#1: CaseMode := Sensitive

#2: InputMode := Word

tuition by college B and college A

#3: ta :∈ Real (0, ∞)

#4: tb :∈ Real (0, ∞)

Time discount factor

#5: δ :∈ Real (0, 1)

interest rate

#6: r :∈ Real (0, ∞)

consumer ability index a

#7: a :∈ Real [0, 1]

earning wage parameters college 1 and 2 and nondegree

#8: μa :∈ Real (0, ∞)

#9: μb :∈ Real (0, ∞)

#10: μn :∈ Real (0, ∞)

prob getting a degree job

#11: ρ :∈ Real (0, 1)

Enrollment capacity constraints

#12: ka :∈ Real (0, ∞)

#13: kb :∈ Real (0, ∞)

*** Section 3, equation (2) expected utility U(a)

(2): enrolled in A

#14: $\delta\cdot(\rho\cdot\mu a\cdot a + (1 - \rho)\cdot\mu n\cdot a) - (ta - c) - \delta\cdot c\cdot(1 + r)$

(2): enrolled in B

#15: $\delta\cdot(\rho\cdot\mu b\cdot a + (1 - \rho)\cdot\mu n\cdot a) - (tb - c) - \delta\cdot c\cdot(1 + r)$

(2) not enrolled

#16: $\mu n\cdot a + \delta\cdot\mu n\cdot a$

*** Section 4: Equilibrium tuition with enrollment capacity constraints

eq (3) abar and ahat

#17: $\delta\cdot(\rho\cdot\mu b\cdot abar + (1 - \rho)\cdot\mu n\cdot abar) - (tb - c) - \delta\cdot c\cdot(1 + r) = \mu n\cdot abar + \delta\cdot\mu n\cdot abar$

#18: $\mathrm{SOLVE}(\delta\cdot(\rho\cdot\mu b\cdot abar + (1 - \rho)\cdot\mu n\cdot abar) - (tb - c) - \delta\cdot c\cdot(1 + r) = \mu n\cdot abar + \delta\cdot\mu n\cdot abar, abar)$

#19:
$$abar = \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}$$

#20: $\delta\cdot(\rho\cdot\mu a\cdot ahat + (1 - \rho)\cdot\mu n\cdot ahat) - (ta - c) - \delta\cdot c\cdot(1 + r)$

#21: $\delta\cdot(\rho\cdot\mu a\cdot ahat + (1 - \rho)\cdot\mu n\cdot ahat) - (ta - c) - \delta\cdot c\cdot(1 + r) = \delta\cdot(\rho\cdot\mu b\cdot ahat + (1 - \rho)\cdot\mu n\cdot ahat) - (tb - c) - \delta\cdot c\cdot(1 + r)$

#22: $\mathrm{SOLVE}(\delta\cdot(\rho\cdot\mu a\cdot ahat + (1 - \rho)\cdot\mu n\cdot ahat) - (ta - c) - \delta\cdot c\cdot(1 + r) = \delta\cdot(\rho\cdot\mu b\cdot ahat + (1 - \rho)\cdot\mu n\cdot ahat) - (tb - c) - \delta\cdot c\cdot(1 + r), ahat)$

#23:
$$ahat = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}$$

try ahat - abar =

#24: $$\frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}$$

#25: $$\frac{c\cdot\delta\cdot\rho\cdot(\mu a - \mu b)\cdot(r\cdot\delta + \delta - 1) + ta\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + tb\cdot(\delta\cdot\rho\cdot(\mu a - \mu n) - \mu n)}{\delta\cdot\rho\cdot(\mu b - \mu a)\cdot(\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n)}$$

equation (4) capacities

#26: $ka = 1 - ahat$

#27: $kb = ahat - abar$

#28: $$ka = 1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}$$

#29: $$kb = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}$$

#30: $$\mathrm{SOLVE}\left(\left[ka = 1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)},\ kb = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}\right], [ta, tb]\right)$$

eq (5): equilibrium ta and tb

#31: $$[ta = - c\cdot(r\cdot\delta + \delta - 1) + ka\cdot(\mu n - \delta\cdot\rho\cdot(\mu a - \mu n)) + kb\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + \delta\cdot\rho\cdot(\mu a - \mu n) - \mu n \wedge$$
$$tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) - c\cdot(r\cdot\delta + \delta - 1)]$$

