#1: CaseMode := Sensitive

#2: InputMode := Word

tuition by college B and college A

#3: ta :∈ Real (0, ∞)

#4: tb :∈ Real (0, ∞)

Time discount factor

#5: δ :∈ Real (0, 1)

interest rate

#6: r :∈ Real (0, ∞)

consumer ability index a

#7: a :∈ Real [0, 1]

earning wage parameters college 1 and 2 and nondegree

#8: μa :∈ Real (0, ∞)

#9: μb :∈ Real (0, ∞)

#10: μn :∈ Real (0, ∞)

prob getting a degree job

#11: ρ :∈ Real (0, 1)

Enrollment capacity constraints

#12: ka :∈ Real (0, ∞)

#13: kb :∈ Real (0, ∞)

*** Section 3, equation (2) expected utility U(a)

(2): enrolled in A

#14: $\delta\cdot(\rho\cdot\mu a\cdot a + (1 - \rho)\cdot\mu n\cdot a) - (ta - c) - \delta\cdot c\cdot(1 + r)$

(2): enrolled in B

#15: $\delta\cdot(\rho\cdot\mu b\cdot a + (1 - \rho)\cdot\mu n\cdot a) - (tb - c) - \delta\cdot c\cdot(1 + r)$

(2) not enrolled

#16: $\mu n\cdot a + \delta\cdot\mu n\cdot a$

*** Section 4: Equilibrium tuition with enrollment capacity constraints

eq (3) abar and ahat

#17: $\delta\cdot(\rho\cdot\mu b\cdot abar + (1 - \rho)\cdot\mu n\cdot abar) - (tb - c) - \delta\cdot c\cdot(1 + r) = \mu n\cdot abar + \delta\cdot\mu n\cdot abar$

#18: SOLVE($\delta\cdot(\rho\cdot\mu b\cdot abar + (1 - \rho)\cdot\mu n\cdot abar) - (tb - c) - \delta\cdot c\cdot(1 + r) = \mu n\cdot abar + \delta\cdot\mu n\cdot abar$, abar)

#19: $$abar = \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}$$

#20: $\delta\cdot(\rho\cdot\mu a\cdot ahat + (1 - \rho)\cdot\mu n\cdot ahat) - (ta - c) - \delta\cdot c\cdot(1 + r)$

#21: $\delta\cdot(\rho\cdot\mu a\cdot ahat + (1 - \rho)\cdot\mu n\cdot ahat) - (ta - c) - \delta\cdot c\cdot(1 + r) = \delta\cdot(\rho\cdot\mu b\cdot ahat + (1 - \rho)\cdot\mu n\cdot ahat) - (tb - c) - \delta\cdot c\cdot(1 + r)$

#22: SOLVE($\delta\cdot(\rho\cdot\mu a\cdot ahat + (1 - \rho)\cdot\mu n\cdot ahat) - (ta - c) - \delta\cdot c\cdot(1 + r) = \delta\cdot(\rho\cdot\mu b\cdot ahat + (1 - \rho)\cdot\mu n\cdot ahat) - (tb - c) - \delta\cdot c\cdot(1 + r)$, ahat)

#23: $$ahat = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}$$

try ahat - abar =

#24: $$\frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}$$

#25: $$\frac{c\cdot\delta\cdot\rho\cdot(\mu a - \mu b)\cdot(r\cdot\delta + \delta - 1) + ta\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + tb\cdot(\delta\cdot\rho\cdot(\mu a - \mu n) - \mu n)}{\delta\cdot\rho\cdot(\mu b - \mu a)\cdot(\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n)}$$

equation (4) capacities

#26: $ka = 1 - ahat$

#27: $kb = ahat - abar$

#28: $$ka = 1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}$$

#29: $$kb = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}$$

#30: $$\mathrm{SOLVE}\left(\left[ka = 1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)},\ kb = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}\right], [ta, tb]\right)$$

eq (5): equilibrium ta and tb

#31: $$[ta = - c\cdot(r\cdot\delta + \delta - 1) + ka\cdot(\mu n - \delta\cdot\rho\cdot(\mu a - \mu n)) + kb\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + \delta\cdot\rho\cdot(\mu a - \mu n) - \mu n \wedge$$
$$tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) - c\cdot(r\cdot\delta + \delta - 1)]$$

