vat_compete_2024_x_y.dfw

#1: CaseMode :≡ Sensitive

#2: InputMode :≡ Word

degree of market power (transp cost)

#3: $\mu :\in \text{Real } (0, \infty)$

rate of sales tax

#4: $\tau :\in \text{Real } (0, \infty)$

total consumer population

#5: $n :\in \text{Real } (0, \infty)$

Unit costs

#6: $ca :\in \text{Real } [0, \infty)$

#7: $cb :\in \text{Real } [0, \infty)$

prices

#8: $pa :\in \text{Real } (0, \infty)$

#9: $pb :\in \text{Real } (0, \infty)$

#10: $qa :\in \text{Real } (0, \infty)$

#11: $qb :\in \text{Real } (0, \infty)$

A's market share

#12: $xhat :\in \text{Real } (0, 1)$

basic valuations

#13: $va :\in \text{Real } (0, \infty)$

#14: $vb :\in \text{Real } (0, \infty)$

#15: $\Delta v :\in \text{Real } [0, \infty)$

*** Section 2: Price embedded into the price (benchmark model)

eq (1)

#16: $qa = pa\cdot(1 + \tau)$

#17: $qb = pb\cdot(1 + \tau)$

#18: $\text{SOLVE}(qa = pa\cdot(1 + \tau), pa)$

#19: $$pa = \frac{qa}{\tau + 1}$$

#20: $\text{SOLVE}(qb = pb\cdot(1 + \tau), pb)$

#21: $$pb = \frac{qb}{\tau + 1}$$

eq (2) Utility functions

#22: $va - pa\cdot(1 + \tau) - \mu\cdot x$

#23: $va - qa - \mu\cdot x$

#24: $vb - pb\cdot(1 + \tau) - \mu\cdot(1 - x)$

#25: $vb - qb - \mu\cdot(1 - x)$

#26: $va - pa\cdot(1 + \tau) - \mu\cdot x = vb - pb\cdot(1 + \tau) - \mu\cdot(1 - x)$

#27: $\text{SOLVE}(va - pa\cdot(1 + \tau) - \mu\cdot x = vb - pb\cdot(1 + \tau) - \mu\cdot(1 - x), x)$

eq (3)

#28: $$xhat = - \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - va + vb - \mu}{2\cdot\mu}$$

#29: $$xhat = - \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}$$

#30: $$xhat = - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}$$

eq (4) Profit max w.r.t. qa and qb (tax inclusive)

#31: $profita = (pa - ca)\cdot n\cdot xhat$

#32: $profitb = (pb - cb)\cdot n\cdot (1 - xhat)$

#33: $$profita = \left(\frac{qa}{\tau + 1} - ca\right)\cdot n\cdot\left(- \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)$$

#34: $$profitb = \left(\frac{qb}{\tau + 1} - cb\right)\cdot n\cdot\left(1 - - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)$$

Appendix A. eq (A.1)

#35: $$\frac{d}{d\ qa}\left(profita = \left(\frac{qa}{\tau + 1} - ca\right)\cdot n\cdot\left(- \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#36: $$0 = \frac{n\cdot(ca\cdot(\tau + 1) - 2\cdot qa + qb + \Delta v + \mu)}{2\cdot\mu\cdot(\tau + 1)}$$

#37: $$\frac{d}{d\ qa}\frac{d}{d\ qa}\left(profita = \left(\frac{qa}{\tau + 1} - ca\right)\cdot n\cdot\left(- \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#38: $$0 > - \frac{n}{\mu\cdot(\tau + 1)}$$

#39: $$\frac{d}{d\ qb}\left(profitb = \left(\frac{qb}{\tau + 1} - cb\right)\cdot n\cdot\left(1 - - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#40: $$0 = \frac{n\cdot(cb\cdot(\tau + 1) + qa - 2\cdot qb - \Delta v + \mu)}{2\cdot\mu\cdot(\tau + 1)}$$

#41: $$\frac{d}{d\ qb}\frac{d}{d\ qb}\left(profitb = \left(\frac{qb}{\tau + 1} - cb\right)\cdot n\cdot\left(1 - - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#42: $$0 > - \frac{n}{\mu\cdot(\tau + 1)}$$

eq (5)

#43: $$SOLVE\left(\left[0 = \frac{n\cdot(ca\cdot(\tau + 1) - 2\cdot qa + qb + \Delta v + \mu)}{2\cdot\mu\cdot(\tau + 1)},\ 0 = \frac{n\cdot(cb\cdot(\tau + 1) + qa - 2\cdot qb - \Delta v + \mu)}{2\cdot\mu\cdot(\tau + 1)}\right], [qa, qb]\right)$$

#44: $$\left[qaI = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3} \wedge qbI = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3}\right]$$

#45: $$\left[pa\cdot(1 + \tau) = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3} \wedge pb\cdot(1 + \tau) = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3}\right]$$

#46: $$\mathrm{SOLVE}\left(\left[pa\cdot(1 + \tau) = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3} \wedge pb\cdot(1 + \tau) = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3}\right], [pa, pb]\right)$$

#47: $$\left[paI = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)} \wedge pbI = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}\right]$$

Define $\Delta c = ca - cb$

eq (6)

#48: $$xhatI = - \frac{\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu}{6\cdot\mu}$$

#49: $$profitaI = \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

#50: $$profitbI = \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v + 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

Result 1a

#51: $$\frac{d}{d\tau}\left(qaI = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3}\right)$$

