vat_compete_2024_x_y.dfw

#1: CaseMode := Sensitive

#2: InputMode := Word

degree of market power (transp cost)

#3: $\mu :\in \text{Real } (0, \infty)$

rate of sales tax

#4: $\tau :\in \text{Real } (0, \infty)$

total consumer population

#5: $n :\in \text{Real } (0, \infty)$

Unit costs

#6: $ca :\in \text{Real } [0, \infty)$

#7: $cb :\in \text{Real } [0, \infty)$

prices

#8: $pa :\in \text{Real } (0, \infty)$

#9: $pb :\in \text{Real } (0, \infty)$

#10: $qa :\in \text{Real } (0, \infty)$

#11: $qb :\in \text{Real } (0, \infty)$

A's market share

#12: $xhat :\in \text{Real } (0, 1)$

basic valuations

#13: $va :\in \text{Real } (0, \infty)$

#14: $vb :\in \text{Real } (0, \infty)$

#15: $\Delta v :\in \text{Real } [0, \infty)$

*** Section 2: Price embedded into the price (benchmark model)

eq (1)

#16: $qa = pa\cdot(1 + \tau)$

#17: $qb = pb\cdot(1 + \tau)$

#18: $\text{SOLVE}(qa = pa\cdot(1 + \tau), pa)$

#19: $$pa = \frac{qa}{\tau + 1}$$

#20: $\text{SOLVE}(qb = pb\cdot(1 + \tau), pb)$

#21: $$pb = \frac{qb}{\tau + 1}$$

eq (2) Utility functions

#22: $va - pa\cdot(1 + \tau) - \mu\cdot x$

#23: $va - qa - \mu\cdot x$

#24: $vb - pb\cdot(1 + \tau) - \mu\cdot(1 - x)$

#25: $vb - qb - \mu\cdot(1 - x)$

#26: $va - pa\cdot(1 + \tau) - \mu\cdot x = vb - pb\cdot(1 + \tau) - \mu\cdot(1 - x)$

#27: $\text{SOLVE}(va - pa\cdot(1 + \tau) - \mu\cdot x = vb - pb\cdot(1 + \tau) - \mu\cdot(1 - x), x)$

eq (3)

#28: $$xhat = - \frac{pa \cdot (\tau + 1) - pb \cdot (\tau + 1) - va + vb - \mu}{2 \cdot \mu}$$

#29: $$xhat = - \frac{pa \cdot (\tau + 1) - pb \cdot (\tau + 1) - \Delta v - \mu}{2 \cdot \mu}$$

#30: $$xhat = - \frac{qa - qb - \Delta v - \mu}{2 \cdot \mu}$$

eq (4) Profit max w.r.t. qa and qb (tax inclusive)

#31: $profita = (pa - ca) \cdot n \cdot xhat$

#32: $profitb = (pb - cb) \cdot n \cdot (1 - xhat)$

#33: $$profita = \left(\frac{qa}{\tau + 1} - ca \right) \cdot n \cdot \left(- \frac{qa - qb - \Delta v - \mu}{2 \cdot \mu} \right)$$

#34: $$profitb = \left(\frac{qb}{\tau + 1} - cb \right) \cdot n \cdot \left(1 - - \frac{qa - qb - \Delta v - \mu}{2 \cdot \mu} \right)$$

Appendix A. eq (A.1)

#35: $$\frac{d}{d\ qa} \left(profita = \left(\frac{qa}{\tau + 1} - ca \right) \cdot n \cdot \left(- \frac{qa - qb - \Delta v - \mu}{2 \cdot \mu} \right) \right)$$

#36: $$0 = \frac{n \cdot (ca \cdot (\tau + 1) - 2 \cdot qa + qb + \Delta v + \mu)}{2 \cdot \mu \cdot (\tau + 1)}$$

#37: $$\frac{d}{d\ qa}\frac{d}{d\ qa}\left(profita = \left(\frac{qa}{\tau + 1} - ca\right)\cdot n\cdot\left(- \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#38: $$0 > - \frac{n}{\mu\cdot(\tau + 1)}$$

