Random Tariff Wars
Country 1: Producer wine brand A. Country 2: produces wine brand B

#1: CaseMode ≔ Sensitive

#2: InputMode ≔ Word

Unit production cost

#3: c ∈ Real [0, ∞)

population country 1 and 2

#4: n1 ∈ Real (0, ∞)

#5: n2 ∈ Real (0, ∞)

#6: n ∈ Real (0, ∞)

producer prices

#7: pa ∈ Real (0, ∞)

#8: pb ∈ Real (0, ∞)

tariff rates on imports

#9: t1 ∈ Real [0, 1)

#10: t2 ∈ Real [0, 1)

Total government tariff revenues

#11: G1 ∈ Real [0, ∞)

#12: G2 ∈ Real [0, ∞)

per-capita government revenue from tariff

#13: g1 ∈ Real [0, ∞)

#14: g2 ∈ Real [0, ∞)

Income of consumers in country 1 and 2

#15: I1 ∈ Real [0, ∞)

#16: I2 ∈ Real [0, ∞)

differentiation parameter

#17: δ ∈ Real (0, ∞)

eq (1): utility of consumer x in country 1 (buying A domestically, or B imported)

#18: I1 − pa − δ·x

#19: I1 − pb·(1 + t1) − δ·(1 − x)

eq (2): utility of consumer x in country 2 (buying A imported, or B domestic)

#20: I2 − pa·(1 + t2) − δ·x

#21: I2 − pb − δ·(1 − x)

eq (3): Indifferent consumer country 1 and then 2:

#22: I1 − pa − δ·x = I1 − pb·(1 + t1) − δ·(1 − x)

#23: SOLVE(I1 − pa − δ·x = I1 − pb·(1 + t1) − δ·(1 − x), x)

#24: $$xhat1 = -\frac{pa - pb\cdot(t1 + 1) - \delta}{2\cdot\delta}$$

#25: $$xhat1 = \frac{pb\cdot(t1 + 1) - pa}{2\cdot\delta} + \frac{1}{2}$$

#26: $I2 - pa\cdot(1 + t2) - \delta\cdot x = I2 - pb - \delta\cdot(1 - x)$

#27: $\text{SOLVE}(I2 - pa\cdot(1 + t2) - \delta\cdot x = I2 - pb - \delta\cdot(1 - x), x)$

#28: $$xhat2 = -\frac{pa\cdot(t2 + 1) - pb - \delta}{2\cdot\delta}$$

#29: $$xhat2 = \frac{pb - pa\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}$$

quantity produced

#30: $qa1 = n1\cdot xhat1$

#31: $qa2 = n2\cdot xhat2$

#32: $qb2 = n2\cdot(1 - xhat2)$

#33: $qb1 = n1\cdot(1 - xhat1)$

#34: $$qa1 = n1\cdot\left(\frac{pb\cdot(t1 + 1) - pa}{2\cdot\delta} + \frac{1}{2}\right)$$

#35: $$qa2 = n2\cdot\left(\frac{pb - pa\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)$$

#36: $$qb2 = n2\cdot\left(1 - \left(\frac{pb - pa\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)$$

#37: $$qb1 = n1\cdot\left(1 - \left(\frac{pb\cdot(t1 + 1) - pa}{2\cdot\delta} + \frac{1}{2}\right)\right)$$

eq (4) (5): Profit A Profit B

#38: $profita = (pa - c)\cdot(qa1 + qa2)$

#39: $profitb = (pb - c)\cdot(qb1 + qb2)$

#40: $$profita = (pa - c)\cdot\left(n1\cdot\left(\frac{pb\cdot(t1 + 1) - pa}{2\cdot\delta} + \frac{1}{2}\right) + n2\cdot\left(\frac{pb - pa\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)$$

#41: $$profitb = (pb - c)\cdot\left(n1\cdot\left(1 - \left(\frac{pb\cdot(t1 + 1) - pa}{2\cdot\delta} + \frac{1}{2}\right)\right) + n2\cdot\left(1 - \left(\frac{pb - pa\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)\right)$$

Appendix A

#42: $$\frac{d}{d\ pa}\left(profita = (pa - c)\cdot\left(n1\cdot\left(\frac{pb\cdot(t1 + 1) - pa}{2\cdot\delta} + \frac{1}{2}\right) + n2\cdot\left(\frac{pb - pa\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)\right)$$

eq (A.1)

#43: $$0 = \frac{c\cdot(n1 + n2\cdot(t2 + 1)) - n1\cdot(2\cdot pa - pb\cdot(t1 + 1) - \delta) - n2\cdot(2\cdot pa\cdot(t2 + 1) - pb - \delta)}{2\cdot\delta}$$

#44: $$\frac{d}{d\ pa}\frac{d}{d\ pa}\left(profita = (pa - c)\cdot\left(n1\cdot\left(\frac{pb\cdot(t1 + 1) - pa}{2\cdot\delta} + \frac{1}{2}\right) + n2\cdot\left(\frac{pb - pa\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)\right)$$

#45: $$0 > -\frac{n1 + n2\cdot(t2 + 1)}{\delta}$$

#46: $$\frac{d}{d\ pb}\left(profitb = (pb - c)\cdot\left(n1\cdot\left(1 - \left(\frac{pb\cdot(t1 + 1) - pa}{2\cdot\delta} + \frac{1}{2}\right)\right) + n2\cdot\left(1 - \left(\frac{pb - pa\cdot(t2 + 1)}{2\cdot\delta} +\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\frac{1}{2}\right)\right)\right)\right)\right)$$

eq (A.2)

