phi}{tbs} < 0 \vee \delta \geq 2, -\left(-\frac{rk \cdot (\phi - 1) + \phi}{tbs \cdot \delta \cdot \lambda \cdot \phi}\right)^{1/(\delta - 1)} + rd + 1\right)$

$$\#29:\quad qhats = -\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)} + rd + 1$$

need to show q > 0 and q < 1

$$\#30:\quad -\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)} + rd + 1 > 0$$

$$\#31:\quad \text{SOLVE}\left(-\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)} + rd + 1 > 0,\ rd\right)$$

$$\#32:\quad rd > \left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)} - 1$$

$$\#33:\quad -\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)} + rd + 1 < 1$$

$$\#34:\quad \text{SOLVE}\left(-\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)} + rd + 1 < 1,\ rd\right)$$

$$\#35:\quad rd < \left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)}$$

Verifying Result 2:

$$\#36:\quad \frac{d}{d\ rd}\left(qhats = -\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)} + rd + 1\right)$$

$$\#37:\quad 0 < 1$$

$$\#38:\quad \frac{d}{d\ tbs}\left(qhats = -\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)} + rd + 1\right)$$

$$\#39:\quad 0 < \frac{\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)}}{tbs\cdot(\delta-1)}$$

$$\#40:\quad \frac{d}{d\phi}\left(qhats = -\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)} + rd + 1\right)$$

$$\#41:\quad 0 < \frac{rk\cdot\phi^{\delta/(1-\delta)}\cdot\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda}\right)^{1/(\delta-1)}}{(1-\delta)\cdot(rk\cdot(\phi-1)+\phi)}$$

$$\#42:\quad \frac{d}{d\ rk}\left(qhats = -\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)} + rd + 1\right)$$

$$\#43:\quad 0 > \frac{(1-\phi)\cdot\left(-\frac{rk\cdot(\phi-1)+\phi}{tbs\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)}}{(\delta-1)\cdot(rk\cdot(\phi-1)+\phi)}$$

** Subsection 3.2 optimal q under fast bailout

$$\#44:\quad ewf = (rd - \phi\cdot\lambda\cdot tbf\cdot(1 + rd - qtbf)^{\delta}) + ((1-\phi)\cdot rk\cdot(1-q) - (1-\phi)\cdot rd) - \phi\cdot(1 + rd - qtbf)$$

$$\#45:\quad ewf = (rd - \phi\cdot\lambda\cdot tbf\cdot(1 + rd - (q - \sigma\cdot tbf))^{\delta}) + ((1-\phi)\cdot rk\cdot(1-q) - (1-\phi)\cdot rd) - \phi\cdot(1 + rd - (q$$

$$- \sigma \cdot tbf))$$

Appendix A deriving (12)

$$\#46: \quad \frac{d}{dq}\left(ewf = \left(rd - \phi \cdot \lambda \cdot tbf \cdot (1 + rd - (q - \sigma \cdot tbf))^{\delta}\right) + ((1 - \phi) \cdot rk \cdot (1 - q) - (1 - \phi) \cdot rd) - \phi \cdot (1 + rd - (q - \sigma \cdot tbf))\right)$$

$$\#47: \quad 0 = tbf \cdot \delta \cdot \lambda \cdot \phi \cdot (-q + rd + tbf \cdot \sigma + 1)^{\delta - 1} + rk \cdot (\phi - 1) + \phi$$

$$\#48: \quad \frac{d}{dq}\frac{d}{dq}\left(ewf = \left(rd - \phi \cdot \lambda \cdot tbf \cdot (1 + rd - (q - \sigma \cdot tbf))^{\delta}\right) + ((1 - \phi) \cdot rk \cdot (1 - q) - (1 - \phi) \cdot rd) - \phi \cdot (1 + rd - (q - \sigma \cdot tbf))\right)$$

$$\#49: \quad 0 > tbf \cdot \delta \cdot \lambda \cdot \phi \cdot (1 - \delta) \cdot (-q + rd + tbf \cdot \sigma + 1)^{\delta - 2}$$

$$\#50: \quad \text{SOLVE}(0 = tbf \cdot \delta \cdot \lambda \cdot \phi \cdot (-q + rd + tbf \cdot \sigma + 1)^{\delta - 1} + rk \cdot (\phi - 1) + \phi, q)$$

$$\#51: \quad q = \text{IF}\left(\frac{rk \cdot (\phi - 1) + \phi}{tbf} < 0 \vee \delta \geq 2, -\left(-\frac{rk \cdot (\phi - 1) + \phi}{tbf \cdot \delta \cdot \lambda \cdot \phi}\right)^{1/(\delta - 1)} + rd + tbf \cdot \sigma + 1\right)$$

eq (12)

$$\#52: \quad qhatf = -\left(-\frac{rk \cdot (\phi - 1) + \phi}{tbf \cdot \delta \cdot \lambda \cdot \phi}\right)^{1/(\delta - 1)} + rd + tbf \cdot \sigma + 1$$

conditions to ensure 0 < qhatf <1

#53: $$-\left(-\frac{rk\cdot(\phi-1)+\phi}{tbf\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)}+rd+tbf\cdot\sigma+1>0$$

#54: $$\text{SOLVE}\left(-\left(-\frac{rk\cdot(\phi-1)+\phi}{tbf\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)}+rd+tbf\cdot\sigma+1>0,\ rd\right)$$

#55: $$rd>\left(-\frac{rk\cdot(\phi-1)+\phi}{tbf\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)}-tbf\cdot\sigma-1$$

#56: $$-\left(-\frac{rk\cdot(\phi-1)+\phi}{tbf\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)}+rd+tbf\cdot\sigma+1<1$$

#57: $$\text{SOLVE}\left(-\left(-\frac{rk\cdot(\phi-1)+\phi}{tbf\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)}+rd+tbf\cdot\sigma+1<1,\ rd\right)$$

#58: $$rd<\left(-\frac{rk\cdot(\phi-1)+\phi}{tbf\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)}-tbf\cdot\sigma$$

Result 3:

#59: $$\frac{d}{d\sigma}\left(qhatf=-\left(-\frac{rk\cdot(\phi-1)+\phi}{tbf\cdot\delta\cdot\lambda\cdot\phi}\right)^{1/(\delta-1)}+rd+tbf\cdot\sigma+1\right)$$

#60: $$0<tbf$$