#52: $$0 < \frac{2\cdot ca + cb}{3}$$

#53: $$\frac{d}{d\tau}\left(qbI = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3}\right)$$

#54: $$0 < \frac{ca + 2\cdot cb}{3}$$

#55: $$\frac{d}{d\tau}\left(paI = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}\right)$$

#56: $$0 > -\frac{\Delta v + 3\cdot\mu}{3\cdot(\tau + 1)^2}$$

#57: $$\frac{d}{d\tau}\left(pbI = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}\right)$$

#58: $$0 > \frac{\Delta v - 3\cdot\mu}{3\cdot(\tau + 1)^2}$$

by Assumption 2.

restriction xhat > 0 if

#59: $-\dfrac{\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu}{6\cdot\mu} > 0$

#60: $\text{SOLVE}\left(-\dfrac{\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu}{6\cdot\mu} > 0,\ \mu\right)$

#61: $\left(\mu < \dfrac{\Delta c\cdot(\tau + 1) - \Delta v}{3} \wedge \mu < 0\right) \vee \left(\mu > \dfrac{\Delta c\cdot(\tau + 1) - \Delta v}{3} \wedge \mu > 0\right)$

#62: $\mu > \dfrac{\Delta c\cdot(\tau + 1) - \Delta v}{3}$

restriction xhat < 1 if

#63: $-\dfrac{\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu}{6\cdot\mu} < 1$

#64: $\text{SOLVE}\left(-\dfrac{\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu}{6\cdot\mu} < 1,\ \mu\right)$

#65: $\left(\mu < \dfrac{\Delta v - \Delta c\cdot(\tau + 1)}{3} \wedge \mu < 0\right) \vee \left(\mu > \dfrac{\Delta v - \Delta c\cdot(\tau + 1)}{3} \wedge \mu > 0\right)$

#66: $\mu > \dfrac{\Delta v - \Delta c\cdot(\tau + 1)}{3}$

xhatI > 0 if

#67: $\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu < 0$

#68: $\text{SOLVE}(\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu < 0,\ \mu)$

#69: $$\mu > \frac{\Delta c \cdot (\tau + 1) - \Delta v}{3}$$

*** Section 3: Price competition with sales tax separated from price
** Subsection 3.1: Fast-computing consumers: An equivalence result

eq (7)

#70: $\text{profita} = (\text{pa} - \text{ca}) \cdot n \cdot \text{xhat}$

#71: $\text{profitb} = (\text{pb} - \text{cb}) \cdot n \cdot (1 - \text{xhat})$

#72: $$\text{profita} = (\text{pa} - \text{ca}) \cdot n \cdot \left(- \frac{\text{pa} \cdot (\tau + 1) - \text{pb} \cdot (\tau + 1) - \text{va} + \text{vb} - \mu}{2 \cdot \mu} \right)$$

#73: $$\text{profitb} = (\text{pb} - \text{cb}) \cdot n \cdot \left(1 - - \frac{\text{pa} \cdot (\tau + 1) - \text{pb} \cdot (\tau + 1) - \text{va} + \text{vb} - \mu}{2 \cdot \mu} \right)$$

Appendix B, eq (B.1), and Result 2

#74: $$\frac{d}{d\ \text{pa}} \left(\text{profita} = (\text{pa} - \text{ca}) \cdot n \cdot \left(- \frac{\text{pa} \cdot (\tau + 1) - \text{pb} \cdot (\tau + 1) - \text{va} + \text{vb} - \mu}{2 \cdot \mu} \right) \right)$$

#75: $$0 = \frac{n \cdot (\text{ca} \cdot (\tau + 1) - 2 \cdot \text{pa} \cdot (\tau + 1) + \text{pb} \cdot (\tau + 1) + \text{va} - \text{vb} + \mu)}{2 \cdot \mu}$$

#76: $$\frac{d}{d\ \text{pa}} \left(0 = \frac{n \cdot (\text{ca} \cdot (\tau + 1) - 2 \cdot \text{pa} \cdot (\tau + 1) + \text{pb} \cdot (\tau + 1) + \text{va} - \text{vb} + \mu)}{2 \cdot \mu} \right)$$

#77: $$0 > - \frac{n \cdot (\tau + 1)}{\mu}$$

#78: $$\frac{d}{d\ pb}\left(profitb = (pb - cb)\cdot n\cdot\left(1 - -\frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - va + vb - \mu}{2\cdot\mu}\right)\right)$$

#79: $$0 = \frac{n\cdot(cb\cdot(\tau + 1) + pa\cdot(\tau + 1) - 2\cdot pb\cdot(\tau + 1) - va + vb + \mu)}{2\cdot\mu}$$

#80: $$\frac{d}{d\ pb}\frac{d}{d\ pb}\left(profitb = (pb - cb)\cdot n\cdot\left(1 - -\frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - va + vb - \mu}{2\cdot\mu}\right)\right)$$

#81: $$0 > -\frac{n\cdot(\tau + 1)}{\mu}$$

#82: $$\mathrm{SOLVE}\left(\left[0 = \frac{n\cdot(ca\cdot(\tau + 1) - 2\cdot pa\cdot(\tau + 1) + pb\cdot(\tau + 1) + va - vb + \mu)}{2\cdot\mu}, 0 = \frac{n\cdot(cb\cdot(\tau + 1) + pa\cdot(\tau + 1) - 2\cdot pb\cdot(\tau + 1) - va + vb + \mu)}{2\cdot\mu}\right], [pa, pb]\right)$$

