tiprise_2024_1_30.dfw

#1: CaseMode := Sensitive

#2: InputMode := Word

#3: $nt :\in \text{Real } (0, \infty)$

#4: $nht1 :\in \text{Real } (0, \infty)$

#5: $nlt :\in \text{Real } (0, \infty)$

#6: $nlt :\in \text{Real } (0, \infty)$

#7: $nht1 :\in \text{Real } (0, \infty)$

#8: $nlt1 :\in \text{Real } (0, \infty)$

eq (1)

#9: $rht = (1 + \mu h) \cdot rt1$

#10: $rlt = (1 - \mu l) \cdot rt1$

eq (2)

#11: $nt = (1 + \eta - \lambda) \cdot nt1$

eq (3) in paper

#12: $nht = nht1 + \eta \cdot \phi \cdot nt1 - \lambda \cdot nht1$

#13: $nht = nht1 + \eta \cdot \phi \cdot (nht1 + nlt1) - \lambda \cdot nht1$

#14: $$nht = nht1 \cdot (\eta \cdot \phi - \lambda + 1) + nlt1 \cdot \eta \cdot \phi$$

#15: $nlt = nlt1 + \eta \cdot (1 - \phi) \cdot nt1 - \lambda \cdot nlt1$

#16: $nlt = nlt1 + \eta \cdot (1 - \phi) \cdot (nht1 + nlt1) - \lambda \cdot nlt1$

#17: $$nlt = nht1\cdot\eta\cdot(1 - \phi) - nlt1\cdot(\eta\cdot(\phi - 1) + \lambda - 1)$$

verify sum up to nt in eq (2)

#18: $$nht + nlt = nht1 + \eta\cdot\phi\cdot nt1 - \lambda\cdot nht1 + nlt1 + \eta\cdot(1 - \phi)\cdot nt1 - \lambda\cdot nlt1$$

#19: $$nht + nlt = nht1\cdot(1 - \lambda) + nlt1\cdot(1 - \lambda) + nt1\cdot\eta$$

eq (4) avg tipping rate at t

#20: $$rt = \frac{nht\cdot rht + nlt\cdot rlt}{nht + nlt}$$

#21: $$rt = \frac{nht\cdot((1 + \mu h)\cdot rt1) + nlt\cdot((1 - \mu l)\cdot rt1)}{nht + nlt}$$

#22: $$rt = \frac{rt1\cdot(nht\cdot(\mu h + 1) + nlt\cdot(1 - \mu l))}{nht + nlt}$$

Result 1 and eq (5)

rt > rt1 if

#23: $$\frac{nht\cdot(\mu h + 1) + nlt\cdot(1 - \mu l)}{nht + nlt} > 1$$

#24: $$\mathrm{SOLVE}\left(\frac{nht\cdot(\mu h + 1) + nlt\cdot(1 - \mu l)}{nht + nlt} > 1, \mu h\right)$$

#25: $$\mathrm{IF}\left(\frac{nht}{nht + nlt} < 0, \mu h < \frac{nlt\cdot\mu l}{nht}\right) \vee \mathrm{IF}\left(\frac{nht}{nht + nlt} > 0, \mu h > \frac{nlt\cdot\mu l}{nht}\right)$$

eq (6)

#26: $\mu h > \frac{nlt \cdot \mu l}{nht}$

*** Section 3: Model calibrations and simulations

eq (7) symmetric μ

#27: $rt = \frac{rt1 \cdot (nht \cdot (\mu + 1) + nlt \cdot (1 - \mu))}{nht + nlt}$

#28: $rt = \frac{rt1 \cdot (\mu \cdot (nht - nlt) + nht + nlt)}{nht + nlt}$

*** Section 4: Utility formulation

#29: $\alpha :\in \text{Real } (0, 1)$

#30: $\beta :\in \text{Real } (0, 1)$

#31: $\gamma :\in \text{Real } (0, \infty)$

#32: $py :\in \text{Real } (0, \infty)$

#33: $y :\in \text{Real } (0, \infty)$

#34: $\tau :\in \text{Real } (0, \infty)$

#35: $\Delta\tau :\in \text{Real } (0, \infty)$

consumer's objective function, eq (9) in paper, formulating utilities in (13)

