whistle_2024_mm_dd

#1: CaseMode ≔ Sensitive

#2: InputMode ≔ Word

reputation parameter (loss of profits)

#3: $\rho :\in$ Real $(0, \infty)$

producer index: reputation level loss

#4: $t :\in$ Real $(0, 1)$

concavity/convexity of WBs utility function => not used

#5: $\gamma :\in$ Real $(0, \infty)$

Failure probabilities

#6: $\phi r :\in$ Real $(0, 1)$

#7: $\phi s :\in$ Real $(0, 1)$

damage to consumers

#8: $\delta :\in$ Real $(0, \infty)$

price

#9: $p :\in$ Real $[0, \infty)$

production costs (safer and riskier)

#10: $cs :\in$ Real $(0, \infty)$

#11: $cr :\in$ Real $(0, \infty)$

penalty on product failure

#12: $n :\in \text{Real } [0, \infty)$

WB burden parameter

#13: $\beta :\in \text{Real } (0, \infty)$

*** Section 3

eq (1) Utility

operational product (nondefective)

#14: $v - p$

defective

#15: $-p - \delta$

** Subsection 3.1: Production and profit

eq (2): profits

#16: $profits = p - cs$

#17: $profitr = p - cr$

** Subsection 3.2: Optimal production and safety w/o WB

eq (3) exp total surplus

#18: $ets = (1 - \phi s)\cdot v - cs - \phi s\cdot \delta$

#19: $etr = (1 - \phi r)\cdot v - cr - \phi r\cdot \delta$

*** Section 4: Whistleblowers

eq (4) Reproduction as safe using WB info

#20: $etrsw = (1 - \phi s)\cdot v - cr - cs - \phi s\cdot \delta$

eq (5) Restricting etrsw > etr => reproduction of a risky product is benefical

#21: $(1 - \phi s)\cdot v - cr - cs - \phi s\cdot\delta > (1 - \phi r)\cdot v - cr - \phi r\cdot\delta$

#22: $\text{SOLVE}((1 - \phi s)\cdot v - cr - cs - \phi s\cdot\delta > (1 - \phi r)\cdot v - cr - \phi r\cdot\delta, \delta)$

#23: $$\text{IF}\left(\phi r - \phi s < 0, \delta < \frac{cs + v\cdot(\phi s - \phi r)}{\phi r - \phi s}\right) \vee \text{IF}\left(\phi r - \phi s > 0, \delta > \frac{cs + v\cdot(\phi s - \phi r)}{\phi r - \phi s}\right)$$

#24: $\delta w = \dfrac{cs + v\cdot(\phi s - \phi r)}{\phi r - \phi s}$

#25: $$\delta w = \frac{cs}{\phi r - \phi s} - v$$

eq (5) Restricting ets > 0 => production of the safe product is benefical

#26: $(1 - \phi s)\cdot v - cs - \phi s\cdot\delta > 0$

#27: $\text{SOLVE}((1 - \phi s)\cdot v - cs - \phi s\cdot\delta > 0, \delta)$

#28: $$\delta < - \frac{cs + v\cdot(\phi s - 1)}{\phi s}$$

#29: $\delta max < - \dfrac{cs + v\cdot(\phi s - 1)}{\phi s}$

#30: $$\delta max = \frac{v\cdot(1 - \phi s)}{\phi s} - \frac{cs}{\phi s}$$

δmax = 0 if (Figure 1) [Yes]

#31: $0 = \frac{v\cdot(1 - \phi s)}{\phi s} - \frac{cs}{\phi s}$

#32: $\text{SOLVE}\left(0 = \frac{v\cdot(1 - \phi s)}{\phi s} - \frac{cs}{\phi s}, v\right)$

#33: $$v = \frac{cs}{1 - \phi s}$$

#34: $\frac{cs}{\phi r - \phi s} - \frac{cs}{1 - \phi s}$

#35: $$\frac{cs\cdot(\phi r - 1)}{(\phi r - \phi s)\cdot(\phi s - 1)} > 0$$

eq (6) WB utility functions

#36: $u = m - \beta\cdot b$

#37: $0 = m - \beta\cdot b$

#38: $\text{SOLVE}(0 = m - \beta\cdot b, b)$

eq (7) bhat

#39: $$bhat = \frac{m}{\beta}$$

*** Section 5: Optimal compensation

eq (8) social gain from acting on WB info

#40: $etrsw - etr = ((1 - \phi s)\cdot v - cr - cs - \phi s\cdot\delta) - ((1 - \phi r)\cdot v - cr - \phi r\cdot\delta)$

