vat_compete_2024_x_y.dfw

#1: CaseMode :≡ Sensitive

#2: InputMode :≡ Word

degree of market power (transp cost)

#3: $\mu :\in \text{Real } (0, \infty)$

rate of sales tax

#4: $\tau :\in \text{Real } (0, \infty)$

total consumer population

#5: $n :\in \text{Real } (0, \infty)$

prices

#6: $pa :\in \text{Real } (0, \infty)$

#7: $pb :\in \text{Real } (0, \infty)$

#8: $qa :\in \text{Real } (0, \infty)$

#9: $qb :\in \text{Real } (0, \infty)$

A's market share

#10: $xhat :\in \text{Real } (0, 1)$

basic valuations

#11: $va :\in \text{Real } (0, \infty)$

#12: $vb :\in \text{Real } (0, \infty)$

#13: $\Delta v :\in \text{Real } [0, \infty)$

*** Section 2: Price embedded into the price (benchmark model)

eq (1)

#14: $qa = pa \cdot (1 + \tau)$

#15: $qb = pb \cdot (1 + \tau)$

#16: SOLVE($qa = pa \cdot (1 + \tau)$, pa)

#17:
$$pa = \frac{qa}{\tau + 1}$$

#18: SOLVE($qb = pb \cdot (1 + \tau)$, pb)

#19:
$$pb = \frac{qb}{\tau + 1}$$

eq (2) Utility functions

#20: $va - pa \cdot (1 + \tau) - \mu \cdot x$

#21: $va - qa - \mu \cdot x$

#22: $vb - pb \cdot (1 + \tau) - \mu \cdot (1 - x)$

#23: $vb - qb - \mu \cdot (1 - x)$

#24: $va - pa \cdot (1 + \tau) - \mu \cdot x = vb - pb \cdot (1 + \tau) - \mu \cdot (1 - x)$

#25: SOLVE($va - pa \cdot (1 + \tau) - \mu \cdot x = vb - pb \cdot (1 + \tau) - \mu \cdot (1 - x)$, x)

eq (3)

#26:
$$xhat = - \frac{pa \cdot (\tau + 1) - pb \cdot (\tau + 1) - va + vb - \mu}{2 \cdot \mu}$$

#27: $$\text{xhat} = - \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}$$

#28: $$\text{xhat} = - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}$$

eq (4) Profit max w.r.t. qa and qb (tax inclusive)

#29: $$\text{profita} = pa\cdot n\cdot \text{xhat}$$

#30: $$\text{profitb} = pb\cdot n\cdot(1 - \text{xhat})$$

#31: $$\text{profita} = \frac{qa}{\tau + 1}\cdot n\cdot\left(- \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)$$

#32: $$\text{profitb} = \frac{qb}{\tau + 1}\cdot n\cdot\left(1 - - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)$$

Appendix A. eq (A.1)

#33: $$\frac{d}{d\ qa}\left(\text{profita} = \frac{qa}{\tau + 1}\cdot n\cdot\left(- \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#34: $$0 = - \frac{n\cdot(2\cdot qa - qb - \Delta v - \mu)}{2\cdot\mu\cdot(\tau + 1)}$$

#35: $$\frac{d}{d\ qa}\frac{d}{d\ qa}\left(\text{profita} = \frac{qa}{\tau + 1}\cdot n\cdot\left(- \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#36: $$0 > - \frac{n}{\mu\cdot(\tau + 1)}$$

#37: $$\frac{d}{d\ qb}\left(profitb = \frac{qb}{\tau + 1}\cdot n\cdot\left(1 - - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#38: $$0 = \frac{n\cdot(qa - 2\cdot qb - \Delta v + \mu)}{2\cdot\mu\cdot(\tau + 1)}$$

#39: $$\frac{d}{d\ qb}\frac{d}{d\ qb}\left(profitb = \frac{qb}{\tau + 1}\cdot n\cdot\left(1 - - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#40: $$0 > - \frac{n}{\mu\cdot(\tau + 1)}$$