#47: $$0 = \frac{c\cdot(n1\cdot(t1 + 1) + n2) + n1\cdot(pa - 2\cdot pb\cdot(t1 + 1) + \delta) + n2\cdot(pa\cdot(t2 + 1) - 2\cdot pb + \delta)}{2\cdot\delta}$$

#48: $$\frac{d}{d\ pb}\frac{d}{d\ pb}\left(profitb = (pb - c)\cdot\left(n1\cdot\left(1 - \left(\frac{pb\cdot(t1 + 1) - pa}{2\cdot\delta} + \frac{1}{2}\right)\right) + n2\cdot\left(1 - \left(\frac{pb - pa\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)\right)\right)$$

#49: $$0 > -\frac{n1\cdot(t1 + 1) + n2}{\delta}$$

eq (6): Eql prices

#50: $$\text{SOLVE}\left(\left[0 = \frac{c\cdot(n1 + n2\cdot(t2 + 1)) - n1\cdot(2\cdot pa - pb\cdot(t1 + 1) - \delta) - n2\cdot(2\cdot pa\cdot(t2 + 1) - pb - \delta)}{2\cdot\delta}, 0 = \frac{c\cdot(n1\cdot(t1 + 1) + n2) + n1\cdot(pa - 2\cdot pb\cdot(t1 + 1) + \delta) + n2\cdot(pa\cdot(t2 + 1) - 2\cdot pb + \delta)}{2\cdot\delta}\right], [pa, pb]\right)$$

#51: $$\left[pa = \frac{c\cdot(n1\cdot(t1 + 3) + n2\cdot(2\cdot t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1 + n2\cdot(t2 + 1))} \wedge pb =\right.$$

$$\frac{c\cdot(n1\cdot(2\cdot t1 + 3) + n2\cdot(t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1\cdot(t1 + 1) + n2)}\Bigg]$$

Result 1 and Appendix B

eq (B.1)

$$\#52:\quad \frac{d}{d\ t1}\left(pa = \frac{c\cdot(n1\cdot(t1 + 3) + n2\cdot(2\cdot t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1 + n2\cdot(t2 + 1))}\right)$$

$$\#53:\quad 0 < \frac{c\cdot n1}{3\cdot(n1 + n2\cdot(t2 + 1))}$$

$$\#54:\quad \frac{d}{d\ t2}\left(pb = \frac{c\cdot(n1\cdot(2\cdot t1 + 3) + n2\cdot(t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1\cdot(t1 + 1) + n2)}\right)$$

$$\#55:\quad 0 < \frac{c\cdot n2}{3\cdot(n1\cdot(t1 + 1) + n2)}$$

eq (B.2)

$$\#56:\quad \frac{d}{dc}\frac{d}{d\ t1}\left(pa = \frac{c\cdot(n1\cdot(t1 + 3) + n2\cdot(2\cdot t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1 + n2\cdot(t2 + 1))}\right)$$

$$\#57:\quad 0 < \frac{n1}{3\cdot(n1 + n2\cdot(t2 + 1))}$$

$$\#58:\quad \frac{d}{dc}\frac{d}{d\ t2}\left(pb = \frac{c\cdot(n1\cdot(2\cdot t1 + 3) + n2\cdot(t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1\cdot(t1 + 1) + n2)}\right)$$

#59: $$0 < \frac{n2}{3\cdot(n1\cdot(t1 + 1) + n2)}$$

eq (B.3)

#60: $$\frac{d}{d\ t2}\left(pa = \frac{c\cdot(n1\cdot(t1 + 3) + n2\cdot(2\cdot t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1 + n2\cdot(t2 + 1))}\right)$$

#61: $$0 > -\frac{n2\cdot(c\cdot(n1\cdot(t1 + 1) + n2) + 3\cdot\delta\cdot(n1 + n2))}{3\cdot(n1 + n2\cdot(t2 + 1))^2}$$

#62: $$\frac{d}{d\ t1}\left(pb = \frac{c\cdot(n1\cdot(2\cdot t1 + 3) + n2\cdot(t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1\cdot(t1 + 1) + n2)}\right)$$

#63: $$0 > -\frac{n1\cdot(c\cdot(n1 + n2\cdot(t2 + 1)) + 3\cdot\delta\cdot(n1 + n2))}{3\cdot(n1\cdot(t1 + 1) + n2)^2}$$

eq (B.4)

#64: $$\frac{c\cdot(n1\cdot(t1 + 3) + n2\cdot(2\cdot t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1 + n2\cdot(t2 + 1))}\cdot(1 + t2)$$

#65: $$\frac{d}{d\ t2}\left(\frac{c\cdot(n1\cdot(t1 + 3) + n2\cdot(2\cdot t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1 + n2\cdot(t2 + 1))}\cdot(1 + t2)\right)$$

