Random Tariff Wars
Country 1: Producer wine brand A. Country 2: produces wine brand B

#1: CaseMode ≔ Sensitive

#2: InputMode ≔ Word

Unit production cost

#3: c ∈ Real [0, ∞)

#4: ca ∈ Real [0, ∞)

#5: cb ∈ Real [0, ∞)

population country 1 and 2

#6: n1 ∈ Real (0, ∞)

#7: n2 ∈ Real (0, ∞)

#8: n ∈ Real (0, ∞)

producer prices

#9: pa ∈ Real (0, ∞)

#10: pb ∈ Real (0, ∞)

tariff rates on imports, and reciprocal tariff

#11: t1 ∈ Real [0, 1)

#12: t2 ∈ Real [0, 1)

#13: T ∈ Real [0, 1)

Total government tariff revenues

#14: G1 ∈ Real [0, ∞)

#15: G2 ∈ Real [0, ∞)

per-capita government revenue from tariff

#16: g1 ∈ Real [0, ∞)

#17: g2 ∈ Real [0, ∞)

Income of consumers in country 1 and 2

#18: I1 ∈ Real [0, ∞)

#19: I2 ∈ Real [0, ∞)

differentiation parameter

#20: δ ∈ Real (0, ∞)

brand valuations and Δv = va-vb

#21: va ∈ Real [0, ∞)

#22: vb ∈ Real [0, ∞)

#23: Δv ∈ Real [0, ∞)

eq (1): utility of consumer x in country 1 (buying A domestically, or B imported)

#24: I1 + va − pa − δ·x

#25: I1 + vb − pb·(1 + t1) − δ·(1 − x)

eq (2): utility of consumer x in country 2 (buying A imported, or B domestic)

#26: I2 + va − pa·(1 + t2) − δ·x

#27: I2 + vb − pb − δ·(1 − x)

eq (3): Indifferent consumer country 1 and then 2:

#28: $I1 + va - pa - \delta \cdot x = I1 + vb - pb \cdot (1 + t1) - \delta \cdot (1 - x)$

#29: $\text{SOLVE}(I1 + va - pa - \delta \cdot x = I1 + vb - pb \cdot (1 + t1) - \delta \cdot (1 - x), x)$

#30: $$xhat1 = -\frac{pa - pb \cdot (t1 + 1) - va + vb - \delta}{2 \cdot \delta}$$

#31: $$xhat1 = -\frac{pa - pb \cdot (t1 + 1) - \Delta v - \delta}{2 \cdot \delta}$$

#32: $$xhat1 = \frac{1}{2} - \frac{pa - pb \cdot (t1 + 1) - \Delta v}{2 \cdot \delta}$$

#33: $I2 + va - pa \cdot (1 + t2) - \delta \cdot x = I2 + vb - pb - \delta \cdot (1 - x)$

#34: $\text{SOLVE}(I2 + va - pa \cdot (1 + t2) - \delta \cdot x = I2 + vb - pb - \delta \cdot (1 - x), x)$

#35: $$xhat2 = -\frac{pa \cdot (t2 + 1) - pb - va + vb - \delta}{2 \cdot \delta}$$

#36: $$xhat2 = -\frac{pa \cdot (t2 + 1) - pb - \Delta v - \delta}{2 \cdot \delta}$$

#37: $$xhat2 = \frac{1}{2} - \frac{pa \cdot (t2 + 1) - pb - \Delta v}{2 \cdot \delta}$$

quantity produced

#38: $qa1 = n1 \cdot xhat1$

#39: $qa2 = n2 \cdot xhat2$

#40: $qb1 = n1 \cdot (1 - xhat1)$

#41: $qb2 = n2 \cdot (1 - xhat2)$

#42: $$qa1 = n1 \cdot \left(\frac{1}{2} - \frac{pa - pb \cdot (t1 + 1) - \Delta v}{2 \cdot \delta} \right)$$

#43: $$qa2 = n2 \cdot \left(- \frac{pa \cdot (t2 + 1) - pb - \Delta v - \delta}{2 \cdot \delta} \right)$$

#44: $$qb1 = n1 \cdot \left(1 - \left(\frac{1}{2} - \frac{pa - pb \cdot (t1 + 1) - \Delta v}{2 \cdot \delta} \right) \right)$$

#45: $$qb2 = n2 \cdot \left(1 - \left(\frac{1}{2} - \frac{pa \cdot (t2 + 1) - pb - \Delta v}{2 \cdot \delta} \right) \right)$$

profits

#46: $profita = (pa - ca) \cdot (qa1 + qa2)$

#47: $profitb = (pb - cb) \cdot (qb1 + qb2)$

#48: $$profita = (pa - ca) \cdot \left(n1 \cdot \left(\frac{1}{2} - \frac{pa - pb \cdot (t1 + 1) - \Delta v}{2 \cdot \delta} \right) + n2 \cdot \left(- \frac{pa \cdot (t2 + 1) - pb - \Delta v - \delta}{2 \cdot \delta} \right) \right)$$

#49: $$profitb = (pb - cb) \cdot \left(n1 \cdot \left(1 - \left(\frac{1}{2} - \frac{pa - pb \cdot (t1 + 1) - \Delta v}{2 \cdot \delta} \right) \right) + n2 \cdot \left(1 - \left(\frac{1}{2} - \frac{pa \cdot (t2 + 1) - pb - \Delta v}{2 \cdot \delta} \right) \right) \right)$$

Appendix A and eq (6)

#50: $$\frac{d}{d\ pa}\left(profita = (pa - ca)\cdot\left(n1\cdot\left(\frac{1}{2} - \frac{pa - pb\cdot(t1 + 1) - \Delta v}{2\cdot\delta}\right) + n2\cdot\left(-\frac{pa\cdot(t2 + 1) - pb - \Delta v - \delta}{2\cdot\delta}\right)\right)\right)$$

#51: $$0 = \frac{ca\cdot(n1 + n2\cdot(t2 + 1)) - n1\cdot(2\cdot pa - pb\cdot(t1 + 1) - \Delta v - \delta) - n2\cdot(2\cdot pa\cdot(t2 + 1) - pb - \Delta v - \delta)}{2\cdot\delta}$$

#52: $$\frac{d}{d\ pa}\frac{d}{d\ pa}\left(profita = (pa - ca)\cdot\left(n1\cdot\left(\frac{1}{2} - \frac{pa - pb\cdot(t1 + 1) - \Delta v}{2\cdot\delta}\right) + n2\cdot\left(-\frac{pa\cdot(t2 + 1) - pb - \Delta v - \delta}{2\cdot\delta}\right)\right)\right)$$

#53: $$0 > -\frac{n1 + n2\cdot(t2 + 1)}{\delta}$$

#54: $$\frac{d}{d\ pb}\left(profitb = (pb - cb)\cdot\left(n1\cdot\left(1 - \left(\frac{1}{2} - \frac{pa - pb\cdot(t1 + 1) - \Delta v}{2\cdot\delta}\right)\right) + n2\cdot\left(1 - \left(\frac{1}{2} - \frac{pa\cdot(t2 + 1) - pb - \Delta v}{2\cdot\delta}\right)\right)\right)\right)$$

#55: $$0 = \frac{cb\cdot(n1\cdot(t1 + 1) + n2) + n1\cdot(pa - 2\cdot pb\cdot(t1 + 1) - \Delta v + \delta) + n2\cdot(pa\cdot(t2 + 1) - 2\cdot pb - \Delta v + \delta)}{2\cdot\delta}$$

#56: $$\frac{d}{d\ pb}\frac{d}{d\ pb}\left(profitb = (pb - cb)\cdot\left(n1\cdot\left(1 - \left(\frac{1}{2} - \frac{pa - pb\cdot(t1 + 1) - \Delta v}{2\cdot\delta}\right)\right) + n2\cdot\left(1 - \left(\frac{1}{2} -\right.\right.\right.\right.$$

$$\frac{pa\cdot(t2 + 1) - pb - \Delta v}{2\cdot\delta}\Big)\Big)\Big)\Big)$$

#57: $$0 > -\frac{n1\cdot(t1 + 1) + n2}{\delta}$$

#58: SOLVE$\Big(\Big[0 =$

$$\frac{ca\cdot(n1 + n2\cdot(t2 + 1)) - n1\cdot(2\cdot pa - pb\cdot(t1 + 1) - \Delta v - \delta) - n2\cdot(2\cdot pa\cdot(t2 + 1) - pb - \Delta v - \delta)}{2\cdot\delta}, 0$$

$=$

$$\frac{cb\cdot(n1\cdot(t1 + 1) + n2) + n1\cdot(pa - 2\cdot pb\cdot(t1 + 1) - \Delta v + \delta) + n2\cdot(pa\cdot(t2 + 1) - 2\cdot pb - \Delta v + \delta)}{2\cdot\delta}\Big],$$