#83: $$\left[pa = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + va - vb + 3\cdot\mu}{3\cdot(\tau + 1)} \wedge pb = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - va + vb + 3\cdot\mu}{3\cdot(\tau + 1)}\right]$$

#84: $$\left[pa = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)} \wedge pb = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}\right]$$

compare with paI

#85: $$\frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)} - \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#86: $$0$$

Compare with pbI

#87: $$\frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)} - \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#88: $$0$$

** Subsection3.2: Slow-computing consumers (nonequivalence result)

eq (8)

#89: $va - pa - vata - \mu\cdot x$

#90: $vb - pb - vatb - \mu\cdot(1 - x)$

#91: $va - pa - vata - \mu\cdot xhat = vb - pb - vatb - \mu\cdot(1 - xhat)$

eq (9)

#92: SOLVE($va - pa - vata - \mu\cdot xhat = vb - pb - vatb - \mu\cdot(1 - xhat)$, xhat)

#93: $$xhat = - \frac{pa - pb - va + vata - vatb + vb - \mu}{2\cdot\mu}$$

#94: $$xhat = - \frac{pa - pb - \Delta v + vata - vatb - \mu}{2\cdot\mu}$$

eq (10)

#95: $profita = (pa - ca)\cdot n\cdot xhat$

#96: $\text{profitb} = (\text{pb} - \text{cb})\cdot n\cdot(1 - \text{xhat})$

#97: $$\text{profita} = (\text{pa} - \text{ca})\cdot n\cdot\left(- \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)$$

#98: $$\text{profitb} = (\text{pb} - \text{cb})\cdot n\cdot\left(1 - - \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)$$

Appendix C

#99: $$\frac{d}{d\ \text{pa}}\left(\text{profita} = (\text{pa} - \text{ca})\cdot n\cdot\left(- \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#100: $$0 = \frac{n\cdot(\text{ca} - 2\cdot\text{pa} + \text{pb} - \text{vata} + \text{vatb} + \Delta v + \mu)}{2\cdot\mu}$$

#101: $$\frac{d}{d\ \text{pa}}\frac{d}{d\ \text{pa}}\left(\text{profita} = (\text{pa} - \text{ca})\cdot n\cdot\left(- \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#102: $$0 > - \frac{n}{\mu}$$

#103: $$\frac{d}{d\ \text{pb}}\left(\text{profitb} = (\text{pb} - \text{cb})\cdot n\cdot\left(1 - - \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#104: $$0 = \frac{n\cdot(\text{cb} + \text{pa} - 2\cdot\text{pb} + \text{vata} - \text{vatb} - \Delta v + \mu)}{2\cdot\mu}$$

#105: $$\frac{d}{d\ \text{pb}}\frac{d}{d\ \text{pb}}\left(\text{profitb} = (\text{pb} - \text{cb})\cdot n\cdot\left(1 - - \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#106: $$0 > - \frac{n}{\mu}$$

#107: $$\mathrm{SOLVE}\left(\left[0 = \frac{n\cdot(ca - 2\cdot pa + pb - vata + vatb + \Delta v + \mu)}{2\cdot\mu}, 0 = \frac{n\cdot(cb + pa - 2\cdot pb + vata - vatb - \Delta v + \mu)}{2\cdot\mu}\right], [pa, pb]\right)$$

eq (11)

#108: $$\left[pa = \frac{2\cdot ca + cb - vata + vatb + \Delta v + 3\cdot\mu}{3} \wedge pb = \frac{ca + 2\cdot cb + vata - vatb - \Delta v + 3\cdot\mu}{3}\right]$$

eq (12)

#109: $vata = \tau\cdot pa$

#110: $vatb = \tau\cdot pb$

#111: $$\left[pa = \frac{2\cdot ca + cb - \tau\cdot pa + \tau\cdot pb + \Delta v + 3\cdot\mu}{3} \wedge pb = \frac{ca + 2\cdot cb + \tau\cdot pa - \tau\cdot pb - \Delta v + 3\cdot\mu}{3}\right]$$

#112: $$[\,]$$

#113: $$\mathrm{SOLVE}\left(\left[pa = \frac{2\cdot ca + cb - \tau\cdot pa + \tau\cdot pb + \Delta v + 3\cdot\mu}{3}, pb = \frac{ca + 2\cdot cb + \tau\cdot pa - \tau\cdot pb - \Delta v + 3\cdot\mu}{3}\right], [pa, pb]\right)$$

#114: $$\left[paII = \frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3} \wedge pbII = \frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}\right]$$

#115: $$qaII = (1 + \tau)\cdot\frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}$$

#116: $$qbII = (1 + \tau)\cdot\frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}$$

Result 3a Appendix C, eq (C.2)

#117: $$\frac{d}{d\tau}\left(qaII = (1 + \tau)\cdot\frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}\right)$$

#118: $$0 < \frac{ca\cdot(2\cdot\tau^2 + 6\cdot\tau + 5) + 2\cdot cb\cdot(\tau^2 + 3\cdot\tau + 2) + \Delta v + \mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9)}{(2\cdot\tau + 3)^2}$$

#119: $$\frac{d}{d\tau}\left(qbII = (1 + \tau)\cdot\frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}\right)$$

#120: $$0 < \frac{2\cdot ca\cdot(\tau^2 + 3\cdot\tau + 2) + cb\cdot(2\cdot\tau^2 + 6\cdot\tau + 5) - \Delta v + \mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9)}{(2\cdot\tau + 3)^2}$$

by Assumption 2.