#66: $$0 < \frac{c\cdot(n1^2\cdot(t1 + 3) + n1\cdot n2\cdot(4\cdot t2 + 5) + 2\cdot n2^2\cdot(t2^2 + 2\cdot t2 + 1)) + 3\cdot n1\cdot\delta\cdot(n1 + n2)}{3\cdot(n1 + n2\cdot(t2 + 1))^2}$$

eq (B.5)

$$\#67: \quad \frac{c\cdot(n1\cdot(2\cdot t1 + 3) + n2\cdot(t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1\cdot(t1 + 1) + n2)}\cdot(1 + t1)$$

$$\#68: \quad \frac{d}{d\ t1}\left(\frac{c\cdot(n1\cdot(2\cdot t1 + 3) + n2\cdot(t2 + 3)) + 3\cdot\delta\cdot(n1 + n2)}{3\cdot(n1\cdot(t1 + 1) + n2)}\cdot(1 + t1)\right)$$

$$\#69: \quad 0 < \frac{c\cdot(2\cdot n1^2\cdot(t1^2 + 2\cdot t1 + 1) + n1\cdot n2\cdot(4\cdot t1 + 5) + n2^2\cdot(t2 + 3)) + 3\cdot n2\cdot\delta\cdot(n1 + n2)}{3\cdot(n1\cdot(t1 + 1) + n2)^2}$$

eq (7) equilibrium profits:

$$\#70: \quad profita = \frac{(c\cdot(n1\cdot t1 - n2\cdot t2) + 3\cdot\delta\cdot(n1 + n2))^2}{18\cdot\delta\cdot(n1 + n2\cdot(t2 + 1))}$$

$$\#71: \quad profitb = \frac{(c\cdot(n1\cdot t1 - n2\cdot t2) - 3\cdot\delta\cdot(n1 + n2))^2}{18\cdot\delta\cdot(n1\cdot(t1 + 1) + n2)}$$

Result 2 and Appendix C

eq (C.1)

$$\#72: \quad \frac{d}{d\ t1}\left(profita = \frac{(c\cdot(n1\cdot t1 - n2\cdot t2) + 3\cdot\delta\cdot(n1 + n2))^2}{18\cdot\delta\cdot(n1 + n2\cdot(t2 + 1))}\right)$$

#73: $$\frac{c \cdot n1 \cdot (c \cdot (n1 \cdot t1 - n2 \cdot t2) + 3 \cdot \delta \cdot (n1 + n2))}{9 \cdot \delta \cdot (n1 + n2 \cdot (t2 + 1))}$$

#74: $$\frac{d}{d\ t1}\left(\text{profitb} = \frac{(c \cdot (n1 \cdot t1 - n2 \cdot t2) - 3 \cdot \delta \cdot (n1 + n2))^2}{18 \cdot \delta \cdot (n1 \cdot (t1 + 1) + n2)}\right)$$

#75: $$\frac{n1 \cdot (c \cdot (n1 \cdot t1 - n2 \cdot t2) - 3 \cdot \delta \cdot (n1 + n2)) \cdot (c \cdot (n1 \cdot (t1 + 2) + n2 \cdot (t2 + 2)) + 3 \cdot \delta \cdot (n1 + n2))}{18 \cdot \delta \cdot (n1 \cdot (t1 + 1) + n2)^2}$$

#76: $$\frac{d}{d\ t2}\left(\text{profita} = \frac{(c \cdot (n1 \cdot t1 - n2 \cdot t2) + 3 \cdot \delta \cdot (n1 + n2))^2}{18 \cdot \delta \cdot (n1 + n2 \cdot (t2 + 1))}\right)$$

#77: $$-\frac{n2 \cdot (c \cdot (n1 \cdot t1 - n2 \cdot t2) + 3 \cdot \delta \cdot (n1 + n2)) \cdot (c \cdot (n1 \cdot (t1 + 2) + n2 \cdot (t2 + 2)) + 3 \cdot \delta \cdot (n1 + n2))}{18 \cdot \delta \cdot (n1 + n2 \cdot (t2 + 1))^2}$$

#78: $$\frac{d}{d\ t2}\left(\text{profitb} = \frac{(c \cdot (n1 \cdot t1 - n2 \cdot t2) - 3 \cdot \delta \cdot (n1 + n2))^2}{18 \cdot \delta \cdot (n1 \cdot (t1 + 1) + n2)}\right)$$

#79: $$\frac{c \cdot n2 \cdot (3 \cdot \delta \cdot (n1 + n2) - c \cdot (n1 \cdot t1 - n2 \cdot t2))}{9 \cdot \delta \cdot (n1 \cdot (t1 + 1) + n2)}$$

eq (C.2) substituting t1=t2=0 into (C.1) in the same order:

#80: $$0 < \frac{c \cdot n1}{3}$$

#81: $$0 > -\frac{n1\cdot(2\cdot c + 3\cdot\delta)}{6}$$

#82: $$0 > -\frac{n2\cdot(2\cdot c + 3\cdot\delta)}{6}$$

#83: $$0 < \frac{c\cdot n2}{3}$$

eq (8) profits under free trade:

#84: $$\text{profit\_ft} = \frac{\delta\cdot(n1 + n2)}{2}$$

#85: $$\text{profitb\_ft} = \frac{\delta\cdot(n1 + n2)}{2}$$

*** Section 3: Reciprocal tariffs

eq (9): Recall equilibrium prices from #51: substitute T for t1 and t2

#86: $$\left[\text{pa} = \frac{T\cdot c\cdot(n1 + 2\cdot n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n2 + n1 + n2)} \wedge \text{pb} = \frac{T\cdot c\cdot(2\cdot n1 + n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n1 + n1 + n2)}\right]$$

Result 4 and Appendix D.