$[pa, pb]\Big)$

eq (6) equilibrium prices in the general case

#59: $$\Big[pa = \frac{2\cdot ca\cdot(n1 + n2\cdot(t2 + 1)) + cb\cdot(n1\cdot(t1 + 1) + n2) + (n1 + n2)\cdot(\Delta v + 3\cdot\delta)}{3\cdot(n1 + n2\cdot(t2 + 1))} \wedge pb =$$

$$\frac{ca\cdot(n1 + n2\cdot(t2 + 1)) + 2\cdot cb\cdot(n1\cdot(t1 + 1) + n2) + (n1 + n2)\cdot(3\cdot\delta - \Delta v)}{3\cdot(n1\cdot(t1 + 1) + n2)}\Big]$$

eq (7) equilibrium profits in the general case

$$\#60: \quad profita = \frac{(ca\cdot(n1 + n2\cdot(t2 + 1)) - cb\cdot(n1\cdot(t1 + 1) + n2) - (n1 + n2)\cdot(\Delta v + 3\cdot\delta))^2}{18\cdot\delta\cdot(n1 + n2\cdot(t2 + 1))}$$

$$\#61: \quad profitb = \frac{(ca\cdot(n1 + n2\cdot(t2 + 1)) - cb\cdot(n1\cdot(t1 + 1) + n2) + (n1 + n2)\cdot(3\cdot\delta - \Delta v))^2}{18\cdot\delta\cdot(n1\cdot(t1 + 1) + n2)}$$

*** Section 3: Complete symmetry

eq (8): Prices under complete symmetry

$$\#62: \quad \left[pa = \frac{T\cdot c + 2\cdot(c + \delta)}{T + 2} \wedge pb = \frac{T\cdot c + 2\cdot(c + \delta)}{T + 2}\right]$$

eq (8): profits under complete symmetry

$$\#63: \quad profita = \frac{2\cdot n\cdot\delta}{T + 2}$$

$$\#64: \quad profitb = \frac{2\cdot n\cdot\delta}{T + 2}$$

eq (9): equilibrium market shares under complete symmetry

$$\#65: \quad xhat1 = \frac{T^2\cdot c + T\cdot(2\cdot c + 3\cdot\delta) + 2\cdot\delta}{2\cdot\delta\cdot(T + 2)}$$

$$\#66: \quad xhat2 = -\frac{T^2\cdot c + T\cdot(2\cdot c + \delta) - 2\cdot\delta}{2\cdot\delta\cdot(T + 2)}$$

Deriving Assumption 1

xhat1 < 1 if

$$\#67:\quad T^2\cdot c + T\cdot(2\cdot c + 3\cdot\delta) + 2\cdot\delta < 2\cdot\delta\cdot(T + 2)$$

$$\#68:\quad \text{SOLVE}(T^2\cdot c + T\cdot(2\cdot c + 3\cdot\delta) + 2\cdot\delta < 2\cdot\delta\cdot(T + 2), \delta)$$

$$\#69:\quad \delta > \frac{T\cdot c\cdot(T + 2)}{2 - T}$$

xhat2 > 0 if

$$\#70:\quad T^2\cdot c + T\cdot(2\cdot c + \delta) - 2\cdot\delta < 0$$

$$\#71:\quad \text{SOLVE}(T^2\cdot c + T\cdot(2\cdot c + \delta) - 2\cdot\delta < 0, \delta)$$

$$\#72:\quad \delta > \frac{T\cdot c\cdot(T + 2)}{2 - T}$$

xhat2 < 1 if [always]

$$\#73:\quad -\frac{T^2\cdot c + T\cdot(2\cdot c + \delta) - 2\cdot\delta}{2\cdot\delta\cdot(T + 2)} < 1$$

$$\#74:\quad -(T^2\cdot c + T\cdot(2\cdot c + \delta) - 2\cdot\delta) < 2\cdot\delta\cdot(T + 2)$$

$$\#75:\quad \text{SOLVE}(-(T^2\cdot c + T\cdot(2\cdot c + \delta) - 2\cdot\delta) < 2\cdot\delta\cdot(T + 2), \delta)$$

#76: $$\delta > -\frac{T\cdot c\cdot(T+2)}{3\cdot T+2}$$

Result 1, Appendix B complete symmetry

Result 1a

#77: $$\frac{d}{dT}\left(pa = \frac{T\cdot c + 2\cdot(c+\delta)}{T+2}\right)$$

#78: $$0 > -\frac{2\cdot\delta}{(T+2)^2}$$

#79: $$\frac{d}{dT}\left(pb = \frac{T\cdot c + 2\cdot(c+\delta)}{T+2}\right)$$

#80: $$0 > -\frac{2\cdot\delta}{(T+2)^2}$$

Result 1b

#81: $$(1+T)\cdot\frac{T\cdot c + 2\cdot(c+\delta)}{T+2}$$

#82: $$\frac{d}{dT}\left((1+T)\cdot\frac{T\cdot c + 2\cdot(c+\delta)}{T+2}\right)$$

#83: $$0 < \frac{T^2 \cdot c + 4 \cdot T \cdot c + 2 \cdot (2 \cdot c + \delta)}{(T + 2)^2}$$

Result 1c

#84: $$\frac{d}{dT}\left(\text{xhat1} = \frac{T^2 \cdot c + T \cdot (2 \cdot c + 3 \cdot \delta) + 2 \cdot \delta}{2 \cdot \delta \cdot (T + 2)}\right)$$

#85: $$0 < \frac{T^2 \cdot c + 4 \cdot T \cdot c + 4 \cdot (c + \delta)}{2 \cdot \delta \cdot (T + 2)^2}$$

#86: $$\frac{d}{dT}\left(\text{xhat2} = -\frac{T^2 \cdot c + T \cdot (2 \cdot c + \delta) - 2 \cdot \delta}{2 \cdot \delta \cdot (T + 2)}\right)$$

#87: $$0 > -\frac{T^2 \cdot c + 4 \cdot T \cdot c + 4 \cdot (c + \delta)}{2 \cdot \delta \cdot (T + 2)^2}$$

Result 1d

#88: $$\frac{d}{dT}\left(\text{profita} = \frac{2 \cdot n \cdot \delta}{T + 2}\right)$$

#89: $$0 > -\frac{2\cdot n\cdot \delta}{(T+2)^2}$$

#90: $$\frac{d}{dT}\left(\text{profitb} = \frac{2\cdot n\cdot \delta}{T+2}\right)$$

#91: $$0 > -\frac{2\cdot n\cdot \delta}{(T+2)^2}$$

verifications of Figure 2 (top):

#92: $$\text{profita} = \frac{2\cdot n\cdot \delta}{0+2}$$

#93: $$\text{profita} = n\cdot \delta$$

#94: $$\text{profita} = \frac{2\cdot n\cdot \delta}{1+2}$$

#95: $$\text{profita} = \frac{2\cdot n\cdot \delta}{3}$$

#96: $$\frac{d}{dT}\frac{d}{dT}\left(\text{profita} = \frac{2\cdot n\cdot \delta}{T+2}\right)$$

#97: $$0 < \frac{4\cdot n\cdot \delta}{(T+2)^3}$$

verifications of Figure 2 (bottom):

$$\#98:\quad \mathrm{xhat1} = \frac{0^{2}\cdot c + 0\cdot(2\cdot c + 3\cdot\delta) + 2\cdot\delta}{2\cdot\delta\cdot(0+2)}$$

$$\#99:\quad \mathrm{xhat1} = \frac{1}{2}$$

$$\#100:\quad \mathrm{xhat2} = \frac{1}{2}$$

$$\#101:\quad \mathrm{xhat1} = \frac{1^{2}\cdot c + 1\cdot(2\cdot c + 3\cdot\delta) + 2\cdot\delta}{2\cdot\delta\cdot(1+2)}$$

$$\#102:\quad \mathrm{xhat1} = \frac{3\cdot c + 5\cdot\delta}{6\cdot\delta}$$

$$\#103:\quad \mathrm{xhat1} = \frac{c}{2\cdot\delta} + \frac{5}{6}$$

$$\#104:\quad \mathrm{xhat2} = -\frac{1^{2}\cdot c + 1\cdot(2\cdot c + \delta) - 2\cdot\delta}{2\cdot\delta\cdot(1+2)}$$

$$\#105:\quad \mathrm{xhat2} = \frac{\delta - 3\cdot c}{6\cdot\delta}$$

#106: $$xhat2 = \frac{1}{6} - \frac{c}{2\cdot\delta}$$

#107: $$\frac{d}{dT}\frac{d}{dT}\left(xhat1 = \frac{T^2\cdot c + T\cdot(2\cdot c + 3\cdot\delta) + 2\cdot\delta}{2\cdot\delta\cdot(T+2)}\right)$$

#108: $$0 > -\frac{4}{(T+2)^3}$$

#109: $$\frac{d}{dT}\frac{d}{dT}\left(xhat2 = -\frac{T^2\cdot c + T\cdot(2\cdot c + \delta) - 2\cdot\delta}{2\cdot\delta\cdot(T+2)}\right)$$