#41: $$etrsw - etr = -cs + v\cdot(\phi r - \phi s) + \delta\cdot(\phi r - \phi s)$$

#42: $$etrsw - etr = (v + \delta)\cdot(\phi r - \phi s) - cs$$

eq (9) expected benefit max problem

#43: $ebw = bhat\cdot(etrsw - etr - m)$

#44: $$ebw = \frac{m}{\beta}\cdot(((v + \delta)\cdot(\phi r - \phi s) - cs) - m)$$

#45: $$ebw = \frac{m}{\beta}\cdot(-cs - m + (v + \delta)\cdot(\phi r - \phi s))$$

eq (11) m* and bhat* and Appendix 1

#46: $$\frac{d}{dm}\left(ebw = \frac{m}{\beta}\cdot(-cs - m + (v + \delta)\cdot(\phi r - \phi s))\right)$$

eq (A.1)

#47: $$0 = -\frac{cs + 2\cdot m + (v + \delta)\cdot(\phi s - \phi r)}{\beta}$$

#48: $$\frac{d}{dm}\frac{d}{dm}\left(ebw = \frac{m}{\beta}\cdot(-cs - m + (v + \delta)\cdot(\phi r - \phi s))\right)$$

#49: $$0 > -\frac{2}{\beta}$$

#50: $$\mathrm{SOLVE}\left(0 = -\frac{cs + 2\cdot m + (v + \delta)\cdot(\phi s - \phi r)}{\beta}, m\right)$$

eq (11)

#51: $$mstar = - \frac{cs + (v + \delta)\cdot(\phi s - \phi r)}{2}$$

#52: $$bhat = - \frac{cs + (v + \delta)\cdot(\phi s - \phi r)}{2\cdot\beta}$$

Assumption 5 and eq (10) (restriction on β sufficiently high) [also in Appendix A]

bhat < 1 if

#53: $- (cs + (v + \delta)\cdot(\phi s - \phi r)) < 2\cdot\beta$

#54: SOLVE($- (cs + (v + \delta)\cdot(\phi s - \phi r)) < 2\cdot\beta$, β)

#55: $$\beta > - \frac{cs + (v + \delta)\cdot(\phi s - \phi r)}{2}$$

eq (10)

#56: $$\beta w = - \frac{cs + (v + \delta)\cdot(\phi s - \phi r)}{2}$$

*** Section 6: WB as deterrence

eq (12) profits

#57: profits = p − cs

#58: profitsr = p − cr − bhat·cs

Result 2 and equation (13)

#59: p − cs > p − cr − bhat·cs

#60: SOLVE(p − cs > p − cr − bhat·cs, bhat)

#61: $$\text{bhat} > \frac{\text{cs} - \text{cr}}{\text{cs}}$$

#62: $$\text{bhatd} = 1 - \frac{\text{cr}}{\text{cs}}$$

#63: $$1 - \frac{\text{cr}}{\text{cs}} = \frac{\text{m}}{\beta}$$

#64: $$\text{SOLVE}\left(1 - \frac{\text{cr}}{\text{cs}} = \frac{\text{m}}{\beta}, \text{m}\right)$$

#65: $$\text{md} = \frac{\beta\cdot(\text{cs} - \text{cr})}{\text{cs}}$$

eq (14): md > mstar if

#66: $$\frac{\beta\cdot(\text{cs} - \text{cr})}{\text{cs}} > - \frac{\text{cs} + (\text{v} + \delta)\cdot(\phi\text{s} - \phi\text{r})}{2}$$

#67: $$\text{SOLVE}\left(\frac{\beta\cdot(\text{cs} - \text{cr})}{\text{cs}} > - \frac{\text{cs} + (\text{v} + \delta)\cdot(\phi\text{s} - \phi\text{r})}{2}, \beta\right)$$

