

tiprise_2024_1_30.dfw

#1: CaseMode := Sensitive

#2: InputMode := Word

#3: $nt \in \text{Real } (0, \infty)$ #4: $nht1 \in \text{Real } (0, \infty)$ #5: $nlt \in \text{Real } (0, \infty)$ #6: $nlt \in \text{Real } (0, \infty)$ #7: $nht1 \in \text{Real } (0, \infty)$ #8: $nlt1 \in \text{Real } (0, \infty)$

eq (1)

#9: $rht = (1 + \mu h) \cdot rt1$ #10: $rlt = (1 - \mu l) \cdot rt1$

eq (2)

#11: $nt = (1 + \eta - \lambda) \cdot nt1$

eq (3) in paper

#12: $nht = nht1 + \eta \cdot \phi \cdot nt1 - \lambda \cdot nht1$ #13: $nht = nht1 + \eta \cdot \phi \cdot (nht1 + nlt1) - \lambda \cdot nht1$ #14:
$$nht = nht1 \cdot (\eta \cdot \phi - \lambda + 1) + nlt1 \cdot \eta \cdot \phi$$
#15: $nlt = nlt1 + \eta \cdot (1 - \phi) \cdot nt1 - \lambda \cdot nlt1$ #16: $nlt = nlt1 + \eta \cdot (1 - \phi) \cdot (nht1 + nlt1) - \lambda \cdot nlt1$

$$\#17: \quad nlt = nht1 \cdot \eta \cdot (1 - \phi) - nlt1 \cdot (\eta \cdot (\phi - 1) + \lambda - 1)$$

verify sum up to nt in eq (2)

$$\#18: \quad nht + nlt = nht1 + \eta \cdot \phi \cdot nt1 - \lambda \cdot nht1 + nlt1 + \eta \cdot (1 - \phi) \cdot nt1 - \lambda \cdot nlt1$$

$$\#19: \quad nht + nlt = nht1 \cdot (1 - \lambda) + nlt1 \cdot (1 - \lambda) + nt1 \cdot \eta$$

eq (4) avg tipping rate at t

$$\#20: \quad rt = \frac{nht \cdot rht + nlt \cdot rlt}{nht + nlt}$$

$$\#21: \quad rt = \frac{nht \cdot ((1 + \mu_h) \cdot rt1) + nlt \cdot ((1 - \mu_l) \cdot rt1)}{nht + nlt}$$

$$\#22: \quad rt = \frac{rt1 \cdot (nht \cdot (\mu_h + 1) + nlt \cdot (1 - \mu_l))}{nht + nlt}$$

Result 1 and eq (5)

$rt > rt1$ if

$$\#23: \quad \frac{nht \cdot (\mu_h + 1) + nlt \cdot (1 - \mu_l)}{nht + nlt} > 1$$

$$\#24: \quad \text{SOLVE} \left(\frac{nht \cdot (\mu_h + 1) + nlt \cdot (1 - \mu_l)}{nht + nlt} > 1, \mu_h \right)$$

$$\#25: \quad \text{IF} \left(\frac{nht}{nht + nlt} < 0, \mu_h < \frac{nlt \cdot \mu_l}{nht} \right) \vee \text{IF} \left(\frac{nht}{nht + nlt} > 0, \mu_h > \frac{nlt \cdot \mu_l}{nht} \right)$$

$$\#26: \mu_h > \frac{n_{lt} \cdot \mu_l}{n_{ht}}$$

* trying to develop the evolution paths of n_{ht} and n_{lt} from n_{h0} and n_{l0}

$$\#27: n_1 = (1 + \eta - \lambda) \cdot n_0$$

$$\#28: n_2 = (1 + \eta - \lambda) \cdot n_1$$

$$\#29: n_2 = (1 + \eta - \lambda) \cdot ((1 + \eta - \lambda) \cdot n_0)$$

$$\#30: n_2 = n_0 \cdot (\eta - \lambda + 1)^2$$

hence,

$$\#31: n_t = n_0 \cdot (\eta - \lambda + 1)^t$$

next, path of n_{ht}

$$\#32: n_{ht} = n_{ht1} + \eta \cdot \phi \cdot n_{t1} - \lambda \cdot n_{ht1}$$