#41: $$\text{SOLVE}\left(\left[0 = - \frac{n\cdot(2\cdot qa - qb - \Delta v - \mu)}{2\cdot\mu\cdot(\tau + 1)}, 0 = \frac{n\cdot(qa - 2\cdot qb - \Delta v + \mu)}{2\cdot\mu\cdot(\tau + 1)}\right], [qa, qb]\right)$$

eq (5): eql tax-inclusive prices

#42: $$\left[qaI = \frac{\Delta v + 3\cdot\mu}{3} \wedge qbI = \frac{3\cdot\mu - \Delta v}{3}\right]$$

#43: $$paI = \frac{\Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#44: $$pbI = \frac{3\cdot\mu - \Delta v}{3\cdot(\tau + 1)}$$

eq (6)

#45: $$xhatI = \frac{\Delta v + 3\cdot\mu}{6\cdot\mu}$$

#46: $$\text{profitaI} = \frac{n\cdot(\Delta v + 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

#47: $$\text{profitbI} = \frac{n\cdot(\Delta v - 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

Discussion below eq (6)

#48: $$\frac{d}{d\mu}\left(\text{profitb} = \frac{n\cdot(\Delta v - 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}\right)$$

#49: $$0 < \frac{n\cdot(\Delta v + 3\cdot\mu)\cdot(3\cdot\mu - \Delta v)}{18\cdot\mu^2\cdot(\tau + 1)}$$

#50: $$\frac{d}{d\mu}\left(\text{profita} = \frac{n\cdot(\Delta v + 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}\right)$$

#51: $$0 < \frac{n\cdot(\Delta v + 3\cdot\mu)\cdot(3\cdot\mu - \Delta v)}{18\cdot\mu^2\cdot(\tau + 1)}$$

Deriving eq (7)

#52: $g = n\cdot\text{xhat}\cdot\tau\cdot\text{pa} + n\cdot(1 - \text{xhat})\cdot\tau\cdot\text{pb}$

#53: $$gI = \frac{n\cdot\tau\cdot(\Delta v^2 + 9\cdot\mu^2)}{9\cdot\mu\cdot(\tau + 1)}$$

Result 1: parts (a) and (b) are trivial by inspection (differentiation is not needed).
Part (c) has been added to appendix A.

Result 1(c), proved in Appendix A, eq (A.2)

#54: $$\frac{d}{d\tau}\left(gI = \frac{n\cdot\tau\cdot(\Delta v^2 + 9\cdot\mu^2)}{9\cdot\mu\cdot(\tau + 1)}\right)$$

#55: $$0 < \frac{n\cdot(\Delta v^2 + 9\cdot\mu^2)}{9\cdot\mu\cdot(\tau + 1)^2}$$

*** section 3: Price competition with sales tax separated

** subsection 3.1: Single-stage cosnumer decision making

eq (8)

#56: $$profita = pa\cdot n\cdot xhat$$

#57: $$profitb = pb\cdot n\cdot(1 - xhat)$$

#58: $$profita = pa\cdot n\cdot\left(- \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)$$

#59: $$profitb = pb\cdot n\cdot\left(1 - - \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)$$

Appendix B, eq (B.1), Proof of Result 2

#60: $$\frac{d}{d\ pa}\left(profita = pa\cdot n\cdot\left(-\frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#61: $$0 = -\frac{n\cdot(2\cdot pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu)}{2\cdot\mu}$$

#62: $$\frac{d}{d\ pa}\frac{d}{d\ pa}\left(profita = pa\cdot n\cdot\left(-\frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#63: $$0 > -\frac{n\cdot(\tau + 1)}{\mu}$$

#64: $$\frac{d}{d\ pb}\left(profitb = pb\cdot n\cdot\left(1 - -\frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#65: $$0 = \frac{n\cdot(pa\cdot(\tau + 1) - 2\cdot pb\cdot(\tau + 1) - \Delta v + \mu)}{2\cdot\mu}$$

#66: $$\frac{d}{d\ pb}\frac{d}{d\ pb}\left(profitb = pb\cdot n\cdot\left(1 - -\frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#67: $$0 > -\frac{n\cdot(\tau + 1)}{\mu}$$

#68: $$\mathrm{SOLVE}\left(\left[0 = -\frac{n\cdot(2\cdot pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu)}{2\cdot\mu},\ 0 =\right.\right.$$

$$\left.\frac{n\cdot(pa\cdot(\tau + 1) - 2\cdot pb\cdot(\tau + 1) - \Delta v + \mu)}{2\cdot\mu}\right], [pa, pb]\Bigg)$$