#39: $$\frac{d}{d\ qb}\left(profitb = \left(\frac{qb}{\tau + 1} - cb\right)\cdot n\cdot\left(1 - - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#40: $$0 = \frac{n\cdot(cb\cdot(\tau + 1) + qa - 2\cdot qb - \Delta v + \mu)}{2\cdot\mu\cdot(\tau + 1)}$$

#41: $$\frac{d}{d\ qb}\frac{d}{d\ qb}\left(profitb = \left(\frac{qb}{\tau + 1} - cb\right)\cdot n\cdot\left(1 - - \frac{qa - qb - \Delta v - \mu}{2\cdot\mu}\right)\right)$$

#42: $$0 > - \frac{n}{\mu\cdot(\tau + 1)}$$

eq (5)

#43: $$SOLVE\left(\left[0 = \frac{n\cdot(ca\cdot(\tau + 1) - 2\cdot qa + qb + \Delta v + \mu)}{2\cdot\mu\cdot(\tau + 1)},\ 0 = \frac{n\cdot(cb\cdot(\tau + 1) + qa - 2\cdot qb - \Delta v + \mu)}{2\cdot\mu\cdot(\tau + 1)}\right], [qa, qb]\right)$$

#44: $$\left[qaI = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3} \wedge qbI = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3}\right]$$

#45: $$\left[pa\cdot(1+\tau) = \frac{2\cdot ca\cdot(\tau+1) + cb\cdot(\tau+1) + \Delta v + 3\cdot\mu}{3} \wedge pb\cdot(1+\tau) = \frac{ca\cdot(\tau+1) + 2\cdot cb\cdot(\tau+1) - \Delta v + 3\cdot\mu}{3}\right]$$

#46: $$\mathrm{SOLVE}\left(\left[pa\cdot(1+\tau) = \frac{2\cdot ca\cdot(\tau+1) + cb\cdot(\tau+1) + \Delta v + 3\cdot\mu}{3} \wedge pb\cdot(1+\tau) = \frac{ca\cdot(\tau+1) + 2\cdot cb\cdot(\tau+1) - \Delta v + 3\cdot\mu}{3}\right], [pa, pb]\right)$$

#47: $$\left[paI = \frac{2\cdot ca\cdot(\tau+1) + cb\cdot(\tau+1) + \Delta v + 3\cdot\mu}{3\cdot(\tau+1)} \wedge pbI = \frac{ca\cdot(\tau+1) + 2\cdot cb\cdot(\tau+1) - \Delta v + 3\cdot\mu}{3\cdot(\tau+1)}\right]$$

Define $\Delta c = ca - cb$

eq (6)

#48: $$xhatI = -\frac{\Delta c\cdot(\tau+1) - \Delta v - 3\cdot\mu}{6\cdot\mu}$$

#49: $$profitaI = \frac{n\cdot(\Delta c\cdot(\tau+1) - \Delta v - 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau+1)}$$

#50: $$profitbI = \frac{n\cdot(\Delta c\cdot(\tau+1) - \Delta v + 3\cdot\mu)^2}{18\cdot\mu\cdot(\tau+1)}$$

Result 1a

#51: $$\frac{d}{d\tau}\left(qaI = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3}\right)$$

#52: $$0 < \frac{2\cdot ca + cb}{3}$$

#53: $$\frac{d}{d\tau}\left(qbI = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3}\right)$$

#54: $$0 < \frac{ca + 2\cdot cb}{3}$$

#55: $$\frac{d}{d\tau}\left(paI = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}\right)$$

#56: $$0 > -\frac{\Delta v + 3\cdot\mu}{3\cdot(\tau + 1)^2}$$

#57: $$\frac{d}{d\tau}\left(pbI = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}\right)$$

#58: $$0 > \frac{\Delta v - 3\cdot\mu}{3\cdot(\tau + 1)^2}$$

by Assumption 2.

restriction xhat > 0 if

#59: $$- \frac{\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu}{6\cdot\mu} > 0$$

#60: $$\text{SOLVE}\left(- \frac{\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu}{6\cdot\mu} > 0,\ \mu\right)$$

