vat_compete_2024_x_y.dfw

#1: CaseMode := Sensitive

#2: InputMode := Word

degree of market power (transp cost)

#3: $\mu :\in \text{Real } (0, \infty)$

rate of sales tax

#4: $\tau :\in \text{Real } (0, \infty)$

total consumer population

#5: $n :\in \text{Real } (0, \infty)$

prices

#6: $pa :\in \text{Real } (0, \infty)$

#7: $pb :\in \text{Real } (0, \infty)$

#8: $qa :\in \text{Real } (0, \infty)$

#9: $qb :\in \text{Real } (0, \infty)$

A's market share

#10: $xhat :\in \text{Real } (0, 1)$

basic valuations

#11: $va :\in \text{Real } (0, \infty)$

#12: $vb :\in \text{Real } (0, \infty)$

#13: $\Delta v :\in \text{Real } [0, \infty)$

*** Section 2: Price embedded into the price (benchmark model)

eq (1)

#14: $qa = pa\cdot(1 + \tau)$

#15: $qb = pb\cdot(1 + \tau)$

#16: SOLVE($qa = pa\cdot(1 + \tau)$, pa)

#17: $$pa = \frac{qa}{\tau + 1}$$

#18: SOLVE($qb = pb\cdot(1 + \tau)$, pb)

#19: $$pb = \frac{qb}{\tau + 1}$$

eq (2) Utility functions

#20: $va - pa\cdot(1 + \tau) - \mu\cdot x$

#21: $va - qa - \mu\cdot x$

#22: $vb - pb\cdot(1 + \tau) - \mu\cdot(1 - x)$

#23: $vb - qb - \mu\cdot(1 - x)$

#24: $va - pa\cdot(1 + \tau) - \mu\cdot x = vb - pb\cdot(1 + \tau) - \mu\cdot(1 - x)$

#25: SOLVE($va - pa\cdot(1 + \tau) - \mu\cdot x = vb - pb\cdot(1 + \tau) - \mu\cdot(1 - x)$, x)

eq (3)

#26: $$xhat = - \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - va + vb - \mu}{2\cdot\mu}$$

#27: $$\text{xhat} = - \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}$$

#28: $$\text{xhat} = - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}$$

eq (4) Profit max w.r.t. qa and qb (tax inclusive)

#29: $\text{profita} = pa\cdot n\cdot \text{xhat}$

#30: $\text{profitb} = pb\cdot n\cdot(1 - \text{xhat})$

#31: $$\text{profita} = \frac{qa}{\tau + 1}\cdot n\cdot\left(- \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)$$

#32: $$\text{profitb} = \frac{qb}{\tau + 1}\cdot n\cdot\left(1 - - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)$$

Appendix A. eq (A.1)

#33: $$\frac{d}{d\ qa}\left(\text{profita} = \frac{qa}{\tau + 1}\cdot n\cdot\left(- \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#34: $$0 = - \frac{n\cdot(2\cdot qa - qb - \Delta v - \mu)}{2\cdot\mu\cdot(\tau + 1)}$$

#35: $$\frac{d}{d\ qa}\frac{d}{d\ qa}\left(\text{profita} = \frac{qa}{\tau + 1}\cdot n\cdot\left(- \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#36: $$0 > - \frac{n}{\mu\cdot(\tau + 1)}$$

#37: $$\frac{d}{d\,qb}\left(profitb = \frac{qb}{\tau + 1}\cdot n\cdot\left(1 - - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#38: $$0 = \frac{n\cdot(qa - 2\cdot qb - \Delta v + \mu)}{2\cdot\mu\cdot(\tau + 1)}$$

#39: $$\frac{d}{d\,qb}\frac{d}{d\,qb}\left(profitb = \frac{qb}{\tau + 1}\cdot n\cdot\left(1 - - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#40: $$0 > - \frac{n}{\mu\cdot(\tau + 1)}$$

#41: $$\mathrm{SOLVE}\left(\left[0 = - \frac{n\cdot(2\cdot qa - qb - \Delta v - \mu)}{2\cdot\mu\cdot(\tau + 1)},\ 0 = \frac{n\cdot(qa - 2\cdot qb - \Delta v + \mu)}{2\cdot\mu\cdot(\tau + 1)}\right], [qa, qb]\right)$$

eq (5): eql tax-inclusive prices

#42: $$\left[qaI = \frac{\Delta v + 3\cdot\mu}{3} \wedge qbI = \frac{3\cdot\mu - \Delta v}{3}\right]$$