#68: $$\text{IF}\left(\text{cr} - \text{cs} < 0, \beta > \frac{\text{cs}\cdot(\text{cs} + (\text{v} + \delta)\cdot(\phi\text{s} - \phi\text{r}))}{2\cdot(\text{cr} - \text{cs})}\right) \vee \text{IF}\left(\text{cr} - \text{cs} > 0, \beta <\right.$$

$$\left.\frac{cs\cdot(cs + (v + \delta)\cdot(\phi s - \phi r))}{2\cdot(cr - cs)}\right)$$

#69: $$\beta > \frac{cs\cdot(cs + (v + \delta)\cdot(\phi s - \phi r))}{2\cdot(cr - cs)}$$

#70: $$\beta d = \frac{cs\cdot(cs + (v + \delta)\cdot(\phi s - \phi r))}{2\cdot(cr - cs)}$$

Below eq (14) it is argued that βd > βw (specified in (10)

#71: $$\frac{cs\cdot(cs + (v + \delta)\cdot(\phi s - \phi r))}{2\cdot(cr - cs)} - - \frac{cs + (v + \delta)\cdot(\phi s - \phi r)}{2}$$

#72: $$\frac{cr\cdot(cs + (v + \delta)\cdot(\phi s - \phi r))}{2\cdot(cr - cs)} > 0$$

by Assumption 3: ETs > 0 (or δ < δw)

*** Section 7: Reputation and heterogeneous producers

eq (15): profits

#73: $eprofits = p - cs - \phi s\cdot\rho\cdot f$

#74: $eprofitr = p - cr - (1 - bhat)\cdot\phi r\cdot\rho\cdot f - bhat\cdot(cs + \phi s\cdot\rho\cdot f)$

eq (16) fhat

eprofitr > eprofits if

#75: $p - cr - (1 - bhat)\cdot\phi r\cdot\rho\cdot f - bhat\cdot(cs + \phi s\cdot\rho\cdot f) > p - cs - \phi s\cdot\rho\cdot f$

#76: $SOLVE(p - cr - (1 - bhat)\cdot\phi r\cdot\rho\cdot f - bhat\cdot(cs + \phi s\cdot\rho\cdot f) > p - cs - \phi s\cdot\rho\cdot f, f)$

$$\#77:\quad \text{IF}\left(\text{bhat}\cdot(\phi r - \phi s) - \phi r + \phi s < 0,\ f < \frac{\text{bhat}\cdot cs + cr - cs}{\rho\cdot(\text{bhat} - 1)\cdot(\phi r - \phi s)}\right) \vee \text{IF}\left(\text{bhat}\cdot(\phi r - \phi s) - \phi r + \phi s > 0,\ f > \frac{\text{bhat}\cdot cs + cr - cs}{\rho\cdot(\text{bhat} - 1)\cdot(\phi r - \phi s)}\right)$$

$$\#78:\quad f < \frac{\text{bhat}\cdot cs + cr - cs}{\rho\cdot(\text{bhat} - 1)\cdot(\phi r - \phi s)}$$

$$\#79:\quad \text{fhat} = \frac{\text{bhat}\cdot cs + cr - cs}{\rho\cdot(\text{bhat} - 1)\cdot(\phi r - \phi s)}$$

$$\#80:\quad \text{fhat} = \frac{-\ \text{bhat}\cdot cs - cr + cs}{\rho\cdot(1 - \text{bhat})\cdot(\phi r - \phi s)}$$

eq (16) fhat

$$\#81:\quad \text{fhat} = \frac{(1 - \text{bhat})\cdot cs - cr}{\rho\cdot(1 - \text{bhat})\cdot(\phi r - \phi s)}$$

Deriving Assumption 6: fhat < 1 if

$$\#82:\quad \text{fhat} = \frac{0\cdot cs + cr - cs}{\rho\cdot(0 - 1)\cdot(\phi r - \phi s)}$$

$$\#83:\quad \text{fhat} = \frac{cr - cs}{\rho\cdot(\phi s - \phi r)}$$

$$\#84:\quad \text{fhat} = \frac{cs - cr}{\rho\cdot(\phi r - \phi s)}$$

< 1 if

#85: $cs - cr < \rho \cdot (\phi r - \phi s)$

back to eq (16)

#86: $$fhat = \frac{\left(1 - \frac{m}{\beta}\right) \cdot cs - cr}{\rho \cdot \left(1 - \frac{m}{\beta}\right) \cdot (\phi r - \phi s)}$$