#110: $$0 < \frac{4}{(T+2)^3}$$

eq (10): tariff revenue + domestic firm's profit: complete symmetry

#111: $$profitag = T\cdot pb\cdot n\cdot(1 - xhat1) + \frac{2\cdot n\cdot\delta}{T+2}$$

#112: $$profitbg = T\cdot pa\cdot n\cdot xhat2 + \frac{2\cdot n\cdot\delta}{T+2}$$

#113: $$profitag = T\cdot\frac{T\cdot c + 2\cdot(c+\delta)}{T+2}\cdot n\cdot\left(1 - \frac{T^2\cdot c + T\cdot(2\cdot c + 3\cdot\delta) + 2\cdot\delta}{2\cdot\delta\cdot(T+2)}\right) + \frac{2\cdot n\cdot\delta}{T+2}$$

$$\#114:\ \text{profitbg} = T\cdot\frac{T\cdot c + 2\cdot(c+\delta)}{T+2}\cdot n\cdot\left(-\frac{T^2\cdot c + T\cdot(2\cdot c+\delta) - 2\cdot\delta}{2\cdot\delta\cdot(T+2)}\right) + \frac{2\cdot n\cdot\delta}{T+2}$$

Result 2, Appendix C

$$\#115:\ \frac{d}{dT}\left(\text{profitag} = T\cdot\frac{T\cdot c + 2\cdot(c+\delta)}{T+2}\cdot n\cdot\left(1-\frac{T^2\cdot c + T\cdot(2\cdot c+3\cdot\delta) + 2\cdot\delta}{2\cdot\delta\cdot(T+2)}\right) + \frac{2\cdot n\cdot\delta}{T+2}\right)$$

$$\#116:\ -\frac{n\cdot(2\cdot T^4\cdot c^2 + 3\cdot T^3\cdot c\cdot(4\cdot c+\delta) + 6\cdot T^2\cdot c\cdot(4\cdot c+3\cdot\delta) + 4\cdot T\cdot(4\cdot c^2 + 5\cdot c\cdot\delta + 4\cdot\delta^2) - 8\cdot c\cdot\delta)}{2\cdot\delta\cdot(T+2)^3}$$

$$\#117:\ -\frac{n\cdot(2\cdot 0^4\cdot c^2 + 3\cdot 0^3\cdot c\cdot(4\cdot c+\delta) + 6\cdot 0^2\cdot c\cdot(4\cdot c+3\cdot\delta) + 4\cdot 0\cdot(4\cdot c^2 + 5\cdot c\cdot\delta + 4\cdot\delta^2) - 8\cdot c\cdot\delta)}{2\cdot\delta\cdot(0+2)^3}$$

$$\#118:\ 0 < \frac{c\cdot n}{2}$$

$$\#119:\ \frac{d}{dT}\left(\text{profitbg} = T\cdot\frac{T\cdot c + 2\cdot(c+\delta)}{T+2}\cdot n\cdot\left(-\frac{T^2\cdot c + T\cdot(2\cdot c+\delta) - 2\cdot\delta}{2\cdot\delta\cdot(T+2)}\right) + \frac{2\cdot n\cdot\delta}{T+2}\right)$$

$$\#120:\ -\frac{n\cdot(2\cdot T^4\cdot c^2 + 3\cdot T^3\cdot c\cdot(4\cdot c+\delta) + 6\cdot T^2\cdot c\cdot(4\cdot c+3\cdot\delta) + 4\cdot T\cdot(4\cdot c^2 + 5\cdot c\cdot\delta + 4\cdot\delta^2) - 8\cdot c\cdot\delta)}{2\cdot\delta\cdot(T+2)^3}$$

$$\#121:\ -\frac{n\cdot(2\cdot 0^{4}\cdot c^{2}+3\cdot 0^{3}\cdot c\cdot(4\cdot c+\delta)+6\cdot 0^{2}\cdot c\cdot(4\cdot c+3\cdot\delta)+4\cdot 0\cdot(4\cdot c^{2}+5\cdot c\cdot\delta+4\cdot\delta^{2})-8\cdot c\cdot\delta)}{2\cdot\delta\cdot(0+2)^{3}}$$

$$\#122:\quad 0<\frac{c\cdot n}{2}$$

*** Section 4: N1 > N2

eq (11): profits with N1 > N2

$$\#123:\quad \text{profita}=\frac{(T\cdot c\cdot(n1-n2)+3\cdot\delta\cdot(n1+n2))^{2}}{18\cdot\delta\cdot(T\cdot n2+n1+n2)}$$

$$\#124:\quad \text{profitb}=\frac{(T\cdot c\cdot(n1-n2)-3\cdot\delta\cdot(n1+n2))^{2}}{18\cdot\delta\cdot(T\cdot n1+n1+n2)}$$

Result 3 (N1 > N2)

$$\#125:\ \text{profita}-\text{profitb}=\frac{(T\cdot c\cdot(n1-n2)+3\cdot\delta\cdot(n1+n2))^{2}}{18\cdot\delta\cdot(T\cdot n2+n1+n2)}-\frac{(T\cdot c\cdot(n1-n2)-3\cdot\delta\cdot(n1+n2))^{2}}{18\cdot\delta\cdot(T\cdot n1+n1+n2)}$$

#126: profita – profitb =

$$\frac{T\cdot(n1-n2)\cdot(T^2\cdot c^2\cdot(n1-n2)^2+6\cdot T\cdot c\cdot\delta\cdot(n1+n2)^2+3\cdot\delta\cdot(n1+n2)^2\cdot(4\cdot c+3\cdot\delta))}{18\cdot\delta\cdot(T\cdot n1+n1+n2)\cdot(T\cdot n2+n1+n2)} > 0$$

at T=0

#127: profita − profitb =

$$\frac{0\cdot(n1-n2)\cdot(0^2\cdot c^2\cdot(n1-n2)^2+6\cdot 0\cdot c\cdot\delta\cdot(n1+n2)^2+3\cdot\delta\cdot(n1+n2)^2\cdot(4\cdot c+3\cdot\delta))}{18\cdot\delta\cdot(0\cdot n1+n1+n2)\cdot(0\cdot n2+n1+n2)}$$

#128: profita − profitb = 0

#129: $\frac{d}{dT}\left(\text{profita} = \frac{(T\cdot c\cdot(n1-n2)+3\cdot\delta\cdot(n1+n2))^2}{18\cdot\delta\cdot(T\cdot n2+n1+n2)}\right)$

#130: $$\frac{(T\cdot c\cdot n2\cdot(n1-n2)+(n1+n2)\cdot(2\cdot c\cdot(n1-n2)-3\cdot n2\cdot\delta))\cdot(T\cdot c\cdot(n1-n2)+3\cdot\delta\cdot(n1+n2))}{18\cdot\delta\cdot(T\cdot n2+n1+n2)^2}$$

#131: $$\frac{(0\cdot c\cdot n2\cdot(n1-n2)+(n1+n2)\cdot(2\cdot c\cdot(n1-n2)-3\cdot n2\cdot\delta))\cdot(0\cdot c\cdot(n1-n2)+3\cdot\delta\cdot(n1+n2))}{18\cdot\delta\cdot(0\cdot n2+n1+n2)^2}$$

#132: $$\frac{2\cdot c\cdot(n1-n2)-3\cdot n2\cdot\delta}{6}$$

> 0 if

#133: $2 \cdot c \cdot (n1 - n2) - 3 \cdot n2 \cdot \delta > 0$

#134: $$\delta < \frac{2 \cdot c \cdot (n1 - n2)}{3 \cdot n2}$$

#135: $$\frac{d}{dT}\left(\text{profitb} = \frac{(T \cdot c \cdot (n1 - n2) - 3 \cdot \delta \cdot (n1 + n2))^2}{18 \cdot \delta \cdot (T \cdot n1 + n1 + n2)}\right)$$

#136: $$\frac{(T \cdot c \cdot n1 \cdot (n1 - n2) + (n1 + n2) \cdot (2 \cdot c \cdot (n1 - n2) + 3 \cdot n1 \cdot \delta)) \cdot (T \cdot c \cdot (n1 - n2) - 3 \cdot \delta \cdot (n1 + n2))}{18 \cdot \delta \cdot (T \cdot n1 + n1 + n2)^2}$$

#137: $$-\frac{2 \cdot c \cdot (n1 - n2) + 3 \cdot n1 \cdot \delta}{6}$$

#138: $$\frac{(0 \cdot c \cdot n1 \cdot (n1 - n2) + (n1 + n2) \cdot (2 \cdot c \cdot (n1 - n2) + 3 \cdot n1 \cdot \delta)) \cdot (0 \cdot c \cdot (n1 - n2) - 3 \cdot \delta \cdot (n1 + n2))}{18 \cdot \delta \cdot (0 \cdot n1 + n1 + n2)^2}$$

#139: $$-\frac{2 \cdot c \cdot (n1 - n2) + 3 \cdot n1 \cdot \delta}{6}$$

< 0 if [always]