#43: $$paI = \frac{\Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#44: $$pbI = \frac{3\cdot\mu - \Delta v}{3\cdot(\tau + 1)}$$

eq (6)

#45: $$xhatI = \frac{\Delta v + 3\cdot\mu}{6\cdot\mu}$$

#46: $$\text{profitaI} = \frac{n\cdot(\Delta v + 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

#47: $$\text{profitbI} = \frac{n\cdot(\Delta v - 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

Discussion below eq (6)

#48: $$\frac{d}{d\mu}\left(\text{profitb} = \frac{n\cdot(\Delta v - 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}\right)$$

#49: $$0 < \frac{n\cdot(\Delta v + 3\cdot\mu)\cdot(3\cdot\mu - \Delta v)}{18\cdot\mu^2\cdot(\tau + 1)}$$

#50: $$\frac{d}{d\mu}\left(\text{profita} = \frac{n\cdot(\Delta v + 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}\right)$$

#51: $$0 < \frac{n\cdot(\Delta v + 3\cdot\mu)\cdot(3\cdot\mu - \Delta v)}{18\cdot\mu^2\cdot(\tau + 1)}$$

Deriving eq (7)

#52: $g = n\cdot\text{xhat}\cdot\tau\cdot\text{pa} + n\cdot(1 - \text{xhat})\cdot\tau\cdot\text{pb}$

#53: $$gI = \frac{n\cdot\tau\cdot(\Delta v^2 + 9\cdot\mu^2)}{9\cdot\mu\cdot(\tau + 1)}$$

*** section 3: Price competition with sales tax separated

** subsection 3.1: Single-stage cosnumer decision making

eq (8)

#54: $profita = pa\cdot n\cdot xhat$

#55: $profitb = pb\cdot n\cdot(1 - xhat)$

#56: $$profita = pa\cdot n\cdot\left(- \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)$$

#57: $$profitb = pb\cdot n\cdot\left(1 - - \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)$$

Appendix B, eq (B.1), Proof of Result 1

#58: $$\frac{d}{d\ pa}\left(profita = pa\cdot n\cdot\left(- \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#59: $$0 = - \frac{n\cdot(2\cdot pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu)}{2\cdot\mu}$$

#60: $$\frac{d}{d\ pa}\frac{d}{d\ pa}\left(profita = pa\cdot n\cdot\left(- \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#61: $$0 > - \frac{n\cdot(\tau + 1)}{\mu}$$

#62: $$\frac{d}{d\ pb}\left(profitb = pb\cdot n\cdot\left(1 - - \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#63: $$0 = \frac{n\cdot(pa\cdot(\tau + 1) - 2\cdot pb\cdot(\tau + 1) - \Delta v + \mu)}{2\cdot\mu}$$

#64: $$\frac{d}{d\ pb}\frac{d}{d\ pb}\left(profitb = pb\cdot n\cdot\left(1 - - \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#65: $$0 > - \frac{n\cdot(\tau + 1)}{\mu}$$

#66: $$SOLVE\left(\left[0 = - \frac{n\cdot(2\cdot pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - \Delta v - \mu)}{2\cdot\mu}, 0 = \frac{n\cdot(pa\cdot(\tau + 1) - 2\cdot pb\cdot(\tau + 1) - \Delta v + \mu)}{2\cdot\mu}\right], [pa, pb]\right)$$

The solution below is the same as eq (5) above, proving Result 1:

#67: $$\left[pa = \frac{\Delta v + 3\cdot\mu}{3\cdot(\tau + 1)} \wedge pb = \frac{3\cdot\mu - \Delta v}{3\cdot(\tau + 1)}\right]$$

** Subsection 3.2: Two-stage decision process with prices separated from sales tax

eq (9) utility

#68: $va - pa - vata - \mu\cdot x$

#69: $vb - pb - vatb - \mu\cdot(1 - x)$

#70: $vb - pb - vatb - \mu\cdot(1 - x) = va - pa - vata - \mu\cdot x$

#71: $\mathrm{SOLVE}(vb - pb - vatb - \mu\cdot(1 - x) = va - pa - vata - \mu\cdot x,\ x)$

eq (10)