The solution below is the same as eq (5) above, proving Result 2:

#69: $$\left[pa = \frac{\Delta v + 3\cdot\mu}{3\cdot(\tau + 1)} \wedge pb = \frac{3\cdot\mu - \Delta v}{3\cdot(\tau + 1)}\right]$$

** Subsection 3.2: Two-stage decision process with prices separated from sales tax

eq (9) utility

#70: $va - pa - vata - \mu\cdot x$

#71: $vb - pb - vatb - \mu\cdot(1 - x)$

#72: $vb - pb - vatb - \mu\cdot(1 - x) = va - pa - vata - \mu\cdot x$

#73: $\mathrm{SOLVE}(vb - pb - vatb - \mu\cdot(1 - x) = va - pa - vata - \mu\cdot x, x)$

eq (10)

#74: $$xhat = - \frac{pa - pb - va + vata - vatb + vb - \mu}{2\cdot\mu}$$

#75: $$xhat = - \frac{pa - pb - \Delta v + vata - vatb - \mu}{2\cdot\mu}$$

eq (11) profit max w/ two stages

#76: $profita = pa\cdot n\cdot xhat$

#77: $profitb = pb\cdot n\cdot(1 - xhat)$

#78: $$\text{profita} = \text{pa}\cdot n\cdot\left(-\frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)$$

#79: $$\text{profitb} = \text{pb}\cdot n\cdot\left(1 - -\frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)$$

Appendix C, eq (C.1)

#80: $$\frac{d}{d\ \text{pa}}\left(\text{profita} = \text{pa}\cdot n\cdot\left(-\frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#81: $$0 = -\frac{n\cdot(2\cdot\text{pa} - \text{pb} + \text{vata} - \text{vatb} - \Delta v - \mu)}{2\cdot\mu}$$

#82: $$\frac{d}{d\ \text{pa}}\frac{d}{d\ \text{pa}}\left(\text{profita} = \text{pa}\cdot n\cdot\left(-\frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#83: $$0 > -\frac{n}{\mu}$$

#84: $$\frac{d}{d\ \text{pb}}\left(\text{profitb} = \text{pb}\cdot n\cdot\left(1 - -\frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#85: $$0 = \frac{n\cdot(\text{pa} - 2\cdot\text{pb} + \text{vata} - \text{vatb} - \Delta v + \mu)}{2\cdot\mu}$$

#86: $$\frac{d}{d\ \text{pb}}\frac{d}{d\ \text{pb}}\left(\text{profitb} = \text{pb}\cdot n\cdot\left(1 - -\frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#87: $$0 > -\frac{n}{\mu}$$

eq (12) eq price two stage as functions of vata and vatb

#88: $$\text{SOLVE}\left(\left[0 = -\frac{n\cdot(2\cdot pa - pb + vata - vatb - \Delta v - \mu)}{2\cdot\mu},\ 0 = \frac{n\cdot(pa - 2\cdot pb + vata - vatb - \Delta v + \mu)}{2\cdot\mu}\right],\ [pa,\ pb]\right)$$

#89: $$\left[pa = -\frac{vata - vatb - \Delta v - 3\cdot\mu}{3} \wedge pb = \frac{vata - vatb - \Delta v + 3\cdot\mu}{3}\right]$$

eq (13) eql prices with two stage

#90: $vata = \tau\cdot pa$

#91: $vatb = \tau\cdot pb$

#92: $$\left[pa = -\frac{\tau\cdot pa - \tau\cdot pb - \Delta v - 3\cdot\mu}{3} \wedge pb = \frac{\tau\cdot pa - \tau\cdot pb - \Delta v + 3\cdot\mu}{3}\right]$$

#93: $$\text{SOLVE}\left(\left[pa = -\frac{\tau\cdot pa - \tau\cdot pb - \Delta v - 3\cdot\mu}{3} \wedge pb = \frac{\tau\cdot pa - \tau\cdot pb - \Delta v + 3\cdot\mu}{3}\right],\ [pa,\ pb]\right)$$

#94: $$\left[paII = \frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3} \wedge pbII = \frac{\mu\cdot(2\cdot\tau + 3) - \Delta v}{2\cdot\tau + 3}\right]$$