#36: $u = I - y \cdot py \cdot (1 + \tau + \Delta\tau) + y^{\alpha} + \gamma \cdot (\tau + \Delta\tau)^{\beta}$

Appendix A eq (A.1) and (A.2) FOCs

#37: $$\frac{d}{dy}\left(u = I - y \cdot py \cdot (1 + \tau + \Delta\tau) + y^{\alpha} + \gamma \cdot (\tau + \Delta\tau)^{\beta}\right)$$

#38: $$0 = \alpha \cdot y^{\alpha - 1} - py \cdot (\Delta\tau + \tau + 1)$$

#39: $$\frac{d}{d\ \Delta\tau}\left(u = I - y \cdot py \cdot (1 + \tau + \Delta\tau) + y^{\alpha} + \gamma \cdot (\tau + \Delta\tau)^{\beta}\right)$$

#40: $$0 = \beta \cdot \gamma \cdot (\Delta\tau + \tau)^{\beta - 1} - py \cdot y$$

Appendix A: SOCs

#41: $$\frac{d}{dy}\frac{d}{dy}\left(u = I - y \cdot py \cdot (1 + \tau + \Delta\tau) + y^{\alpha} + \gamma \cdot (\tau + \Delta\tau)^{\beta}\right)$$

#42: $$0 > \alpha \cdot y^{\alpha - 2} \cdot (\alpha - 1)$$

#43: $$\frac{d}{d\ \Delta\tau}\frac{d}{d\ \Delta\tau}\left(u = I - y \cdot py \cdot (1 + \tau + \Delta\tau) + y^{\alpha} + \gamma \cdot (\tau + \Delta\tau)^{\beta}\right)$$

#44: $$0 > \beta \cdot \gamma \cdot (\Delta\tau + \tau)^{\beta - 2} \cdot (\beta - 1)$$

#45: $$\frac{d}{d\ \Delta\tau}\frac{d}{dy}\left(u = I - y \cdot py \cdot (1 + \tau + \Delta\tau) + y^{\alpha} + \gamma \cdot (\tau + \Delta\tau)^{\beta}\right)$$

#46: $$-py$$

#47: $$\frac{d}{dy}\frac{d}{d\ \Delta\tau}\left(u = I - y \cdot py \cdot (1 + \tau + \Delta\tau) + y^{\alpha} + \gamma \cdot (\tau + \Delta\tau)^{\beta}\right)$$

#48: $-py$

#49: $H = (\alpha\cdot y^{\alpha - 2}\cdot(\alpha - 1))\cdot(\beta\cdot\gamma\cdot(\Delta\tau + \tau)^{\beta - 2}\cdot(\beta - 1)) - (-py)^2$

#50: $$H = \alpha\cdot\beta\cdot\gamma\cdot y^{\alpha - 2}\cdot(\Delta\tau + \tau)^{\beta - 2}\cdot(\alpha - 1)\cdot(\beta - 1) - py^2$$

Solving (A.2) for y yields eq (A.4)

#51: $\mathrm{SOLVE}(0 = \beta\cdot\gamma\cdot(\Delta\tau + \tau)^{\beta - 1} - py\cdot y,\ y)$

#52: $$y = \frac{\beta\cdot\gamma\cdot(\Delta\tau + \tau)^{\beta - 1}}{py}$$

Subst y from (A.4) into FOC (A.1) yields

#53: $0 = \alpha\cdot\left(\frac{\beta\cdot\gamma\cdot(\Delta\tau + \tau)^{\beta - 1}}{py}\right)^{\alpha - 1} - py\cdot(\Delta\tau + \tau + 1)$

#54: $\mathrm{SOLVE}\left(0 = \alpha\cdot\left(\frac{\beta\cdot\gamma\cdot(\Delta\tau + \tau)^{\beta - 1}}{py}\right)^{\alpha - 1} - py\cdot(\Delta\tau + \tau + 1),\ \gamma\right)$

eq (14)

#55: $$\gamma = \frac{py^{\alpha/(\alpha - 1)}\cdot(\Delta\tau + \tau)^{1 - \beta}\cdot\left(\frac{\Delta\tau + \tau + 1}{\alpha}\right)^{1/(\alpha - 1)}}{\beta}$$