#72: $$xhat = - \frac{pa - pb - va + vata - vatb + vb - \mu}{2\cdot\mu}$$

#73: $$xhat = - \frac{pa - pb - \Delta v + vata - vatb - \mu}{2\cdot\mu}$$

eq (11) profit max w/ two stages

#74: $profita = pa\cdot n\cdot xhat$

#75: $profitb = pb\cdot n\cdot(1 - xhat)$

#76: $$profita = pa\cdot n\cdot\left(- \frac{pa - pb - \Delta v + vata - vatb - \mu}{2\cdot\mu}\right)$$

#77: $$profitb = pb\cdot n\cdot\left(1 - - \frac{pa - pb - \Delta v + vata - vatb - \mu}{2\cdot\mu}\right)$$

Appendix C, eq (C.1)

#78: $$\frac{d}{d\ pa}\left(profita = pa\cdot n\cdot\left(- \frac{pa - pb - \Delta v + vata - vatb - \mu}{2\cdot\mu}\right)\right)$$

#79: $$0 = - \frac{n\cdot(2\cdot pa - pb + vata - vatb - \Delta v - \mu)}{2\cdot\mu}$$

#80: $$\frac{d}{d\ pa}\frac{d}{d\ pa}\left(profita = pa\cdot n\cdot\left(-\frac{pa - pb - \Delta v + vata - vatb - \mu}{2\cdot\mu}\right)\right)$$

#81: $$0 > -\frac{n}{\mu}$$

#82: $$\frac{d}{d\ pb}\left(profitb = pb\cdot n\cdot\left(1 - -\frac{pa - pb - \Delta v + vata - vatb - \mu}{2\cdot\mu}\right)\right)$$

#83: $$0 = \frac{n\cdot(pa - 2\cdot pb + vata - vatb - \Delta v + \mu)}{2\cdot\mu}$$

#84: $$\frac{d}{d\ pb}\frac{d}{d\ pb}\left(profitb = pb\cdot n\cdot\left(1 - -\frac{pa - pb - \Delta v + vata - vatb - \mu}{2\cdot\mu}\right)\right)$$

#85: $$0 > -\frac{n}{\mu}$$

eq (12) eq price two stage as functions of vata and vatb

#86: $$\mathrm{SOLVE}\left(\left[0 = -\frac{n\cdot(2\cdot pa - pb + vata - vatb - \Delta v - \mu)}{2\cdot\mu},\ 0 = \frac{n\cdot(pa - 2\cdot pb + vata - vatb - \Delta v + \mu)}{2\cdot\mu}\right], [pa, pb]\right)$$

#87: $$\left[pa = -\frac{vata - vatb - \Delta v - 3\cdot\mu}{3} \wedge pb = \frac{vata - vatb - \Delta v + 3\cdot\mu}{3}\right]$$

eq (13) eql prices with two stage

#88: $vata = \tau \cdot pa$

#89: $vatb = \tau \cdot pb$

#90: $$\left[pa = -\frac{\tau \cdot pa - \tau \cdot pb - \Delta v - 3 \cdot \mu}{3} \wedge pb = \frac{\tau \cdot pa - \tau \cdot pb - \Delta v + 3 \cdot \mu}{3}\right]$$

#91: $$\text{SOLVE}\left(\left[pa = -\frac{\tau \cdot pa - \tau \cdot pb - \Delta v - 3 \cdot \mu}{3} \wedge pb = \frac{\tau \cdot pa - \tau \cdot pb - \Delta v + 3 \cdot \mu}{3}\right], [pa, pb]\right)$$

#92: $$\left[paII = \frac{\Delta v + \mu \cdot (2 \cdot \tau + 3)}{2 \cdot \tau + 3} \wedge pbII = \frac{\mu \cdot (2 \cdot \tau + 3) - \Delta v}{2 \cdot \tau + 3}\right]$$