#87: $$fhat = \frac{cr \cdot \beta + cs \cdot (m - \beta)}{\rho \cdot (m - \beta) \cdot (\phi r - \phi s)}$$

#88: $$fhat = \frac{- cr \cdot \beta + cs \cdot (\beta - m)}{\rho \cdot (\beta - m) \cdot (\phi r - \phi s)}$$

fhat > 0 if

#89: $- cr \cdot \beta + cs \cdot (\beta - m) > 0$

#90: $\mathrm{SOLVE}(- cr \cdot \beta + cs \cdot (\beta - m) > 0, m)$

#91: $$m < \frac{\beta \cdot (cs - cr)}{cs}$$

Result 4 and Appendix B
eq (B.1)

#92: $$\frac{d}{dm} \left(fhat = \frac{- cr \cdot \beta + cs \cdot (\beta - m)}{\rho \cdot (\beta - m) \cdot (\phi r - \phi s)} \right)$$

#93: $$0 > \frac{cr\cdot\beta}{\rho\cdot(m - \beta)^2\cdot(\phi s - \phi r)}$$

eq (B.2)

#94: $$\frac{d}{dm}\frac{d}{dm}\left(fhat = \frac{- cr\cdot\beta + cs\cdot(\beta - m)}{\rho\cdot(\beta - m)\cdot(\phi r - \phi s)}\right)$$

#95: $$0 > \frac{2\cdot cr\cdot\beta}{\rho\cdot(m - \beta)^3\cdot(\phi r - \phi s)}$$

eq (17) social gain from WB

#96: $$\left(v + \delta + \frac{\rho\cdot fhat}{2}\right)\cdot(\phi r - \phi s) - cs - m$$

eq (18) EB_W (maximization problem)

#97: $$ebw = bhat\cdot\left(\left(v + \delta + \frac{\rho\cdot fhat}{2}\right)\cdot(\phi r - \phi s) - cs - m\right)$$

#98: $$ebw = \frac{m}{\beta}\cdot\left(\left(v + \delta + \frac{\rho\cdot\frac{- cr\cdot\beta + cs\cdot(\beta - m)}{\rho\cdot(\beta - m)\cdot(\phi r - \phi s)}}{2}\right)\cdot(\phi r - \phi s) - cs - m\right)$$

#99: $$ebw = \frac{m}{\beta}\cdot\left(\left(v + \delta + \frac{cr\cdot\beta + cs\cdot(m - \beta)}{2\cdot(m - \beta)\cdot(\phi r - \phi s)}\right)\cdot(\phi r - \phi s) - cs - m\right)$$

#100: $$\mathrm{ebw} = \frac{m}{\beta}\cdot\left(\left(v + \delta + \frac{-\,cr\cdot\beta + cs\cdot(\beta - m)}{2\cdot(\beta - m)\cdot(\phi r - \phi s)}\right)\cdot(\phi r - \phi s) - cs - m\right)$$

Appendix C showing strict concavity of EB_W

#101: $$\frac{d}{dm}\left(\mathrm{ebw} = \frac{m}{\beta}\cdot\left(\left(v + \delta + \frac{cr\cdot\beta + cs\cdot(m - \beta)}{2\cdot(m - \beta)\cdot(\phi r - \phi s)}\right)\cdot(\phi r - \phi s) - cs - m\right)\right)$$

#102: $$0 = -\,\frac{cr\cdot\beta^{2} + (cs + 2\cdot(2\cdot m + v\cdot(\phi s - \phi r) + \delta\cdot(\phi s - \phi r)))\cdot(m^{2} - 2\cdot m\cdot\beta + \beta^{2})}{2\cdot\beta\cdot(m - \beta)^{2}}$$

#103: $$0 = -\,\frac{cr\cdot\beta^{2} + (cs + 2\cdot(2\cdot m + v\cdot(\phi s - \phi r) + \delta\cdot(\phi s - \phi r)))\cdot(m - \beta)^{2}}{2\cdot\beta\cdot(m - \beta)^{2}}$$

proving strict concavity

#104: $$\frac{d}{dm}\,\frac{d}{dm}\left(\mathrm{ebw} = \frac{m}{\beta}\cdot\left(\left(v + \delta + \frac{cr\cdot\beta + cs\cdot(m - \beta)}{2\cdot(m - \beta)\cdot(\phi r - \phi s)}\right)\cdot(\phi r - \phi s) - cs - m\right)\right)$$