#140: $2 \cdot c \cdot (n1 - n2) + 3 \cdot n1 \cdot \delta > 0$

#141: $$\delta > \frac{2 \cdot c \cdot (n2 - n1)}{3 \cdot n1}$$

equilibrium prices N1 > N2

#142: $$\left[pa = \frac{T\cdot c\cdot(n1 + 2\cdot n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n2 + n1 + n2)} \wedge pb = \frac{T\cdot c\cdot(2\cdot n1 + n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n1 + n1 + n2)}\right]$$

#143: $$pa - pb = \frac{T\cdot c\cdot(n1 + 2\cdot n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n2 + n1 + n2)} - \frac{T\cdot c\cdot(2\cdot n1 + n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n1 + n1 + n2)}$$

#144: $$pa - pb = \frac{T\cdot(n1 + n2)\cdot(n1 - n2)\cdot(T\cdot c + 2\cdot c + 3\cdot\delta)}{3\cdot(T\cdot n1 + n1 + n2)\cdot(T\cdot n2 + n1 + n2)} > 0$$

market shares to be used in the R-code assuming $\Delta v = 0$

#145: $$xhat1 = -\frac{pa - pb\cdot(t1 + 1) - \delta}{2\cdot\delta}$$

#146: $$xhat2 = -\frac{pa\cdot(t2 + 1) - pb - \delta}{2\cdot\delta}$$

#147: $$\frac{d}{dT}\left(pa = \frac{T\cdot c\cdot(n1 + 2\cdot n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n2 + n1 + n2)}\right)$$

#148: $$\frac{\frac{(n1 + n2)\cdot(c\cdot(n1 - n2) - 3\cdot n2\cdot\delta)}{2}}{3\cdot(T\cdot n2 + n1 + n2)}$$

> 0 if

#149: $(n1 + n2)\cdot(c\cdot(n1 - n2) - 3\cdot n2\cdot\delta) > 0$

#150: SOLVE((n1 + n2)·(c·(n1 − n2) − 3·n2·δ) > 0, δ)

$$\#151: \quad \delta < \frac{c\cdot(n1 - n2)}{3\cdot n2}$$

the above is more restrictive than the condition for profita rises with T

$$\#152: \quad \frac{d}{dT}\left(pb = \frac{T\cdot c\cdot(2\cdot n1 + n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n1 + n1 + n2)}\right)$$

$$\#153: \quad - \frac{(n1 + n2)\cdot(c\cdot(n1 - n2) + 3\cdot n1\cdot\delta)}{3\cdot(T\cdot n1 + n1 + n2)^2}$$

always < 0.

*** Section 5: Other symmetry (last section on integrated markets)
** Section 5.1: Production cost asymmetry: Suppose ca > cb

eq (12) profits

$$\#154: \quad profita = \frac{n\cdot(T\cdot(ca - cb) + 2\cdot(ca - cb - 3\cdot\delta))^2}{18\cdot\delta\cdot(T + 2)}$$

$$\#155: \quad profitb = \frac{n\cdot(T\cdot(ca - cb) + 2\cdot(ca - cb + 3\cdot\delta))^2}{18\cdot\delta\cdot(T + 2)}$$

$$\#156: \quad profita = \frac{n\cdot(T\cdot\Delta c + 2\cdot(\Delta c - 3\cdot\delta))^2}{18\cdot\delta\cdot(T + 2)}$$

$$\#157:\ \text{profitb} = \frac{n\cdot(T\cdot\Delta c + 2\cdot(\Delta c + 3\cdot\delta))^2}{18\cdot\delta\cdot(T+2)}$$

profita − profitb =

$$\#158:\ \frac{n\cdot(T\cdot\Delta c + 2\cdot(\Delta c - 3\cdot\delta))^2}{18\cdot\delta\cdot(T+2)} - \frac{n\cdot(T\cdot\Delta c + 2\cdot(\Delta c + 3\cdot\delta))^2}{18\cdot\delta\cdot(T+2)}$$

$$\#159:\ -\frac{4\cdot n\cdot\Delta c}{3} < 0$$

Result 4 and Appendix E (ca > cb under market integrated markets)

eq (E.1)

$$\#160:\ \left[pa = \frac{T\cdot(2\cdot ca + cb) + 2\cdot(2\cdot ca + cb + 3\cdot\delta)}{3\cdot(T+2)} \wedge pb = \frac{T\cdot(ca + 2\cdot cb) + 2\cdot(ca + 2\cdot cb + 3\cdot\delta)}{3\cdot(T+2)}\right]$$

eq (E.2)

$$\#161:\ \text{xhat1} = \frac{T^2\cdot(ca + 2\cdot cb) + T\cdot(ca + 5\cdot cb + 9\cdot\delta) - 2\cdot(ca - cb - 3\cdot\delta)}{6\cdot\delta\cdot(T+2)}$$

$$\#162:\ \text{xhat2} = -\frac{T^2\cdot(2\cdot ca + cb) + T\cdot(5\cdot ca + cb + 3\cdot\delta) + 2\cdot(ca - cb - 3\cdot\delta)}{6\cdot\delta\cdot(T+2)}$$

at T=0

#163: $$xhat1 = -\frac{ca - cb - 3\cdot\delta}{6\cdot\delta}$$

#164: $$xhat2 = -\frac{ca - cb - 3\cdot\delta}{6\cdot\delta}$$

both > 0 if

#165: $ca - cb - 3\cdot\delta < 0$

#166: SOLVE(ca − cb − 3·δ < 0, δ)

#167: $$\delta > \frac{\Delta c}{3}$$

#168: $$\text{SOLVE}\left(\delta > \frac{\Delta c}{3}, \Delta c\right)$$

eq (E.3)

#169: $$\Delta c < 3\cdot\delta$$

Deriving eq (E.4)

#170: $$\frac{d}{dT}\left(\text{profita} = \frac{n\cdot(T\cdot\Delta c + 2\cdot(\Delta c - 3\cdot\delta))^2}{18\cdot\delta\cdot(T+2)}\right)$$

#171: $$\frac{n\cdot(T\cdot\Delta c + 2\cdot(\Delta c + 3\cdot\delta))\cdot(T\cdot\Delta c + 2\cdot(\Delta c - 3\cdot\delta))}{18\cdot\delta\cdot(T+2)^2}$$

$$\#172: \frac{d}{dT}\left(\text{profitb} = \frac{n\cdot(T\cdot\Delta c + 2\cdot(\Delta c + 3\cdot\delta))^2}{18\cdot\delta\cdot(T+2)}\right)$$

$$\#173: \frac{n\cdot(T\cdot\Delta c + 2\cdot(\Delta c + 3\cdot\delta))\cdot(T\cdot\Delta c + 2\cdot(\Delta c - 3\cdot\delta))}{18\cdot\delta\cdot(T+2)^2}$$

< 0 if

$$\#174: T\cdot\Delta c + 2\cdot(\Delta c - 3\cdot\delta) < 0$$

$$\#175: 0\cdot\Delta c + 2\cdot(\Delta c - 3\cdot\delta) < 0$$

$$\#176: \Delta c - 3\cdot\delta < 0$$

** Subsection 5.2: va > vb under integrated markets

eq (13) profits

$$\#177: \text{profita} = \frac{2\cdot n\cdot(\Delta v + 3\cdot\delta)^2}{9\cdot\delta\cdot(T+2)}$$

$$\#178: \text{profitb} = \frac{2\cdot n\cdot(\Delta v - 3\cdot\delta)^2}{9\cdot\delta\cdot(T+2)}$$

profita – profitb =

$$\#179: \frac{2\cdot n\cdot(\Delta v + 3\cdot\delta)^2}{9\cdot\delta\cdot(T+2)} - \frac{2\cdot n\cdot(\Delta v - 3\cdot\delta)^2}{9\cdot\delta\cdot(T+2)}$$

#180: $$\frac{8\cdot n\cdot \Delta v}{3\cdot (T+2)}$$

*** Section 6: Segmented markets

eq (14): market shares

#181: $$xhat1 = \frac{1}{2} - \frac{pa1 - pb1\cdot (T+1) - \Delta v}{2\cdot \delta}$$

#182: $$xhat2 = \frac{1}{2} - \frac{pa2\cdot (T+1) - pb2 - \Delta v}{2\cdot \delta}$$

eq (15) profit A under general segmented markets

#183: profita = (pa1 – ca)·n1·x1 + (pa2 – ca)·n2·x2

eq (16) profit B under general segmented markets

#184: profitb = (pb1 – cb)·n1·(1 – x1) + (pb2 – cb)·n2·(1 – x2)

eq (17) and Appendix F

#185: $$profita = (pa1 - ca)\cdot n1\cdot\left(\frac{1}{2} - \frac{pa1 - pb1\cdot (T+1) - \Delta v}{2\cdot \delta}\right) + (pa2 - ca)\cdot n2\cdot\left(\frac{1}{2} - \frac{pa2\cdot (T+1) - pb2 - \Delta v}{2\cdot \delta}\right)$$

eq (F.1)