$$\#33: n_{h1} = n_{h0} + \eta \cdot \phi \cdot n_0 - \lambda \cdot n_{h0}$$

$$\#34: n_{h1} = n_{h0} \cdot (1 - \lambda) + n_0 \cdot \eta \cdot \phi$$

$$\#35: n_{h2} = n_{h1} \cdot (1 - \lambda) + n_1 \cdot \eta \cdot \phi$$

$$\#36: n_{h2} = (n_{h0} + \eta \cdot \phi \cdot n_0 - \lambda \cdot n_{h0}) \cdot (1 - \lambda) + ((1 + \eta - \lambda) \cdot n_0) \cdot \eta \cdot \phi$$

$$\#37: n_{h2} = n_0 \cdot \eta \cdot \phi \cdot (\eta - 2 \cdot (\lambda - 1)) + n_{h0} \cdot (\lambda - 1)^2$$

$$\#38: n_{h3} = n_{h2} \cdot (1 - \lambda) + n_2 \cdot \eta \cdot \phi$$

$$\#39: n_{h3} = (n_0 \cdot \eta \cdot \phi \cdot (\eta - 2 \cdot (\lambda - 1)) + n_{h0} \cdot (\lambda - 1)^2) \cdot (1 - \lambda) + (n_0 \cdot (\eta - \lambda + 1)^2) \cdot \eta \cdot \phi$$

$$\#40: \quad nh3 = n0 \cdot \eta \cdot \phi \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) + nh0 \cdot (1 - \lambda)^3$$

$$\#41: \quad nh4 = nh3 \cdot (1 - \lambda) + n3 \cdot \eta \cdot \phi$$

$$\#42: \quad nh4 = (n0 \cdot \eta \cdot \phi \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) + nh0 \cdot (1 - \lambda)^3) \cdot (1 - \lambda) + (n0 \cdot (\eta - \lambda + 1)^3) \cdot \eta \cdot \phi$$

$$\#43: \quad nh4 = n0 \cdot \eta \cdot \phi \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + nh0 \cdot (\lambda - 1)^4$$

$$\#44: \quad nh5 = nh4 \cdot (1 - \lambda) + n4 \cdot \eta \cdot \phi$$

$$\#45: \quad nh5 = n0 \cdot \eta \cdot \phi \cdot (\eta^5 + 5 \cdot \eta^4 \cdot (1 - \lambda) + \eta^3 \cdot (\lambda - 1) \cdot (10 \cdot \lambda - 11) + 2 \cdot \eta^2 \cdot (\lambda - 1)^2 \cdot (7 - 5 \cdot \lambda) + \eta \cdot (\lambda - 1)^3 \cdot (5 \cdot \lambda - 11) - (\lambda - 5) \cdot (\lambda - 1)^4) + nh0 \cdot (1 - \lambda)^5$$

$$\#46: \quad nh5 = (n0 \cdot \eta \cdot \phi \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + nh0 \cdot (\lambda - 1)^4) \cdot (1 - \lambda) + (n0 \cdot (\eta - \lambda + 1)^4) \cdot \eta \cdot \phi$$

$$\#47: \quad nh5 = n0 \cdot \eta \cdot \phi \cdot (\eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4) + nh0 \cdot (1 - \lambda)^5$$

$$\#48: \quad nh6 = nh5 \cdot (1 - \lambda) + n5 \cdot \eta \cdot \phi$$

$$\#49: \quad nh6 = (n0 \cdot \eta \cdot \phi \cdot (\eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4) + nh0 \cdot (1 - \lambda)^5) \cdot (1 - \lambda) + (n0 \cdot (\eta - \lambda + 1)^6) \cdot \eta \cdot \phi$$

$$\#50: \quad nh6 = n0 \cdot \eta \cdot \phi \cdot (\eta^6 + 6 \cdot \eta^5 \cdot (1 - \lambda) + \eta^4 \cdot (\lambda - 1) \cdot (15 \cdot \lambda - 16) + 5 \cdot \eta^3 \cdot (\lambda - 1)^2 \cdot (5 - 4 \cdot \lambda) + 5 \cdot \eta^2 \cdot (\lambda -$$

$$1)^3 \cdot (3 \cdot \lambda - 5) + 2 \cdot \eta \cdot (\lambda - 1)^4 \cdot (8 - 3 \cdot \lambda) + (\lambda - 6) \cdot (\lambda - 1)^5 + nh0 \cdot (\lambda - 1)^6$$

developing nlt for t=1..5

$$\#51: \quad nlt = nlt1 + \eta \cdot (1 - \phi) \cdot nt1 - \lambda \cdot nlt1$$