#93: $$\left[qaII = \frac{(1 + \tau) \cdot (\Delta v + \mu \cdot (2 \cdot \tau + 3))}{2 \cdot \tau + 3} \wedge qbII = \frac{(1 + \tau) \cdot (\mu \cdot (2 \cdot \tau + 3) - \Delta v)}{2 \cdot \tau + 3}\right]$$

#94: $$\left[qaII = \frac{(\Delta v + \mu \cdot (2 \cdot \tau + 3)) \cdot (\tau + 1)}{2 \cdot \tau + 3} \wedge qbII = \frac{(\mu \cdot (2 \cdot \tau + 3) - \Delta v) \cdot (\tau + 1)}{2 \cdot \tau + 3}\right]$$

eqs (14)

#95: $$vata = \tau \cdot \frac{\Delta v + \mu \cdot (2 \cdot \tau + 3)}{2 \cdot \tau + 3}$$

#96: $$vatb = \tau \cdot \frac{\mu \cdot (2 \cdot \tau + 3) - \Delta v}{2 \cdot \tau + 3}$$

#97: $$xhatII = \frac{\Delta v + \mu \cdot (2 \cdot \tau + 3)}{2 \cdot \mu \cdot (2 \cdot \tau + 3)}$$

#98: $$\text{profitaII} = \frac{n\cdot(\Delta v + \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

#99: $$\text{profitbII} = \frac{n\cdot(\Delta v - \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

eq (15) gov't revenue with two stage decision making

#100: $$\text{gII} = \frac{n\cdot\tau\cdot(\Delta v^2 + \mu^2\cdot(2\cdot\tau + 3)^2)}{\mu\cdot(2\cdot\tau + 3)^2}$$

*** Section 4: Comparing the two pricing structure.

eq (16) and Result 2 (price comparisons)

#101: $$\text{paII} - \text{pa} = \frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3} - \frac{\Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#102: $$\text{paII} - \text{pa} = \frac{\tau\cdot(\Delta v + 3\cdot\mu\cdot(2\cdot\tau + 3))}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)} > 0$$

#103: $$\text{pbII} - \text{pb} = \frac{\mu\cdot(2\cdot\tau + 3) - \Delta v}{2\cdot\tau + 3} - \frac{3\cdot\mu - \Delta v}{3\cdot(\tau + 1)}$$

#104: $$\text{pbII} - \text{pb} = \frac{\tau\cdot(3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v)}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)}$$

> 0 if [Yes!]

#105: $3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v > 0$

#106: SOLVE($3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v > 0$, Δv)

#107: $$\Delta v < 3\cdot\mu\cdot(2\cdot\tau + 3)$$

Assumption 2 implies that $\Delta v < \mu$. Hence, it is sufficient to show

#108: $\mu < 3\cdot\mu\cdot(2\cdot\tau + 3)$

#109: SOLVE($\mu < 3\cdot\mu\cdot(2\cdot\tau + 3)$, μ)

#110: $$\mu > 0$$

Proving Result 2(b): Added to the first part of Appendix D

#111: $$\frac{d}{d\tau}\left(qaII = \frac{(\Delta v + \mu\cdot(2\cdot\tau + 3))\cdot(\tau + 1)}{2\cdot\tau + 3}\right)$$

eq (D.1)

#112: $$0 < \frac{\Delta v + \mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9)}{(2\cdot\tau + 3)^2}$$

#113: $$\frac{d}{d\tau}\left(qbII = \frac{(\mu\cdot(2\cdot\tau + 3) - \Delta v)\cdot(\tau + 1)}{2\cdot\tau + 3}\right)$$

eq (D.2)

#114: $$\frac{\mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9) - \Delta v}{(2\cdot\tau + 3)^2}$$

> 0 if

#115: $\mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9) - \Delta v > 0$

#116: $\mathrm{SOLVE}(\mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9) - \Delta v > 0, \Delta v)$

#117: $$\Delta v < \mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9)$$

eq (17) and Result 3

#118: $gII - g = \dfrac{n\cdot\tau\cdot(\Delta v^2 + \mu^2\cdot(2\cdot\tau + 3)^2)}{\mu\cdot(2\cdot\tau + 3)^2} - \dfrac{n\cdot\tau\cdot(\Delta v^2 + 9\cdot\mu^2)}{9\cdot\mu\cdot(\tau + 1)}$

#119: $$gII - g = \frac{n\cdot\tau^2\cdot(9\cdot\mu^2\cdot(2\cdot\tau + 3)^2 - \Delta v^2\cdot(4\cdot\tau + 3))}{9\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

> 0 Appendix D if

#120: $9\cdot\mu^2\cdot(2\cdot\tau + 3)^2 - \Delta v^2\cdot(4\cdot\tau + 3) > 0$

#121: $\mathrm{SOLVE}(9\cdot\mu^2\cdot(2\cdot\tau + 3)^2 - \Delta v^2\cdot(4\cdot\tau + 3) > 0, \Delta v)$

eq (D.3)

#122: $$- \frac{3\cdot\mu\cdot(2\cdot\tau + 3)}{\sqrt{(4\cdot\tau + 3)}} < \Delta v < \frac{3\cdot\mu\cdot(2\cdot\tau + 3)}{\sqrt{(4\cdot\tau + 3)}}$$

By Assumption 2, $\Delta v < \mu$, hence it is sufficient to show that: eq (D.4)

#123: $$\frac{3\cdot\mu\cdot(2\cdot\tau + 3)}{\sqrt{(4\cdot\tau + 3)}} - \mu > 0$$

or that [holds for every $\tau \geq 0$]

#124: $$\frac{3\cdot(2\cdot\tau + 3)}{\sqrt{(4\cdot\tau + 3)}} - 1 > 0$$

Result 4 and Appendix E

#125: $$profitaII - profita = \frac{n\cdot(\Delta v + \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2} - \frac{n\cdot(\Delta v + 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

#126: $$profitaII - profita = - \frac{n\cdot\tau\cdot(\Delta v^2\cdot(4\cdot\tau + 3) - 6\cdot\Delta v\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

evalutated at $\Delta v = 0$,

#127: $$profitaII - profita = - \frac{n\cdot\tau\cdot(0^2\cdot(4\cdot\tau + 3) - 6\cdot 0\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

#128: $$\text{profitaII} - \text{profita} = \frac{n\cdot\mu\cdot\tau}{2\cdot(\tau + 1)} > 0$$

#129: $$\text{profitbII} - \text{profitb} = \frac{n\cdot(\Delta v - \mu\cdot(2\cdot\tau + 3))^2}{2\cdot\mu\cdot(2\cdot\tau + 3)^2} - \frac{n\cdot(\Delta v - 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau + 1)}$$

#130: $$\text{profitbII} - \text{profitb} = - \frac{n\cdot\tau\cdot(\Delta v^2\cdot(4\cdot\tau + 3) + 6\cdot\Delta v\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

evaluated at $\Delta v = 0$,

#131: $$\text{profitbII} - \text{profitb} = - \frac{n\cdot\tau\cdot(0^2\cdot(4\cdot\tau + 3) + 6\cdot0\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

#132: $$\text{profitbII} - \text{profitb} = \frac{n\cdot\mu\cdot\tau}{2\cdot(\tau + 1)} > 0$$

evaluted at $\Delta v = \mu$ (the other extreme):

#133: $$\text{profitaII} - \text{profita} = - \frac{n\cdot\tau\cdot(\mu^2\cdot(4\cdot\tau + 3) - 6\cdot\mu\cdot\mu\cdot(2\cdot\tau + 3) - 9\cdot\mu^2\cdot(2\cdot\tau + 3)^2)}{18\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

#134: $$\text{profitaII} - \text{profita} = \frac{2\cdot n\cdot \mu\cdot \tau\cdot (9\cdot \tau^2 + 29\cdot \tau + 24)}{9\cdot (\tau + 1)\cdot (2\cdot \tau + 3)^2} > 0$$

#135: $$\text{profitbII} - \text{profitb} = - \frac{n\cdot \tau\cdot (\mu^2\cdot (4\cdot \tau + 3) + 6\cdot \mu\cdot \mu\cdot (2\cdot \tau + 3) - 9\cdot \mu^2\cdot (2\cdot \tau + 3)^2)}{18\cdot \mu\cdot (\tau + 1)\cdot (2\cdot \tau + 3)^2}$$

#136: $$\text{profitbII} - \text{profitb} = \frac{2\cdot n\cdot \mu\cdot \tau\cdot (9\cdot \tau^2 + 23\cdot \tau + 15)}{9\cdot (\tau + 1)\cdot (2\cdot \tau + 3)^2} > 0$$

so, look at the trend between Δv=0 and Δv=μ (look for monotonicity , i.e., derivative is either all >0 or all <0

#137: $$\frac{d}{d\ \Delta v}\left(\text{profitbII} - \text{profitb} = - \frac{n\cdot \tau\cdot (\Delta v^2\cdot (4\cdot \tau + 3) + 6\cdot \Delta v\cdot \mu\cdot (2\cdot \tau + 3) - 9\cdot \mu^2\cdot (2\cdot \tau + 3)^2)}{18\cdot \mu\cdot (\tau + 1)\cdot (2\cdot \tau + 3)^2}\right)$$

#138: $$- \frac{n\cdot \tau\cdot (\Delta v\cdot (4\cdot \tau + 3) + 3\cdot \mu\cdot (2\cdot \tau + 3))}{9\cdot \mu\cdot (\tau + 1)\cdot (2\cdot \tau + 3)^2} < 0$$

therefore profitbII is higher all over the range of Δv

#139: $$\frac{d}{d\ \Delta v}\left(\text{profitaII} - \text{profita} = - \frac{n\cdot \tau\cdot (\Delta v^2\cdot (4\cdot \tau + 3) - 6\cdot \Delta v\cdot \mu\cdot (2\cdot \tau + 3) - 9\cdot \mu^2\cdot (2\cdot \tau + 3)^2)}{18\cdot \mu\cdot (\tau + 1)\cdot (2\cdot \tau + 3)^2}\right)$$

#140: $$\frac{n\cdot\tau\cdot(3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v\cdot(4\cdot\tau + 3))}{9\cdot\mu\cdot(\tau + 1)\cdot(2\cdot\tau + 3)^2}$$

> 0 if

#141: $3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v\cdot(4\cdot\tau + 3) > 0$

#142: SOLVE($3\cdot\mu\cdot(2\cdot\tau + 3) - \Delta v\cdot(4\cdot\tau + 3) > 0$, Δv)

#143: $$\Delta v < \frac{3\cdot\mu\cdot(2\cdot\tau + 3)}{4\cdot\tau + 3}$$

#144: $$\frac{3\cdot\mu\cdot(2\cdot\tau + 3)}{4\cdot\tau + 3} - \mu$$

#145: $$\frac{2\cdot\mu\cdot(\tau + 3)}{4\cdot\tau + 3} > 0$$

therefore profitaII is higher all over the range of Δv

*** Section 5: Welfare analysis

eq (18), first-best x^*

#146: $va - \mu\cdot xhatstar = vb - \mu\cdot(1 - xhatstar)$

#147: SOLVE($va - \mu\cdot xhatstar = vb - \mu\cdot(1 - xhatstar)$, xhatstar)

#148: $$xhatstar = \frac{va - vb + \mu}{2\cdot\mu}$$

eq (18)

#149: $\text{xhatstar} = \dfrac{\Delta v + \mu}{2\cdot\mu}$

recall xhatI and xhatII (start deriving Result 5)

#150: $\text{xhatI} = \dfrac{\Delta v + 3\cdot\mu}{6\cdot\mu}$

#151: $\text{xhatII} = \dfrac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)}$

#152: $\text{xhatstar} - \text{xhatI} = \dfrac{\Delta v + \mu}{2\cdot\mu} - \dfrac{\Delta v + 3\cdot\mu}{6\cdot\mu}$

eq (19)

#153: $$\text{xhatstar} - \text{xhatI} = \frac{\Delta v}{3\cdot\mu}$$

#154: $\text{xhatI} - \text{xhatII} = \dfrac{\Delta v + 3\cdot\mu}{6\cdot\mu} - \dfrac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)}$

#155: $$\text{xhatI} - \text{xhatII} = \frac{\Delta v\cdot\tau}{3\cdot\mu\cdot(2\cdot\tau + 3)}$$

In-text (below Result 5), derivation of Result 5(b)

#156: $\dfrac{d}{d\tau}\left(\text{xhatI} - \text{xhatII} = \dfrac{\Delta v\cdot\tau}{3\cdot\mu\cdot(2\cdot\tau + 3)}\right)$

#157: $$0 < \frac{\Delta v}{\mu \cdot (2 \cdot \tau + 3)^2}$$

#158: $$\frac{d}{d\ \Delta v}\left(xhatI - xhatII = \frac{\Delta v \cdot \tau}{3 \cdot \mu \cdot (2 \cdot \tau + 3)}\right)$$

#159: $$0 < \frac{\tau}{3 \cdot \mu \cdot (2 \cdot \tau + 3)}$$

eq (20) total welfare

#160: $$\int_0^{xhat} (va - \mu \cdot x)\ dx$$

#161: $$\int_{xhat}^{1} (vb - \mu \cdot (1 - x))\ dx$$

#162: $$w = n \cdot \int_0^{xhat} (va - \mu \cdot x)\ dx + n \cdot \int_{xhat}^{1} (vb - \mu \cdot (1 - x))\ dx$$

#163: $$w = n \cdot \left(va \cdot xhat - \frac{2 \cdot vb \cdot (xhat - 1) + \mu \cdot (2 \cdot xhat^2 - 2 \cdot xhat + 1)}{2}\right)$$

#164: $$w = \frac{n \cdot (2 \cdot va \cdot xhat + 2 \cdot vb \cdot (1 - xhat) - \mu \cdot (2 \cdot xhat^2 - 2 \cdot xhat + 1))}{2}$$

eq (21) three welfare levels

#165:

$$wI = \frac{n\cdot\left(2\cdot va\cdot\frac{\Delta v + 3\cdot\mu}{6\cdot\mu} + 2\cdot vb\cdot\left(1 - \frac{\Delta v + 3\cdot\mu}{6\cdot\mu}\right) - \mu\cdot\left(2\cdot\left(\frac{\Delta v + 3\cdot\mu}{6\cdot\mu}\right)^2 - 2\cdot\frac{\Delta v + 3\cdot\mu}{6\cdot\mu} + 1\right)\right)}{2}$$

#166:

$$wI = \frac{n\cdot(6\cdot va\cdot(\Delta v + 3\cdot\mu) + 6\cdot vb\cdot(3\cdot\mu - \Delta v) - \Delta v^2 - 9\cdot\mu^2)}{36\cdot\mu}$$

#167:

$$wII = \frac{n\cdot\left(2\cdot va\cdot\frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)} + 2\cdot vb\cdot\left(1 - \frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)}\right) - \mu\cdot\left(2\cdot\left(\frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)}\right)^2 - 2\cdot\frac{\Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\mu\cdot(2\cdot\tau + 3)} + 1\right)\right)}{2}$$

#168:

$$wII = n\cdot\left(\frac{va\cdot(\Delta v + \mu\cdot(2\cdot\tau + 3))}{2\cdot\mu\cdot(2\cdot\tau + 3)} + \frac{vb\cdot(\mu\cdot(2\cdot\tau + 3) - \Delta v)}{2\cdot\mu\cdot(2\cdot\tau + 3)} - \frac{\Delta v^2 + \mu^2\cdot(2\cdot\tau + 3)^2}{4\cdot\mu\cdot(2\cdot\tau + 3)^2}\right)$$

#169:

$$\mathrm{wstar} = \frac{n\cdot\left(2\cdot va\cdot\frac{\Delta v + \mu}{2\cdot\mu} + 2\cdot vb\cdot\left(1 - \frac{\Delta v + \mu}{2\cdot\mu}\right) - \mu\cdot\left(2\cdot\left(\frac{\Delta v + \mu}{2\cdot\mu}\right)^2 - 2\cdot\frac{\Delta v + \mu}{2\cdot\mu} + 1\right)\right)}{2}$$

#170:

$$\mathrm{wstar} = \frac{n\cdot(2\cdot va\cdot(\Delta v + \mu) + 2\cdot vb\cdot(\mu - \Delta v) - \Delta v^2 - \mu^2)}{4\cdot\mu}$$