Result 3b Appendix C, eq (C.3)

#121: $$\frac{d}{d\tau}\left(paII = \frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}\right)$$

#122: $$0 > - \frac{ca - cb + 2\cdot\Delta v}{(2\cdot\tau + 3)^2}$$

if ca ≥ cb (Assumption 1b).

#123: $$\frac{d}{d\tau}\left(pbII = \frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}\right)$$

#124: $$0 < \frac{ca - cb + 2\cdot\Delta v}{(2\cdot\tau + 3)^2}$$

since (Assumption 1b) ca ≥ cb

eq (13)

#125: $vataII = \tau\cdot paII$

#126: $$vataII = \tau\cdot\frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}$$

#127: $vatbII = \tau\cdot pbII$

#128: $$vatbII = \tau\cdot\frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}$$

#129: $$\text{xhat} = - \frac{\text{pa} - \text{pb} - \Delta v + \tau\cdot\text{paII} - \tau\cdot\text{pbII} - \mu}{2\cdot\mu}$$

#130: $$\text{xhatII} = - \frac{\Delta c\cdot(\tau + 1) - \Delta v - \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)}$$

#131: $$\text{profitaII} = \frac{n\cdot(\text{ca}\cdot(\tau + 1) - \text{cb}\cdot(\tau + 1) - \Delta v - \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

#132: $$\text{profitbII} = \frac{n\cdot(\text{ca}\cdot(\tau + 1) - \text{cb}\cdot(\tau + 1) - \Delta v + \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

#133: $$\text{profitaII} = \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v - \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

#134: $$\text{profitbII} = \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v + \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

*** Section 4: Cmparing market outcomes under the two pricing structures

eq (14) and Result 4

paII − paI =

#135: $$\frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3} - \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#136: $$-\frac{\tau\cdot(ca\cdot(\tau + 1) - cb\cdot(\tau + 1) - \Delta v - 3\cdot\mu\cdot(2\cdot\tau + 3))}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)}$$

#137: $$-\frac{\tau\cdot(\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu\cdot(2\cdot\tau + 3))}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)} > 0$$

if Δv > 2 Δc [Assumption 1c]

pbII - pbI =

#138: $$\frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3} - \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#139: $$\frac{\tau\cdot(ca\cdot(\tau + 1) - cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu\cdot(2\cdot\tau + 3))}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)} > 0$$

#140: $$\frac{\tau\cdot(\Delta c\cdot(\tau + 1) - \Delta v + 3\cdot\mu\cdot(2\cdot\tau + 3))}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)} > 0$$

by assumption 2

Result 4c

#141: $$profitaII = \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v - \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

$$\#142:\ \text{profitbII} = \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v + \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

$$\#143:\ \text{profitaI} = \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

$$\#144:\ \text{profitbI} = \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v + 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

$$\#145:\ \text{profitaII} - \text{profitaI} = \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v - \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2} - \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

$$\#146:\ \text{profitaII} - \text{profitaI} = -$$

$$\frac{n\cdot\tau\cdot(\Delta c^2\cdot(\tau + 1)^2\cdot(4\cdot\tau + 3) + 2\cdot\Delta c\cdot(\tau + 1)\cdot(3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v\cdot(4\cdot\tau + 3)) + \Delta v^2\cdot(4\cdot\tau + 3) - 6\cdot\Delta\sim}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}\sim$$

$$v\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^{2}\cdot(2\cdot\tau + 3)^{2})$$

try Δv=Δc=0

#147:

$$\frac{n\cdot\mu\cdot\tau}{2\cdot(\tau + 1)} > 0$$

#148:

$$\text{profitbII} - \text{profitbI} = \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v + \mu\cdot(2\cdot\tau + 3))^{2}}{2\cdot\mu\cdot(2\cdot\tau + 3)^{2}} - \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v + 3\cdot\mu)^{2}}{18\cdot\mu\cdot(\tau + 1)}$$

#149: profitbII − profitbI = −

$$\frac{n\cdot\tau\cdot(\Delta c^{2}\cdot(\tau + 1)^{2}\cdot(4\cdot\tau + 3) - 2\cdot\Delta c\cdot(\tau + 1)\cdot(\Delta v\cdot(4\cdot\tau + 3) + 3\cdot\mu\cdot(2\cdot\tau + 3)) + \Delta v^{2}\cdot(4\cdot\tau + 3) + 6\cdot\Delta\sim}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^{2}}\sim$$

$$v\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^{2}\cdot(2\cdot\tau + 3)^{2})$$

try Δv=Δc=0

#150: $$\text{profitbII} - \text{profitbI} = \frac{n\cdot\mu\cdot\tau}{2\cdot(\tau + 1)} > 0$$

compare total profit for any Δv and Δc subject to Assumption 1c => no nice results. Ignore.

