vat_compete_2024_x_y.dfw

#1: CaseMode := Sensitive

#2: InputMode := Word

(transp cost, differentiation parameter): Note μ is used below for section 2. but then I switched the entire paper to λ (location)

#3: $\mu :\in \text{Real }(0, \infty)$

#4: $\lambda \in \text{Real } (0, \infty)$

rate of sales tax

#5: τ :∈ Real (0, ∞)

total consumer population

#6: n :∈ Real (0, ∞)

Unit costs

#7: ca :∈ Real [0, ∞)

#8: cb :∈ Real [0, ∞)

prices

#9: pa :∈ Real (0, ∞)

#10: pb :∈ Real (0, ∞)

#11: qa :∈ Real (0, ∞)

#12: qb :∈ Real (0, ∞)

A's market share

Date: 4/13/2025

#13: xhat :∈ Real (0, 1)

basic valuations

#14: va :∈ Real (0, ∞)

#15: vb :∈ Real (0, ∞)

#16: Δv :∈ Real [0, ∞)

fraction of slow-computing consumers

#17: $\sigma :\in \text{Real} [0, 1]$

*** Section 2: Price embedded into the price (benchmark model)

eq (1)

#18: $qa = pa \cdot (1 + \tau)$

#19: $qb = pb \cdot (1 + \tau)$

#20: SOLVE(qa = $pa \cdot (1 + \tau)$, pa)

#21: $pa = \frac{qa}{\tau + 1}$

#22: SOLVE(qb = $pb \cdot (1 + \tau)$, pb)

#23: $pb = \frac{qb}{T + 1}$

eq (2) Utility functions

#24: $va - pa \cdot (1 + \tau) - \mu \cdot x$

#25: $va - qa - \mu \cdot x$

#26:
$$vb - pb \cdot (1 + \tau) - \mu \cdot (1 - x)$$

#27:
$$vb - qb - \mu \cdot (1 - x)$$

#28:
$$va - pa \cdot (1 + \tau) - \mu \cdot x = vb - pb \cdot (1 + \tau) - \mu \cdot (1 - x)$$

#29: SOLVE(va - pa·(1 +
$$\tau$$
) - μ ·x = vb - pb·(1 + τ) - μ ·(1 - x), x)

eq (3)

#30:
$$xhat = -\frac{pa \cdot (\tau + 1) - pb \cdot (\tau + 1) - va + vb - \mu}{2 \cdot \mu}$$

#31: xhat =
$$-\frac{pa \cdot (\tau + 1) - pb \cdot (\tau + 1) - \Delta v - \mu}{2 \cdot \mu}$$

#32: xhat =
$$-\frac{\text{qa - qb - }\Delta \text{V - }\mu}{2 \cdot \mu}$$

eq (4) Profit max w.r.t. qa and qb (tax inclusive)

#33: profita =
$$(pa - ca) \cdot n \cdot xhat$$

#34: profitb = (pb - cb)
$$\cdot n \cdot (1 - xhat)$$

#35: profita =
$$\left(\frac{qa}{\tau + 1} - ca\right) \cdot n \cdot \left(-\frac{qa - qb - \Delta v - \mu}{2 \cdot \mu}\right)$$

#36: profitb =
$$\left(\frac{qb}{\tau + 1} - cb\right) \cdot n \cdot \left(1 - - \frac{qa - qb - \Delta v - \mu}{2 \cdot \mu}\right)$$

Appendix A. eq (A.1)

#37:
$$\frac{d}{d \ qa} \left(profita = \left(\frac{qa}{\tau + 1} - ca \right) \cdot n \cdot \left(- \frac{qa - qb - \Delta v - \mu}{2 \cdot \mu} \right) \right)$$

#38:
$$0 = \frac{n \cdot (ca \cdot (\tau + 1) - 2 \cdot qa + qb + \Delta v + \mu)}{2 \cdot \mu \cdot (\tau + 1)}$$