$$\#52: \quad n11 = n10 + \eta \cdot (1 - \phi) \cdot n0 - \lambda \cdot n10$$

$$\#53: \quad n11 = n0 \cdot \eta \cdot (1 - \phi) + n10 \cdot (1 - \lambda)$$

$$\#54: \quad n12 = n11 + \eta \cdot (1 - \phi) \cdot n1 - \lambda \cdot n11$$

$$\#55: \quad n12 = (n0 \cdot \eta \cdot (1 - \phi) + n10 \cdot (1 - \lambda)) + \eta \cdot (1 - \phi) \cdot (n0 \cdot (\eta - \lambda + 1)^1) - \lambda \cdot (n0 \cdot \eta \cdot (1 - \phi) + n10 \cdot (1 - \lambda))$$

$$\#56: \quad n12 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - 2 \cdot (\lambda - 1)) + n10 \cdot (\lambda^2 - 2 \cdot \lambda + 1)$$

$$\#57: \quad n12 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - 2 \cdot (\lambda - 1)) + n10 \cdot (\lambda - 1)^2$$

$$\#58: \quad n13 = n12 + \eta \cdot (1 - \phi) \cdot n2 - \lambda \cdot n12$$

$$\#59: \quad n13 = (n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - 2 \cdot (\lambda - 1)) + n10 \cdot (\lambda^2 - 2 \cdot \lambda + 1)) + \eta \cdot (1 - \phi) \cdot (n0 \cdot (\eta - \lambda + 1)^2) - \lambda \cdot (n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - 2 \cdot (\lambda - 1)) + n10 \cdot (\lambda^2 - 2 \cdot \lambda + 1))$$

$$\#60: \quad n13 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) - n10 \cdot (\lambda^3 - 3 \cdot \lambda^2 + 3 \cdot \lambda - 1)$$

$$\#61: \quad n13 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) - n10 \cdot (\lambda - 1)^3$$

$$\#62: \quad n14 = n13 + \eta \cdot (1 - \phi) \cdot n3 - \lambda \cdot n13$$

$$\begin{aligned} \#63: \quad n\lambda4 &= (n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) - n\lambda0 \cdot (\lambda - 1)^3) + \eta \cdot (1 - \phi) \cdot (n0 \cdot (\eta - \lambda + 1)^3 \\ &\quad - \lambda \cdot (n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) - n\lambda0 \cdot (\lambda - 1)^3) \end{aligned}$$

$$\#64: \quad n\lambda4 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + n\lambda0 \cdot (\lambda - 1)^4$$

$$\#65: \quad n\lambda5 = n\lambda4 + \eta \cdot (1 - \phi) \cdot n4 - \lambda \cdot n\lambda4$$

$$\begin{aligned} \#66: \quad n\lambda5 &= (n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + n\lambda0 \cdot (\lambda - 1)^4) + \eta \cdot (1 - \\ &\quad \phi) \cdot (n0 \cdot (\eta - \lambda + 1)^4 - \lambda \cdot (n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + n\lambda0 \cdot (\lambda \\ &\quad - 1)^4) \end{aligned}$$

$$\#67: \quad n\lambda5 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4) + n\lambda0 \cdot (1 - \lambda)^5$$

verifying nht + nlt = nt for t = 1..5 [Yes]

$$\#68: \quad nh1 + n\lambda1 = nh0 \cdot (1 - \lambda) + n0 \cdot \eta \cdot \phi + n0 \cdot \eta \cdot (1 - \phi) + n\lambda0 \cdot (1 - \lambda)$$

$$\#69: \quad nh1 + n\lambda1 = n0 \cdot \eta + nh0 \cdot (1 - \lambda) + n\lambda0 \cdot (1 - \lambda)$$

$$\#70: \quad nh2 + n\lambda2 = n0 \cdot \eta \cdot \phi \cdot (\eta - 2 \cdot (\lambda - 1)) + nh0 \cdot (\lambda - 1)^2 + n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - 2 \cdot (\lambda - 1)) + n\lambda0 \cdot (\lambda - 1)^2$$

$$\#71: \quad nh2 + n\lambda2 = n0 \cdot \eta \cdot (\eta - 2 \cdot (\lambda - 1)) + nh0 \cdot (\lambda - 1)^2 + n\lambda0 \cdot (\lambda - 1)^2$$

subst n0 for nh0+n\lambda0

$$\#72: \quad nh2 + n\lambda2 = n0 \cdot \eta \cdot (\eta - 2 \cdot (\lambda - 1)) + n0 \cdot (\lambda - 1)^2$$

$$\#73: \quad nh2 + n12 = n0 \cdot (\eta^2 + 2 \cdot \eta \cdot (1 - \lambda) + (\lambda - 1)^2)$$