#61: $$\left(\mu < \frac{\Delta c\cdot(\tau + 1) - \Delta v}{3} \wedge \mu < 0\right) \vee \left(\mu > \frac{\Delta c\cdot(\tau + 1) - \Delta v}{3} \wedge \mu > 0\right)$$

#62: $$\mu > \frac{\Delta c\cdot(\tau + 1) - \Delta v}{3}$$

restriction xhat < 1 if

#63: $$- \frac{\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu}{6\cdot\mu} < 1$$

#64: $$\text{SOLVE}\left(- \frac{\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu}{6\cdot\mu} < 1,\ \mu\right)$$

#65: $$\left(\mu < \frac{\Delta v - \Delta c\cdot(\tau + 1)}{3} \wedge \mu < 0\right) \vee \left(\mu > \frac{\Delta v - \Delta c\cdot(\tau + 1)}{3} \wedge \mu > 0\right)$$

#66: $$\mu > \frac{\Delta v - \Delta c\cdot(\tau + 1)}{3}$$

xhatI > 0 if

#67: $\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu < 0$

#68: $\text{SOLVE}(\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu < 0,\ \mu)$

#69: $$\mu > \frac{\Delta c\cdot(\tau + 1) - \Delta v}{3}$$

*** Section 3: Price competition with sales tax separated from price
** Subsection 3.1: Fast-computing consumers: An equivalence result

eq (7)

#70: $profita = (pa - ca)\cdot n\cdot xhat$

#71: $profitb = (pb - cb)\cdot n\cdot(1 - xhat)$

#72: $$profita = (pa - ca)\cdot n\cdot\left(- \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - va + vb - \mu}{2\cdot\mu}\right)$$

#73: $$profitb = (pb - cb)\cdot n\cdot\left(1 - - \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - va + vb - \mu}{2\cdot\mu}\right)$$

Appendix B, eq (B.1), and Result 2

#74: $$\frac{d}{d\ pa}\left(profita = (pa - ca)\cdot n\cdot\left(- \frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - va + vb - \mu}{2\cdot\mu}\right)\right)$$

#75: $$0 = \frac{n\cdot(ca\cdot(\tau + 1) - 2\cdot pa\cdot(\tau + 1) + pb\cdot(\tau + 1) + va - vb + \mu)}{2\cdot\mu}$$

#76: $$\frac{d}{d\ pa}\left(0 = \frac{n\cdot(ca\cdot(\tau + 1) - 2\cdot pa\cdot(\tau + 1) + pb\cdot(\tau + 1) + va - vb + \mu)}{2\cdot\mu}\right)$$

#77: $$0 > - \frac{n\cdot(\tau + 1)}{\mu}$$

#78: $$\frac{d}{d\ pb}\left(profitb = (pb - cb)\cdot n\cdot\left(1 - -\frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - va + vb - \mu}{2\cdot\mu}\right)\right)$$

#79: $$0 = \frac{n\cdot(cb\cdot(\tau + 1) + pa\cdot(\tau + 1) - 2\cdot pb\cdot(\tau + 1) - va + vb + \mu)}{2\cdot\mu}$$

#80: $$\frac{d}{d\ pb}\frac{d}{d\ pb}\left(profitb = (pb - cb)\cdot n\cdot\left(1 - -\frac{pa\cdot(\tau + 1) - pb\cdot(\tau + 1) - va + vb - \mu}{2\cdot\mu}\right)\right)$$

#81: $$0 > -\frac{n\cdot(\tau + 1)}{\mu}$$

#82: $$\text{SOLVE}\left(\left[0 = \frac{n\cdot(ca\cdot(\tau + 1) - 2\cdot pa\cdot(\tau + 1) + pb\cdot(\tau + 1) + va - vb + \mu)}{2\cdot\mu}, 0 = \frac{n\cdot(cb\cdot(\tau + 1) + pa\cdot(\tau + 1) - 2\cdot pb\cdot(\tau + 1) - va + vb + \mu)}{2\cdot\mu}\right], [pa, pb]\right)$$

#83: $$\left[pa = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + va - vb + 3\cdot\mu}{3\cdot(\tau + 1)} \wedge pb = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - va + vb + 3\cdot\mu}{3\cdot(\tau + 1)}\right]$$