#151: $$\text{totalprofitII} - \text{totalprofitI} = \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v - \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2} + \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v + \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2} - \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)} - \frac{n\cdot(\Delta c\cdot(\tau + 1) - \Delta v + 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

#152: $$\text{totalprofitII} - \text{totalprofitI} = - \frac{n\cdot\tau\cdot(\Delta c^2\cdot(\tau + 1)^2\cdot(4\cdot\tau + 3) - 2\cdot\Delta c\cdot\Delta v\cdot(\tau + 1)\cdot(4\cdot\tau + 3) + \Delta v^2\cdot(4\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{9\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

> 0 if

#153: $\Delta c^2\cdot(\tau + 1)^2\cdot(4\cdot\tau + 3) - 2\cdot\Delta c\cdot\Delta v\cdot(\tau + 1)\cdot(4\cdot\tau + 3) + \Delta v^2\cdot(4\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2 > 0$

#154: $\text{SOLVE}(\Delta c^2\cdot(\tau + 1)^2\cdot(4\cdot\tau + 3) - 2\cdot\Delta c\cdot\Delta v\cdot(\tau + 1)\cdot(4\cdot\tau + 3) + \Delta v^2\cdot(4\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2 > 0, \mu)$

#155: $$\left(\left(\mu < \frac{\sqrt{(4\cdot\tau + 3)}\cdot(\Delta v - \Delta c\cdot(\tau + 1))}{3\cdot(2\cdot\tau + 3)} \wedge \Delta c\cdot(\tau + 1) - \Delta v \le 0 \wedge \mu > \frac{\sqrt{(4\cdot\tau + 3)}\cdot(\Delta c\cdot(\tau + 1) - \Delta v)}{3\cdot(2\cdot\tau + 3)}\right) \vee \left(\mu < \frac{\sqrt{(4\cdot\tau + 3)}\cdot(\Delta c\cdot(\tau + 1) - \Delta v)}{3\cdot(2\cdot\tau + 3)} \wedge \mu > \frac{\sqrt{(4\cdot\tau + 3)}\cdot(\Delta v - \Delta c\cdot(\tau + 1))}{3\cdot(2\cdot\tau + 3)} \wedge \Delta c\cdot(\tau + 1) - \Delta v \ge 0\right)\right) \wedge \Delta c^2\cdot(\tau + 1)^2 - 2\cdot\Delta c\cdot\Delta v\cdot(\tau + 1) + \Delta v^2 > 0$$

*** Section 5: 2-stage with (no) regret

eq (15): First-stage utility

#156: $ua1 = va - pa - \mu\cdot x$

#157: $ub1 = vb - pb - \mu\cdot(1 - x)$

eq (16): Second-stage utility

#158: $uaa = va - qa$

#159: $uab = vb - qb - \mu\cdot 1$

#160: $ubb = vb - qb$

#161: $uba = va - qa - \mu\cdot 1$

eq (17): First stage xhat

#162: $va - pa - \mu\cdot x = vb - pb - \mu\cdot(1 - x)$

#163: SOLVE($va - pa - \mu\cdot x = vb - pb - \mu\cdot(1 - x)$, x)

#164: $$xhat1 = - \frac{pa - pb - va + vb - \mu}{2\cdot\mu}$$

#165: $xhat1 = - \dfrac{pa - pb - \Delta v - \mu}{2\cdot\mu}$

eq (18) copy from (11) special case with vata = vatb =0

#166: $\left[pa = \dfrac{2\cdot ca + cb - 0 + 0 + \Delta v + 3\cdot\mu}{3} \wedge pb = \dfrac{ca + 2\cdot cb + 0 - 0 - \Delta v + 3\cdot\mu}{3}\right]$

#167: $$\left[paIII = \frac{2\cdot ca + cb + \Delta v + 3\cdot\mu}{3} \wedge pbIII = \frac{ca + 2\cdot cb - \Delta v + 3\cdot\mu}{3}\right]$$

Result 5: proof and Appendix D

#168: $qbIII = (1 + \tau)\cdot\dfrac{ca + 2\cdot cb - \Delta v + 3\cdot\mu}{3}$

#169: $qaIII = (1 + \tau)\cdot\dfrac{2\cdot ca + cb + \Delta v + 3\cdot\mu}{3}$

eq (D.1) uaa $\geq$ uab if

#170: $va - \dfrac{(1 + \tau)\cdot(2\cdot ca + cb + \Delta v + 3\cdot\mu)}{3} \geq vb - \dfrac{(1 + \tau)\cdot(ca + 2\cdot cb - \Delta v + 3\cdot\mu)}{3} - \mu\cdot 1$

#171: $\text{SOLVE}\left(va - \frac{(1 + \tau)\cdot(2\cdot ca + cb + \Delta v + 3\cdot\mu)}{3} \ge vb - \frac{(1 + \tau)\cdot(ca + 2\cdot cb - \Delta v + 3\cdot\mu)}{3} - \mu\cdot 1, \mu\right)$

#172: $$\mu \ge \frac{ca\cdot(\tau + 1) - cb\cdot(\tau + 1) - 3\cdot va + 3\cdot vb + 2\cdot\Delta v\cdot(\tau + 1)}{3}$$

#173: $\mu \ge \frac{\Delta c\cdot(\tau + 1) - 3\cdot\Delta v + 2\cdot\Delta v\cdot(\tau + 1)}{3}$

eq (D.2) if

#174: $$\mu \ge \frac{\Delta c\cdot(\tau + 1) + \Delta v\cdot(2\cdot\tau - 1)}{3}$$

now by Assumption 2: $\mu > \Delta v$, so it is sufficient to show

eq (D.3)

#175: $\Delta v - \frac{\Delta c\cdot(\tau + 1) + \Delta v\cdot(2\cdot\tau - 1)}{3}$

#176: $$- \frac{\Delta c\cdot(\tau + 1) + 2\cdot\Delta v\cdot(\tau - 2)}{3}$$

> 0 if

#177: $\Delta c\cdot(\tau + 1) + 2\cdot\Delta v\cdot(\tau - 2) < 0$

#178: $\text{SOLVE}(\Delta c\cdot(\tau + 1) + 2\cdot\Delta v\cdot(\tau - 2) < 0, \Delta v)$