#32: $$ta - tb = ka\cdot\delta\cdot\rho\cdot(\mu b - \mu a) + \delta\cdot\rho\cdot(\mu a - \mu b)$$

#33: $$ta - tb = (- c\cdot(r\cdot\delta + \delta - 1) + ka\cdot(\mu n - \delta\cdot\rho\cdot(\mu a - \mu n)) + kb\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + \delta\cdot\rho\cdot(\mu a - \mu n) -$$

$\mu n) - ((ka + kb - 1)\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) - c\cdot(r\cdot\delta + \delta - 1))$

Result 1

#34: $\frac{d}{dc}(ta = - c\cdot(r\cdot\delta + \delta - 1) + ka\cdot(\mu n - \delta\cdot\rho\cdot(\mu a - \mu n)) + kb\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + \delta\cdot\rho\cdot(\mu a - \mu n) - \mu n)$

#35: $0 < - r\cdot\delta - \delta + 1$

#36: $\frac{d}{dc}(tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) - c\cdot(r\cdot\delta + \delta - 1))$

#37: $0 < - r\cdot\delta - \delta + 1$

#38: $\frac{d}{dr}(ta = - c\cdot(r\cdot\delta + \delta - 1) + ka\cdot(\mu n - \delta\cdot\rho\cdot(\mu a - \mu n)) + kb\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + \delta\cdot\rho\cdot(\mu a - \mu n) - \mu n)$

#39: $0 > - c\cdot\delta$

#40: $\frac{d}{dr}(tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) - c\cdot(r\cdot\delta + \delta - 1))$

#41: $0 > - c\cdot\delta$

*** Section 5: Introducing loan defaults

equation (6): Utility function [modifying (2)] enrolled in A

#42: $\delta\cdot(\rho\cdot\mu a\cdot a) - (ta - c) - \delta\cdot\rho\cdot c\cdot(1 + r)$

enrolled in B

#43: $\delta\cdot(\rho\cdot\mu b\cdot a) - (tb - c) - \delta\cdot\rho\cdot c\cdot(1 + r)$

not enrolled

#44: $\mu n\cdot a + \delta\cdot\mu n\cdot a$

equation (7): abar and ahat

#45: $\delta\cdot(\rho\cdot\mu b\cdot abar) - (tb - c) - \delta\cdot\rho\cdot c\cdot(1 + r) = \mu n\cdot abar + \delta\cdot\mu n\cdot abar$

#46: SOLVE($\delta\cdot(\rho\cdot\mu b\cdot abar) - (tb - c) - \delta\cdot\rho\cdot c\cdot(1 + r) = \mu n\cdot abar + \delta\cdot\mu n\cdot abar$, abar)

#47:
$$abar = \frac{c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + tb}{\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n}$$

#48: $\delta\cdot(\rho\cdot\mu a\cdot ahat) - (ta - c) - \delta\cdot\rho\cdot c\cdot(1 + r) = \delta\cdot(\rho\cdot\mu b\cdot ahat) - (tb - c) - \delta\cdot\rho\cdot c\cdot(1 + r)$

#49: SOLVE($\delta\cdot(\rho\cdot\mu a\cdot ahat) - (ta - c) - \delta\cdot\rho\cdot c\cdot(1 + r) = \delta\cdot(\rho\cdot\mu b\cdot ahat) - (tb - c) - \delta\cdot\rho\cdot c\cdot(1 + r)$, ahat)

#50:
$$ahat = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}$$

deriving equation (8)

#51: $ka = 1 - ahat$

#52: $kb = ahat - abar$

#53: $ka = 1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}$

#54: $$kb = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + tb}{\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n}$$

#55: $$\text{SOLVE}\left(\left[ka = 1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)},\ kb = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + tb}{\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n}\right], [ta, tb]\right)$$

#56: $$[ta = - c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + ka\cdot(\mu n - \delta\cdot(\mu a\cdot\rho - \mu n)) + kb\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) + \delta\cdot(\mu a\cdot\rho - \mu n) - \mu n \wedge tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) - c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1)]$$

Deriving (A.3) [abar and ahat under default] => not finished, not in the paper

#57: $$abar = \frac{c\cdot\delta\cdot(r + 1)\cdot(\rho - 1) + (ka + kb - 1)\cdot(\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n)}{\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)}$$