#105: $$\frac{cr\cdot\beta^{2} - 2\cdot(m^{3} - 3\cdot m^{2}\cdot\beta + 3\cdot m\cdot\beta^{2} - \beta^{3})}{\beta\cdot(m - \beta)^{3}}$$

#106: $$0 > \frac{cr\cdot\beta^{2} - 2\cdot(m - \beta)^{3}}{\beta\cdot(m - \beta)^{3}}$$

Below: trying to extract m from the FOC ==> ugly, not useable!

#107: $cr\cdot\beta^2 + (cs + 2\cdot(2\cdot m + v\cdot(\phi s - \phi r) + \delta\cdot(\phi s - \phi r)))\cdot(m^2 - 2\cdot m\cdot\beta + \beta^2) = 0$

#108: $\mathrm{SOLVE}(cr\cdot\beta^2 + (cs + 2\cdot(2\cdot m + v\cdot(\phi s - \phi r) + \delta\cdot(\phi s - \phi r)))\cdot(m^2 - 2\cdot m\cdot\beta + \beta^2) = 0,\ m)$

#109:

$$m = \frac{\left|cs - 2\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi s))\right|\cdot \mathrm{COS}\left(\frac{\mathrm{ACOS}\left(-\frac{216\cdot cr\cdot\beta^2 + cs^3 - 6\cdot cs^2\cdot(v\cdot(\phi r - \phi s) - \sim}{\sim}\right)\sim}{\sim}\right)\sim}{\sim}$$

$$\frac{\frac{2\cdot\beta + \delta\cdot(\phi r - \phi s)) + 12\cdot cs\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi s))^2 - 8\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi\sim}{\left|cs - 2\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi s))\right|^3}\sim}{3}\sim$$

$$\frac{\sim}{6}\sim$$

$$s))^{3}$$

$$- \frac{cs - 2\cdot(v\cdot(\phi r - \phi s) + 4\cdot\beta + \delta\cdot(\phi r - \phi s))}{12} \vee m = -$$

$$\frac{\left|cs - 2\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi s))\right|\cdot \mathrm{SIN}\left(\frac{\mathrm{ASIN}\left(\frac{216\cdot cr\cdot\beta^{2} + cs^{3} - 6\cdot cs^{2}\cdot(v\cdot(\phi r - \phi s) - 2\cdot \sim}{\sim}\right.}{\sim}\right.}{\sim}$$

$$\frac{\frac{\beta + \delta\cdot(\phi r - \phi s)) + 12\cdot cs\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi s))^{2} - 8\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi s)\sim}{\left|cs - 2\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi s))\right|^{3}}\sim}{3}\sim}{6}\sim$$

$$\left.\left.\frac{\left.\frac{)^{3}}{\quad}\right)}{\quad} + \frac{\pi}{3}\right)\right/ \quad - \frac{cs - 2\cdot(v\cdot(\phi r - \phi s) + 4\cdot\beta + \delta\cdot(\phi r - \phi s))}{12} \lor m =$$

$$\frac{|cs - 2\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi s))|\cdot \mathrm{SIN}\left(\frac{\mathrm{ASIN}\left(\frac{216\cdot cr\cdot\beta^{2} + cs^{3} - 6\cdot cs^{2}\cdot(v\cdot(\phi r - \phi s) - 2\cdot \sim}{\sim}\right.}{\sim}\right.}{\sim}$$

$$\frac{\frac{\beta + \delta\cdot(\phi r - \phi s)) + 12\cdot cs\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi s))^{2} - 8\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi s)\sim}{|cs - 2\cdot(v\cdot(\phi r - \phi s) - 2\cdot\beta + \delta\cdot(\phi r - \phi s))|^{3}} \sim}{\frac{3}{6}} \sim$$

$$\left.\left.\frac{\left.\frac{)^{3}}{\quad}\right)}{\quad}\right)}{\quad} - \frac{cs - 2\cdot(v\cdot(\phi r - \phi s) + 4\cdot\beta + \delta\cdot(\phi r - \phi s))}{12}$$