#84: $$\left[pa = \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)} \wedge pb = \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}\right]$$

compare with paI

#85: $$\frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)} - \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#86: $$0$$

Compare with pbI

#87: $$\frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)} - \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#88: $$0$$

** Subsection3.2: Slow-computing consumers (nonequivalence result)

eq (8)

#89: $va - pa - vata - \mu\cdot x$

#90: $vb - pb - vatb - \mu\cdot(1 - x)$

#91: $va - pa - vata - \mu\cdot xhat = vb - pb - vatb - \mu\cdot(1 - xhat)$

eq (9)

#92: SOLVE(va - pa - vata - μ·xhat = vb - pb - vatb - μ·(1 - xhat), xhat)

#93: $$xhat = - \frac{pa - pb - va + vata - vatb + vb - \mu}{2\cdot\mu}$$

#94: $$xhat = - \frac{pa - pb - \Delta v + vata - vatb - \mu}{2\cdot\mu}$$

eq (10)

#95: $profita = (pa - ca)\cdot n\cdot xhat$

#96: $\text{profitb} = (\text{pb} - \text{cb})\cdot n\cdot(1 - \text{xhat})$

#97: $$\text{profita} = (\text{pa} - \text{ca})\cdot n\cdot\left(- \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)$$

#98: $$\text{profitb} = (\text{pb} - \text{cb})\cdot n\cdot\left(1 - - \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)$$

Appendix C

#99: $$\frac{d}{d\ \text{pa}}\left(\text{profita} = (\text{pa} - \text{ca})\cdot n\cdot\left(- \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#100: $$0 = \frac{n\cdot(\text{ca} - 2\cdot\text{pa} + \text{pb} - \text{vata} + \text{vatb} + \Delta v + \mu)}{2\cdot\mu}$$

#101: $$\frac{d}{d\ \text{pa}}\frac{d}{d\ \text{pa}}\left(\text{profita} = (\text{pa} - \text{ca})\cdot n\cdot\left(- \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#102: $$0 > - \frac{n}{\mu}$$

#103: $$\frac{d}{d\ \text{pb}}\left(\text{profitb} = (\text{pb} - \text{cb})\cdot n\cdot\left(1 - - \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#104: $$0 = \frac{n\cdot(\text{cb} + \text{pa} - 2\cdot\text{pb} + \text{vata} - \text{vatb} - \Delta v + \mu)}{2\cdot\mu}$$

#105: $$\frac{d}{d\ \text{pb}}\frac{d}{d\ \text{pb}}\left(\text{profitb} = (\text{pb} - \text{cb})\cdot n\cdot\left(1 - - \frac{\text{pa} - \text{pb} - \Delta v + \text{vata} - \text{vatb} - \mu}{2\cdot\mu}\right)\right)$$

#106: $$0 > - \frac{n}{\mu}$$

#107: $$\mathrm{SOLVE}\left(\left[0 = \frac{n\cdot(ca - 2\cdot pa + pb - vata + vatb + \Delta v + \mu)}{2\cdot\mu}, 0 = \frac{n\cdot(cb + pa - 2\cdot pb + vata - vatb - \Delta v + \mu)}{2\cdot\mu}\right], [pa, pb]\right)$$

eq (11)

#108: $$\left[pa = \frac{2\cdot ca + cb - vata + vatb + \Delta v + 3\cdot\mu}{3} \wedge pb = \frac{ca + 2\cdot cb + vata - vatb - \Delta v + 3\cdot\mu}{3}\right]$$

eq (12)

#109: $vata = \tau\cdot pa$

#110: $vatb = \tau\cdot pb$

#111: $$\left[pa = \frac{2\cdot ca + cb - \tau\cdot pa + \tau\cdot pb + \Delta v + 3\cdot\mu}{3} \wedge pb = \frac{ca + 2\cdot cb + \tau\cdot pa - \tau\cdot pb - \Delta v + 3\cdot\mu}{3}\right]$$