$$\#74: \quad nh2 + n12 = n0 \cdot (\eta - \lambda + 1)^2$$

$$\#75: \quad nh3 + n13 = n0 \cdot \eta \cdot \phi \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) + nh0 \cdot (1 - \lambda)^3 + n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) - n10 \cdot (\lambda - 1)^3$$

$$\#76: \quad nh3 + n13 = n0 \cdot \eta \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) + (1 - \lambda)^3 \cdot (nh0 + n10)$$

$$\#77: \quad nh3 + n13 = n0 \cdot \eta \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) + (1 - \lambda)^3 \cdot n0$$

$$\#78: \quad nh3 + n13 = n0 \cdot (\eta^3 + 3 \cdot \eta^2 \cdot (1 - \lambda) + 3 \cdot \eta \cdot (\lambda - 1)^2 - (\lambda - 1)^3)$$

$$\#79: \quad nh3 + n13 = n0 \cdot (\eta - \lambda + 1)^3$$

$$\#80: \quad nh4 + n14 = (n0 \cdot \eta \cdot \phi \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + nh0 \cdot (\lambda - 1)^4) + n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + n10 \cdot (\lambda - 1)^4$$

$$\#81: \quad nh4 + n14 = n0 \cdot \eta \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + (nh0 + n10) \cdot (\lambda - 1)^4$$

$$\#82: \quad nh4 + n14 = n0 \cdot \eta \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + n0 \cdot (\lambda - 1)^4$$

$$\#83: \quad nh4 + n14 = n0 \cdot (\eta - \lambda + 1)^4$$

$$\begin{aligned} \#84: \quad nh5 + n15 = & n0 \cdot \eta \cdot \phi \cdot (\eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4) + nh0 \cdot (1 - \\ & \lambda)^5 + n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4) + n10 \cdot (1 - \\ & \lambda)^5 \end{aligned}$$

$$\begin{aligned} \#85: \quad nh5 + n15 = & n0 \cdot \eta \cdot (\eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4) + (1 - \lambda)^5 \cdot (nh0 \\ & + n10) \end{aligned}$$

$$\#86: \quad nh5 + n15 = n0 \cdot \eta \cdot (\eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4) + (1 - \lambda)^5 \cdot n0$$

$$\#87: \quad nh5 + n15 = n0 \cdot (\eta - \lambda + 1)^5$$

** try to expand the path to see if easier to infer general path w.r.t time 0

$$\#88: \quad nh2 = (nh0 + n10) \cdot \eta \cdot \phi \cdot (\eta - 2 \cdot (\lambda - 1))^2 + nh0 \cdot (\lambda - 1)^2$$

$$\#89: \quad nh2 = nh0 \cdot (\eta^2 \cdot \phi + 2 \cdot \eta \cdot \phi \cdot (1 - \lambda) + (\lambda - 1)^2) + n10 \cdot \eta \cdot \phi \cdot (\eta - 2 \cdot (\lambda - 1))$$

$$\#90: \quad nh3 = (nh0 + n10) \cdot \eta \cdot \phi \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) + nh0 \cdot (1 - \lambda)^3$$

$$\begin{aligned} \#91: \quad nh3 = & nh0 \cdot (\eta^3 \cdot \phi + 3 \cdot \eta^2 \cdot \phi \cdot (1 - \lambda) + 3 \cdot \eta \cdot \phi \cdot (\lambda - 1)^2 - (\lambda - 1)^3) + n10 \cdot \eta \cdot \phi \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - \\ & 1)^2) \end{aligned}$$

$$\#92: \quad nh4 = (nh0 + n10) \cdot \eta \cdot \phi \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + nh0 \cdot (\lambda - 1)^4$$

$$\#93: \quad nh4 = nh0 \cdot (\eta^4 \cdot \phi + 4 \cdot \eta^3 \cdot \phi \cdot (1 - \lambda) + 6 \cdot \eta^2 \cdot \phi \cdot (\lambda - 1)^2 + 4 \cdot \eta \cdot \phi \cdot (1 - \lambda)^3 + (\lambda - 1)^4) + n10 \cdot \eta \cdot \phi \cdot (\eta^2 + 4 \cdot \eta \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3)$$

$$\#94: \quad n15 = (nh0 + n10) \cdot \eta \cdot (1 - \phi) \cdot (\eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4) + n10 \cdot (1 - \lambda)^5$$

$$\#95: \quad n15 = nh0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4) - n10 \cdot (\eta^5 \cdot (\phi - 1) + 5 \cdot \eta^4 \cdot (1 - \phi) \cdot (\lambda - 1) + 10 \cdot \eta^3 \cdot (\lambda - 1)^2 \cdot (\phi - 1) + 10 \cdot \eta^2 \cdot (1 - \phi) \cdot (\lambda - 1)^3 + 5 \cdot \eta \cdot (\lambda - 1)^4 \cdot (\phi - 1) + (\lambda - 1)^5)$$