Result 3 parts (a) and (b), at the end of Appendix C

#95: $$\frac{d}{d\tau}\left(paII = \frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}\right)$$

#96: $$0 > -\frac{2\cdot\Delta v}{(2\cdot\tau + 3)^2}$$

#97: $$\frac{d}{d\tau}\left(pbII = \frac{\mu\cdot(2\cdot\tau + 3) - \Delta v}{2\cdot\tau + 3}\right)$$

#98: $$0 < \frac{2\cdot\Delta v}{(2\cdot\tau + 3)^2}$$

#99: $$\left[qaII = \frac{(1 + \tau)\cdot(\Delta v + \mu\cdot(2\cdot\tau + 3))}{2\cdot\tau + 3} \wedge qbII = \frac{(1 + \tau)\cdot(\mu\cdot(2\cdot\tau + 3) - \Delta v)}{2\cdot\tau + 3}\right]$$

#100: $$\left[qaII = \frac{(\Delta v + \mu\cdot(2\cdot\tau + 3))\cdot(\tau + 1)}{2\cdot\tau + 3} \wedge qbII = \frac{(\mu\cdot(2\cdot\tau + 3) - \Delta v)\cdot(\tau + 1)}{2\cdot\tau + 3}\right]$$

#101: $$\frac{d}{d\tau}\left(qaII = \frac{(\Delta v + \mu\cdot(2\cdot\tau + 3))\cdot(\tau + 1)}{2\cdot\tau + 3}\right)$$

#102: $$0 < \frac{\Delta v + \mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9)}{(2\cdot\tau + 3)^2}$$

#103: $$\frac{d}{d\tau}\left(qbII = \frac{(\mu\cdot(2\cdot\tau + 3) - \Delta v)\cdot(\tau + 1)}{2\cdot\tau + 3}\right)$$

#104: $$\frac{\mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9) - \Delta v}{(2\cdot\tau + 3)^2} > 0$$

eqs (14)

#105: $$vata = \tau\cdot\frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}$$

#106: $$vatb = \tau\cdot\frac{\mu\cdot(2\cdot\tau + 3) - \Delta v}{2\cdot\tau + 3}$$

#107: $$xhatII = \frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)}$$

#108: $$profitaII = \frac{n\cdot(\Delta v + \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

#109: $$profitbII = \frac{n\cdot(\Delta v - \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

eq (15) gov't revenue with two stage decision making

#110: $$gII = \frac{n\cdot\tau\cdot(\Delta v^2 + \mu^2\cdot(2\cdot\tau + 3)^2)}{\mu\cdot(2\cdot\tau + 3)^2}$$

Proof of Result 3(c) at the end of Appendix C eq (c.4)

$$\#111: \frac{d}{d\tau}\left(gII = \frac{n\cdot\tau\cdot(\Delta v^2 + \mu^2\cdot(2\cdot\tau + 3)^2)}{\mu\cdot(2\cdot\tau + 3)^2}\right)$$

$$\#112: \frac{n\cdot(\mu^2\cdot(8\cdot\tau^3 + 36\cdot\tau^2 + 54\cdot\tau + 27) - \Delta v^2\cdot(2\cdot\tau - 3))}{\mu\cdot(2\cdot\tau + 3)^3}$$

> 0 if

$$\#113: \mu^2\cdot(8\cdot\tau^3 + 36\cdot\tau^2 + 54\cdot\tau + 27) - \Delta v^2\cdot(2\cdot\tau - 3) > 0$$

because $\tau < 1$ and hence $2\tau - 3 < 0$

*** Section 4: Comparing the two pricing structure.

eq (16) and Result 4 (price comparisons)

$$\#114: paII - pa = \frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3} - \frac{\Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

$$\#115: paII - paI = \frac{\tau\cdot(\Delta v + 3\cdot\mu\cdot(2\cdot\tau + 3))}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)} > 0$$

$$\#116: pbII - pb = \frac{\mu\cdot(2\cdot\tau + 3) - \Delta v}{2\cdot\tau + 3} - \frac{3\cdot\mu - \Delta v}{3\cdot(\tau + 1)}$$

#117: $$pbII - pbI = \frac{\tau\cdot(3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v)}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)}$$

> 0 if [Yes!]

#118: $3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v > 0$

#119: SOLVE($3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v > 0$, Δv)

#120: $$\Delta v < 3\cdot\mu\cdot(2\cdot\tau + 3)$$

Assumption 2 implies that $\Delta v < \mu$. Hence, it is sufficient to show

#121: $\mu < 3\cdot\mu\cdot(2\cdot\tau + 3)$

#122: SOLVE($\mu < 3\cdot\mu\cdot(2\cdot\tau + 3)$, μ)

#123: $$\mu > 0$$

Proving Result 4(b): Added to the first part of Appendix D

#124: $\frac{d}{d\tau}\left(qaII = \frac{(\Delta v + \mu\cdot(2\cdot\tau + 3))\cdot(\tau + 1)}{2\cdot\tau + 3}\right)$

eq (D.1)

#125: $$0 < \frac{\Delta v + \mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9)}{(2\cdot\tau + 3)^2}$$

#126: $\frac{d}{d\tau}\left(qbII = \frac{(\mu\cdot(2\cdot\tau + 3) - \Delta v)\cdot(\tau + 1)}{2\cdot\tau + 3}\right)$

eq (D.2)

#127: $$\frac{\mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9) - \Delta v}{(2\cdot\tau + 3)^2}$$

> 0 if

#128: $\mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9) - \Delta v > 0$

#129: $\text{SOLVE}(\mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9) - \Delta v > 0, \Delta v)$

#130: $$\Delta v < \mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9)$$

eq (17) and Result 5

#131: $$gII - g = \frac{n\cdot\tau\cdot(\Delta v^2 + \mu^2\cdot(2\cdot\tau + 3)^2)}{\mu\cdot(2\cdot\tau + 3)^2} - \frac{n\cdot\tau\cdot(\Delta v^2 + 9\cdot\mu^2)}{9\cdot\mu\cdot(\tau + 1)}$$

#132: $$gII - g = \frac{n\cdot\tau^2\cdot(9\cdot\mu^2\cdot(2\cdot\tau + 3)^2 - \Delta v^2\cdot(4\cdot\tau + 3))}{9\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

> 0 Appendix D if

#133: $9\cdot\mu^2\cdot(2\cdot\tau + 3)^2 - \Delta v^2\cdot(4\cdot\tau + 3) > 0$

#134: $\text{SOLVE}(9\cdot\mu^2\cdot(2\cdot\tau + 3)^2 - \Delta v^2\cdot(4\cdot\tau + 3) > 0, \Delta v)$

eq (D.3)

#135: $$- \frac{3\cdot\mu\cdot(2\cdot\tau + 3)}{\sqrt{(4\cdot\tau + 3)}} < \Delta v < \frac{3\cdot\mu\cdot(2\cdot\tau + 3)}{\sqrt{(4\cdot\tau + 3)}}$$

By Assumption 2, $\Delta v < \mu$, hence it is sufficient to show that: eq (D.4)

#136: $$\frac{3\cdot\mu\cdot(2\cdot\tau + 3)}{\sqrt{(4\cdot\tau + 3)}} - \mu > 0$$

or that [holds for every $\tau \geq 0$]

#137: $$\frac{3\cdot(2\cdot\tau + 3)}{\sqrt{(4\cdot\tau + 3)}} - 1 > 0$$

Result 6 and Appendix E

#138: $$\text{profitaII} - \text{profita} = \frac{n\cdot(\Delta v + \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2} - \frac{n\cdot(\Delta v + 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

eq (E.1)

#139: $$\text{profitaII} - \text{profita} = - \frac{n\cdot\tau\cdot(\Delta v^2\cdot(4\cdot\tau + 3) - 6\cdot\Delta v\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

evalutated at $\Delta v = 0$,

#140: $\text{profitaII} - \text{profita} = -\dfrac{n\cdot\tau\cdot(0^2\cdot(4\cdot\tau + 3) - 6\cdot 0\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$

#141: $\text{profitaII} - \text{profita} = \dfrac{n\cdot\mu\cdot\tau}{2\cdot(\tau + 1)} > 0$

#142: $\text{profitbII} - \text{profitb} = \dfrac{n\cdot(\Delta v - \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2} - \dfrac{n\cdot(\Delta v - 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$

Also eq (E.1)

#143: $\text{profitbII} - \text{profitb} = -\dfrac{n\cdot\tau\cdot(\Delta v^2\cdot(4\cdot\tau + 3) + 6\cdot\Delta v\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$

evaluated at $\Delta v = 0$,

#144: $\text{profitbII} - \text{profitb} = -\dfrac{n\cdot\tau\cdot(0^2\cdot(4\cdot\tau + 3) + 6\cdot 0\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$

#145: $\text{profitbII} - \text{profitb} = \dfrac{n\cdot\mu\cdot\tau}{2\cdot(\tau + 1)} > 0$

evaluted at $\Delta v = \mu$ (the other extreme): eq (E.3)

#146: $$\text{profitaII} - \text{profita} = - \frac{n\cdot\tau\cdot(\mu^2\cdot(4\cdot\tau + 3) - 6\cdot\mu\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

#147: $$\text{profitaII} - \text{profita} = \frac{2\cdot n\cdot\mu\cdot\tau\cdot(9\cdot\tau^2 + 29\cdot\tau + 24)}{9\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2} > 0$$

#148: $$\text{profitbII} - \text{profitb} = - \frac{n\cdot\tau\cdot(\mu^2\cdot(4\cdot\tau + 3) + 6\cdot\mu\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

#149: $$\text{profitbII} - \text{profitb} = \frac{2\cdot n\cdot\mu\cdot\tau\cdot(9\cdot\tau^2 + 23\cdot\tau + 15)}{9\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2} > 0$$

so, look at the trend between Δv=0 and Δv=μ (look for monotonicity , i.e., derivative is either all >0 or all <0

#150: $$\frac{d}{d\ \Delta v}\left(\text{profitbII} - \text{profitb} = - \frac{n\cdot\tau\cdot(\Delta v^2\cdot(4\cdot\tau + 3) + 6\cdot\Delta v\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}\right)$$

eq (E.4)

#151: $$- \frac{n\cdot\tau\cdot(\Delta v\cdot(4\cdot\tau + 3) + 3\cdot\mu\cdot(2\cdot\tau + 3))}{9\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2} < 0$$

therefore profitbII is higher all over the range of Δv

#152: $$\frac{d}{d\ \Delta v}\left(\text{profitaII} - \text{profita} = - \frac{n\cdot\tau\cdot(\Delta v^2\cdot(4\cdot\tau + 3) - 6\cdot\Delta v\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}\right)$$

Also eq (E.4)

#153: $$\frac{n\cdot\tau\cdot(3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v\cdot(4\cdot\tau + 3))}{9\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

> 0 if

#154: $3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v\cdot(4\cdot\tau + 3) > 0$

#155: $\text{SOLVE}(3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v\cdot(4\cdot\tau + 3) > 0, \Delta v)$

eq (E.5)

#156: $$\Delta v < \frac{3\cdot\mu\cdot(2\cdot\tau + 3)}{4\cdot\tau + 3}$$

#157: $$\frac{3\cdot\mu\cdot(2\cdot\tau + 3)}{4\cdot\tau + 3} - \mu$$

#158: $$\frac{2\cdot\mu\cdot(\tau + 3)}{4\cdot\tau + 3} > 0$$

therefore profitaII is higher all over the range of Δv

*** Section 5: Welfare analysis

eq (18), first-best x_star

#159: $va - \mu \cdot xhatstar = vb - \mu \cdot (1 - xhatstar)$

#160: $SOLVE(va - \mu \cdot xhatstar = vb - \mu \cdot (1 - xhatstar), xhatstar)$

#161: $$xhatstar = \frac{va - vb + \mu}{2 \cdot \mu}$$

eq (18)

#162: $xhatstar = \dfrac{\Delta v + \mu}{2 \cdot \mu}$

recall xhatI and xhatII (start deriving Result 8)

#163: $xhatI = \dfrac{\Delta v + 3 \cdot \mu}{6 \cdot \mu}$

#164: $xhatII = \dfrac{\Delta v + \mu \cdot (2 \cdot \tau + 3)}{2 \cdot \mu \cdot (2 \cdot \tau + 3)}$

eqs (19)

#165: $xhatstar - xhatI = \dfrac{\Delta v + \mu}{2 \cdot \mu} - \dfrac{\Delta v + 3 \cdot \mu}{6 \cdot \mu}$

eq (19)

#166: $$xhatstar - xhatI = \frac{\Delta v}{3 \cdot \mu}$$

#167: $\text{xhatI} - \text{xhatII} = \dfrac{\Delta v + 3\cdot\mu}{6\cdot\mu} - \dfrac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)}$

#168: $$\text{xhatI} - \text{xhatII} = \frac{\Delta v\cdot\tau}{3\cdot\mu\cdot(2\cdot\tau + 3)}$$

In-text (below Result 8), derivation of Result 8(b)

#169: $\dfrac{d}{d\tau}\left(\text{xhatI} - \text{xhatII} = \dfrac{\Delta v\cdot\tau}{3\cdot\mu\cdot(2\cdot\tau + 3)}\right)$

#170: $$0 < \frac{\Delta v}{\mu\cdot(2\cdot\tau + 3)^2}$$

#171: $\dfrac{d}{d\ \Delta v}\left(\text{xhatI} - \text{xhatII} = \dfrac{\Delta v\cdot\tau}{3\cdot\mu\cdot(2\cdot\tau + 3)}\right)$

#172: $$0 < \frac{\tau}{3\cdot\mu\cdot(2\cdot\tau + 3)}$$

eq (20) total welfare

#173: $w = n\cdot\int_0^{\text{xhat}} (va - \mu\cdot x)\ dx + n\cdot\int_{\text{xhat}}^{1} (vb - \mu\cdot(1 - x))\ dx$

#174: $$w = n\cdot\left(va\cdot\text{xhat} - \frac{2\cdot vb\cdot(\text{xhat} - 1) + \mu\cdot(2\cdot\text{xhat}^2 - 2\cdot\text{xhat} + 1)}{2}\right)$$

#175: $$w = \frac{n\cdot(2\cdot va\cdot xhat + 2\cdot vb\cdot(1 - xhat) - \mu\cdot(2\cdot xhat^2 - 2\cdot xhat + 1))}{2}$$

eq (21) three welfare levels

#176: $$wI = \frac{n\cdot\left(2\cdot va\cdot\frac{\Delta v + 3\cdot\mu}{6\cdot\mu} + 2\cdot vb\cdot\left(1 - \frac{\Delta v + 3\cdot\mu}{6\cdot\mu}\right) - \mu\cdot\left(2\cdot\left(\frac{\Delta v + 3\cdot\mu}{6\cdot\mu}\right)^2 - 2\cdot\frac{\Delta v + 3\cdot\mu}{6\cdot\mu} + 1\right)\right)}{2}$$

#177: $$wI = \frac{n\cdot(6\cdot va\cdot(\Delta v + 3\cdot\mu) + 6\cdot vb\cdot(3\cdot\mu - \Delta v) - \Delta v^2 - 9\cdot\mu^2)}{36\cdot\mu}$$

#178: $$wII = \frac{n\cdot\left(2\cdot va\cdot\frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)} + 2\cdot vb\cdot\left(1 - \frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)}\right) - \mu\cdot\left(2\cdot\left(\frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)}\right)^2 - 2\cdot\frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)} + 1\right)\right)}{2}$$

#179:
$$wII = n\cdot\left(\frac{va\cdot(\Delta v + \mu\cdot(2\cdot\tau + 3))}{2\cdot\mu\cdot(2\cdot\tau + 3)} + \frac{vb\cdot(\mu\cdot(2\cdot\tau + 3) - \Delta v)}{2\cdot\mu\cdot(2\cdot\tau + 3)} - \frac{\Delta v^2 + \mu^2\cdot(2\cdot\tau + 3)^2}{4\cdot\mu\cdot(2\cdot\tau + 3)^2}\right)$$

#180:
$$wstar = \frac{n\cdot\left(2\cdot va\cdot\frac{\Delta v + \mu}{2\cdot\mu} + 2\cdot vb\cdot\left(1 - \frac{\Delta v + \mu}{2\cdot\mu}\right) - \mu\cdot\left(2\cdot\left(\frac{\Delta v + \mu}{2\cdot\mu}\right)^2 - 2\cdot\frac{\Delta v + \mu}{2\cdot\mu} + 1\right)\right)}{2}$$

#181:
$$wstar = \frac{n\cdot(2\cdot va\cdot(\Delta v + \mu) + 2\cdot vb\cdot(\mu - \Delta v) - \Delta v^2 - \mu^2)}{4\cdot\mu}$$