#39:
$$\frac{d}{d \ qa} \frac{d}{d \ qa} \left(profita = \left(\frac{qa}{\tau + 1} - ca \right) \cdot n \cdot \left(- \frac{qa - qb - \Delta v - \mu}{2 \cdot \mu} \right) \right)$$

#40:
$$0 > -\frac{n}{\mu \cdot (\tau + 1)}$$

#41:
$$\frac{d}{d \ qb} \left(profitb = \left(\frac{qb}{\tau + 1} - cb \right) \cdot n \cdot \left(1 - - \frac{qa - qb - \Delta v - \mu}{2 \cdot \mu} \right) \right)$$

#42:
$$0 = \frac{n \cdot (cb \cdot (\tau + 1) + qa - 2 \cdot qb - \Delta v + \mu)}{2 \cdot \mu \cdot (\tau + 1)}$$

#43:
$$\frac{d}{d \ qb} \frac{d}{d \ qb} \left(profitb = \left(\frac{qb}{\tau + 1} - cb \right) \cdot n \cdot \left(1 - - \frac{qa - qb - \Delta v - \mu}{2 \cdot \mu} \right) \right)$$

#44:
$$0 > -\frac{n}{\mu \cdot (\tau + 1)}$$

eq (5)

$$\#45: \quad \text{SOLVE} \Bigg[\Bigg[0 = \frac{ n \cdot (\text{ca} \cdot (\tau + 1) - 2 \cdot \text{qa} + \text{qb} + \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)}, \quad 0 = \frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg], \quad [\text{qa}, \text{qa}] \Bigg] \Bigg] = \frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg], \quad [\text{qa}, \text{qa}] \Bigg] \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg] \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - 2 \cdot \text{qb} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg] \Bigg[-\frac{ n \cdot (\text{cb} \cdot (\tau + 1) + \text{qa} - \Delta \text{v} + \mu)}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\tau + 1) + \text{qa} - \Delta \text{v} + \mu}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg[-\frac{ n \cdot (\tau + 1) + \text{qa} - \Delta \text{v} + \mu}{ 2 \cdot \mu \cdot (\tau + 1)} \Bigg] \Bigg] \Bigg[-\frac{$$

#46:
$$\left[qaI = \frac{2 \cdot ca \cdot (\tau + 1) + cb \cdot (\tau + 1) + \Delta v + 3 \cdot \mu}{3} \wedge qbI = \frac{ca \cdot (\tau + 1) + 2 \cdot cb \cdot (\tau + 1) - \Delta v + 3 \cdot \mu}{3} \right]$$

#47:
$$\left[pa \cdot (1 + \tau) = \frac{2 \cdot ca \cdot (\tau + 1) + cb \cdot (\tau + 1) + \Delta v + 3 \cdot \mu}{3} \wedge pb \cdot (1 + \tau) = \frac{2 \cdot ca \cdot (\tau + 1) + cb \cdot (\tau + 1) + \Delta v + 3 \cdot \mu}{3} \right]$$

$$\frac{\operatorname{ca}\cdot(\tau+1)+2\cdot\operatorname{cb}\cdot(\tau+1)-\Delta v+3\cdot\mu}{3}$$

$$\frac{\operatorname{ca}\cdot(\tau+1)+2\cdot\operatorname{cb}\cdot(\tau+1)-\Delta v+3\cdot\mu}{3},\ [\operatorname{pa},\ \operatorname{pb}]$$

#49:
$$\left[paI = \frac{2 \cdot ca \cdot (\tau + 1) + cb \cdot (\tau + 1) + \Delta v + 3 \cdot \mu}{3 \cdot (\tau + 1)} \wedge pbI = \frac{ca \cdot (\tau + 1) + 2 \cdot cb \cdot (\tau + 1) - \Delta v + 3 \cdot \mu}{3 \cdot (\tau + 1)} \right]$$

Define $\Delta c = ca - cb$

eq (6)

#50:
$$xhatI = -\frac{\Delta c \cdot (\tau + 1) - \Delta v - 3 \cdot \mu}{6 \cdot \mu}$$