#186: $$\frac{d}{d\ pa1}\left(profita = (pa1 - ca)\cdot n1\cdot\left(\frac{1}{2} - \frac{pa1 - pb1\cdot (T+1) - \Delta v}{2\cdot \delta}\right) + (pa2 - ca)\cdot n2\cdot\left(\frac{1}{2} -\right.\right.$$

$$\left.\left.\frac{pa2\cdot(T+1)-pb2-\Delta v}{2\cdot\delta}\right)\right)$$

#187: $$0 = \frac{n1\cdot(T\cdot pb1 + ca - 2\cdot pa1 + pb1 + \Delta v + \delta)}{2\cdot\delta}$$

#188: $$\frac{d}{d\ pa2}\left(profita = (pa1-ca)\cdot n1\cdot\left(\frac{1}{2} - \frac{pa1 - pb1\cdot(T+1) - \Delta v}{2\cdot\delta}\right) + (pa2-ca)\cdot n2\cdot\left(\frac{1}{2} - \left.\left.\frac{pa2\cdot(T+1)-pb2-\Delta v}{2\cdot\delta}\right)\right)\right)$$

#189: $$0 = \frac{n2\cdot(T\cdot(ca-2\cdot pa2) + ca - 2\cdot pa2 + pb2 + \Delta v + \delta)}{2\cdot\delta}$$

#190: $$profitb = (pb1-cb)\cdot n1\cdot\left(1 - \left(\frac{1}{2} - \frac{pa1 - pb1\cdot(T+1) - \Delta v}{2\cdot\delta}\right)\right) + (pb2-cb)\cdot n2\cdot\left(1 - \left(\frac{1}{2} - \left.\left.\frac{pa2\cdot(T+1)-pb2-\Delta v}{2\cdot\delta}\right)\right)\right)$$

#191: $$\frac{d}{d\ pb1}\left(profitb = (pb1-cb)\cdot n1\cdot\left(1 - \left(\frac{1}{2} - \frac{pa1 - pb1\cdot(T+1) - \Delta v}{2\cdot\delta}\right)\right) + (pb2-cb)\cdot n2\cdot\left(1 - \left(\frac{1}{2} - \left.\left.\frac{pa2\cdot(T+1)-pb2-\Delta v}{2\cdot\delta}\right)\right)\right)\right)$$

#192: $$0 = \frac{n1\cdot(T\cdot(cb-2\cdot pb1) + cb + pa1 - 2\cdot pb1 - \Delta v + \delta)}{2\cdot\delta}$$

$$\#193:\ \frac{d}{d\ pb2}\left(profitb = (pb1 - cb)\cdot n1\cdot\left(1 - \left(\frac{1}{2} - \frac{pa1 - pb1\cdot(T + 1) - \Delta v}{2\cdot\delta}\right)\right) + (pb2 - cb)\cdot n2\cdot\left(1 - \left(\frac{1}{2} - \frac{pa2\cdot(T + 1) - pb2 - \Delta v}{2\cdot\delta}\right)\right)\right)$$

$$\#194:\quad 0 = \frac{n2\cdot(T\cdot pa2 + cb + pa2 - 2\cdot pb2 - \Delta v + \delta)}{2\cdot\delta}$$

eq (17): Equilibrium prices under general market segmentation

$$\#195:\ SOLVE\left(\left[0 = \frac{n1\cdot(T\cdot pb1 + ca - 2\cdot pa1 + pb1 + \Delta v + \delta)}{2\cdot\delta},\ 0 = \frac{n2\cdot(T\cdot(ca - 2\cdot pa2) + ca - 2\cdot pa2 + pb2 + \Delta v + \delta)}{2\cdot\delta},\ 0 = \frac{n1\cdot(T\cdot(cb - 2\cdot pb1) + cb + pa1 - 2\cdot pb1 - \Delta v + \delta)}{2\cdot\delta},\ 0 = \frac{n2\cdot(T\cdot pa2 + cb + pa2 - 2\cdot pb2 - \Delta v + \delta)}{2\cdot\delta}\right],\ [pa1,\ pa2,\ pb1,\ pb2]\right)$$

$$\#196:\ \left[pa1 = \frac{T\cdot cb + 2\cdot ca + cb + \Delta v + 3\cdot\delta}{3} \wedge pa2 = \frac{2\cdot T\cdot ca + 2\cdot ca + cb + \Delta v + 3\cdot\delta}{3\cdot(T + 1)} \wedge pb1 = \frac{2\cdot T\cdot cb + ca + 2\cdot cb - \Delta v + 3\cdot\delta}{3\cdot(T + 1)} \wedge pb2 = \frac{T\cdot ca + ca + 2\cdot cb - \Delta v + 3\cdot\delta}{3}\right]$$

$$\#197:\ \left[pa1 = \frac{2\cdot ca + cb\cdot(T + 1) + \Delta v + 3\cdot\delta}{3} \wedge pa2 = \frac{2\cdot ca\cdot(T + 1) + cb + \Delta v + 3\cdot\delta}{3\cdot(T + 1)} \wedge pb1 = \frac{ca + 2\cdot cb\cdot(T + 1) - \Delta v + 3\cdot\delta}{3\cdot(T + 1)} \wedge pb2 = \frac{ca\cdot(T + 1) + 2\cdot cb - \Delta v + 3\cdot\delta}{3}\right]$$

SOC:

$$\#198:\ \frac{d}{d\ pa1}\left(\frac{d}{d\ pa1}\left(profita = (pa1 - ca)\cdot n1\cdot\left(\frac{1}{2} - \frac{pa1 - pb1\cdot(T + 1) - \Delta v}{2\cdot\delta}\right) + (pa2 - ca)\cdot n2\cdot\left(\frac{1}{2} - \frac{pa2\cdot(T + 1) - pb2 - \Delta v}{2\cdot\delta}\right)\right)\right)$$

$$\#199:\ 0 > -\frac{n1}{\delta}$$

$$\#200:\ \frac{d}{d\ pa2}\left(\frac{d}{d\ pa2}\left(profita = (pa1 - ca)\cdot n1\cdot\left(\frac{1}{2} - \frac{pa1 - pb1\cdot(T + 1) - \Delta v}{2\cdot\delta}\right) + (pa2 - ca)\cdot n2\cdot\left(\frac{1}{2} - \frac{pa2\cdot(T + 1) - pb2 - \Delta v}{2\cdot\delta}\right)\right)\right)$$

$$\#201:\ 0 > -\frac{n2\cdot(T + 1)}{\delta}$$

$$\#202:\ \frac{d}{d\ pa2}\left(\frac{d}{d\ pa1}\left(profita = (pa1 - ca)\cdot n1\cdot\left(\frac{1}{2} - \frac{pa1 - pb1\cdot(T + 1) - \Delta v}{2\cdot\delta}\right) + (pa2 - ca)\cdot n2\cdot\left(\frac{1}{2} -\right.\right.\right.$$

$$\left.\left.\frac{pa2\cdot(T+1)-pb2-\Delta v}{2\cdot\delta}\right)\right)$$

#203: $0 = 0$

#204: $$\frac{d}{d\,pb2}\,\frac{d}{d\,pb1}\left(profitb = (pb1-cb)\cdot n1\cdot\left(1-\left(\frac{1}{2}-\frac{pa1-pb1\cdot(T+1)-\Delta v}{2\cdot\delta}\right)\right)+(pb2-cb)\cdot n2\cdot\left(1-\left(\frac{1}{2}-\frac{pa2\cdot(T+1)-pb2-\Delta v}{2\cdot\delta}\right)\right)\right)$$

#205: $0 = 0$

#206: $$\frac{d}{d\,pb1}\,\frac{d}{d\,pb1}\left(profitb = (pb1-cb)\cdot n1\cdot\left(1-\left(\frac{1}{2}-\frac{pa1-pb1\cdot(T+1)-\Delta v}{2\cdot\delta}\right)\right)+(pb2-cb)\cdot n2\cdot\left(1-\left(\frac{1}{2}-\frac{pa2\cdot(T+1)-pb2-\Delta v}{2\cdot\delta}\right)\right)\right)$$

#207: $$0 > -\frac{n1\cdot(T+1)}{\delta}$$

#208: $$\frac{d}{d\,pb2}\,\frac{d}{d\,pb2}\left(profitb = (pb1-cb)\cdot n1\cdot\left(1-\left(\frac{1}{2}-\frac{pa1-pb1\cdot(T+1)-\Delta v}{2\cdot\delta}\right)\right)+(pb2-cb)\cdot n2\cdot\left(1-\left(\frac{1}{2}-\frac{pa2\cdot(T+1)-pb2-\Delta v}{2\cdot\delta}\right)\right)\right)$$