** try solving system of equations

$$\#96: \quad \text{SOLVE}([nh_t = nh_{t1} + \eta \cdot \phi \cdot nt1 - \lambda \cdot nh_{t1}, n1_t = n1_{t1} + \eta \cdot (1 - \phi) \cdot nt1 - \lambda \cdot n1_{t1}], [nh_{t1}, n1_{t1}])$$

$$\#97: \quad \left[nh_{t1} = \frac{nh_t - nt1 \cdot \eta \cdot \phi}{1 - \lambda} \wedge n1_{t1} = \frac{n1_t + nt1 \cdot \eta \cdot (\phi - 1)}{1 - \lambda} \right]$$

$$\#98: \quad \text{SOLVE}([nh_t = nh_{t1} \cdot (\eta \cdot \phi - \lambda + 1) + n1_{t1} \cdot \eta \cdot \phi, n1_t = nh_{t1} \cdot \eta \cdot (1 - \phi) - n1_{t1} \cdot (\eta \cdot (\phi - 1) + \lambda - 1)], [nh_{t1}, n1_{t1}])$$

$$\#99: \quad \left[nh_{t1} = \frac{nh_t \cdot (\eta \cdot (\phi - 1) + \lambda - 1) + n1_t \cdot \eta \cdot \phi}{(\eta - \lambda + 1) \cdot (\lambda - 1)} \wedge n1_{t1} = \frac{nh_t \cdot \eta \cdot (\phi - 1) + n1_t \cdot (\eta \cdot \phi - \lambda + 1)}{(1 - \lambda) \cdot (\eta - \lambda + 1)} \right]$$

*** redoing from eq (3)

$$\#100: nt = n0 \cdot (\eta - \lambda + 1)^t$$

eq (4)

$$\#101: nht = nht1 + \eta \cdot \phi \cdot nt1 - \lambda \cdot nht1$$

$$\#102: nlt = nlt1 + \eta \cdot (1 - \phi) \cdot nt1 - \lambda \cdot nlt1$$

$$\#103: nht = nht1 + \eta \cdot \phi \cdot (n0 \cdot (\eta - \lambda + 1)^{t-1}) - \lambda \cdot nht1$$

$$\#104: nlt = nlt1 + \eta \cdot (1 - \phi) \cdot (n0 \cdot (\eta - \lambda + 1)^{t-1}) - \lambda \cdot nlt1$$

$$\#105: nht = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1)^{t-1} + nht1 \cdot (1 - \lambda)$$

$$\#106: nlt = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1)^{t-1} + nlt1 \cdot (1 - \lambda)$$

verify summing up

$$\#107: nht + nlt = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1)^{t-1} + nht1 \cdot (1 - \lambda) + n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1)^{t-1} + nlt1 \cdot (1 - \lambda)$$

$$\#108: nht + nlt = n0 \cdot \eta \cdot (\eta - \lambda + 1)^{t-1} + nht1 \cdot (1 - \lambda) + nlt1 \cdot (1 - \lambda)$$

$$\#109: nht + nlt = n0 \cdot \eta \cdot (\eta - \lambda + 1)^{t-1} + nt1 \cdot (1 - \lambda)$$

$$\#110: nht + nlt = n0 \cdot \eta \cdot (\eta - \lambda + 1)^{t-1} + (n0 \cdot (\eta - \lambda + 1)^{t-1}) \cdot (1 - \lambda)$$

$$\#111: nht + nlt = n0 \cdot (\eta - \lambda + 1)^t$$

* working nht and nlt forward

$$\#112: nht = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1)^{t-1} + nht1 \cdot (1 - \lambda)$$

$$\#113: nlt = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1)^{t-1} + nlt1 \cdot (1 - \lambda)$$

t = 1

$$\#114: nh1 = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1)^{1-1} + nh0 \cdot (1 - \lambda)$$

$$\#115: nh1 = n0 \cdot \eta \cdot \phi + nh0 \cdot (1 - \lambda)$$

$$\#116: nlt1 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1)^{1-1} + nlt0 \cdot (1 - \lambda)$$