** Subection 5.1 Variety of tipped services [no complex derivations, just simulations in R]

** Section 5.2 Variety in the utility function

start deriving eq (17) utility function

#56: y

#57: $\int_0^{shat + \Delta s} yst \cdot py \cdot (1 + \tau)\, ds$

#58: $\int_{shat + \Delta s}^{1} ysu \cdot py\, ds$

#59: $\int_0^{shat + \Delta s} \frac{yst^{\alpha}}{\alpha}\, ds$

#60: $\int_{shat + \Delta s}^{1} \frac{ysu^{\alpha}}{\alpha}\, ds$

#61: $u = I - \int_0^{shat + \Delta s} yst \cdot py \cdot (1 + \tau)\, ds - \int_{shat + \Delta s}^{1} ysu \cdot py\, ds + \int_0^{shat + \Delta s} \frac{yst^{\alpha}}{\alpha}\, ds + \int_{shat + \Delta s}^{1}$

$$\frac{ysu^{\alpha}}{\alpha}\ ds$$

#62: $$u = I - py \cdot yst \cdot (shat + \Delta s) \cdot (\tau + 1) - \int_{shat + \Delta s}^{1} ysu \cdot py\ ds + \int_{0}^{shat + \Delta s} \frac{yst^{\alpha}}{\alpha}\ ds + \int_{shat + \Delta s}^{1} \frac{ysu^{\alpha}}{\alpha}\ ds$$

#63: $$u = I - \int_{0}^{shat + \Delta s} yst \cdot py \cdot (1 + \tau)\ ds - - py \cdot ysu \cdot (shat + \Delta s - 1) + \int_{0}^{shat + \Delta s} \frac{yst^{\alpha}}{\alpha}\ ds + \int_{shat + \Delta s}^{1} \frac{ysu^{\alpha}}{\alpha}\ ds$$

$$\#64:\quad u = I - \int_0^{shat + \Delta s} yst \cdot py \cdot (1 + \tau)\, ds - - py \cdot ysu \cdot (shat + \Delta s - 1) + \frac{yst^{\alpha} \cdot (shat + \Delta s)}{\alpha} + \int_{shat + \Delta s}^{1} \frac{ysu^{\alpha}}{\alpha}\, ds$$

$$\#65:\quad u = I - \int_0^{shat + \Delta s} yst \cdot py \cdot (1 + \tau)\, ds - - py \cdot ysu \cdot (shat + \Delta s - 1) + \frac{yst^{\alpha} \cdot (shat + \Delta s)}{\alpha} + - \frac{ysu^{\alpha} \cdot (shat + \Delta s - 1)}{\alpha}$$

eq (17)

$$\#66:\quad u = I - py \cdot yst \cdot (shat + \Delta s) \cdot (\tau + 1) - - py \cdot ysu \cdot (shat + \Delta s - 1) + \frac{yst^{\alpha} \cdot (shat + \Delta s)}{\alpha} + - \frac{ysu^{\alpha} \cdot (shat + \Delta s - 1)}{\alpha} + \frac{\gamma \cdot (shat + \Delta s)^{\beta}}{\beta}$$

Appendix B FOC

eq (B.1)

#67: $$\frac{d}{d\ yst}\left(u = I - py\cdot yst\cdot(shat + \Delta s)\cdot(\tau + 1) - - py\cdot ysu\cdot(shat + \Delta s - 1) + \frac{yst^{\alpha}\cdot(shat + \Delta s)}{\alpha} + - \frac{ysu^{\alpha}\cdot(shat + \Delta s - 1)}{\alpha} + \frac{\gamma\cdot(shat + \Delta s)^{\beta}}{\beta}\right)$$

#68: $$0 = yst^{\alpha - 1}\cdot(shat + \Delta s) - py\cdot(shat + \Delta s)\cdot(\tau + 1)$$

eq (B.2)

#69: $$\frac{d}{d\ ysu}\left(u = I - py\cdot yst\cdot(shat + \Delta s)\cdot(\tau + 1) - - py\cdot ysu\cdot(shat + \Delta s - 1) + \frac{yst^{\alpha}\cdot(shat + \Delta s)}{\alpha} + - \frac{ysu^{\alpha}\cdot(shat + \Delta s - 1)}{\alpha} + \frac{\gamma\cdot(shat + \Delta s)^{\beta}}{\beta}\right)$$