#58: $$ahat = 1 - ka$$

ahat − abar =

#59: $$1 - ka - \frac{c\cdot\delta\cdot(r + 1)\cdot(\rho - 1) + (ka + kb - 1)\cdot(\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n)}{\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)}$$

#60: $$\frac{c\cdot\delta\cdot(r + 1)\cdot(\rho - 1) + ka\cdot\delta\cdot\mu n\cdot(\rho - 1) + kb\cdot(\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n) + \delta\cdot\mu n\cdot(1 - \rho)}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}$$

ta − tb =

#61: $$(- c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + ka\cdot(\mu n - \delta\cdot(\mu a\cdot\rho - \mu n)) + kb\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) + \delta\cdot(\mu a\cdot\rho - \mu n) - \mu n) - ((ka + kb - 1)\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) - c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1))$$

#62: $$ka\cdot\delta\cdot(\mu b\cdot\rho - \mu a\cdot\rho) + \delta\cdot(\mu a\cdot\rho - \mu b\cdot\rho)$$

#63: $$\delta\cdot\rho\cdot(ka - 1)\cdot(\mu b - \mu a)$$

Result 2:

#64: $$\frac{d}{dc}\left(ta = -c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + ka\cdot(\mu n - \delta\cdot(\mu a\cdot\rho - \mu n)) + kb\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) + \delta\cdot(\mu a\cdot\rho - \mu n) - \mu n\right)$$

#65: $$0 < -r\cdot\delta\cdot\rho - \delta\cdot\rho + 1$$

#66: $$\frac{d}{dc}\left(tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) - c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1)\right)$$

#67: $$0 < -r\cdot\delta\cdot\rho - \delta\cdot\rho + 1$$

#68: SOLVE$(0 < -r\cdot\delta\cdot\rho - \delta\cdot\rho + 1, r)$

#69: $$r < \frac{1 - \delta\cdot\rho}{\delta\cdot\rho}$$

below I use Assumption 2(a) to show that the above < holds by Assumption 2(a).

#70: $$\frac{1 - \delta\cdot\rho}{\delta\cdot\rho} - \frac{1 - \delta}{\delta}$$

#71: $$\frac{1 - \rho}{\delta\cdot\rho} > 0$$

#72: $$\frac{d}{dr}\left(ta = -c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + ka\cdot(\mu n - \delta\cdot(\mu a\cdot\rho - \mu n)) + kb\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) + \delta\cdot(\mu a\cdot\rho - \mu n)\right.$$

$- \mu n)$

#73: $$0 > - c\cdot\delta\cdot\rho$$

#74: $$\frac{d}{dr}\,(tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) - c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1))$$

#75: $$0 > - c\cdot\delta\cdot\rho$$

*** Section 6: Profit and ability maximizng colleges.

#76: $$ga = ta\cdot(1 - ahat) + \frac{1 - ahat}{2}$$

#77: $$gb = tb\cdot(ahat - abar) + \frac{ahat - abar}{2}$$

#78: $$ga = ta\cdot\left(1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}\right) + \frac{1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}}{2}$$

#79: $$gb = tb\cdot\left(\frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}\right) + \frac{\frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}}{2}$$

#80: $$\frac{d}{d\ ta}\left(ga = ta\cdot\left(1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}\right) + \frac{1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}}{2}\right)$$

eq (A.3)

#81:
$$0 = \frac{4\cdot ta - 2\cdot tb + 2\cdot\delta\cdot\rho\cdot(\mu b - \mu a) + 1}{2\cdot\delta\cdot\rho\cdot(\mu b - \mu a)}$$

#82:
$$\frac{d}{d\ ta}\frac{d}{d\ ta}\left(ga = ta\cdot\left(1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}\right) + \frac{1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}}{2}\right)$$

#83:
$$0 > \frac{2}{\delta\cdot\rho\cdot(\mu b - \mu a)}$$

#84:
$$\frac{d}{d\ tb}\left(gb = tb\cdot\left(\frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}\right) + \frac{\frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}}{2}\right)$$

equation (A.4)

#85:
$$0 = \frac{2\cdot c\cdot\delta\cdot\rho\cdot(\mu a - \mu b)\cdot(r\cdot\delta + \delta - 1) + 2\cdot ta\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + (4\cdot tb + 1)\cdot(\delta\cdot\rho\cdot(\mu a - \mu n) - \mu n)}{2\cdot\delta\cdot\rho\cdot(\mu b - \mu a)\cdot(\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n)}$$