$$\#117: nlt1 = n0 \cdot \eta \cdot (1 - \phi) + nlt0 \cdot (1 - \lambda)$$

t = 2

$$\#118: nh2 = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1)^{2-1} + nh1 \cdot (1 - \lambda)$$

$$\#119: nh2 = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1) + nh1 \cdot (1 - \lambda)$$

$$\#120: nh2 = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1) + (n0 \cdot \eta \cdot \phi + nh0 \cdot (1 - \lambda)) \cdot (1 - \lambda)$$

$$\#121: nh2 = n0 \cdot \eta \cdot \phi \cdot (\eta - 2 \cdot \lambda + 2) + nh0 \cdot (\lambda - 1)^2$$

$$\#122: nlt2 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1)^{2-1} + nlt1 \cdot (1 - \lambda)$$

$$\#123: nlt2 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1) + nlt1 \cdot (1 - \lambda)$$

$$\#124: nlt2 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - 2 \cdot (\lambda - 1)) + nlt0 \cdot (\lambda - 1)^2$$

t = 3

$$\#125: nh3 = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1)^{3-1} + nh2 \cdot (1 - \lambda)$$

$$\#126: nh3 = n0 \cdot \eta \cdot \phi \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot \lambda^2 - 6 \cdot \lambda + 3) + nh0 \cdot (1 - \lambda)^3$$

$$\#127: nh3 = n0 \cdot \eta \cdot \phi \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) + nh0 \cdot (1 - \lambda)^3$$

$$\#128: n13 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1)^{3-1} + n12 \cdot (1 - \lambda)$$

$$\#129: n13 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda^2 - 2 \cdot \lambda + 1)) + n10 \cdot (1 - \lambda)^3$$

$$\#130: n13 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2) + n10 \cdot (1 - \lambda)^3$$

t = 4

$$\#131: nh4 = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1)^{4-1} + nh3 \cdot (1 - \lambda)$$

$$\#132: nh4 = n0 \cdot \eta \cdot \phi \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + nh0 \cdot (\lambda - 1)^4$$

$$\#133: n14 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1)^{4-1} + n13 \cdot (1 - \lambda)$$

$$\#134: n14 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3) + n10 \cdot (\lambda - 1)^4$$

t = 5

$$\#135: nh5 = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1)^{5-1} + nh4 \cdot (1 - \lambda)$$

$$\#136: \quad nh5 = n0 \cdot \eta \cdot \phi \cdot (\eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4) + nh0 \cdot (1 - \lambda)^5$$

$$\#137: \quad n15 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1)^{5-1} + n14 \cdot (1 - \lambda)$$

$$\#138: \quad n15 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4) + n10 \cdot (1 - \lambda)^5$$

t = 6

$$\#139: \quad nh6 = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1)^{6-1} + nh5 \cdot (1 - \lambda)$$

$$\#140: \quad nh6 = n0 \cdot \eta \cdot \phi \cdot (\eta^5 + 6 \cdot \eta^4 \cdot (1 - \lambda) + 15 \cdot \eta^3 \cdot (\lambda - 1)^2 + 20 \cdot \eta^2 \cdot (1 - \lambda)^3 + 15 \cdot \eta \cdot (\lambda - 1)^4 - 6 \cdot (\lambda - 1)^5) + nh0 \cdot (\lambda - 1)^6$$

$$\#141: \quad n16 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1)^{6-1} + n15 \cdot (1 - \lambda)$$

$$\#142: \quad n16 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^5 + 6 \cdot \eta^4 \cdot (1 - \lambda) + 15 \cdot \eta^3 \cdot (\lambda - 1)^2 + 20 \cdot \eta^2 \cdot (1 - \lambda)^3 + 15 \cdot \eta \cdot (\lambda - 1)^4 - 6 \cdot (\lambda - 1)^5) + n10 \cdot (\lambda - 1)^6$$

t = 7

$$\#143: \quad nh7 = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1)^{7-1} + nh6 \cdot (1 - \lambda)$$

$$\#144: \quad nh7 = n0 \cdot \eta \cdot \phi \cdot (\eta^6 + 7 \cdot \eta^5 \cdot (1 - \lambda) + 21 \cdot \eta^4 \cdot (\lambda - 1)^2 + 35 \cdot \eta^3 \cdot (1 - \lambda)^3 + 35 \cdot \eta^2 \cdot (\lambda - 1)^4 + 21 \cdot \eta \cdot (1 - \lambda)^5)$$