#87: $$\frac{T\cdot c\cdot(n1 + 2\cdot n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n2 + n1 + n2)}\cdot(1 + T)$$

eq (D.1)

$$\#88:\quad \frac{d}{dT}\left(\frac{T\cdot c\cdot(n1 + 2\cdot n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n2 + n1 + n2)}\cdot(1 + T)\right)$$

$$\#89:\quad 0 < \frac{T^2\cdot c\cdot n2\cdot(n1 + 2\cdot n2) + 2\cdot T\cdot c\cdot(n1 + n2)\cdot(n1 + 2\cdot n2) + (n1 + n2)\cdot(2\cdot c\cdot(2\cdot n1 + n2) + 3\cdot n1\cdot\delta)}{3\cdot(T\cdot n2 + n1 + n2)^2}$$

$$\#90:\quad \frac{T\cdot c\cdot(2\cdot n1 + n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n1 + n1 + n2)}\cdot(1 + T)$$

eq (D.2)

$$\#91:\quad \frac{d}{dT}\left(\frac{T\cdot c\cdot(2\cdot n1 + n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n1 + n1 + n2)}\cdot(1 + T)\right)$$

$$\#92:\quad 0 < \frac{T^2\cdot c\cdot n1\cdot(2\cdot n1 + n2) + 2\cdot T\cdot c\cdot(n1 + n2)\cdot(2\cdot n1 + n2) + (n1 + n2)\cdot(2\cdot c\cdot(n1 + 2\cdot n2) + 3\cdot n2\cdot\delta)}{3\cdot(T\cdot n1 + n1 + n2)^2}$$

eq (D.3)

$$\#93:\quad \frac{d}{dT}\left(pb = \frac{T\cdot c\cdot(2\cdot n1 + n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n1 + n1 + n2)}\right)$$

$$\#94:\quad 0 > -\frac{(n1 + n2)\cdot(c\cdot(n1 - n2) + 3\cdot n1\cdot\delta)}{3\cdot(T\cdot n1 + n1 + n2)^2}$$

eq (D.4)

$$\#95:\quad \frac{d}{dT}\left(pa = \frac{T\cdot c\cdot(n1 + 2\cdot n2) + 3\cdot(c + \delta)\cdot(n1 + n2)}{3\cdot(T\cdot n2 + n1 + n2)}\right)$$

$$\#96:\quad \frac{(n1 + n2)\cdot(c\cdot(n1 - n2) - 3\cdot n2\cdot\delta)}{3\cdot(T\cdot n2 + n1 + n2)^2}$$

< 0 if

#97: $c\cdot(n1 - n2) - 3\cdot n2\cdot\delta < 0$

#98: SOLVE(c·(n1 − n2) − 3·n2·δ < 0, δ)

$$\#99:\quad \delta > \frac{c\cdot(n1 - n2)}{3\cdot n2}$$

eq (10): Profits under t1=t2=T

$$\#100:\quad profita = \frac{(T\cdot c\cdot(n1 - n2) + 3\cdot\delta\cdot(n1 + n2))^2}{18\cdot\delta\cdot(T\cdot n2 + n1 + n2)}$$

$$\#101:\quad profitb = \frac{(T\cdot c\cdot(n1 - n2) - 3\cdot\delta\cdot(n1 + n2))^2}{18\cdot\delta\cdot(T\cdot n1 + n1 + n2)}$$

Result 5 and Appendix E (n1=n2)

$$\#102:\quad \frac{d}{dT}\left(profita = \frac{(T\cdot c\cdot(n1 - n2) + 3\cdot\delta\cdot(n1 + n2))^2}{18\cdot\delta\cdot(T\cdot n2 + n1 + n2)}\right)$$

$$\#103: \quad \frac{(T\cdot c\cdot n2\cdot(n1 - n2) + (n1 + n2)\cdot(2\cdot c\cdot(n1 - n2) - 3\cdot n2\cdot\delta))\cdot(T\cdot c\cdot(n1 - n2) + 3\cdot\delta\cdot(n1 + n2))}{18\cdot\delta\cdot(T\cdot n2 + n1 + n2)^2}$$

eqs (E.1)

$$\#104: \quad 0 > -\frac{2\cdot n\cdot\delta}{(T + 2)^2}$$

$$\#105: \quad \frac{d}{dT}\frac{d}{dT}\left(\text{profita} = \frac{(T\cdot c\cdot(n1 - n2) + 3\cdot\delta\cdot(n1 + n2))^2}{18\cdot\delta\cdot(T\cdot n2 + n1 + n2)}\right)$$

$$\#106: \quad \frac{(n1 + n2)^2\cdot(c^2\cdot(n1 - n2)^2 + 6\cdot c\cdot n2\cdot\delta\cdot(n2 - n1) + 9\cdot n2^2\cdot\delta^2)}{9\cdot\delta\cdot(T\cdot n2 + n1 + n2)^3}$$

$$\#107: \quad 0 < \frac{4\cdot n\cdot\delta}{(T + 2)^3}$$

$$\#108: \quad \frac{d}{dT}\left(\text{profitb} = \frac{(T\cdot c\cdot(n1 - n2) - 3\cdot\delta\cdot(n1 + n2))^2}{18\cdot\delta\cdot(T\cdot n1 + n1 + n2)}\right)$$

$$\#109: \quad \frac{(T\cdot c\cdot n1\cdot(n1 - n2) + (n1 + n2)\cdot(2\cdot c\cdot(n1 - n2) + 3\cdot n1\cdot\delta))\cdot(T\cdot c\cdot(n1 - n2) - 3\cdot\delta\cdot(n1 + n2))}{18\cdot\delta\cdot(T\cdot n1 + n1 + n2)^2}$$

#110: $$0 > -\frac{2\cdot n\cdot \delta}{(T+2)^2}$$

#111: $$\frac{d}{dT}\frac{d}{dT}\left(\text{profitb} = \frac{(T\cdot c\cdot(n1-n2)-3\cdot\delta\cdot(n1+n2))^2}{18\cdot\delta\cdot(T\cdot n1+n1+n2)}\right)$$