Result 9 and Appendix F

#182:
$$wstar - wI = \frac{n\cdot(2\cdot va\cdot(\Delta v + \mu) + 2\cdot vb\cdot(\mu - \Delta v) - \Delta v^2 - \mu^2)}{4\cdot\mu} - \frac{n\cdot(6\cdot va\cdot(\Delta v + 3\cdot\mu) + 6\cdot vb\cdot(3\cdot\mu - \Delta v) - \Delta v^2 - 9\cdot\mu^2)}{36\cdot\mu}$$

eq (F.1)

#183:
$$wstar - wI = \frac{n\cdot\Delta v\cdot(3\cdot va - 3\cdot vb - 2\cdot\Delta v)}{9\cdot\mu}$$

$$\#184:\ wI - wII = \frac{n\cdot(6\cdot va\cdot(\Delta v + 3\cdot\mu) + 6\cdot vb\cdot(3\cdot\mu - \Delta v) - \Delta v^2 - 9\cdot\mu^2)}{36\cdot\mu} - n\cdot\left(\frac{va\cdot(\Delta v + \mu\cdot(2\cdot\tau + 3))}{2\cdot\mu\cdot(2\cdot\tau + 3)} + \frac{vb\cdot(\mu\cdot(2\cdot\tau + 3) - \Delta v)}{2\cdot\mu\cdot(2\cdot\tau + 3)} - \frac{\Delta v^2 + \mu^2\cdot(2\cdot\tau + 3)^2}{4\cdot\mu\cdot(2\cdot\tau + 3)^2}\right)$$

$$\#185:\ wI - wII = \frac{n\cdot\Delta v\cdot\tau\cdot(3\cdot va\cdot(2\cdot\tau + 3) - 3\cdot vb\cdot(2\cdot\tau + 3) - \Delta v\cdot(\tau + 3))}{9\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

$$\#186:\ wI - wII = \frac{n\cdot\Delta v\cdot\tau\cdot(3\cdot\Delta v\cdot(2\cdot\tau + 3) - \Delta v\cdot(\tau + 3))}{9\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

$$\#187:\ wI - wII = \frac{n\cdot\Delta v^2\cdot\tau\cdot(5\cdot\tau + 6)}{9\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

Second part of Result 9 (Appendix F) showing that the above gap increases (basically WII declines) with a higher τ

$$\#188:\ \frac{d}{d\tau}\left(wI - wII = \frac{n\cdot\Delta v^2\cdot\tau\cdot(5\cdot\tau + 6)}{9\cdot\mu\cdot(2\cdot\tau + 3)^2}\right)$$

eq (F.2)

#189: $$0 < \frac{2\cdot n\cdot \Delta v^2\cdot(\tau + 1)}{\mu\cdot(2\cdot\tau + 3)^3}$$

Another way is to differentiate wII w.r.t. τ

#190: $$\frac{d}{d\tau}\left(wII = n\cdot\left(\frac{va\cdot(\Delta v + \mu\cdot(2\cdot\tau + 3))}{2\cdot\mu\cdot(2\cdot\tau + 3)} + \frac{vb\cdot(\mu\cdot(2\cdot\tau + 3) - \Delta v)}{2\cdot\mu\cdot(2\cdot\tau + 3)} - \frac{\Delta v^2 + \mu^2\cdot(2\cdot\tau + 3)^2}{4\cdot\mu\cdot(2\cdot\tau + 3)^2}\right)\right)$$

#191: $$-\frac{n\cdot\Delta v\cdot(va\cdot(2\cdot\tau + 3) - vb\cdot(2\cdot\tau + 3) - \Delta v)}{\mu\cdot(2\cdot\tau + 3)^3}$$

< 0 if [Yes]

#192: $n\cdot\Delta v\cdot(va\cdot(2\cdot\tau + 3) - vb\cdot(2\cdot\tau + 3) - \Delta v) > 0$

#193: $n\cdot\Delta v\cdot(\Delta v\cdot(2\cdot\tau + 3) - \Delta v) > 0$

#194: $$n\cdot\Delta v^2\cdot(2\cdot\tau + 2) > 0$$

xxxxxxxxxxxxxxxxxxx