#179: $$\text{IF}\left(\tau < 2, \Delta v > \frac{\Delta c\cdot(\tau + 1)}{2\cdot(2 - \tau)}\right) \vee \text{IF}\left(\tau > 2, \Delta v < \frac{\Delta c\cdot(\tau + 1)}{2\cdot(2 - \tau)}\right)$$

$$\#180:\ \Delta v > \frac{\Delta c\cdot(\tau + 1)}{2\cdot(2 - \tau)}$$

holds since $\tau < 1$ and Assumption 1c in which $\Delta v > \Delta c\ (1+\tau)$.

eq (D.4) ubb $\geq$ uba if

$$\#181:\ vb - (1 + \tau)\cdot\frac{ca + 2\cdot cb - \Delta v + 3\cdot\mu}{3} \geq va - (1 + \tau)\cdot\frac{2\cdot ca + cb + \Delta v + 3\cdot\mu}{3} - \mu\cdot 1$$

eq (D.5) if

$$\#182:\ \mathrm{SOLVE}\left(vb - (1 + \tau)\cdot\frac{ca + 2\cdot cb - \Delta v + 3\cdot\mu}{3} \geq va - (1 + \tau)\cdot\frac{2\cdot ca + cb + \Delta v + 3\cdot\mu}{3} - \mu\cdot 1,\ \mu\right)$$

$$\#183:\quad \mu \geq - \frac{ca\cdot(\tau + 1) - cb\cdot(\tau + 1) - 3\cdot va + 3\cdot vb + 2\cdot\Delta v\cdot(\tau + 1)}{3}$$

$$\#184:\ \mu \geq - \frac{\Delta c\cdot(\tau + 1) - 3\cdot\Delta v + 2\cdot\Delta v\cdot(\tau + 1)}{3}$$

$$\#185:\quad \mu \geq - \frac{\Delta c\cdot(\tau + 1) + \Delta v\cdot(2\cdot\tau - 1)}{3}$$

by Assumption 2, $\mu > \Delta v$, so it is sufficient to show

$$\#186:\ \Delta v - - \frac{\Delta c\cdot(\tau + 1) - 3\cdot\Delta v + 2\cdot\Delta v\cdot(\tau + 1)}{3}$$

$$\#187:\quad \frac{(\Delta c + 2\cdot\Delta v)\cdot(\tau + 1)}{3} > 0$$

eq (19)

#188: $$paIII - paI = \frac{2\cdot ca + cb + \Delta v + 3\cdot \mu}{3} - \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot \mu}{3\cdot(\tau + 1)}$$

#189: $$paIII - paI = \frac{\tau\cdot(\Delta v + 3\cdot \mu)}{3\cdot(\tau + 1)} > 0$$

#190: $$pbIII - pbI = \frac{ca + 2\cdot cb - \Delta v + 3\cdot \mu}{3} - \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot \mu}{3\cdot(\tau + 1)}$$

#191: $$pbIII - pbI = \frac{\tau\cdot(3\cdot \mu - \Delta v)}{3\cdot(\tau + 1)}$$

*** New Section 6: Local monopolies

Recall eq (2), now adding reservation utility eq (20)

#192: $va - pa\cdot(1 + \tau) - \mu\cdot x$

#193: $va - qa - \mu\cdot x$

#194: $vb - pb\cdot(1 + \tau) - \mu\cdot(1 - x)$

#195: $vb - qb - \mu\cdot(1 - x)$

consumer indiff between buying A and not buying any brand

#196: $va - pa\cdot(1 + \tau) - \mu\cdot x = 0$

#197: $\mathrm{SOLVE}(va - pa\cdot(1 + \tau) - \mu\cdot x = 0, x)$

eq (21)

#198: $$xa = \frac{va - pa\cdot(\tau + 1)}{\mu}$$

#199: $$xa = \frac{va - qa}{\mu}$$

consumer indiff between buying B and not buying any brand

#200: SOLVE(vb − pb·(1 + τ) − μ·(1 − x), x)

#201: $$xb = \frac{pb\cdot(\tau + 1) - vb + \mu}{\mu}$$

#202: $$xb = \frac{qb - vb + \mu}{\mu}$$

** Subsection 6.1 Tax-inclusive monopoly pricing

eq (22) profits

#203: profita = n·(pa − ca)·xa

#204: $$profita = n\cdot\left(\frac{qa}{1 + \tau} - ca\right)\cdot\frac{va - qa}{\mu}$$

#205: profitb = n·(pb − cb)·(1 − xb)

#206: $$profitb = n\cdot\left(\frac{qb}{1 + \tau} - cb\right)\cdot\left(1 - \frac{qb - vb + \mu}{\mu}\right)$$

Deriving equilibrium I: eq (23) and Appendix E

#207: $\frac{d}{d\ qa}\left(profita = n\cdot\left(\frac{qa}{1 + \tau} - ca\right)\cdot\frac{va - qa}{\mu}\right)$

#208: $$0 = \frac{n\cdot(ca\cdot(\tau + 1) - 2\cdot qa + va)}{\mu\cdot(\tau + 1)}$$

#209: $\frac{d}{d\ qa}\frac{d}{d\ qa}\left(profita = n\cdot\left(\frac{qa}{1 + \tau} - ca\right)\cdot\frac{va - qa}{\mu}\right)$