#112: $$0 = \frac{(n1+n2)^2\cdot(c^2\cdot(n1-n2)^2+6\cdot c\cdot n1\cdot\delta\cdot(n1-n2)+9\cdot n1^2\cdot\delta^2)}{9\cdot\delta\cdot(T\cdot n1+n1+n2)^3}$$

#113: $$0 < \frac{4\cdot n\cdot\delta}{(T+2)^3}$$

eq (E.2) market shares under t1=t2=T

#114: $$\left[\text{pa} = \frac{T\cdot c\cdot(n+2\cdot n)+3\cdot(c+\delta)\cdot(n+n)}{3\cdot(T\cdot n+n+n)} \wedge \text{pb} = \frac{T\cdot c\cdot(2\cdot n+n)+3\cdot(c+\delta)\cdot(n+n)}{3\cdot(T\cdot n+n+n)}\right]$$

#115: $$\left[\text{pa} = \frac{T\cdot c+2\cdot(c+\delta)}{T+2} \wedge \text{pb} = \frac{T\cdot c+2\cdot(c+\delta)}{T+2}\right]$$

#116: $$\text{xhat1} = \frac{T^2\cdot c+T\cdot(2\cdot c+3\cdot\delta)+2\cdot\delta}{2\cdot\delta\cdot(T+2)}$$

#117: $$\text{xhat2} = -\frac{T^2\cdot c + T\cdot(2\cdot c + \delta) - 2\cdot\delta}{2\cdot\delta\cdot(T + 2)}$$

eq (E.3)

#118: $$\frac{d}{dT}\left(\text{xhat1} = \frac{T^2\cdot c + T\cdot(2\cdot c + 3\cdot\delta) + 2\cdot\delta}{2\cdot\delta\cdot(T + 2)}\right)$$

#119: $$0 < \frac{T^2\cdot c + 4\cdot T\cdot c + 4\cdot(c + \delta)}{2\cdot\delta\cdot(T + 2)^2}$$

#120: $$\frac{d}{dT}\left(\text{xhat2} = -\frac{T^2\cdot c + T\cdot(2\cdot c + \delta) - 2\cdot\delta}{2\cdot\delta\cdot(T + 2)}\right)$$

#121: $$0 > -\frac{T^2\cdot c + 4\cdot T\cdot c + 4\cdot(c + \delta)}{2\cdot\delta\cdot(T + 2)^2}$$

eq (E.4)

#122: $$\frac{d}{dT}\frac{d}{dT}\left(\text{xhat1} = \frac{T^2\cdot c + T\cdot(2\cdot c + 3\cdot\delta) + 2\cdot\delta}{2\cdot\delta\cdot(T + 2)}\right)$$

$$\#123: \quad 0 > -\frac{4}{(T+2)^3}$$

$$\#124: \quad \frac{d}{dT}\frac{d}{dT}\left(xhat2 = -\frac{T^2\cdot c + T\cdot(2\cdot c + \delta) - 2\cdot\delta}{2\cdot\delta\cdot(T+2)}\right)$$

$$\#125: \quad 0 > \frac{4}{(T+2)^3}$$

*** Section 4: Segmented international markets

eq (11) market shares (from #24 above)

$$\#126: \quad xhat1 = \frac{pb1\cdot(t1+1) - pa1}{2\cdot\delta} + \frac{1}{2}$$

$$\#127: \quad xhat2 = \frac{pb2 - pa2\cdot(t2+1)}{2\cdot\delta} + \frac{1}{2}$$

eq (12) profits

#128: profita = (pa1 – c)·n1·xhat1 + (pa2 – c)·n2·xhat2

#129: profitb = (pb1 – c)·n1·(1 – xhat1) + (pb2 – c)·n2·(1 – xhat2)

Appendix F

$$\#130: \quad profita = (pa1 - c)\cdot n1\cdot\left(\frac{pb1\cdot(t1+1) - pa1}{2\cdot\delta} + \frac{1}{2}\right) + (pa2 - c)\cdot n2\cdot\left(\frac{pb2 - pa2\cdot(t2+1)}{2\cdot\delta} + \frac{1}{2}\right)$$

$$\#131:\ \text{profitb} = (pb1 - c)\cdot n1\cdot\left(1 - \left(\frac{pb1\cdot(t1 + 1) - pa1}{2\cdot\delta} + \frac{1}{2}\right)\right) + (pb2 - c)\cdot n2\cdot\left(1 - \left(\frac{pb2 - pa2\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)$$

eq (F.1)

$$\#132:\ \frac{d}{d\ pa1}\left(\text{profita} = (pa1 - c)\cdot n1\cdot\left(\frac{pb1\cdot(t1 + 1) - pa1}{2\cdot\delta} + \frac{1}{2}\right) + (pa2 - c)\cdot n2\cdot\left(\frac{pb2 - pa2\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)$$

$$\#133:\ 0 = \frac{n1\cdot(c - 2\cdot pa1 + pb1\cdot(t1 + 1) + \delta)}{2\cdot\delta}$$

$$\#134:\ \frac{d}{d\ pa2}\left(\text{profita} = (pa1 - c)\cdot n1\cdot\left(\frac{pb1\cdot(t1 + 1) - pa1}{2\cdot\delta} + \frac{1}{2}\right) + (pa2 - c)\cdot n2\cdot\left(\frac{pb2 - pa2\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)$$

$$\#135:\ 0 = \frac{n2\cdot(c\cdot(t2 + 1) - 2\cdot pa2\cdot(t2 + 1) + pb2 + \delta)}{2\cdot\delta}$$

$$\#136:\ \frac{d}{d\ pa1}\ \frac{d}{d\ pa1}\left(\text{profita} = (pa1 - c)\cdot n1\cdot\left(\frac{pb1\cdot(t1 + 1) - pa1}{2\cdot\delta} + \frac{1}{2}\right) + (pa2 -\right.$$

$$c)\cdot n2\cdot\left(\frac{pb2 - pa2\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)$$