#112: $$[\,]$$

#113: $$\mathrm{SOLVE}\left(\left[pa = \frac{2\cdot ca + cb - \tau\cdot pa + \tau\cdot pb + \Delta v + 3\cdot\mu}{3}, pb = \frac{ca + 2\cdot cb + \tau\cdot pa - \tau\cdot pb - \Delta v + 3\cdot\mu}{3}\right], [pa, pb]\right)$$

#114: $$\left[paII = \frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3} \wedge pbII = \frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}\right]$$

#115: $$qaII = (1 + \tau)\cdot\frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}$$

#116: $$qbII = (1 + \tau)\cdot\frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}$$

Result 3a Appendix C, eq (C.2)

#117: $$\frac{d}{d\tau}\left(qaII = (1 + \tau)\cdot\frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}\right)$$

#118: $$0 < \frac{ca\cdot(2\cdot\tau^2 + 6\cdot\tau + 5) + 2\cdot cb\cdot(\tau^2 + 3\cdot\tau + 2) + \Delta v + \mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9)}{(2\cdot\tau + 3)^2}$$

#119: $$\frac{d}{d\tau}\left(qbII = (1 + \tau)\cdot\frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}\right)$$

#120: $$0 < \frac{2\cdot ca\cdot(\tau^2 + 3\cdot\tau + 2) + cb\cdot(2\cdot\tau^2 + 6\cdot\tau + 5) - \Delta v + \mu\cdot(4\cdot\tau^2 + 12\cdot\tau + 9)}{(2\cdot\tau + 3)^2}$$

by Assumption 2.

Result 3b Appendix C, eq (C.3)

#121: $$\frac{d}{d\tau}\left(paII = \frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}\right)$$

#122: $$0 > -\frac{ca - cb + 2\cdot\Delta v}{(2\cdot\tau + 3)^2}$$

if ca ≥ cb (Assumption 1b).

#123: $$\frac{d}{d\tau}\left(pbII = \frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}\right)$$

#124: $$0 < \frac{ca - cb + 2\cdot\Delta v}{(2\cdot\tau + 3)^2}$$

since (Assumption 1b) ca ≥ cb

eq (13)

#125: $vataII = \tau\cdot paII$

#126: $$vataII = \tau\cdot\frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}$$

#127: $vatbII = \tau\cdot pbII$

#128: $$vatbII = \tau\cdot\frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3}$$

#129: $xhat = - \frac{pa - pb - \Delta v + \tau \cdot paII - \tau \cdot pbII - \mu}{2 \cdot \mu}$

#130: $xhatII = - \frac{\Delta c \cdot (\tau + 1) - \Delta v - \mu \cdot (2 \cdot \tau + 3)}{2 \cdot \mu \cdot (2 \cdot \tau + 3)}$

#131: $profitaII = \frac{n \cdot (ca \cdot (\tau + 1) - cb \cdot (\tau + 1) - \Delta v - \mu \cdot (2 \cdot \tau + 3))^2}{2 \cdot \mu \cdot (2 \cdot \tau + 3)^2}$

#132: $profitbII = \frac{n \cdot (ca \cdot (\tau + 1) - cb \cdot (\tau + 1) - \Delta v + \mu \cdot (2 \cdot \tau + 3))^2}{2 \cdot \mu \cdot (2 \cdot \tau + 3)^2}$

#133: $profitaII = \frac{n \cdot (\Delta c \cdot (\tau + 1) - \Delta v - \mu \cdot (2 \cdot \tau + 3))^2}{2 \cdot \mu \cdot (2 \cdot \tau + 3)^2}$

#134: $profitbII = \frac{n \cdot (\Delta c \cdot (\tau + 1) - \Delta v + \mu \cdot (2 \cdot \tau + 3))^2}{2 \cdot \mu \cdot (2 \cdot \tau + 3)^2}$

*** Section 4: Cmparing market outcomes under the two pricing structures

eq (14) and Result 4

paII − paI =

#135: $$\frac{ca\cdot(\tau + 2) + cb\cdot(\tau + 1) + \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3} - \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#136: $$- \frac{\tau\cdot(ca\cdot(\tau + 1) - cb\cdot(\tau + 1) - \Delta v - 3\cdot\mu\cdot(2\cdot\tau + 3))}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)}$$