#210: $$0 > -\frac{2\cdot n}{\mu\cdot(\tau + 1)}$$

#211: $\frac{d}{d\ qb}\left(profitb = n\cdot\left(\frac{qb}{1 + \tau} - cb\right)\cdot\left(1 - \frac{qb - vb + \mu}{\mu}\right)\right)$

#212: $$0 = \frac{n\cdot(cb\cdot(\tau + 1) - 2\cdot qb + vb)}{\mu\cdot(\tau + 1)}$$

#213: $\frac{d}{d\ qb}\frac{d}{d\ qb}\left(profitb = n\cdot\left(\frac{qb}{1 + \tau} - cb\right)\cdot\left(1 - \frac{qb - vb + \mu}{\mu}\right)\right)$

#214: $$0 > -\frac{2\cdot n}{\mu\cdot(\tau + 1)}$$

#215: $\mathrm{SOLVE}\left(\left[0 = \frac{n\cdot(ca\cdot(\tau + 1) - 2\cdot qa + va)}{\mu\cdot(\tau + 1)},\ 0 = \frac{n\cdot(cb\cdot(\tau + 1) - 2\cdot qb + vb)}{\mu\cdot(\tau + 1)}\right],\ [qa,\ qb]\right)$

eq (23)

#216: $$\left[qaI = \frac{ca\cdot(\tau + 1) + va}{2} \wedge qbI = \frac{cb\cdot(\tau + 1) + vb}{2}\right]$$

#217: $$profitaI = \frac{n\cdot(ca\cdot(\tau + 1) - va)^2}{4\cdot\mu\cdot(\tau + 1)}$$

#218: $$profitbI = \frac{n\cdot(cb\cdot(\tau + 1) - vb)^2}{4\cdot\mu\cdot(\tau + 1)}$$

eq (E.2) market shares

#219: $$xaI = \frac{va - ca\cdot(\tau + 1)}{2\cdot\mu}$$

#220: $$xbI = \frac{cb\cdot(\tau + 1) - vb + 2\cdot\mu}{2\cdot\mu}$$

eq (24) number of unserved consumers

#221: $nuI = n\cdot(xbI - xaI)$

#222: $$nuI = n\cdot\left(\frac{cb\cdot(\tau + 1) - vb + 2\cdot\mu}{2\cdot\mu} - \frac{va - ca\cdot(\tau + 1)}{2\cdot\mu}\right)$$

#223: $$nuI = \frac{n\cdot(ca\cdot(\tau + 1) + cb\cdot(\tau + 1) - va - vb + 2\cdot\mu)}{2\cdot\mu}$$

nuI > 0 if [holds by Assumption 4b]

#224: $ca\cdot(\tau + 1) + cb\cdot(\tau + 1) - va - vb + 2\cdot\mu > 0$

#225: SOLVE(ca·(τ + 1) + cb·(τ + 1) − va − vb + 2·μ > 0, μ)

#226: $$\mu > - \frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 1) - va - vb}{2}$$

** Subsection 6.2 Monopoly pricing with sales tax separted from price and low computing consumers.

modify eq (8)

#227: va − pa − vata − μ·xa = 0

#228: vb − pb − vatb − μ·(1 − xb) = 0

eq (25)

#229: SOLVE(va − pa − vata − μ·xa = 0, xa)

#230: $$xa = - \frac{pa - va + vata}{\mu}$$

#231: SOLVE(vb − pb − vatb − μ·(1 − xb) = 0, xb)

#232: $$xb = \frac{pb + vatb - vb + \mu}{\mu}$$

eq (26) profit functions

#233: profita = n·(pa − ca)·xa

#234: profitb = n·(pb − cb)·(1 − xb)

#235: $$profita = n\cdot(pa - ca)\cdot\left(- \frac{pa - va + vata}{\mu}\right)$$

#236: $$\text{profitb} = n\cdot(pb - cb)\cdot\left(1 - \frac{pb + vatb - vb + \mu}{\mu}\right)$$

Appendix F, deriving eq (28)

#237: $$\frac{d}{d\ pa}\left(\text{profita} = n\cdot(pa - ca)\cdot\left(- \frac{pa - va + vata}{\mu}\right)\right)$$

#238: $$0 = \frac{n\cdot(ca - 2\cdot pa + va - vata)}{\mu}$$

#239: $$\frac{d}{d\ pa}\frac{d}{d\ pa}\left(\text{profita} = n\cdot(pa - ca)\cdot\left(- \frac{pa - va + vata}{\mu}\right)\right)$$

#240: $$0 > - \frac{2\cdot n}{\mu}$$

#241: $$\frac{d}{d\ pb}\left(\text{profitb} = n\cdot(pb - cb)\cdot\left(1 - \frac{pb + vatb - vb + \mu}{\mu}\right)\right)$$

#242: $$0 = \frac{n\cdot(cb - 2\cdot pb - vatb + vb)}{\mu}$$

#243: $$\frac{d}{d\ pb}\frac{d}{d\ pb}\left(\text{profitb} = n\cdot(pb - cb)\cdot\left(1 - \frac{pb + vatb - vb + \mu}{\mu}\right)\right)$$

#244: $$0 > - \frac{2\cdot n}{\mu}$$

#245: $\mathrm{SOLVE}\left(\left[0 = \frac{n\cdot(ca - 2\cdot pa + va - vata)}{\mu}, 0 = \frac{n\cdot(cb - 2\cdot pb - vatb + vb)}{\mu}\right], [pa, pb]\right)$

#246: $\left[pa = \frac{ca + va - vata}{2} \wedge pb = \frac{cb - vatb + vb}{2}\right]$