#32: $$ta - tb = ka\cdot\delta\cdot\rho\cdot(\mu b - \mu a) + \delta\cdot\rho\cdot(\mu a - \mu b)$$

#33: $$ta - tb = (- c\cdot(r\cdot\delta + \delta - 1) + ka\cdot(\mu n - \delta\cdot\rho\cdot(\mu a - \mu n)) + kb\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + \delta\cdot\rho\cdot(\mu a - \mu n) -$$

$\mu n) - ((ka + kb - 1)\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) - c\cdot(r\cdot\delta + \delta - 1))$

Result 1

#34: $\frac{d}{dc}(ta = - c\cdot(r\cdot\delta + \delta - 1) + ka\cdot(\mu n - \delta\cdot\rho\cdot(\mu a - \mu n)) + kb\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + \delta\cdot\rho\cdot(\mu a - \mu n) -$

$\mu n)$

#35: $0 < - r\cdot\delta - \delta + 1$

#36: $\frac{d}{dc}(tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) - c\cdot(r\cdot\delta + \delta - 1))$

#37: $0 < - r\cdot\delta - \delta + 1$

#38: $\frac{d}{dr}(ta = - c\cdot(r\cdot\delta + \delta - 1) + ka\cdot(\mu n - \delta\cdot\rho\cdot(\mu a - \mu n)) + kb\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) + \delta\cdot\rho\cdot(\mu a - \mu n) -$

$\mu n)$

#39: $0 > - c\cdot\delta$

#40: $\frac{d}{dr}(tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)) - c\cdot(r\cdot\delta + \delta - 1))$

#41: $0 > - c\cdot\delta$

*** Section 5: Introducing loan defaults

equation (6): Utility function [modifying (2)] enrolled in A

#42: $\delta\cdot(\rho\cdot\mu a\cdot a) - (ta - c) - \delta\cdot\rho\cdot c\cdot(1 + r)$

enrolled in B

#43: $\delta\cdot(\rho\cdot\mu b\cdot a) - (tb - c) - \delta\cdot\rho\cdot c\cdot(1 + r)$

not enrolled

#44: $\mu n\cdot a + \delta\cdot\mu n\cdot a$

equation (7): abar and ahat

#45: $\delta\cdot(\rho\cdot\mu b\cdot abar) - (tb - c) - \delta\cdot\rho\cdot c\cdot(1 + r) = \mu n\cdot abar + \delta\cdot\mu n\cdot abar$

#46: $\mathrm{SOLVE}(\delta\cdot(\rho\cdot\mu b\cdot abar) - (tb - c) - \delta\cdot\rho\cdot c\cdot(1 + r) = \mu n\cdot abar + \delta\cdot\mu n\cdot abar,\ abar)$

#47: $$abar = \frac{c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + tb}{\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n}$$

#48: $\delta\cdot(\rho\cdot\mu a\cdot ahat) - (ta - c) - \delta\cdot\rho\cdot c\cdot(1 + r) = \delta\cdot(\rho\cdot\mu b\cdot ahat) - (tb - c) - \delta\cdot\rho\cdot c\cdot(1 + r)$

#49: $\mathrm{SOLVE}(\delta\cdot(\rho\cdot\mu a\cdot ahat) - (ta - c) - \delta\cdot\rho\cdot c\cdot(1 + r) = \delta\cdot(\rho\cdot\mu b\cdot ahat) - (tb - c) - \delta\cdot\rho\cdot c\cdot(1 + r),\ ahat)$

#50: $$ahat = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}$$

deriving equation (8)

#51: $ka = 1 - ahat$

#52: $kb = ahat - abar$

#53: $$ka = 1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}$$

#54: $$kb = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + tb}{\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n}$$

#55: $$\text{SOLVE}\left(\left[ka = 1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)},\ kb = \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + tb}{\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n}\right], [ta, tb]\right)$$

#56: $$[ta = - c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + ka\cdot(\mu n - \delta\cdot(\mu a\cdot\rho - \mu n)) + kb\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) + \delta\cdot(\mu a\cdot\rho - \mu n) - \mu n \wedge tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) - c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1)]$$

Deriving (A.3) [abar and ahat under default] => not finished, not in the paper

#57: $$abar = \frac{c\cdot\delta\cdot(r + 1)\cdot(\rho - 1) + (ka + kb - 1)\cdot(\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n)}{\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)}$$

#58: $$ahat = 1 - ka$$

ahat − abar =

#59: $$1 - ka - \frac{c\cdot\delta\cdot(r + 1)\cdot(\rho - 1) + (ka + kb - 1)\cdot(\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n)}{\mu n - \delta\cdot\rho\cdot(\mu b - \mu n)}$$

#60: $$\frac{c\cdot\delta\cdot(r + 1)\cdot(\rho - 1) + ka\cdot\delta\cdot\mu n\cdot(\rho - 1) + kb\cdot(\delta\cdot(\mu b\cdot\rho - \mu n) - \mu n) + \delta\cdot\mu n\cdot(1 - \rho)}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}$$

ta − tb =

#61: $$(- c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + ka\cdot(\mu n - \delta\cdot(\mu a\cdot\rho - \mu n)) + kb\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) + \delta\cdot(\mu a\cdot\rho - \mu n) - \mu n) - ((ka + kb - 1)\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) - c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1))$$

#62: $$ka\cdot\delta\cdot(\mu b\cdot\rho - \mu a\cdot\rho) + \delta\cdot(\mu a\cdot\rho - \mu b\cdot\rho)$$

#63: $$\delta\cdot\rho\cdot(ka - 1)\cdot(\mu b - \mu a)$$

Result 2:

#64: $$\frac{d}{dc}\left(ta = -c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + ka\cdot(\mu n - \delta\cdot(\mu a\cdot\rho - \mu n)) + kb\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) + \delta\cdot(\mu a\cdot\rho - \mu n) - \mu n\right)$$

#65: $$0 < -r\cdot\delta\cdot\rho - \delta\cdot\rho + 1$$

#66: $$\frac{d}{dc}\left(tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) - c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1)\right)$$

#67: $$0 < -r\cdot\delta\cdot\rho - \delta\cdot\rho + 1$$

#68: SOLVE($0 < -r\cdot\delta\cdot\rho - \delta\cdot\rho + 1$, r)

#69: $$r < \frac{1 - \delta\cdot\rho}{\delta\cdot\rho}$$

below I use Assumption 2(a) to show that the above < holds by Assumption 2(a).

#70: $$\frac{1 - \delta\cdot\rho}{\delta\cdot\rho} - \frac{1 - \delta}{\delta}$$

#71: $$\frac{1 - \rho}{\delta\cdot\rho} > 0$$

#72: $$\frac{d}{dr}\left(ta = -c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1) + ka\cdot(\mu n - \delta\cdot(\mu a\cdot\rho - \mu n)) + kb\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) + \delta\cdot(\mu a\cdot\rho - \mu n)\right.$$

$- \mu n)$

#73: $0 > - c\cdot\delta\cdot\rho$

#74: $\frac{d}{dr}\,(tb = (ka + kb - 1)\cdot(\mu n - \delta\cdot(\mu b\cdot\rho - \mu n)) - c\cdot(r\cdot\delta\cdot\rho + \delta\cdot\rho - 1))$

#75: $0 > - c\cdot\delta\cdot\rho$

*** Section 6: Profit and ability maximizng colleges.
2022_10_31.dfw start fixing error in s_loan_7.tex with respect to median quaility.
I also add weights, new parameter α.
Should I also experiment with average quality? => Quardratic may be to complicated

equations (10) and (11):

#76: $ga = \alpha\cdot ta\cdot(1 - ahat) + \frac{(1 - \alpha)\cdot(1 + ahat)}{2}$

#77: $gb = \alpha\cdot tb\cdot(ahat - abar) + \frac{(1 - \alpha)\cdot(ahat + abar)}{2}$

#78: $ga = \alpha\cdot ta\cdot\left(1 - \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}\right) + \frac{(1 - \alpha)\cdot\left(1 + \frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)}\right)}{2}$

#79: $gb = \alpha\cdot tb\cdot\left(\frac{ta - tb}{\delta\cdot\rho\cdot(\mu a - \mu b)} - \frac{c\cdot(r\cdot\delta + \delta - 1) + tb}{\delta\cdot\rho\cdot(\mu b - \mu n) - \mu n}\right) +$

$$\frac{(1-\alpha)\cdot\left(\dfrac{ta-tb}{\delta\cdot\rho\cdot(\mu a-\mu b)}+\dfrac{c\cdot(r\cdot\delta+\delta-1)+tb}{\delta\cdot\rho\cdot(\mu b-\mu n)-\mu n}\right)}{2}$$