#137: $$0 > -\frac{n1}{\delta}$$

#138: $$\frac{d}{d\ pa2}\frac{d}{d\ pa2}\left(profita = (pa1 - c)\cdot n1\cdot\left(\frac{pb1\cdot(t1 + 1) - pa1}{2\cdot\delta} + \frac{1}{2}\right) + (pa2 - c)\cdot n2\cdot\left(\frac{pb2 - pa2\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)$$

#139: $$0 > -\frac{n2\cdot(t2 + 1)}{\delta}$$

#140: $$\frac{d}{d\ pa2}\frac{d}{d\ pa1}\left(profita = (pa1 - c)\cdot n1\cdot\left(\frac{pb1\cdot(t1 + 1) - pa1}{2\cdot\delta} + \frac{1}{2}\right) + (pa2 - c)\cdot n2\cdot\left(\frac{pb2 - pa2\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)$$

#141: $$0 = 0$$

eq (F.2)

#142: $$\frac{d}{d\ pb1}\left(profitb = (pb1 - c)\cdot n1\cdot\left(1 - \left(\frac{pb1\cdot(t1 + 1) - pa1}{2\cdot\delta} + \frac{1}{2}\right)\right) + (pb2 - c)\cdot n2\cdot\left(1 -\right.\right.$$

$$\left(\frac{pb2 - pa2\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)\right)$$

#143: $$0 = \frac{n1\cdot(c\cdot(t1 + 1) + pa1 - 2\cdot pb1\cdot(t1 + 1) + \delta)}{2\cdot\delta}$$

#144: $$\frac{d}{d\ pb2}\left(profitb = (pb1 - c)\cdot n1\cdot\left(1 - \left(\frac{pb1\cdot(t1 + 1) - pa1}{2\cdot\delta} + \frac{1}{2}\right)\right) + (pb2 - c)\cdot n2\cdot\left(1 - \left(\frac{pb2 - pa2\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)\right)$$

#145: $$0 = \frac{n2\cdot(c + pa2\cdot(t2 + 1) - 2\cdot pb2 + \delta)}{2\cdot\delta}$$

#146: $$\frac{d}{d\ pb1}\frac{d}{d\ pb1}\left(profitb = (pb1 - c)\cdot n1\cdot\left(1 - \left(\frac{pb1\cdot(t1 + 1) - pa1}{2\cdot\delta} + \frac{1}{2}\right)\right) + (pb2 - c)\cdot n2\cdot\left(1 - \left(\frac{pb2 - pa2\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)\right)$$

#147: $$0 > -\frac{n1\cdot(t1 + 1)}{\delta}$$

#148: $$\frac{d}{d\ pb2}\frac{d}{d\ pb2}\left(profitb = (pb1 - c)\cdot n1\cdot\left(1 - \left(\frac{pb1\cdot(t1 + 1) - pa1}{2\cdot\delta} + \frac{1}{2}\right)\right) + (pb2 - c)\cdot n2\cdot\left(1 -\right.\right.$$

$$\left(\frac{pb2 - pa2\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)\right)$$

#149: $$0 > -\frac{n2}{\delta}$$

#150: $$\frac{d}{d\ pb2}\frac{d}{d\ pb1}\left(profitb = (pb1 - c)\cdot n1\cdot\left(1 - \left(\frac{pb1\cdot(t1 + 1) - pa1}{2\cdot\delta} + \frac{1}{2}\right)\right) + (pb2 - c)\cdot n2\cdot\left(1 - \left(\frac{pb2 - pa2\cdot(t2 + 1)}{2\cdot\delta} + \frac{1}{2}\right)\right)\right)$$

#151: $$0 = 0$$

eq (14)

#152: $$SOLVE\left(\left[0 = \frac{n1\cdot(c - 2\cdot pa1 + pb1\cdot(t1 + 1) + \delta)}{2\cdot\delta}, 0 = \frac{n2\cdot(c\cdot(t2 + 1) - 2\cdot pa2\cdot(t2 + 1) + pb2 + \delta)}{2\cdot\delta}, 0 = \frac{n1\cdot(c\cdot(t1 + 1) + pa1 - 2\cdot pb1\cdot(t1 + 1) + \delta)}{2\cdot\delta}, 0 = \frac{n2\cdot(c + pa2\cdot(t2 + 1) - 2\cdot pb2 + \delta)}{2\cdot\delta}\right], [pa1, pa2, pb1, pb2]\right)$$

#153: $$\left[pa1 = \frac{c\cdot(t1 + 3) + 3\cdot\delta}{3} \wedge pa2 = \frac{c\cdot(2\cdot t2 + 3) + 3\cdot\delta}{3\cdot(t2 + 1)} \wedge pb1 = \frac{c\cdot(2\cdot t1 + 3) + 3\cdot\delta}{3\cdot(t1 + 1)} \wedge pb2 =\right.$$

$$\frac{c\cdot(t2 + 3) + 3\cdot\delta}{3}\Bigg]$$

eq (15) equilibrium market shares

#154: $$xhat1 = \frac{c\cdot t1}{6\cdot\delta} + \frac{1}{2}$$

#155: $$xhat2 = \frac{1}{2} - \frac{c\cdot t2}{6\cdot\delta}$$

#156: $$\frac{1}{2} - \frac{c\cdot t2}{6\cdot\delta} > 0$$

#157: $$\text{SOLVE}\left(\frac{1}{2} - \frac{c\cdot t2}{6\cdot\delta} > 0,\ t2\right)$$