#51:
$$\operatorname{profitaI} = \frac{n \cdot (\Delta c \cdot (\tau + 1) - \Delta v - 3 \cdot \mu)}{18 \cdot \mu \cdot (\tau + 1)}$$

#52:
$$\operatorname{profitbI} = \frac{n \cdot (\Delta c \cdot (\tau + 1) - \Delta v + 3 \cdot \mu)}{18 \cdot \mu \cdot (\tau + 1)}$$

Result 1a

#53:
$$\frac{d}{d\tau} \left(qaI = \frac{2 \cdot ca \cdot (\tau + 1) + cb \cdot (\tau + 1) + \Delta v + 3 \cdot \mu}{3} \right)$$

#54:
$$0 < \frac{2 \cdot \text{ca} + \text{cb}}{3}$$

#55:
$$\frac{d}{d\tau} \left(qbI = \frac{ca \cdot (\tau + 1) + 2 \cdot cb \cdot (\tau + 1) - \Delta v + 3 \cdot \mu}{3} \right)$$

#56:
$$0 < \frac{\mathsf{ca} + 2 \cdot \mathsf{cb}}{3}$$

#57:
$$\frac{d}{d\tau} \left(paI = \frac{2 \cdot ca \cdot (\tau + 1) + cb \cdot (\tau + 1) + \Delta v + 3 \cdot \mu}{3 \cdot (\tau + 1)} \right)$$

#58:
$$0 > -\frac{\Delta v + 3 \cdot \mu}{2}$$

$$3 \cdot (\tau + 1)$$

#59:
$$\frac{d}{d\tau} \left(pbI = \frac{ca \cdot (\tau + 1) + 2 \cdot cb \cdot (\tau + 1) - \Delta v + 3 \cdot \mu}{3 \cdot (\tau + 1)} \right)$$

#60: $0 > \frac{\Delta v - 3 \cdot \mu}{2}$ $3 \cdot (\tau + 1)$

by Assumption 2.

restriction xhat > 0 if

#61:
$$-\frac{\Delta c \cdot (\tau + 1) - \Delta v - 3 \cdot \mu}{6 \cdot \mu} > 0$$

#62: SOLVE
$$\left(-\frac{\Delta c \cdot (\tau + 1) - \Delta v - 3 \cdot \mu}{6 \cdot \mu} > 0, \mu\right)$$

#63:
$$\left(\mu < \frac{\Delta c \cdot (\tau + 1) - \Delta v}{3} \wedge \mu < 0\right) \vee \left(\mu > \frac{\Delta c \cdot (\tau + 1) - \Delta v}{3} \wedge \mu > 0\right)$$

#64:
$$\mu > \frac{\Delta c \cdot (\tau + 1) - \Delta v}{3}$$

restriction xhat < 1 if

#65:
$$-\frac{\Delta c \cdot (\tau + 1) - \Delta v - 3 \cdot \mu}{6 \cdot \mu} < 1$$

#66: SOLVE
$$\left(-\frac{\Delta c \cdot (\tau + 1) - \Delta v - 3 \cdot \mu}{6 \cdot \mu} < 1, \mu\right)$$

#67:
$$\left(\mu < \frac{\Delta v - \Delta c \cdot (\tau + 1)}{3} \wedge \mu < 0\right) \vee \left(\mu > \frac{\Delta v - \Delta c \cdot (\tau + 1)}{3} \wedge \mu > 0\right)$$

#68:
$$\mu > \frac{\Delta v - \Delta c \cdot (\tau + 1)}{3}$$

xhatI > 0 if

#69:
$$\Delta c \cdot (\tau + 1) - \Delta v - 3 \cdot \mu < 0$$

#70: SOLVE(
$$\Delta c \cdot (\tau + 1) - \Delta v - 3 \cdot \mu < 0, \mu$$
)