#70: $$0 = py\cdot(shat + \Delta s - 1) - ysu^{\alpha - 1}\cdot(shat + \Delta s - 1)$$

eq (B.3)

#71: $$\frac{d}{d\ shat}\left(u = I - py\cdot yst\cdot(shat + \Delta s)\cdot(\tau + 1) - - py\cdot ysu\cdot(shat + \Delta s - 1) + \frac{yst^{\alpha}\cdot(shat + \Delta s)}{\alpha} + -\right.$$

$$\frac{ysu^{\alpha}\cdot(shat + \Delta s - 1)}{\alpha} + \frac{\gamma\cdot(shat + \Delta s)^{\beta}}{\beta}\Bigg)$$

#72: $$0 = \gamma\cdot(shat + \Delta s)^{\beta - 1} + \frac{yst^{\alpha}}{\alpha} - \frac{ysu^{\alpha}}{\alpha} + py\cdot(ysu - yst\cdot(\tau + 1))$$

Deriving (A.4) from (B.1) and (B.2_

#73: $SOLVE(0 = yst^{\alpha - 1}\cdot(shat + \Delta s) - py\cdot(shat + \Delta s)\cdot(\tau + 1), yst)$

#74: $$yst = (py\cdot(\tau + 1))^{1/(\alpha - 1)}$$

#75: $SOLVE(0 = py\cdot(shat + \Delta s - 1) - ysu^{\alpha - 1}\cdot(shat + \Delta s - 1), ysu)$

#76: $$ysu = py^{1/(\alpha - 1)}$$

subs into (B.3) to obtain eq (22)

#77: $$0 = \gamma\cdot(shat + \Delta s)^{\beta - 1} + \frac{((py\cdot(\tau + 1))^{1/(\alpha - 1)})^{\alpha}}{\alpha} - \frac{(py^{1/(\alpha - 1)})^{\alpha}}{\alpha} + py\cdot(py^{1/(\alpha - 1)} - (py\cdot(\tau + 1))^{1/(\alpha - 1)}\cdot(\tau + 1))$$

#78: $$\text{SOLVE}\left(0 = \gamma\cdot(\text{shat} + \Delta s)^{\beta - 1} + \frac{((py\cdot(\tau + 1))^{1/(\alpha - 1)})^{\alpha}}{\alpha} - \frac{(py^{1/(\alpha - 1)})^{\alpha}}{\alpha} + py\cdot(py^{1/(\alpha - 1)} - (py\cdot(\tau + 1))^{1/(\alpha - 1)}\cdot(\tau + 1)), \gamma\right)$$

eq (22) => simulations in Figure 5

#79: $$\gamma = \frac{py^{\alpha/(\alpha - 1)}\cdot(\text{shat} + \Delta s)^{1 - \beta}\cdot(\alpha - 1)\cdot((\tau + 1)^{\alpha/(\alpha - 1)} - 1)}{\alpha}$$

#80: $$\text{SOLVE}\left(0 = \gamma\cdot(\text{shat} + \Delta s)^{\beta - 1} + \frac{((py\cdot(\tau + 1))^{1/(\alpha - 1)})^{\alpha}}{\alpha} - \frac{(py^{1/(\alpha - 1)})^{\alpha}}{\alpha} + py\cdot(py^{1/(\alpha - 1)} - (py\cdot(\tau + 1))^{1/(\alpha - 1)}\cdot(\tau + 1)), \Delta s\right)$$

#81: $$\Delta s = \text{IF}\left((\tau + 1)^{\alpha/(\alpha - 1)} < 1, py^{\alpha/((\alpha - 1)\cdot(\beta - 1))}\cdot\left(\frac{(\alpha - 1)\cdot((\tau + 1)^{\alpha/(\alpha - 1)} - 1)}{\alpha\cdot\gamma}\right)^{1/(\beta - 1)} - \text{shat}\right)$$

I use this in (22) i.e. solving directly for Δs as a function of γ

#82: $$\Delta s = py^{\alpha/((\alpha - 1)\cdot(\beta - 1))}\cdot\left(\frac{(\alpha - 1)\cdot((\tau + 1)^{\alpha/(\alpha - 1)} - 1)}{\alpha\cdot\gamma}\right)^{1/(\beta - 1)} - shat$$