#158: $$t2 < \frac{3\cdot\delta}{c}$$

#159: $$\frac{c\cdot t1}{6\cdot\delta} + \frac{1}{2} < 1$$

#160: $$\text{SOLVE}\left(\frac{c\cdot t1}{6\cdot\delta} + \frac{1}{2} < 1,\ t1\right)$$

#161: $$t1 < \frac{3\cdot\delta}{c}$$

Assumption 2 ensures xhat1 and xhat2 are between 0 and 1

Result 7a and Appendix G

$$\#162: \frac{d}{d\ t1}\left(pa1 = \frac{c\cdot(t1 + 3) + 3\cdot\delta}{3}\right)$$

$$\#163: \quad 0 < \frac{c}{3}$$

$$\#164: \frac{d}{d\ t2}\left(pa2 = \frac{c\cdot(2\cdot t2 + 3) + 3\cdot\delta}{3\cdot(t2 + 1)}\right)$$

$$\#165: \quad 0 > -\frac{c + 3\cdot\delta}{3\cdot(t2 + 1)^2}$$

$$\#166: \frac{d}{d\ t1}\left(pb1 = \frac{c\cdot(2\cdot t1 + 3) + 3\cdot\delta}{3\cdot(t1 + 1)}\right)$$

$$\#167: \quad 0 > -\frac{c + 3\cdot\delta}{3\cdot(t1 + 1)^2}$$

$$\#168: \frac{d}{d\ t2}\left(pb2 = \frac{c\cdot(t2 + 3) + 3\cdot\delta}{3}\right)$$

$$\#169: \quad 0 < \frac{c}{3}$$

Result 7b & equation (G.2)

$$\#170: \frac{c\cdot(2\cdot t2 + 3) + 3\cdot\delta}{3\cdot(t2 + 1)}\cdot(1 + t2)$$

$$\#171: \frac{d}{d\ t2}\left(\frac{c\cdot(2\cdot t2 + 3) + 3\cdot\delta}{3\cdot(t2 + 1)}\cdot(1 + t2)\right)$$

$$\#172: \qquad 0 < \frac{2\cdot c}{3}$$

$$\#173: \frac{c\cdot(2\cdot t1 + 3) + 3\cdot\delta}{3\cdot(t1 + 1)}\cdot(1 + t1)$$

$$\#174: \frac{d}{d\ t1}\left(\frac{c\cdot(2\cdot t1 + 3) + 3\cdot\delta}{3\cdot(t1 + 1)}\cdot(1 + t1)\right)$$

$$\#175: \qquad 0 < \frac{2\cdot c}{3}$$

eq (16): profits under segmented markets

#176: profita =

$$\frac{c^2\cdot(n1\cdot t1^2\cdot(t2 + 1) + n2\cdot t2^2) + 6\cdot c\cdot\delta\cdot(n1\cdot t1\cdot(t2 + 1) - n2\cdot t2) + 9\cdot\delta^2\cdot(n1\cdot(t2 + 1) + n2)}{18\cdot\delta\cdot(t2 + 1)}$$

#177: profitb =

$$\frac{c^2\cdot(n1\cdot t1^2 + n2\cdot t2^2\cdot(t1+1)) + 6\cdot c\cdot\delta\cdot(n2\cdot t2\cdot(t1+1) - n1\cdot t1) + 9\cdot\delta^2\cdot(n1 + n2\cdot(t1+1))}{18\cdot\delta\cdot(t1+1)}$$

Result 8a and Appendix G

#178: $\frac{d}{d\ t1}\left(\text{profita} = \frac{c^2\cdot(n1\cdot t1^2\cdot(t2+1) + n2\cdot t2^2) + 6\cdot c\cdot\delta\cdot(n1\cdot t1\cdot(t2+1) - n2\cdot t2) + 9\cdot\delta^2\cdot(n1\cdot(t2+1) + n2)}{18\cdot\delta\cdot(t2+1)}\right)$

#179: $0 < \frac{c\cdot n1\cdot(c\cdot t1 + 3\cdot\delta)}{9\cdot\delta}$

#180: $\frac{d}{d\ t2}\left(\text{profitb} = \frac{c^2\cdot(n1\cdot t1^2 + n2\cdot t2^2\cdot(t1+1)) + 6\cdot c\cdot\delta\cdot(n2\cdot t2\cdot(t1+1) - n1\cdot t1) + 9\cdot\delta^2\cdot(n1 + n2\cdot(t1+1))}{18\cdot\delta\cdot(t1+1)}\right)$

#181: $0 < \frac{c\cdot n2\cdot(c\cdot t2 + 3\cdot\delta)}{9\cdot\delta}$

$$\#182: \frac{d}{d\ t2}\left[\text{profita} = \frac{c^2\cdot(n1\cdot t1^2\cdot(t2+1)+n2\cdot t2^2)+6\cdot c\cdot\delta\cdot(n1\cdot t1\cdot(t2+1)-n2\cdot t2)+9\cdot\delta^2\cdot(n1\cdot(t2+1)+n2)}{18\cdot\delta\cdot(t2+1)}\right]$$

$$\#183: \frac{n2\cdot(c^2\cdot t2\cdot(t2+2)-6\cdot c\cdot\delta-9\cdot\delta^2)}{18\cdot\delta\cdot(t2+1)^2}$$