#137: $$- \frac{\tau\cdot(\Delta c\cdot(\tau + 1) - \Delta v - 3\cdot\mu\cdot(2\cdot\tau + 3))}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)} > 0$$

if Δv > 2 Δc [Assumption 1c]

pbII - pbI =

#138: $$\frac{ca\cdot(\tau + 1) + cb\cdot(\tau + 2) - \Delta v + \mu\cdot(2\cdot\tau + 3)}{2\cdot\tau + 3} - \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#139: $$\frac{\tau\cdot(ca\cdot(\tau + 1) - cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu\cdot(2\cdot\tau + 3))}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)} > 0$$

#140: $$\frac{\tau\cdot(\Delta c\cdot(\tau + 1) - \Delta v + 3\cdot\mu\cdot(2\cdot\tau + 3))}{3\cdot(\tau + 1)\cdot(2\cdot\tau + 3)} > 0$$

by assumption 2

*** Section 5: 2-stage with (no) regret

eq (15): First-stage utility

#141: $ua1 = va - pa - \mu\cdot x$

#142: $ub1 = vb - pb - \mu\cdot(1 - x)$

eq (16): Second-stage utility

#143: $uaa = va - qa$

#144: $uab = vb - qb - \mu \cdot 1$

#145: $ubb = vb - qb$

#146: $uba = va - qa - \mu \cdot 1$

eq (17): First stage xhat

#147: $va - pa - \mu \cdot x = vb - pb - \mu \cdot (1 - x)$

#148: SOLVE$(va - pa - \mu \cdot x = vb - pb - \mu \cdot (1 - x), x)$

#149: $$xhat1 = - \frac{pa - pb - va + vb - \mu}{2 \cdot \mu}$$

#150: $$xhat1 = - \frac{pa - pb - \Delta v - \mu}{2 \cdot \mu}$$

eq (18) copy from (11) special case with vata = vatb =0

#151: $$\left[pa = \frac{2 \cdot ca + cb - 0 + 0 + \Delta v + 3 \cdot \mu}{3} \wedge pb = \frac{ca + 2 \cdot cb + 0 - 0 - \Delta v + 3 \cdot \mu}{3} \right]$$

#152: $$\left[paIII = \frac{2 \cdot ca + cb + \Delta v + 3 \cdot \mu}{3} \wedge pbIII = \frac{ca + 2 \cdot cb - \Delta v + 3 \cdot \mu}{3} \right]$$

Result 5: proof and Appendix D

#153: $$qbIII = (1 + \tau) \cdot \frac{ca + 2 \cdot cb - \Delta v + 3 \cdot \mu}{3}$$

#154: $qaIII = (1 + \tau)\cdot\frac{2\cdot ca + cb + \Delta v + 3\cdot\mu}{3}$

eq (D.1) uaa ≥ uab if

#155: $va - \frac{(1 + \tau)\cdot(2\cdot ca + cb + \Delta v + 3\cdot\mu)}{3} \ge vb - \frac{(1 + \tau)\cdot(ca + 2\cdot cb - \Delta v + 3\cdot\mu)}{3} - \mu\cdot 1$

#156: $\mathrm{SOLVE}\left(va - \frac{(1 + \tau)\cdot(2\cdot ca + cb + \Delta v + 3\cdot\mu)}{3} \ge vb - \frac{(1 + \tau)\cdot(ca + 2\cdot cb - \Delta v + 3\cdot\mu)}{3} - \mu\cdot 1, \mu\right)$

#157: $\mu \ge \frac{ca\cdot(\tau + 1) - cb\cdot(\tau + 1) - 3\cdot va + 3\cdot vb + 2\cdot\Delta v\cdot(\tau + 1)}{3}$

#158: $\mu \ge \frac{\Delta c\cdot(\tau + 1) - 3\cdot\Delta v + 2\cdot\Delta v\cdot(\tau + 1)}{3}$

eq (D.2) if

#159: $\mu \ge \frac{\Delta c\cdot(\tau + 1) + \Delta v\cdot(2\cdot\tau - 1)}{3}$

now by Assumption 2: μ > Δv, so it is sufficient to show

#160: $\Delta v - \frac{\Delta c\cdot(\tau + 1) + \Delta v\cdot(2\cdot\tau - 1)}{3}$