$$+ 7 \cdot (\lambda - 1)^6 + nh0 \cdot (1 - \lambda)^7$$

$$\#145: n17 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1)^{7-1} + n16 \cdot (1 - \lambda)$$

$$\#146: n17 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^6 + 7 \cdot \eta^5 \cdot (1 - \lambda) + 21 \cdot \eta^4 \cdot (\lambda - 1)^2 + 35 \cdot \eta^3 \cdot (1 - \lambda)^3 + 35 \cdot \eta^2 \cdot (\lambda - 1)^4 + 21 \cdot \eta \cdot (1 - \lambda)^5 + 7 \cdot (\lambda - 1)^6) + n10 \cdot (1 - \lambda)^7$$

$$t = 8$$

$$\#147: nh8 = n0 \cdot \eta \cdot \phi \cdot (\eta - \lambda + 1)^{8-1} + nh7 \cdot (1 - \lambda)$$

$$\#148: nh8 = n0 \cdot \eta \cdot \phi \cdot (\eta^7 + 8 \cdot \eta^6 \cdot (1 - \lambda) + 28 \cdot \eta^5 \cdot (\lambda - 1)^2 + 56 \cdot \eta^4 \cdot (1 - \lambda)^3 + 70 \cdot \eta^3 \cdot (\lambda - 1)^4 + 56 \cdot \eta^2 \cdot (1 - \lambda)^5 + 28 \cdot \eta \cdot (\lambda - 1)^6 - 8 \cdot (\lambda - 1)^7) + nh0 \cdot (\lambda - 1)^8$$

$$\#149: n18 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta - \lambda + 1)^{8-1} + n17 \cdot (1 - \lambda)$$

$$\#150: n18 = n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^7 + 8 \cdot \eta^6 \cdot (1 - \lambda) + 28 \cdot \eta^5 \cdot (\lambda - 1)^2 + 56 \cdot \eta^4 \cdot (1 - \lambda)^3 + 70 \cdot \eta^3 \cdot (\lambda - 1)^4 + 56 \cdot \eta^2 \cdot (1 - \lambda)^5 + 28 \cdot \eta \cdot (\lambda - 1)^6 - 8 \cdot (\lambda - 1)^7) + n10 \cdot (\lambda - 1)^8$$

$$tesing \ nh8 + n18 =$$

$$\#151: n0 \cdot \eta \cdot \phi \cdot (\eta^7 + 8 \cdot \eta^6 \cdot (1 - \lambda) + 28 \cdot \eta^5 \cdot (\lambda - 1)^2 + 56 \cdot \eta^4 \cdot (1 - \lambda)^3 + 70 \cdot \eta^3 \cdot (\lambda - 1)^4 + 56 \cdot \eta^2 \cdot (1 - \lambda)^5 +$$