#86:
$$\frac{d}{d\ tb}\frac{d}{d\ tb}\left(gb = tb\cdot\left(\frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}\right) + \right.$$

$$\left.\frac{\dfrac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \dfrac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}}{2}\right)$$

equation (A.5)

#87: $$0 > \frac{2\cdot(\delta\cdot\rho\cdot(\mu a - \mu n) - \mu n)}{\delta\cdot\rho\cdot(\mu b - \mu a)\cdot(\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n)}$$

by Assumption 1(c)

#88: $$\mathrm{SOLVE}\left(\left[0 = \frac{4\cdot ta - 2\cdot tb + 2\cdot\delta\cdot\rho\cdot(\mu b - \mu a) + 1}{2\cdot\delta\cdot\rho\cdot(\mu b - \mu a)},\ 0 = \right.\right.$$

$$\left.\frac{2\cdot c\cdot\delta\cdot\rho\cdot(\mu a - \mu b)\cdot(r\cdot\delta + \delta - 1) + 2\cdot ta\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + (4\cdot tb + 1)\cdot(\delta\cdot\rho\cdot(\mu a - \mu n) - \mu n)}{2\cdot\delta\cdot\rho\cdot(\mu b - \mu a)\cdot(\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n)}\right]\sim$$

$$\left., [ta, tb]\right)$$

Equations (12), (13) and (14)

#89: $$\Big[ta =$$

$$\frac{2\cdot c\cdot\delta\cdot\rho\cdot(\mu a - \mu b)\cdot(r\cdot\delta + \delta - 1) + 4\cdot\delta^{2}\cdot\rho^{2}\cdot(\mu a - \mu b)\cdot(\mu n - \mu a) + \delta\cdot\rho\cdot(\mu a\cdot(4\cdot\mu n + 3) - \mu n\cdot(4\cdot\mu b +\sim}{2\cdot(3\cdot\mu n - \delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n))}\sim$$

$$\frac{\text{3)) - 3·μn}}{} \wedge tb =$$

$$\frac{4·c·δ·ρ·(μa - μb)·(r·δ + δ - 1) + 2·δ^2·ρ^2·(μa - μb)·(μn - μb) + δ·ρ·(2·μa·(μn + 1) + μb·(1 - 2·\sim}{2·(3·μn - δ·ρ·(4·μa - μb - 3·μn))\qquad\sim}$$

$$\left.\frac{\text{μn) - 3·μn) - 3·μn}}{}\right]$$

Result 3:

$$\#90: \quad \frac{d}{dc}\left(ta = \right.$$

$$\frac{2·c·δ·ρ·(μa - μb)·(r·δ + δ - 1) + 4·δ^2·ρ^2·(μa - μb)·(μn - μa) + δ·ρ·(μa·(4·μn + 3) - μn·(4·μb +\sim}{2·(3·μn - δ·ρ·(4·μa - μb - 3·μn))\qquad\sim}$$

$$\left.\frac{\text{3)) - 3·μn}}{}\right)$$

$$\#91: \qquad 0 < \frac{δ·ρ·(μb - μa)·(r·δ + δ - 1)}{δ·ρ·(4·μa - μb - 3·μn) - 3·μn}$$

#92: $$\frac{d}{dc}\left(tb = \frac{4\cdot c\cdot \delta\cdot \rho\cdot (\mu a - \mu b)\cdot (r\cdot \delta + \delta - 1) + 2\cdot \delta^{2}\cdot \rho^{2}\cdot (\mu a - \mu b)\cdot (\mu n - \mu b) + \delta\cdot \rho\cdot (2\cdot \mu a\cdot (\mu n + 1) + \mu b\cdot (1 - 2\cdot \mu n) - 3\cdot \mu n) - 3\cdot \mu n}{2\cdot (3\cdot \mu n - \delta\cdot \rho\cdot (4\cdot \mu a - \mu b - 3\cdot \mu n))}\right)$$

#93: $$0 = \frac{2\cdot \delta\cdot \rho\cdot (\mu b - \mu a)\cdot (r\cdot \delta + \delta - 1)}{\delta\cdot \rho\cdot (4\cdot \mu a - \mu b - 3\cdot \mu n) - 3\cdot \mu n}$$

#94: $$\frac{d}{dr}\left(ta = \frac{2\cdot c\cdot \delta\cdot \rho\cdot (\mu a - \mu b)\cdot (r\cdot \delta + \delta - 1) + 4\cdot \delta^{2}\cdot \rho^{2}\cdot (\mu a - \mu b)\cdot (\mu n - \mu a) + \delta\cdot \rho\cdot (\mu a\cdot (4\cdot \mu n + 3) - \mu n\cdot (4\cdot \mu b + 3)) - 3\cdot \mu n}{2\cdot (3\cdot \mu n - \delta\cdot \rho\cdot (4\cdot \mu a - \mu b - 3\cdot \mu n))}\right)$$

#95: $$0 > \frac{c\cdot \delta^{2}\cdot \rho\cdot (\mu b - \mu a)}{\delta\cdot \rho\cdot (4\cdot \mu a - \mu b - 3\cdot \mu n) - 3\cdot \mu n}$$

#96: $$\frac{d}{dr}\left(tb = \frac{4\cdot c\cdot\delta\cdot\rho\cdot(\mu a - \mu b)\cdot(r\cdot\delta + \delta - 1) + 2\cdot\delta^2\cdot\rho^2\cdot(\mu a - \mu b)\cdot(\mu n - \mu b) + \delta\cdot\rho\cdot(2\cdot\mu a\cdot(\mu n + 1) + \mu b\cdot(1 - 2\cdot\sim \mu n) - 3\cdot\mu n) - 3\cdot\mu n}{2\cdot(3\cdot\mu n - \delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n))}\right)$$

#97: $$0 > \frac{2\cdot c\cdot\delta^2\cdot\rho\cdot(\mu b - \mu a)}{\delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n) - 3\cdot\mu n}$$

need to show that abar < ahat

#98: $$abar = \frac{2\cdot c\cdot(r\cdot\delta + \delta - 1)\cdot(\delta\cdot\rho\cdot(2\cdot\mu a + \mu b - 3\cdot\mu n) - 3\cdot\mu n) + 2\cdot\delta^2\cdot\rho^2\cdot(\mu a - \mu b)\cdot(\mu b - \mu n) - \delta\cdot\rho\cdot(2\cdot\mu a\cdot(\mu n\sim + 1) + \mu b\cdot(1 - 2\cdot\mu n) - 3\cdot\mu n) + 3\cdot\mu n}{2\cdot(\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n)\cdot(\delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n) - 3\cdot\mu n)}$$

#99:

$$\text{ahat} = \frac{2\cdot c\cdot(r\cdot\delta + \delta - 1) + 2\cdot\delta\cdot\rho\cdot(2\cdot\mu a - \mu b - \mu n) - 2\cdot\mu n - 1}{2\cdot(\delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n) - 3\cdot\mu n)}$$

ahat - abar = [hard to evaluate: use simulations]

#100:

$$\frac{2\cdot c\cdot(r\cdot\delta + \delta - 1) + 2\cdot\delta\cdot\rho\cdot(2\cdot\mu a - \mu b - \mu n) - 2\cdot\mu n - 1}{2\cdot(\delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n) - 3\cdot\mu n)} - \frac{2\cdot c\cdot(r\cdot\delta + \delta - 1)\cdot(\delta\cdot\rho\cdot(2\cdot\mu a + \mu b - 3\cdot\mu n) - 3\cdot\mu n) + 2\cdot\delta^{2}\cdot\rho^{2}\cdot(\mu a - \mu b)\cdot(\mu b - \mu n) - \delta\cdot\rho\cdot(2\cdot\mu a\cdot(\mu n + 1) + \mu b\cdot(1 - 2\cdot\mu n) - 3\cdot\mu n) + 3\cdot\mu n}{2\cdot(\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n)\cdot(\delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n) - 3\cdot\mu n)}$$

#101:

$$\frac{(2\cdot c\cdot(r\cdot\delta + \delta - 1) + \delta\cdot\rho\cdot(\mu n - \mu b) + \mu n - 1)\cdot(\delta\cdot\rho\cdot(\mu a - \mu n) - \mu n)}{(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n))\cdot(\delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n) - 3\cdot\mu n)}$$