#161: $-\frac{\Delta c\cdot(\tau + 1) + 2\cdot\Delta v\cdot(\tau - 2)}{3}$

show that this is > 0 if

#162: $\Delta c\cdot(\tau + 1) + 2\cdot\Delta v\cdot(\tau - 2) < 0$

#163: $\mathrm{SOLVE}(\Delta c\cdot(\tau + 1) + 2\cdot\Delta v\cdot(\tau - 2) < 0, \Delta v)$

#164: $$\mathrm{IF}\left(\tau < 2, \Delta v > \frac{\Delta c\cdot(\tau + 1)}{2\cdot(2 - \tau)}\right) \vee \mathrm{IF}\left(\tau > 2, \Delta v < \frac{\Delta c\cdot(\tau + 1)}{2\cdot(2 - \tau)}\right)$$

#165: $\Delta v > \dfrac{\Delta c\cdot(\tau + 1)}{2\cdot(2 - \tau)}$

now by Assumption 1c Δv > 2 Δc

#166: $2\cdot\Delta c - \dfrac{\Delta c\cdot(\tau + 1)}{2\cdot(2 - \tau)}$

#167: $$\Delta c\cdot\left(\frac{3}{2\cdot(\tau - 2)} + \frac{5}{2}\right)$$

#168: $$\Delta c\cdot\left(\frac{1.5}{\tau - 2} + 2.5\right)$$

eq (D.3) ubb ≥ uba if

#169: $vb - (1 + \tau)\cdot\dfrac{ca + 2\cdot cb - \Delta v + 3\cdot\mu}{3} \ge va - (1 + \tau)\cdot\dfrac{2\cdot ca + cb + \Delta v + 3\cdot\mu}{3} - \mu\cdot 1$

eq (D.4) if

#170: $\mathrm{SOLVE}\left(vb - (1 + \tau)\cdot\dfrac{ca + 2\cdot cb - \Delta v + 3\cdot\mu}{3} \ge va - (1 + \tau)\cdot\dfrac{2\cdot ca + cb + \Delta v + 3\cdot\mu}{3} - \mu\cdot 1, \mu\right)$

#171: $$\mu \ge -\frac{ca\cdot(\tau + 1) - cb\cdot(\tau + 1) - 3\cdot va + 3\cdot vb + 2\cdot\Delta v\cdot(\tau + 1)}{3}$$

#172: $$\mu \ge -\frac{\Delta c\cdot(\tau + 1) - 3\cdot\Delta v + 2\cdot\Delta v\cdot(\tau + 1)}{3}$$

#173: $$\mu \ge -\frac{\Delta c\cdot(\tau + 1) + \Delta v\cdot(2\cdot\tau - 1)}{3}$$

by Assumption 2, $\mu > \Delta v$, so it is sufficient to show

#174: $$\Delta v - -\frac{\Delta c\cdot(\tau + 1) - 3\cdot\Delta v + 2\cdot\Delta v\cdot(\tau + 1)}{3}$$

#175: $$\frac{(\Delta c + 2\cdot\Delta v)\cdot(\tau + 1)}{3} > 0$$

eq (19)

#176: $$paIII - paI = \frac{2\cdot ca + cb + \Delta v + 3\cdot\mu}{3} - \frac{2\cdot ca\cdot(\tau + 1) + cb\cdot(\tau + 1) + \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#177: $$paIII - paI = \frac{\tau\cdot(\Delta v + 3\cdot\mu)}{3\cdot(\tau + 1)} > 0$$

#178: $$pbIII - pbI = \frac{ca + 2\cdot cb - \Delta v + 3\cdot\mu}{3} - \frac{ca\cdot(\tau + 1) + 2\cdot cb\cdot(\tau + 1) - \Delta v + 3\cdot\mu}{3\cdot(\tau + 1)}$$

#179: $$pbIII - pbI = \frac{\tau\cdot(3\cdot\mu - \Delta v)}{3\cdot(\tau + 1)}$$