< 0 if [Assumption 2]

$$\#184: c^2\cdot t2\cdot(t2+2)-6\cdot c\cdot\delta-9\cdot\delta^2<0$$

$$\#185: \text{SOLVE}(c^2\cdot t2\cdot(t2+2)-6\cdot c\cdot\delta-9\cdot\delta^2<0,\ t2)$$

$$\#186: \left(c\neq 0\wedge\frac{3\cdot\delta}{c}<t2<-\frac{2\cdot c+3\cdot\delta}{c}\right)\vee\left(c\neq 0\wedge-\frac{2\cdot c+3\cdot\delta}{c}<t2<\frac{3\cdot\delta}{c}\right)$$

$$\#187: \frac{d}{d\ t1}\left[\text{profitb} = \frac{c^2\cdot(n1\cdot t1^2+n2\cdot t2^2\cdot(t1+1))+6\cdot c\cdot\delta\cdot(n2\cdot t2\cdot(t1+1)-n1\cdot t1)+9\cdot\delta^2\cdot(n1+n2\cdot(t1+1))}{18\cdot\delta\cdot(t1+1)}\right]$$

$$\#188: \quad \frac{n1\cdot(c^2\cdot t1\cdot(t1+2) - 6\cdot c\cdot\delta - 9\cdot\delta^2)}{18\cdot\delta\cdot(t1+1)^2}$$

< 0 if [Assumption 2]

$$\#189: \quad c^2\cdot t1\cdot(t1+2) - 6\cdot c\cdot\delta - 9\cdot\delta^2 < 0$$

$$\#190: \quad \text{SOLVE}(c^2\cdot t1\cdot(t1+2) - 6\cdot c\cdot\delta - 9\cdot\delta^2 < 0,\ t1)$$

$$\#191: \quad \left(c \neq 0 \wedge \frac{3\cdot\delta}{c} < t1 < -\frac{2\cdot c + 3\cdot\delta}{c}\right) \vee \left(c \neq 0 \wedge -\frac{2\cdot c + 3\cdot\delta}{c} < t1 < \frac{3\cdot\delta}{c}\right)$$

** Section 4.2: Reciprocal tariffs under segmented markets

eq (18) and (19) profits under segmented market and t1=t2=T

#192: profita =

$$\frac{T^3\cdot c^2\cdot n1 + T^2\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n1\cdot\delta) + 3\cdot T\cdot\delta\cdot(2\cdot c\cdot(n1-n2) + 3\cdot n1\cdot\delta) + 9\cdot\delta^2\cdot(n1+n2)}{18\cdot\delta\cdot(T+1)}$$

#193: profitb =

$$\frac{T^3\cdot c^2\cdot n2 + T^2\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n2\cdot\delta) + 3\cdot T\cdot\delta\cdot(3\cdot n2\cdot\delta - 2\cdot c\cdot(n1-n2)) + 9\cdot\delta^2\cdot(n1+n2)^2}{18\cdot\delta\cdot(T+1)}$$

go back to n1 ≠ n2

$$\#194: \frac{d}{dT}\left(\text{profita} = \frac{T^3\cdot c^2\cdot n1 + T^2\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n1\cdot\delta) + 3\cdot T\cdot\delta\cdot(2\cdot c\cdot(n1-n2) + 3\cdot n1\cdot\delta) + 9\cdot\delta^2\cdot(n1+n2)^2}{18\cdot\delta\cdot(T+1)}\right)$$

$$\#195: \frac{2\cdot T^3\cdot c^2\cdot n1 + T^2\cdot c\cdot(c\cdot(4\cdot n1+n2) + 6\cdot n1\cdot\delta) + 2\cdot T\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n1\cdot\delta) + 3\cdot\delta\cdot(2\cdot c\cdot(n1-n2) - 3\cdot n2\cdot\delta)}{18\cdot\delta\cdot(T+1)^2}$$

evaluate at T=0

$$\#196: \frac{2\cdot c\cdot(n1-n2) - 3\cdot n2\cdot\delta}{6}$$

$$\#197: \frac{d}{dT}\left(\text{profitb} =\right.$$

$$\left(\frac{T^3\cdot c^2\cdot n2 + T^2\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n2\cdot\delta) + 3\cdot T\cdot\delta\cdot(3\cdot n2\cdot\delta - 2\cdot c\cdot(n1-n2)) + 9\cdot\delta^2\cdot(n1+n2)}{18\cdot\delta\cdot(T+1)}\right)$$

$$\#198:\ \frac{2\cdot T^3\cdot c^2\cdot n2 + T^2\cdot c\cdot(c\cdot(n1+4\cdot n2) + 6\cdot n2\cdot\delta) + 2\cdot T\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n2\cdot\delta) - 3\cdot\delta\cdot(2\cdot c\cdot(n1-n2) + 3\cdot n1\cdot\delta)}{18\cdot\delta\cdot(T+1)^2}$$

evaluate at T=0

$$\#199:\ \frac{2\cdot 0^3\cdot c^2\cdot n2 + 0^2\cdot c\cdot(c\cdot(n1+4\cdot n2) + 6\cdot n2\cdot\delta) + 2\cdot 0\cdot c\cdot(c\cdot(n1+n2) + 6\cdot n2\cdot\delta) - 3\cdot\delta\cdot(2\cdot c\cdot(n1-n2) + 3\cdot n1\cdot\delta)}{18\cdot\delta\cdot(0+1)^2}$$

$$\#200:\quad 0 > -\frac{2\cdot c\cdot(n1-n2) + 3\cdot n1\cdot\delta}{6}$$

*** The End