Appendix A derivations of ta and tb for section 6:

#80: $$\frac{d}{d\ ta}\left(ga=\alpha\cdot ta\cdot\left(1-\frac{ta-tb}{\delta\cdot\rho\cdot(\mu a-\mu b)}\right)+\frac{(1-\alpha)\cdot\left(1+\dfrac{ta-tb}{\delta\cdot\rho\cdot(\mu a-\mu b)}\right)}{2}\right)$$

#81: $$0=\frac{4\cdot ta\cdot\alpha-2\cdot tb\cdot\alpha+\alpha\cdot(1-2\cdot\delta\cdot\rho\cdot(\mu a-\mu b))-1}{2\cdot\delta\cdot\rho\cdot(\mu b-\mu a)}$$

#82: $$\frac{d}{d\ ta}\,\frac{d}{d\ ta}\left(ga=\alpha\cdot ta\cdot\left(1-\frac{ta-tb}{\delta\cdot\rho\cdot(\mu a-\mu b)}\right)+\frac{(1-\alpha)\cdot\left(1+\dfrac{ta-tb}{\delta\cdot\rho\cdot(\mu a-\mu b)}\right)}{2}\right)$$

#83: $$0>\frac{2\cdot\alpha}{\delta\cdot\rho\cdot(\mu b-\mu a)}$$

#84: $$\frac{d}{d\ tb}\left(gb=\alpha\cdot tb\cdot\left(\frac{ta-tb}{\delta\cdot\rho\cdot(\mu a-\mu b)}-\frac{c\cdot(r\cdot\delta+\delta-1)+tb}{\delta\cdot\rho\cdot(\mu b-\mu n)-\mu n}\right)+\right.$$

$$\left.\frac{(1-\alpha)\cdot\left(\dfrac{ta-tb}{\delta\cdot\rho\cdot(\mu a-\mu b)}+\dfrac{c\cdot(r\cdot\delta+\delta-1)+tb}{\delta\cdot\rho\cdot(\mu b-\mu n)-\mu n}\right)}{2}\right)$$

#85: $0 =$

$$\frac{2\cdot c\cdot\alpha\cdot\delta\cdot\rho\cdot(\mu a-\mu b)\cdot(r\cdot\delta+\delta-1)+2\cdot ta\cdot\alpha\cdot(\mu n-\delta\cdot\rho\cdot(\mu b-\mu n))+4\cdot tb\cdot\alpha\cdot(\delta\cdot\rho\cdot(\mu a-\mu n)-\mu n)+\sim}{2\cdot\delta\cdot\rho\cdot(\mu b-\mu a)\cdot(\delta\cdot\rho\cdot(\mu b-\mu n)-\mu n)\quad\sim}$$

$$\frac{(\alpha-1)\cdot(\delta\cdot\rho\cdot(\mu a-2\cdot\mu b+\mu n)+\mu n)}{}$$

#86: $$\frac{d}{d\ tb}\frac{d}{d\ tb}\left(gb=\alpha\cdot tb\cdot\left(\frac{ta-tb}{\delta\cdot\rho\cdot(\mu a-\mu b)}-\frac{c\cdot(r\cdot\delta+\delta-1)+tb}{\delta\cdot\rho\cdot(\mu b-\mu n)-\mu n}\right)+\frac{(1-\alpha)\cdot\left(\dfrac{ta-tb}{\delta\cdot\rho\cdot(\mu a-\mu b)}+\dfrac{c\cdot(r\cdot\delta+\delta-1)+tb}{\delta\cdot\rho\cdot(\mu b-\mu n)-\mu n}\right)}{2}\right)$$

#87: $$\frac{2\cdot\alpha\cdot(\delta\cdot\rho\cdot(\mu a-\mu n)-\mu n)}{\delta\cdot\rho\cdot(\mu b-\mu a)\cdot(\delta\cdot\rho\cdot(\mu b-\mu n)-\mu n)}$$

Verify that < 0 using Assumption 1c

#88: $\delta\cdot\rho\cdot(\mu a-\mu n)-\mu n>0$

if

#89: SOLVE(δ·ρ·(μa − μn) − μn > 0, μn)

#90: $$\mu n < \frac{\delta \cdot \mu a \cdot \rho}{\delta \cdot \rho + 1}$$

#91: δ·ρ·(μb − μn) − μn > 0

if

#92: SOLVE(δ·ρ·(μb − μn) − μn > 0, μn)

#93: $$\mu n < \frac{\delta \cdot \mu b \cdot \rho}{\delta \cdot \rho + 1}$$

the last condition implies also the conditions 2 lines above it.