#71:
$$\mu > \frac{\Delta c \cdot (\tau + 1) - \Delta v}{3}$$

*** Section 3: Price competition with sales tax separated from price

eq (7) utility functions

buys A (fast-computing)

#72:
$$va - pa \cdot (1 + \tau) - \lambda \cdot x$$

buys A (slow computing)

#73:
$$va - pa - \lambda \cdot x$$

buy B (fast)

#74:
$$vb - pb \cdot (1 + \tau) - \lambda \cdot (1 - x)$$

buy B (slow)

#75:
$$vb - pb - \lambda \cdot (1 - x)$$

eqs (8): indiff consumers

xhat_s

#76:
$$va - pa - \lambda \cdot xhats = vb - pb - \lambda \cdot (1 - xhats)$$

#77: SOLVE(va - pa - $\lambda \cdot x$ hats = vb - pb - $\lambda \cdot (1 - x$ hats), xhats)

#78:
$$xhats = -\frac{pa - pb - va + vb - \lambda}{2 \cdot \lambda}$$

#79:
$$xhats = \frac{1}{2} - \frac{pa - pb - \Delta v}{2 \cdot \lambda}$$

xhatf

#80:
$$va - pa \cdot (1 + \tau) - \lambda \cdot x = vb - pb \cdot (1 + \tau) - \lambda \cdot (1 - x)$$

#81: SOLVE(va - pa·(1 +
$$\tau$$
) - λ ·x = vb - pb·(1 + τ) - λ ·(1 - x), x)

#82:
$$xhatf = -\frac{pa \cdot (\tau + 1) - pb \cdot (\tau + 1) - va + vb - \lambda}{2 \cdot \lambda}$$

#83:
$$xhatf = \frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda}$$

eq (9) profit max

#84: profita =
$$(pa - ca) \cdot n \cdot ((1 - \sigma) \cdot xhatf + \sigma \cdot xhats)$$

#85: profitb = (pb - cb)·n·((1 -
$$\sigma$$
)·(1 - xhatf) + σ ·(1 - xhats))

eq (10) and Appendix B: eql prices

#86: profita =
$$(pa - ca) \cdot n \cdot \left((1 - \sigma) \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{pa - pb - \Delta v}{2 \cdot \lambda} \right) \right)$$

#87: profitb =
$$(pb - cb) \cdot n \cdot \left((1 - \sigma) \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{pa - pb - \Delta v}{2 \cdot \lambda} \right) \right) \right)$$

#88:
$$\frac{d}{d pa} \left(profita = (pa - ca) \cdot n \cdot \left((1 - \sigma) \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{pa - pb - \Delta v}{2 \cdot \lambda} \right) \right) \right)$$

#89:
$$0 = -\frac{n \cdot (ca \cdot (\sigma \cdot \tau - \tau - 1) - 2 \cdot pa \cdot (\sigma \cdot \tau - \tau - 1) + pb \cdot (\sigma \cdot \tau - \tau - 1) - \Delta v - \lambda)}{2 \cdot \lambda}$$

#90:
$$\frac{d}{d pa} \frac{d}{d pa} \left(profita = (pa - ca) \cdot n \cdot \left((1 - \sigma) \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) + \sigma \cdot \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right)$$

$$\frac{pa - pb - \Delta v}{2 \cdot \lambda}$$

#91:
$$0 > \frac{n \cdot (\sigma \cdot \tau - \tau - 1)}{\lambda}$$

#92:
$$0 > \frac{n \cdot (\tau \cdot (\sigma - 1) - 1)}{\lambda}$$

#93:
$$\frac{d}{d pb} \left(profitb = (pb - cb) \cdot n \cdot \left((1 - \sigma) \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right)$$

$$\frac{pa - pb - \Delta v}{2 \cdot \lambda}$$

#94:
$$0 = -\frac{n \cdot (\mathsf{cb} \cdot (\sigma \cdot \tau - \tau - 1) + \mathsf{pa} \cdot (\sigma \cdot \tau - \tau - 1) - 2 \cdot \mathsf{pb} \cdot (\sigma \cdot \tau - \tau - 1) + \Delta \mathsf{v} - \lambda)}{2 \cdot \lambda}$$