#247: $\left[pa = \frac{ca + va - \tau\cdot pa}{2} \wedge pb = \frac{cb - \tau\cdot pb + vb}{2}\right]$

#248: $\mathrm{SOLVE}\left(\left[pa = \frac{ca + va - \tau\cdot pa}{2} \wedge pb = \frac{cb - \tau\cdot pb + vb}{2}\right], [pa, pb]\right)$

eq (27)

#249: $\left[paII = \frac{ca + va}{\tau + 2} \wedge pbII = \frac{cb + vb}{\tau + 2}\right]$

eq (F.2)

#250: $xa = - \frac{\frac{ca + va}{\tau + 2} - va + \tau\cdot\frac{ca + va}{\tau + 2}}{\mu}$

#251: $xaII = \frac{va - ca\cdot(\tau + 1)}{\mu\cdot(\tau + 2)}$

#252: $xb = \frac{\frac{cb + vb}{\tau + 2} + \tau\cdot\frac{cb + vb}{\tau + 2} - vb + \mu}{\mu}$

#253: $$\mathrm{xbII} = \frac{cb\cdot(\tau + 1) - vb + \mu\cdot(\tau + 2)}{\mu\cdot(\tau + 2)}$$

xaII and xbII > 0 by Assumption 4a

profit part of eq (27)

#254: $$\mathrm{profitaII} = \frac{n\cdot(ca\cdot(\tau + 1) - va)^2}{\mu\cdot(\tau + 2)^2}$$

#255: $$\mathrm{profitbII} = \frac{n\cdot(cb\cdot(\tau + 1) - vb)^2}{\mu\cdot(\tau + 2)^2}$$

Deriving (28) unserved consumers

#256: $$\mathrm{nuII} = n\cdot(\mathrm{xbII} - \mathrm{xaII}) = \frac{n\cdot(cb\cdot(\tau + 1) - vb + \mu\cdot(\tau + 2))}{\mu\cdot(\tau + 2)} - \frac{va - ca\cdot(\tau + 1)}{\mu\cdot(\tau + 2)}$$

#257: $$\mathrm{nuII} = \frac{n\cdot(ca\cdot(\tau + 1) + cb\cdot(\tau + 1) - va - vb + \mu\cdot(\tau + 2))}{\mu\cdot(\tau + 2)}$$

> 0 if [holds by assumption 4b)

#258: $ca\cdot(\tau + 1) + cb\cdot(\tau + 1) - va - vb + \mu\cdot(\tau + 2) > 0$

#259: SOLVE($ca\cdot(\tau + 1) + cb\cdot(\tau + 1) - va - vb + \mu\cdot(\tau + 2) > 0, \mu$)

#260: $$\mu > - \frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 1) - va - vb}{\tau + 2}$$

** Subsection 6.3: Comparing monoply outcomes under the 2 pricing structures:

eq (29)

#261: $$paII - paI = \frac{ca + va}{\tau + 2} - \frac{\frac{ca\cdot(\tau + 1) + va}{2}}{1 + \tau}$$

#262: $$paII - paI = \frac{\tau\cdot(va - ca\cdot(\tau + 1))}{2\cdot(\tau + 1)\cdot(\tau + 2)}$$

> 0 by Assumption 4a

#263: $$pbII - pbI = \frac{cb + vb}{\tau + 2} - \frac{\frac{cb\cdot(\tau + 1) + vb}{2}}{1 + \tau}$$

#264: $$pbII - pbI = \frac{\tau\cdot(vb - cb\cdot(\tau + 1))}{2\cdot(\tau + 1)\cdot(\tau + 2)}$$

> 0 by Assumption 4a

#265: $$profitaII - profitaI = \frac{n\cdot(ca\cdot(\tau + 1) - va)^2}{\mu\cdot(\tau + 2)^2} - \frac{n\cdot(ca\cdot(\tau + 1) - va)^2}{4\cdot\mu\cdot(\tau + 1)}$$

#266: $$profitaII - profitaI = - \frac{n\cdot\tau^2\cdot(ca\cdot(\tau + 1) - va)^2}{4\cdot\mu\cdot(\tau + 1)\cdot(\tau + 2)^2} < 0$$

#267: $\text{profitbII} - \text{profitbI} = \dfrac{n\cdot(cb\cdot(\tau + 1) - vb)^2}{\mu\cdot(\tau + 2)^2} - \dfrac{n\cdot(cb\cdot(\tau + 1) - vb)^2}{4\cdot\mu\cdot(\tau + 1)}$

#268: $\text{profitbII} - \text{profitbI} = - \dfrac{n\cdot\tau^2\cdot(cb\cdot(\tau + 1) - vb)^2}{4\cdot\mu\cdot(\tau + 1)\cdot(\tau + 2)^2} < 0$

eq (31): comparing number of unserved consumers

#269: $\text{nuII} - \text{nuI} = \dfrac{n\cdot(ca\cdot(\tau + 1) + cb\cdot(\tau + 1) - va - vb + \mu\cdot(\tau + 2))}{\mu\cdot(\tau + 2)} - \dfrac{n\cdot(ca\cdot(\tau + 1) + cb\cdot(\tau + 1) - va - vb + 2\cdot\mu)}{2\cdot\mu}$

#270: $\text{nuII} - \text{nuI} = - \dfrac{n\cdot\tau\cdot(ca\cdot(\tau + 1) + cb\cdot(\tau + 1) - va - vb)}{2\cdot\mu\cdot(\tau + 2)}$

> 0 by Assumption 4b.