Solving for equations (12) and (13): Equlibriuam ta and tb

#94: $$\text{SOLVE}\left(\left[0 = \frac{4 \cdot ta \cdot \alpha - 2 \cdot tb \cdot \alpha + \alpha \cdot (1 - 2 \cdot \delta \cdot \rho \cdot (\mu a - \mu b)) - 1}{2 \cdot \delta \cdot \rho \cdot (\mu b - \mu a)}, 0 = \frac{2 \cdot c \cdot \alpha \cdot \delta \cdot \rho \cdot (\mu a - \mu b) \cdot (r \cdot \delta + \delta - 1) + 2 \cdot ta \cdot \alpha \cdot (\mu n - \delta \cdot \rho \cdot (\mu b - \mu n)) + 4 \cdot tb \cdot \alpha \cdot (\delta \cdot \rho \cdot (\mu a - \mu n) - \mu n) +\sim}{2 \cdot \delta \cdot \rho \cdot (\mu b - \mu a) \cdot (\delta \cdot \rho \cdot (\mu b - \mu n) - \mu n) \sim} \frac{(\alpha - 1) \cdot (\delta \cdot \rho \cdot (\mu a - 2 \cdot \mu b + \mu n) + \mu n)}{}\right], [ta, tb]\right)$$

#95: $$\left[ta = \right.$$

$$\frac{2\cdot c\cdot\alpha\cdot\delta\cdot\rho\cdot(\mu a - \mu b)\cdot(r\cdot\delta + \delta - 1) - \alpha\cdot(4\cdot\delta^2\cdot\rho^2\cdot(\mu a - \mu b)\cdot(\mu a - \mu n) - \delta\cdot\rho\cdot(\mu a\cdot(4\cdot\mu n + 3) - 2\cdot\mu b\cdot \sim}{2\cdot\alpha\cdot(3\cdot\mu n - \delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n)) \quad \sim}$$

$$\frac{(2\cdot\mu n + 1) - \mu n) + \mu n) - \delta\cdot\rho\cdot(3\cdot\mu a - 2\cdot\mu b - \mu n) + \mu n}{} \wedge tb =$$

$$\frac{4\cdot c\cdot\alpha\cdot\delta\cdot\rho\cdot(\mu a - \mu b)\cdot(r\cdot\delta + \delta - 1) - \alpha\cdot(2\cdot\delta^2\cdot\rho^2\cdot(\mu a - \mu b)\cdot(\mu b - \mu n) - \delta\cdot\rho\cdot(2\cdot\mu a\cdot(\mu n + 1) - \mu b\cdot(2\sim}{2\cdot\alpha\cdot(3\cdot\mu n - \delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n)) \quad \sim}$$

$$\left.\frac{\cdot\mu n + 3) + \mu n) - \mu n) - \delta\cdot\rho\cdot(2\cdot\mu a - 3\cdot\mu b + \mu n) - \mu n}{}\right]$$

Define λ as part of the denominator in the aove 2 equations

#96: $\lambda = 3\cdot\mu n - \delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n)$

< 0 since Assumption 1c implies

#97: $3\cdot\mu n - \delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n) < 0$

#98: SOLVE($3\cdot\mu n - \delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n) < 0$, μn)

#99: $$\mu n < \frac{\delta\cdot\rho\cdot(4\cdot\mu a - \mu b)}{3\cdot(\delta\cdot\rho + 1)}$$

Result 3:

#100: $$\frac{d}{dc}\left(ta = \frac{2\cdot c\cdot \alpha\cdot \delta\cdot \rho\cdot (\mu a - \mu b)\cdot (r\cdot \delta + \delta - 1) - \alpha\cdot (4\cdot \delta^2\cdot \rho^2\cdot (\mu a - \mu b)\cdot (\mu a - \mu n) - \delta\cdot \rho\cdot (\mu a\cdot (4\cdot \mu n + 3) - 2\cdot \mu b\cdot (2\cdot \mu n + 1) - \mu n) + \mu n) - \delta\cdot \rho\cdot (3\cdot \mu a - 2\cdot \mu b - \mu n) + \mu n}{2\cdot \alpha\cdot (3\cdot \mu n - \delta\cdot \rho\cdot (4\cdot \mu a - \mu b - 3\cdot \mu n))}\right)$$

#101: $$\frac{\delta\cdot \rho\cdot (\mu b - \mu a)\cdot (r\cdot \delta + \delta - 1)}{\delta\cdot \rho\cdot (4\cdot \mu a - \mu b - 3\cdot \mu n) - 3\cdot \mu n}$$

> 0 by Assumption 2a, Assumption 1, and the denominator is $-\lambda > 0$

#102: $r\cdot \delta + \delta - 1 < 0$

#103: SOLVE($r\cdot \delta + \delta - 1 < 0$, r)

#104: $$r < \frac{1 - \delta}{\delta}$$

#105: $$\frac{d}{dc}\left(tb = \right.$$

$$\frac{4\cdot c\cdot\alpha\cdot\delta\cdot\rho\cdot(\mu a - \mu b)\cdot(r\cdot\delta + \delta - 1) - \alpha\cdot(2\cdot\delta^{2}\cdot\rho^{2}\cdot(\mu a - \mu b)\cdot(\mu b - \mu n) - \delta\cdot\rho\cdot(2\cdot\mu a\cdot(\mu n + 1) - \mu b\cdot(2\cdot\mu n + 3) + \mu n) - \mu n) - \delta\cdot\rho\cdot(2\cdot\mu a - 3\cdot\mu b + \mu n) - \mu n}{2\cdot\alpha\cdot(3\cdot\mu n - \delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n))}\Bigg)$$

#106: $$\frac{2\cdot\delta\cdot\rho\cdot(\mu b - \mu a)\cdot(r\cdot\delta + \delta - 1)}{\delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n) - 3\cdot\mu n} > 0$$

#107: $$\frac{d}{dr}\left(ta = \frac{2\cdot c\cdot\alpha\cdot\delta\cdot\rho\cdot(\mu a - \mu b)\cdot(r\cdot\delta + \delta - 1) - \alpha\cdot(4\cdot\delta^{2}\cdot\rho^{2}\cdot(\mu a - \mu b)\cdot(\mu a - \mu n) - \delta\cdot\rho\cdot(\mu a\cdot(4\cdot\mu n + 3) - 2\cdot\mu b\cdot(2\cdot\mu n + 1) - \mu n) + \mu n) - \delta\cdot\rho\cdot(3\cdot\mu a - 2\cdot\mu b - \mu n) + \mu n}{2\cdot\alpha\cdot(3\cdot\mu n - \delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n))}\right)$$

#108: $$0 > \frac{c\cdot\delta^{2}\cdot\rho\cdot(\mu b - \mu a)}{\delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n) - 3\cdot\mu n}$$

#109: $$\frac{d}{dr}\left(tb = \right.$$

$$\frac{4\cdot c\cdot\alpha\cdot\delta\cdot\rho\cdot(\mu a - \mu b)\cdot(r\cdot\delta + \delta - 1) - \alpha\cdot(2\cdot\delta^2\cdot\rho^2\cdot(\mu a - \mu b)\cdot(\mu b - \mu n) - \delta\cdot\rho\cdot(2\cdot\mu a\cdot(\mu n + 1) - \mu b\cdot(2\sim}{2\cdot\alpha\cdot(3\cdot\mu n - \delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n)) \quad \sim}$$

$$\left.\frac{\cdot\mu n + 3) + \mu n) - \mu n) - \delta\cdot\rho\cdot(2\cdot\mu a - 3\cdot\mu b + \mu n) - \mu n}{}\right)$$

#110: $$0 > \frac{2\cdot c\cdot\delta^2\cdot\rho\cdot(\mu b - \mu a)}{\delta\cdot\rho\cdot(4\cdot\mu a - \mu b - 3\cdot\mu n) - 3\cdot\mu n}$$