#95:
$$\frac{d}{d \ pb} \frac{d}{d \ pb} \left(\text{profitb} = (pb - cb) \cdot n \cdot \left((1 - \sigma) \cdot \left(1 - \left(\frac{1}{2} - \frac{(pa - pb) \cdot (\tau + 1) - \Delta v}{2 \cdot \lambda} \right) \right) + \sigma \cdot \left(1 - \left(\frac{1}{2} - \frac{pa - pb - \Delta v}{2 \cdot \lambda} \right) \right) \right)$$

#96:
$$0 > \frac{n \cdot (\sigma \cdot \tau - \tau - 1)}{\lambda}$$

#97:
$$0 > \frac{n \cdot (\tau \cdot (\sigma - 1) - 1)}{\lambda}$$

#98: SOLVE
$$\left[0 = -\frac{n \cdot (\text{ca} \cdot (\sigma \cdot \tau - \tau - 1) - 2 \cdot \text{pa} \cdot (\sigma \cdot \tau - \tau - 1) + \text{pb} \cdot (\sigma \cdot \tau - \tau - 1) - \Delta v - \lambda)}{2 \cdot \lambda}, \quad 0 = -\frac{n \cdot (\text{cb} \cdot (\sigma \cdot \tau - \tau - 1) + \text{pa} \cdot (\sigma \cdot \tau - \tau - 1) - 2 \cdot \text{pb} \cdot (\sigma \cdot \tau - \tau - 1) + \Delta v - \lambda)}{2 \cdot \lambda} \right], \quad [\text{pa, pb}]$$

eq (10)

#99:
$$\left[pae = \frac{2 \cdot ca \cdot (\sigma \cdot \tau - \tau - 1) + cb \cdot (\sigma \cdot \tau - \tau - 1) - \Delta v - 3 \cdot \lambda}{3 \cdot (\sigma \cdot \tau - \tau - 1)} \wedge pbe = \frac{3 \cdot (\sigma \cdot \tau - \tau - 1)}{3 \cdot (\sigma \cdot \tau - \tau - 1)} \right]$$

$$\frac{\mathsf{ca}\cdot(\sigma\cdot\tau-\tau-1)+2\cdot\mathsf{cb}\cdot(\sigma\cdot\tau-\tau-1)+\Delta\mathsf{v}-3\cdot\lambda}{3\cdot(\sigma\cdot\tau-\tau-1)}\bigg]$$

eq (11): eql profits

#100:
$$\operatorname{profitae} = -\frac{n \cdot (\operatorname{ca} \cdot (\sigma \cdot \tau - \tau - 1) - \operatorname{cb} \cdot (\sigma \cdot \tau - \tau - 1) + \Delta v + 3 \cdot \lambda)}{18 \cdot \lambda \cdot (\sigma \cdot \tau - \tau - 1)}$$

#101:
$$\operatorname{profitbe} = -\frac{n \cdot (\operatorname{ca} \cdot (\sigma \cdot \tau - \tau - 1) - \operatorname{cb} \cdot (\sigma \cdot \tau - \tau - 1) + \Delta v - 3 \cdot \lambda)}{18 \cdot \lambda \cdot (\sigma \cdot \tau - \tau - 1)}$$

Result 2: setting $\sigma=0$

#102:
$$\left[pae_zero = \frac{2 \cdot ca \cdot (\tau + 1) + cb \cdot (\tau + 1) + \Delta v + 3 \cdot \lambda}{3 \cdot (\tau + 1)} \wedge pbe_zero = \frac{ca \cdot (\tau + 1) + 2 \cdot cb \cdot (\tau + 1) - \Delta v + 3 \cdot \lambda}{3 \cdot (\tau + 1)} \right]$$

compare it to pa_I and pb_I

#103:
$$\left[paI = \frac{2 \cdot ca \cdot (\tau + 1) + cb \cdot (\tau + 1) + \Delta v + 3 \cdot \lambda}{3 \cdot (\tau + 1)} \wedge pbI = \frac{ca \cdot (\tau + 1) + 2 \cdot cb \cdot (\tau + 1) - \Delta v + 3 \cdot \lambda}{3 \cdot (\tau + 1)} \right]$$