$$\begin{aligned}
& 28 \cdot \eta \cdot (\lambda - 1)^6 - 8 \cdot (\lambda - 1)^7 + nh0 \cdot (\lambda - 1)^8 + n0 \cdot \eta \cdot (1 - \phi) \cdot (\eta^7 + 8 \cdot \eta^6 \cdot (1 - \lambda) + 28 \cdot \eta^5 \cdot (\lambda - 1)^2 + \\
& 56 \cdot \eta^4 \cdot (1 - \lambda)^3 + 70 \cdot \eta^3 \cdot (\lambda - 1)^4 + 56 \cdot \eta^2 \cdot (1 - \lambda)^5 + 28 \cdot \eta \cdot (\lambda - 1)^6 - 8 \cdot (\lambda - 1)^7 + n10 \cdot (\lambda - 1)^8 \\
\#152: & n0 \cdot \eta \cdot (\eta^7 + 8 \cdot \eta^6 \cdot (1 - \lambda) + 28 \cdot \eta^5 \cdot (\lambda - 1)^2 + 56 \cdot \eta^4 \cdot (1 - \lambda)^3 + 70 \cdot \eta^3 \cdot (\lambda - 1)^4 + 56 \cdot \eta^2 \cdot (1 - \lambda)^5 + \\
& 28 \cdot \eta \cdot (\lambda - 1)^6 - 8 \cdot (\lambda - 1)^7 + nh0 \cdot (\lambda - 1)^8 + n10 \cdot (\lambda - 1)^8 \\
\#153: & (nh0 + n10) \cdot \eta \cdot (\eta^7 + 8 \cdot \eta^6 \cdot (1 - \lambda) + 28 \cdot \eta^5 \cdot (\lambda - 1)^2 + 56 \cdot \eta^4 \cdot (1 - \lambda)^3 + 70 \cdot \eta^3 \cdot (\lambda - 1)^4 + 56 \cdot \eta^2 \cdot (1 - \\
& \lambda)^5 + 28 \cdot \eta \cdot (\lambda - 1)^6 - 8 \cdot (\lambda - 1)^7 + nh0 \cdot (\lambda - 1)^8 + n10 \cdot (\lambda - 1)^8 \\
\#154: & nh0 \cdot (\eta^8 + 8 \cdot \eta^7 \cdot (1 - \lambda) + 28 \cdot \eta^6 \cdot (\lambda - 1)^2 + 56 \cdot \eta^5 \cdot (1 - \lambda)^3 + 70 \cdot \eta^4 \cdot (\lambda - 1)^4 + 56 \cdot \eta^3 \cdot (1 - \lambda)^5 + \\
& 28 \cdot \eta^2 \cdot (\lambda - 1)^6 + 8 \cdot \eta \cdot (1 - \lambda)^7 + (\lambda - 1)^8) + n10 \cdot (\eta^8 + 8 \cdot \eta^7 \cdot (1 - \lambda) + 28 \cdot \eta^6 \cdot (\lambda - 1)^2 + 56 \cdot \eta^5 \cdot (1 - \\
& \lambda)^3 + 70 \cdot \eta^4 \cdot (\lambda - 1)^4 + 56 \cdot \eta^3 \cdot (1 - \lambda)^5 + 28 \cdot \eta^2 \cdot (\lambda - 1)^6 + 8 \cdot \eta \cdot (1 - \lambda)^7 + (\lambda - 1)^8) \\
\#155: & n0 \cdot (\eta^8 + 8 \cdot \eta^7 \cdot (1 - \lambda) + 28 \cdot \eta^6 \cdot (\lambda - 1)^2 + 56 \cdot \eta^5 \cdot (1 - \lambda)^3 + 70 \cdot \eta^4 \cdot (\lambda - 1)^4 + 56 \cdot \eta^3 \cdot (1 - \lambda)^5 + \\
& 28 \cdot \eta^2 \cdot (\lambda - 1)^6 + 8 \cdot \eta \cdot (1 - \lambda)^7 + (\lambda - 1)^8) \\
\#156: & n0 \cdot (\eta - \lambda + 1)^8
\end{aligned}$$

** trying to identify patterns xxx inside the n0 η ϕ (xxx)

t=2

$$\#157: \eta - \lambda + 1$$

t=3

$$\#158: \eta^2 + 3 \cdot \eta \cdot (1 - \lambda) + 3 \cdot (\lambda - 1)^2$$

t=4

$$\#159: \eta^3 + 4 \cdot \eta^2 \cdot (1 - \lambda) + 6 \cdot \eta \cdot (\lambda - 1)^2 - 4 \cdot (\lambda - 1)^3$$

t=5

$$\#160: \eta^4 + 5 \cdot \eta^3 \cdot (1 - \lambda) + 10 \cdot \eta^2 \cdot (\lambda - 1)^2 + 10 \cdot \eta \cdot (1 - \lambda)^3 + 5 \cdot (\lambda - 1)^4$$

t=6

$$\#161: \eta^5 + 6 \cdot \eta^4 \cdot (1 - \lambda) + 15 \cdot \eta^3 \cdot (\lambda - 1)^2 + 20 \cdot \eta^2 \cdot (1 - \lambda)^3 + 15 \cdot \eta \cdot (\lambda - 1)^4 - 6 \cdot (\lambda - 1)^5$$

t=7

$$\#162: \eta^6 + 7 \cdot \eta^5 \cdot (1 - \lambda) + 21 \cdot \eta^4 \cdot (\lambda - 1)^2 + 35 \cdot \eta^3 \cdot (1 - \lambda)^3 + 35 \cdot \eta^2 \cdot (\lambda - 1)^4 + 21 \cdot \eta \cdot (1 - \lambda)^5 + 7 \cdot (\lambda - 1)^6$$

t=8

$$\#163: \eta^7 + 8 \cdot \eta^6 \cdot (1 - \lambda) + 28 \cdot \eta^5 \cdot (\lambda - 1)^2 + 56 \cdot \eta^4 \cdot (1 - \lambda)^3 + 70 \cdot \eta^3 \cdot (\lambda - 1)^4 + 56 \cdot \eta^2 \cdot (1 - \lambda)^5 + 28 \cdot \eta \cdot (\lambda - 1)^6 - 8 \cdot (\lambda - 1)^7$$