Result 6 and Appendix F

#171:

$$\mathrm{wstar} - \mathrm{wI} = \frac{n\cdot(2\cdot va\cdot(\Delta v + \mu) + 2\cdot vb\cdot(\mu - \Delta v) - \Delta v^2 - \mu^2)}{4\cdot\mu} - \frac{n\cdot(6\cdot va\cdot(\Delta v + 3\cdot\mu) + 6\cdot vb\cdot(3\cdot\mu - \Delta v) - \Delta v^2 - 9\cdot\mu^2)}{36\cdot\mu}$$

eq (F.1)

#172:

$$\mathrm{wstar} - \mathrm{wI} = \frac{n\cdot\Delta v\cdot(3\cdot va - 3\cdot vb - 2\cdot\Delta v)}{9\cdot\mu}$$

#173:

$$\mathrm{wI} - \mathrm{wII} = \frac{n\cdot(6\cdot va\cdot(\Delta v + 3\cdot\mu) + 6\cdot vb\cdot(3\cdot\mu - \Delta v) - \Delta v^2 - 9\cdot\mu^2)}{36\cdot\mu} - n\cdot\left(\frac{va\cdot(\Delta v + \mu\cdot(2\cdot\tau + 3))}{2\cdot\mu\cdot(2\cdot\tau + 3)} + \right.$$

$$\left.\frac{vb\cdot(\mu\cdot(2\cdot\tau + 3) - \Delta v)}{2\cdot\mu\cdot(2\cdot\tau + 3)} - \frac{\Delta v^2 + \mu^2\cdot(2\cdot\tau + 3)^2}{4\cdot\mu\cdot(2\cdot\tau + 3)^2}\right)$$

#174: $$wI - wII = \frac{n\cdot\Delta v\cdot\tau\cdot(3\cdot va\cdot(2\cdot\tau + 3) - 3\cdot vb\cdot(2\cdot\tau + 3) - \Delta v\cdot(\tau + 3))}{9\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

#175: $$wI - wII = \frac{n\cdot\Delta v\cdot\tau\cdot(3\cdot\Delta v\cdot(2\cdot\tau + 3) - \Delta v\cdot(\tau + 3))}{9\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

#176: $$wI - wII = \frac{n\cdot\Delta v^2\cdot\tau\cdot(5\cdot\tau + 6)}{9\cdot\mu\cdot(2\cdot\tau + 3)^2}$$

Second part of Result 7 (Appendix F) showing that the above gap increases (basically WII declines) with a higher τ

#177: $$\frac{d}{d\tau}\left(wI - wII = \frac{n\cdot\Delta v^2\cdot\tau\cdot(5\cdot\tau + 6)}{9\cdot\mu\cdot(2\cdot\tau + 3)^2}\right)$$

#178: $$0 < \frac{2\cdot n\cdot\Delta v^2\cdot(\tau + 1)}{\mu\cdot(2\cdot\tau + 3)^3}$$

Another way is to differentiate wII w.r.t. τ

#179: $$\frac{d}{d\tau}\left(wII = n\cdot\left(\frac{va\cdot(\Delta v + \mu\cdot(2\cdot\tau + 3))}{2\cdot\mu\cdot(2\cdot\tau + 3)} + \frac{vb\cdot(\mu\cdot(2\cdot\tau + 3) - \Delta v)}{2\cdot\mu\cdot(2\cdot\tau + 3)} - \frac{\Delta v^2 + \mu^2\cdot(2\cdot\tau + 3)^2}{4\cdot\mu\cdot(2\cdot\tau + 3)^2}\right)\right)$$

#180: $$-\frac{n\cdot\Delta v\cdot(va\cdot(2\cdot\tau + 3) - vb\cdot(2\cdot\tau + 3) - \Delta v)}{\mu\cdot(2\cdot\tau + 3)^3}$$

< 0 if [Yes]

#181: $n\cdot\Delta v\cdot(va\cdot(2\cdot\tau + 3) - vb\cdot(2\cdot\tau + 3) - \Delta v) > 0$

#182: $n\cdot\Delta v\cdot(\Delta v\cdot(2\cdot\tau + 3) - \Delta v) > 0$

#183: $$n\cdot\Delta v^2\cdot(2\cdot\tau + 2) > 0$$

xxxxxxxxxxxxxxxxxxx