#104: pae_zero - paI =
$$\frac{2 \cdot \text{ca} \cdot (\tau + 1) + \text{cb} \cdot (\tau + 1) + \Delta v + 3 \cdot \lambda}{3 \cdot (\tau + 1)} - \frac{2 \cdot \text{ca} \cdot (\tau + 1) + \text{cb} \cdot (\tau + 1) + \Delta v + 3 \cdot \lambda}{3 \cdot (\tau + 1)}$$

#106: pbe_zero - pbI =
$$\frac{\text{ca} \cdot (\tau + 1) + 2 \cdot \text{cb} \cdot (\tau + 1) - \Delta v + 3 \cdot \lambda}{3 \cdot (\tau + 1)} - \frac{\text{ca} \cdot (\tau + 1) + 2 \cdot \text{cb} \cdot (\tau + 1) - \Delta v + 3 \cdot \lambda}{3 \cdot (\tau + 1)}$$

#107:

$$pbe_zero - pbI = 0$$

Result 3 & Appendix B

#108:
$$\frac{d}{d\sigma} \left(pae = \frac{2 \cdot ca \cdot (\sigma \cdot \tau - \tau - 1) + cb \cdot (\sigma \cdot \tau - \tau - 1) - \Delta v - 3 \cdot \lambda}{3 \cdot (\sigma \cdot \tau - \tau - 1)} \right)$$

eq (B.2)

#109:

$$0 < \frac{\tau \cdot (\Delta v + 3 \cdot \lambda)}{2}$$
$$3 \cdot (\sigma \cdot \tau - \tau - 1)$$

$$\#110: \ \frac{d}{d\sigma} \left(pbe \ = \ \frac{ca \cdot (\sigma \cdot \tau \ - \ \tau \ - \ 1) \ + \ 2 \cdot cb \cdot (\sigma \cdot \tau \ - \ \tau \ - \ 1) \ + \ \Delta v \ - \ 3 \cdot \lambda}{3 \cdot (\sigma \cdot \tau \ - \ \tau \ - \ 1)} \right)$$

#111:

$$0 < \frac{\tau \cdot (3 \cdot \lambda - \Delta v)}{3 \cdot (\sigma \cdot \tau - \tau - 1)}$$

eq (B.3)

#112:
$$\frac{d}{d\sigma} \left(\text{profitae} = -\frac{n \cdot (\text{ca} \cdot (\sigma \cdot \tau - \tau - 1) - \text{cb} \cdot (\sigma \cdot \tau - \tau - 1) + \Delta v + 3 \cdot \lambda)}{18 \cdot \lambda \cdot (\sigma \cdot \tau - \tau - 1)} \right)$$

#113:

$$\frac{\mathsf{n} \cdot \mathsf{T} \cdot (\mathsf{ca} \cdot (\sigma \cdot \mathsf{T} - \mathsf{T} - 1) - \mathsf{cb} \cdot (\sigma \cdot \mathsf{T} - \mathsf{T} - 1) - \Delta \mathsf{V} - 3 \cdot \lambda) \cdot (\mathsf{ca} \cdot (\sigma \cdot \mathsf{T} - \mathsf{T} - 1) - \mathsf{cb} \cdot (\sigma \cdot \mathsf{T} - \mathsf{T} - 1) + \Delta \mathsf{V} \sim}{2} \sim \\ + 3 \cdot \lambda)$$

#114:
$$\frac{d}{d\sigma} \left(\text{profitbe} = -\frac{n \cdot (\text{ca} \cdot (\sigma \cdot \tau - \tau - 1) - \text{cb} \cdot (\sigma \cdot \tau - \tau - 1) + \Delta v - 3 \cdot \lambda)}{18 \cdot \lambda \cdot (\sigma \cdot \tau - \tau - 1)} \right)$$