#209: $$0 > -\frac{n2}{\delta}$$

eq (18) market shares under general segmented markets

$$\#210: \quad xhat1 = \frac{T\cdot cb - ca + cb + \Delta v + 3\cdot\delta}{6\cdot\delta}$$

$$\#211: \quad xhat1 = \frac{T\cdot cb - ca + cb + \Delta v}{6\cdot\delta} + \frac{1}{2}$$

$$\#212: \quad xhat1 = -\frac{ca - cb\cdot(T + 1) - \Delta v}{6\cdot\delta} + \frac{1}{2}$$

$$\#213: \quad xhat2 = -\frac{T\cdot ca + ca - cb - \Delta v - 3\cdot\delta}{6\cdot\delta}$$

$$\#214: \quad xhat2 = \frac{1}{2} - \frac{T\cdot ca + ca - cb - \Delta v}{6\cdot\delta}$$

$$\#215: \quad xhat2 = \frac{1}{2} - \frac{ca\cdot(T + 1) - cb - \Delta v}{6\cdot\delta}$$

** Subsection 6.1: Symmetry

eq (19) prices under symmetry

$$\#216: \left[pa1 = \frac{T\cdot c + 3\cdot c + 3\cdot\delta}{3} \wedge pa2 = \frac{2\cdot T\cdot c + 3\cdot(c + \delta)}{3\cdot(T + 1)} \wedge pb1 = \frac{2\cdot T\cdot c + 3\cdot(c + \delta)}{3\cdot(T + 1)} \wedge pb2 = \frac{T\cdot c + 3\cdot c + 3\cdot\delta}{3}\right]$$

#217: $$xhat1 = \frac{T\cdot c + 3\cdot\delta}{6\cdot\delta}$$

#218: $$xhat2 = \frac{3\cdot\delta - T\cdot c}{6\cdot\delta}$$

#219: $$profita = \frac{n\cdot(c^2 + 6\cdot c\cdot\delta + 9\cdot\delta^2)}{18\cdot\delta\cdot(T+1)} + \frac{n\cdot(T^2\cdot c^2 + T\cdot c\cdot(c + 6\cdot\delta) - c^2 - 6\cdot c\cdot\delta + 9\cdot\delta^2)}{18\cdot\delta}$$

#220: $$profita = \frac{n\cdot(T^3\cdot c^2 + 2\cdot T^2\cdot c\cdot(c + 3\cdot\delta) + 9\cdot T\cdot\delta^2 + 18\cdot\delta^2)}{18\cdot\delta\cdot(T+1)}$$

#221: $$profitb = \frac{n\cdot(c^2 + 6\cdot c\cdot\delta + 9\cdot\delta^2)}{18\cdot\delta\cdot(T+1)} + \frac{n\cdot(T^2\cdot c^2 + T\cdot c\cdot(c + 6\cdot\delta) - c^2 - 6\cdot c\cdot\delta + 9\cdot\delta^2)}{18\cdot\delta}$$

#222: $$profitb = \frac{n\cdot(T^3\cdot c^2 + 2\cdot T^2\cdot c\cdot(c + 3\cdot\delta) + 9\cdot T\cdot\delta^2 + 18\cdot\delta^2)}{18\cdot\delta\cdot(T+1)}$$

Result 6 and Appendix G

Result 6a

#223: $$\frac{d}{dT}\left(pa1 = \frac{T\cdot c + 3\cdot c + 3\cdot\delta}{3}\right)$$

#224: $$0 < \frac{c}{3}$$

$$\#225: \frac{d}{dT}\left(pb2 = \frac{T\cdot c + 3\cdot c + 3\cdot\delta}{3}\right)$$

$$\#226: \quad 0 < \frac{c}{3}$$

$$\#227: \frac{d}{dT}\left(pa2 = \frac{2\cdot T\cdot c + 3\cdot(c + \delta)}{3\cdot(T + 1)}\right)$$

$$\#228: \quad 0 > -\frac{c + 3\cdot\delta}{3\cdot(T + 1)^2}$$

$$\#229: \frac{d}{dT}\left(pb1 = \frac{2\cdot T\cdot c + 3\cdot(c + \delta)}{3\cdot(T + 1)}\right)$$

$$\#230: \quad 0 > -\frac{c + 3\cdot\delta}{3\cdot(T + 1)^2}$$

Result 6b, tariff-inclusive prices

$$\#231: (1 + T)\cdot\frac{2\cdot T\cdot c + 3\cdot(c + \delta)}{3\cdot(T + 1)}$$

$$\#232: \frac{d}{dT}\left((1 + T)\cdot\frac{2\cdot T\cdot c + 3\cdot(c + \delta)}{3\cdot(T + 1)}\right)$$

$$\#233: \quad 0 < \frac{2\cdot c}{3}$$

Result 6c

$$\#234: \frac{d}{dT}\left(\text{profita} = \frac{n\cdot(T^3\cdot c^2 + 2\cdot T^2\cdot c\cdot(c + 3\cdot\delta) + 9\cdot T^2\cdot\delta^2 + 18\cdot\delta^2)}{18\cdot\delta\cdot(T + 1)}\right)$$

$$\#235: \frac{n\cdot(2\cdot T^3\cdot c^2 + T^2\cdot c\cdot(5\cdot c + 6\cdot\delta) + 4\cdot T\cdot c\cdot(c + 3\cdot\delta) - 9\cdot\delta^2)^2}{18\cdot\delta\cdot(T + 1)^2}$$

$$\#236: \frac{n\cdot(2\cdot 0^3\cdot c^2 + 0^2\cdot c\cdot(5\cdot c + 6\cdot\delta) + 4\cdot 0\cdot c\cdot(c + 3\cdot\delta) - 9\cdot\delta^2)^2}{18\cdot\delta\cdot(0 + 1)^2}$$

$$\#237: 0 > -\frac{n\cdot\delta}{2}$$

$$\#238: \frac{d}{dT}\left(\text{profitb} = \frac{n\cdot(T^3\cdot c^2 + 2\cdot T^2\cdot c\cdot(c + 3\cdot\delta) + 9\cdot T^2\cdot\delta^2 + 18\cdot\delta^2)}{18\cdot\delta\cdot(T + 1)}\right)$$

$$\#239: \frac{n\cdot(2\cdot T^3\cdot c^2 + T^2\cdot c\cdot(5\cdot c + 6\cdot\delta) + 4\cdot T\cdot c\cdot(c + 3\cdot\delta) - 9\cdot\delta^2)^2}{18\cdot\delta\cdot(T + 1)^2}$$

$$\#240: 0 > -\frac{n\cdot\delta}{2}$$

** Subsection 6.2: n1 > n2 under segmented markets

$$\#241:\ \left[pa1 = \frac{T\cdot c + 3\cdot c + 3\cdot\delta}{3} \wedge pa2 = \frac{2\cdot T\cdot c + 3\cdot(c+\delta)}{3\cdot(T+1)} \wedge pb1 = \frac{2\cdot T\cdot c + 3\cdot(c+\delta)}{3\cdot(T+1)} \wedge pb2 = \frac{T\cdot c + 3\cdot c + 3\cdot\delta}{3} \right]$$

$$\#242:\quad xhat1 = \frac{T\cdot c + 3\cdot\delta}{6\cdot\delta}$$

$$\#243:\quad xhat2 = \frac{3\cdot\delta - T\cdot c}{6\cdot\delta}$$

$$\#244:\ profita = \frac{n2\cdot(c^2 + 6\cdot c\cdot\delta + 9\cdot\delta^2)}{18\cdot\delta\cdot(T+1)} + \frac{T^2\cdot c^2\cdot n1 + T\cdot c\cdot(c\cdot n2 + 6\cdot n1\cdot\delta) - c^2\cdot n2 - 6\cdot c\cdot n2\cdot\delta + 9\cdot n1\cdot\delta^2}{18\cdot\delta}$$

$$\#245:\ profita = \frac{T^3\cdot c^2\cdot n1 + T^2\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n1\cdot\delta) + 3\cdot T\cdot\delta\cdot(2\cdot c\cdot(n1-n2) + 3\cdot n1\cdot\delta) + 9\cdot\delta^2\cdot(n1+n2)}{18\cdot\delta\cdot(T+1)}$$

$$\#246:\ profitb = \frac{n1\cdot(c^2 + 6\cdot c\cdot\delta + 9\cdot\delta^2)}{18\cdot\delta\cdot(T+1)} + \frac{T^2\cdot c^2\cdot n2 + T\cdot c\cdot(c\cdot n1 + 6\cdot n2\cdot\delta) - c^2\cdot n1 - 6\cdot c\cdot n1\cdot\delta + 9\cdot n2\cdot\delta^2}{18\cdot\delta}$$

#247: profitb =

$$\frac{T^3\cdot c^2\cdot n2 + T^2\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n2\cdot\delta) + 3\cdot T\cdot\delta\cdot(3\cdot n2\cdot\delta - 2\cdot c\cdot(n1-n2)) + 9\cdot\delta^2\cdot(n1+n2)}{18\cdot\delta\cdot(T+1)}$$

Result 7b and Appendix H

eq (H.1)

#248: $\frac{d}{dT}\left(\text{profita} = \frac{T^3\cdot c^2\cdot n1 + T^2\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n1\cdot\delta) + 3\cdot T\cdot\delta\cdot(2\cdot c\cdot(n1-n2) + 3\cdot n1\cdot\delta) + 9\cdot\delta^2\cdot(n1+n2)}{18\cdot\delta\cdot(T+1)}\right)$

#249: $$\frac{2\cdot T^3\cdot c^2\cdot n1 + T^2\cdot c\cdot(c\cdot(4\cdot n1+n2) + 6\cdot n1\cdot\delta) + 2\cdot T\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n1\cdot\delta) + 3\cdot\delta\cdot(2\cdot c\cdot(n1-n2) -}{18\cdot\delta\cdot(T+1)^2}$$

$$3\cdot n2\cdot\delta)$$

$$\#250: \frac{d}{dT}\left(\text{profitb} = \frac{T^3\cdot c^2\cdot n2 + T^2\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n2\cdot\delta) + 3\cdot T\cdot\delta\cdot(3\cdot n2\cdot\delta - 2\cdot c\cdot(n1-n2)) + 9\cdot\delta^2\cdot(n1+n2)}{18\cdot\delta\cdot(T+1)}\right)$$

$$\#251: \frac{2\cdot T^3\cdot c^2\cdot n2 + T^2\cdot c\cdot(c\cdot(n1+4\cdot n2) + 6\cdot n2\cdot\delta) + 2\cdot T\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n2\cdot\delta) - 3\cdot\delta\cdot(2\cdot c\cdot(n1-n2) + 3\cdot n1\cdot\delta)}{18\cdot\delta\cdot(T+1)^2}$$

A at T=0

$$\#252: \frac{2\cdot c\cdot(n1-n2) - 3\cdot n2\cdot\delta}{6}$$

> 0 if

#253: $2\cdot c\cdot(n1-n2) - 3\cdot n2\cdot\delta > 0$

#254: SOLVE($2\cdot c\cdot(n1-n2) - 3\cdot n2\cdot\delta > 0$, δ)

$$\#255: \delta < \frac{2\cdot c\cdot(n1-n2)}{3\cdot n2}$$

B at T=0

#256: $$-\frac{2\cdot c\cdot(n1-n2)+3\cdot n1\cdot\delta}{6}$$

< 0 if [always holds]

#257: $2\cdot c\cdot(n1-n2)+3\cdot n1\cdot\delta > 0$

#258: SOLVE(2·c·(n1 − n2) + 3·n1·δ > 0, δ)

#259: $$\delta > \frac{2\cdot c\cdot(n2-n1)}{3\cdot n1}$$

** Subsection 6.3: ca > cb under segmented markets

#260: $$\left[pa1 = \frac{T\cdot cb+2\cdot ca+cb+3\cdot\delta}{3} \wedge pa2 = \frac{2\cdot T\cdot ca+2\cdot ca+cb+3\cdot\delta}{3\cdot(T+1)} \wedge pb1 = \frac{2\cdot T\cdot cb+ca+2\cdot cb+3\cdot\delta}{3\cdot(T+1)} \wedge pb2 = \frac{T\cdot ca+ca+2\cdot cb+3\cdot\delta}{3}\right]$$

#261: $$xhat1 = \frac{T\cdot cb-ca+cb+3\cdot\delta}{6\cdot\delta}$$

#262: $$xhat2 = -\frac{T\cdot ca+ca-cb-3\cdot\delta}{6\cdot\delta}$$

eq (23) profita under ca > cb and segmented markets

#263: $$profita = \frac{n\cdot(cb+3\cdot\delta)^2}{18\cdot\delta\cdot(T+1)} +$$

$$\frac{n\cdot\left(T^2\cdot cb^2 + T\cdot\left(ca^2 - 2\cdot ca\cdot cb + 2\cdot cb\cdot(cb + 3\cdot\delta)\right) + 2\cdot ca^2 - 4\cdot ca\cdot(cb + 3\cdot\delta) + cb^2 + 6\cdot cb\cdot\delta + 9\cdot\delta^2\right)}{18\cdot\delta}$$

)

#264: profita =

$$\frac{n\cdot\left(T^3\cdot cb^2 + T^2\cdot\left(ca^2 - 2\cdot ca\cdot cb + 3\cdot cb\cdot(cb + 2\cdot\delta)\right) + 3\cdot T\cdot(ca - cb - \delta)\cdot(ca - cb - 3\cdot\delta) + 2\cdot(ca - cb - 3\cdot\delta)^2\right)}{18\cdot\delta\cdot(T + 1)}$$

)

eq (24) profitb under ca > cb and segmented markets

#265: profitb =

$$\frac{n\cdot\left(T^3\cdot ca^2 + T^2\cdot\left(3\cdot ca^2 + 2\cdot ca\cdot(3\cdot\delta - cb) + cb^2\right) + 3\cdot T\cdot(ca - cb + \delta)\cdot(ca - cb + 3\cdot\delta) + 2\cdot(ca - cb)^2\right)}{18\cdot\delta\cdot(T + 1)}$$

$$\left. \frac{}{+ 3\cdot\delta)^2)} \right.$$

eq (25)

#266: profita − profitb =

$$\frac{n\cdot(T^3\cdot cb^2 + T^2\cdot(ca^2 - 2\cdot ca\cdot cb + 3\cdot cb\cdot(cb + 2\cdot\delta)) + 3\cdot T\cdot(ca - cb - \delta)\cdot(ca - cb - 3\cdot\delta) + 2\cdot(ca - cb - 3\cdot\delta)^2)}{18\cdot\delta\cdot(T + 1)} -$$

$$\frac{n\cdot(T^3\cdot ca^2 + T^2\cdot(3\cdot ca^2 + 2\cdot ca\cdot(3\cdot\delta - cb) + cb^2) + 3\cdot T\cdot(ca - cb + \delta)\cdot(ca - cb + 3\cdot\delta) + 2\cdot(ca - cb + 3\cdot\delta)^2)}{18\cdot\delta\cdot(T + 1)}$$

#267: $$\text{profita} - \text{profitb} = \frac{n\cdot(cb - ca)\cdot(T^3\cdot(ca + cb) + 2\cdot T^2\cdot(ca + cb + 3\cdot\delta) + 24\cdot T\cdot\delta + 24\cdot\delta)}{18\cdot\delta\cdot(T + 1)}$$

Result 8 and Appendix I

proving Result 8a

eq (I.1): xhat1 and xhat2 when T=0

#268: $$xhat1 = -\frac{ca - cb - 3\cdot\delta}{6\cdot\delta}$$

> 0 if

#269: $$xhat2 = -\frac{ca - cb - 3\cdot\delta}{6\cdot\delta}$$

#270: $\Delta c - 3\cdot\delta < 0$

#271: SOLVE($\Delta c - 3\cdot\delta < 0$, δ)

eq (I.2)

#272: $$\delta > \frac{\Delta c}{3}$$

#273: $$\frac{d}{dT}\left(profita = \frac{n\cdot(T^3\cdot cb^2 + T^2\cdot(ca^2 - 2\cdot ca\cdot cb + 3\cdot cb\cdot(cb + 2\cdot\delta)) + 3\cdot T\cdot(ca - cb - \delta)\cdot(ca - cb - 3\cdot\delta) + 2\cdot(ca - cb - 3\cdot\delta)^2)}{18\cdot\delta\cdot(T + 1)}\right)$$

eq (I.3)

$$\#274: \frac{n\cdot(2\cdot T^3\cdot cb^2 + T^2\cdot(ca^2 - 2\cdot ca\cdot cb + 6\cdot cb\cdot(cb + \delta)) + 2\cdot T\cdot(ca^2 - 2\cdot ca\cdot cb + 3\cdot cb\cdot(cb + 2\cdot\delta)) + (ca - cb - 3\cdot\delta)\cdot(ca - cb + 3\cdot\delta))}{18\cdot\delta\cdot(T + 1)^2}$$

$$\#275: \frac{d}{dT}\left(\text{profitb} = \frac{n\cdot(T^3\cdot ca^2 + T^2\cdot(3\cdot ca^2 + 2\cdot ca\cdot(3\cdot\delta - cb) + cb^2) + 3\cdot T\cdot(ca - cb + \delta)\cdot(ca - cb + 3\cdot\delta) + 2\cdot(ca - cb + 3\cdot\delta)^2)}{18\cdot\delta\cdot(T + 1)^2}\right)$$

$$\#276: \frac{n\cdot(2\cdot T^3\cdot ca^2 + T^2\cdot(6\cdot ca^2 + 2\cdot ca\cdot(3\cdot\delta - cb) + cb^2) + 2\cdot T\cdot(3\cdot ca^2 + 2\cdot ca\cdot(3\cdot\delta - cb) + cb^2) + (ca - cb)^2\cdot(ca - cb + 3\cdot\delta))}{18\cdot\delta\cdot(T + 1)^2}$$

$$\frac{-3\cdot\delta)\cdot(ca - cb + 3\cdot\delta))}{}$$

eq (I.5)

#277: $$\frac{n\cdot(ca - cb + 3\cdot\delta)\cdot(ca - cb - 3\cdot\delta)}{18\cdot\delta}$$

#278: $$\frac{n\cdot(ca - cb + 3\cdot\delta)\cdot(ca - cb - 3\cdot\delta)}{18\cdot\delta}$$

< 0 if

#279: $\Delta c - 3\cdot\delta < 0$

#280: SOLVE($\Delta c - 3\cdot\delta < 0$, δ)

#281: $$\delta > \frac{\Delta c}{3}$$

Result 8b proof in Appendix I

#282: $$\frac{d}{dT}\,\frac{n\cdot(cb - ca)\cdot(T^3\cdot(ca + cb) + 2\cdot T^2\cdot(ca + cb + 3\cdot\delta) + 24\cdot T\cdot\delta + 24\cdot\delta)}{18\cdot\delta\cdot(T + 1)}$$

#283: $$\frac{T\cdot n\cdot(cb - ca)\cdot(2\cdot T^2\cdot(ca + cb) + T\cdot(5\cdot ca + 5\cdot cb + 6\cdot\delta) + 4\cdot(ca + cb + 3\cdot\delta))}{18\cdot\delta\cdot(T + 1)^2} < 0$$

** Subsection 6.4: va > vb under segmented markets

$$\#284:\ \left[pa1 = \frac{T\cdot c + 3\cdot c + \Delta v + 3\cdot\delta}{3} \wedge pa2 = \frac{2\cdot T\cdot c + 3\cdot c + \Delta v + 3\cdot\delta}{3\cdot(T+1)} \wedge pb1 = \frac{2\cdot T\cdot c + 3\cdot c - \Delta v + 3\cdot\delta}{3\cdot(T+1)} \wedge pb2 = \frac{T\cdot c + 3\cdot c - \Delta v + 3\cdot\delta}{3} \right]$$

$$\#285:\ xhat1 = \frac{T\cdot c + \Delta v + 3\cdot\delta}{6\cdot\delta}$$

$$\#286:\ xhat2 = -\frac{T\cdot c - \Delta v - 3\cdot\delta}{6\cdot\delta}$$

$$\#287:\ profita = \frac{n\cdot(c^2 + 2\cdot c\cdot(\Delta v + 3\cdot\delta) + (\Delta v + 3\cdot\delta)^2)}{18\cdot\delta\cdot(T+1)} + \frac{n\cdot(T^2\cdot c^2 + T\cdot c\cdot(c + 2\cdot(\Delta v + 3\cdot\delta)) - c^2 - 2\cdot c\cdot(\Delta v + 3\cdot\delta) + \Delta v^2 + 6\cdot\Delta v\cdot\delta + 9\cdot\delta^2)}{18\cdot\delta}$$

eq (26) profita under va > vb with segmented markets

$$\#288:\ profita = \frac{n\cdot(T^3\cdot c^2 + 2\cdot T^2\cdot c\cdot(c + \Delta v + 3\cdot\delta) + T\cdot(\Delta v^2 + 6\cdot\Delta v\cdot\delta + 9\cdot\delta^2) + 2\cdot(\Delta v^2 + 6\cdot\Delta v\cdot\delta + 9\cdot\delta^2))}{18\cdot\delta\cdot(T+1)}$$

eq (27) profitb under va > vb segmented markets

$$\#289:\quad \text{profitb} = \frac{n\cdot(T^3\cdot c^2 + 2\cdot T^2\cdot c\cdot(c - \Delta v + 3\cdot\delta) + T\cdot(\Delta v^2 - 6\cdot\Delta v\cdot\delta + 9\cdot\delta^2) + 2\cdot(\Delta v^2 - 6\cdot\Delta v\cdot\delta + 9\cdot\delta^2))}{18\cdot\delta\cdot(T+1)}$$

eq (28)

#290: profita – profitb =

$$\frac{n\cdot(T^3\cdot c^2 + 2\cdot T^2\cdot c\cdot(c + \Delta v + 3\cdot\delta) + T\cdot(\Delta v^2 + 6\cdot\Delta v\cdot\delta + 9\cdot\delta^2) + 2\cdot(\Delta v^2 + 6\cdot\Delta v\cdot\delta + 9\cdot\delta^2))}{18\cdot\delta\cdot(T+1)} -$$

$$\frac{n\cdot(T^3\cdot c^2 + 2\cdot T^2\cdot c\cdot(c - \Delta v + 3\cdot\delta) + T\cdot(\Delta v^2 - 6\cdot\Delta v\cdot\delta + 9\cdot\delta^2) + 2\cdot(\Delta v^2 - 6\cdot\Delta v\cdot\delta + 9\cdot\delta^2))}{18\cdot\delta\cdot(T+1)}$$

$$\#291:\quad \text{profita} - \text{profitb} = \frac{2\cdot n\cdot\Delta v\cdot(T^2\cdot c + 3\cdot T\cdot\delta + 6\cdot\delta)}{9\cdot\delta\cdot(T+1)} > 0$$

Result 9 and Appendix J

< 0 under T=0.

$$\#292:\quad \text{xhat1} = \frac{\Delta v + 3\cdot\delta}{6\cdot\delta}$$

$$\#293:\quad \text{xhat2} = \frac{\Delta v + 3\cdot\delta}{6\cdot\delta}$$

< 1 if

$$\#294: \frac{\Delta v + 3\cdot\delta}{6\cdot\delta} < 1$$

$$\#295: \Delta v + 3\cdot\delta < 6\cdot\delta$$

$$\#296: \text{SOLVE}(\Delta v + 3\cdot\delta < 6\cdot\delta, \delta)$$

eq (J.1) => Not needed, commented out in Appendix J

$$\#297: \delta > \frac{\Delta v}{3}$$

$$\#298: \frac{d}{dT}\left(\text{profita} = \frac{n\cdot(T^3\cdot c^2 + 2\cdot T^2\cdot c\cdot(c + \Delta v + 3\cdot\delta) + T\cdot(\Delta v^2 + 6\cdot\Delta v\cdot\delta + 9\cdot\delta^2) + 2\cdot(\Delta v^2 + 6\cdot\Delta v\cdot\delta + 9\cdot\delta^2))}{18\cdot\delta\cdot(T + 1)}\right)$$

$$\#299: \frac{n\cdot(2\cdot T^3\cdot c^2 + T^2\cdot c\cdot(5\cdot c + 2\cdot(\Delta v + 3\cdot\delta)) + 4\cdot T\cdot c\cdot(c + \Delta v + 3\cdot\delta) - \Delta v^2 - 6\cdot\Delta v\cdot\delta - 9\cdot\delta^2)}{18\cdot\delta\cdot(T + 1)^2}$$

$$\#300: \frac{d}{dT}\left(\text{profitb} =\right.$$

$$\left(\frac{n \cdot (T^3 \cdot c^2 + 2 \cdot T^2 \cdot c \cdot (c - \Delta v + 3 \cdot \delta) + T \cdot (\Delta v^2 - 6 \cdot \Delta v \cdot \delta + 9 \cdot \delta^2) + 2 \cdot (\Delta v^2 - 6 \cdot \Delta v \cdot \delta + 9 \cdot \delta^2))}{18 \cdot \delta \cdot (T + 1)} \right)$$

$$\#301: \quad \frac{n \cdot (2 \cdot T^3 \cdot c^2 + T^2 \cdot c \cdot (5 \cdot c - 2 \cdot (\Delta v - 3 \cdot \delta)) + 4 \cdot T \cdot c \cdot (c - \Delta v + 3 \cdot \delta) - \Delta v^2 + 6 \cdot \Delta v \cdot \delta - 9 \cdot \delta^2)}{18 \cdot \delta \cdot (T + 1)^2}$$

eq (J.5) under T=0 => now eq (J.3)

$$\#302: \quad -\frac{n \cdot (\Delta v^2 + 6 \cdot \Delta v \cdot \delta + 9 \cdot \delta^2)}{18 \cdot \delta}$$

$$\#303: \quad -\frac{n \cdot (\Delta v^2 - 6 \cdot \Delta v \cdot \delta + 9 \cdot \delta^2)}{18 \cdot \delta}$$

$$\#304: \quad 0 > -\frac{n \cdot (\Delta v - 3 \cdot \delta)^2}{18 \cdot \delta}$$

Deriving part 9b

$$\#305: \quad \frac{d}{dT} \left(\text{profita} - \text{profitb} = \frac{2 \cdot n \cdot \Delta v \cdot (T^2 \cdot c + 3 \cdot T \cdot \delta + 6 \cdot \delta)}{9 \cdot \delta \cdot (T + 1)} \right)$$

#306:
$$\frac{2 \cdot n \cdot \Delta v \cdot \left(T^{2} \cdot c + 2 \cdot T \cdot c - 3 \cdot \delta\right)}{9 \cdot \delta \cdot \left(T + 1\right)^{2}}$$

at T=0

#307:
$$-\frac{2 \cdot n \cdot \Delta v}{3} < 0$$