0 < - #115:

$$\frac{\mathsf{n} \cdot \mathsf{r} \cdot (\mathsf{ca} \cdot (\sigma \cdot \mathsf{r} - \mathsf{r} - 1) - \mathsf{cb} \cdot (\sigma \cdot \mathsf{r} - \mathsf{r} - 1) - \Delta \mathsf{v} + 3 \cdot \lambda) \cdot (\mathsf{ca} \cdot (\sigma \cdot \mathsf{r} - \mathsf{r} - 1) - \mathsf{cb} \cdot (\sigma \cdot \mathsf{r} - \mathsf{r} - 1) + \Delta \mathsf{v}}{2} \sim \\ 18 \cdot \lambda \cdot (\sigma \cdot \mathsf{r} - \mathsf{r} - 1) \sim \\ -3 \cdot \lambda)$$

compare to the way it is typed it in the paper, eqs (B.3)

#116: d_profitae_paper =
$$\frac{n \cdot \tau \cdot ((3 \cdot \lambda + \Delta v) - (\Delta c \cdot (1 + \tau - \sigma \cdot \tau)))}{2}$$

$$18 \cdot \lambda \cdot (1 + \tau - \sigma \cdot \tau)$$

compare derivatives for A: [yes!]

#117:
$$\frac{n \cdot \tau \cdot ((3 \cdot \lambda + \Delta v)^{2} - (\Delta c \cdot (1 + \tau - \sigma \cdot \tau))^{2})}{2} - \frac{2}{18 \cdot \lambda \cdot (1 + \tau - \sigma \cdot \tau)}$$

$$n \cdot \tau \cdot (ca \cdot (\sigma \cdot \tau - \tau - 1) - cb \cdot (\sigma \cdot \tau - \tau - 1) - \Delta v - 3 \cdot \lambda) \cdot (ca \cdot (\sigma \cdot \tau - \tau - 1) - cb \cdot (\sigma \cdot \tau - \tau - 1) + \Delta v \sim 2$$

+ 3·λ)

#118:
$$\frac{n \cdot \tau \cdot ((3 \cdot \lambda + \Delta v) - ((ca - cb) \cdot (1 + \tau - \sigma \cdot \tau)))}{2} - \frac{2}{18 \cdot \lambda \cdot (1 + \tau - \sigma \cdot \tau)}$$

0

+ 3·λ)

#119:

compare to the way it is typed it in the paper, eqs (B.3)

#120: d_profitbe_paper =
$$\frac{n \cdot \tau \cdot ((3 \cdot \lambda - \Delta v) - (\Delta c \cdot (1 + \tau - \sigma \cdot \tau)))}{2}$$

$$18 \cdot \lambda \cdot (1 + \tau - \sigma \cdot \tau)$$

#121:
$$\frac{n \cdot \tau \cdot ((3 \cdot \lambda - \Delta v) - (\Delta c \cdot (1 + \tau - \sigma \cdot \tau)))}{2} - \frac{2}{18 \cdot \lambda \cdot (1 + \tau - \sigma \cdot \tau)}$$

$$n \cdot \tau \cdot (ca \cdot (\sigma \cdot \tau - \tau - 1) - cb \cdot (\sigma \cdot \tau - \tau - 1) - \Delta v + 3 \cdot \lambda) \cdot (ca \cdot (\sigma \cdot \tau - \tau - 1) - cb \cdot (\sigma \cdot \tau - \tau - 1) + \Delta v \sim 2$$

$$18 \cdot \lambda \cdot (\sigma \cdot \tau - \tau - 1)$$

3·λ)

#122:
$$\frac{n \cdot \tau \cdot ((3 \cdot \lambda - \Delta v) - ((ca - cb) \cdot (1 + \tau - \sigma \cdot \tau)))}{2} - -$$

3·λ)

#123: