whistle_2024_mm_dd

#1: CaseMode ≔ Sensitive

#2: InputMode ≔ Word

Failure probabilities

#3: φr ∈ Real (0, 1)

#4: φs ∈ Real (0, 1)

damage to consumers

#5: δ ∈ Real (0, ∞)

price

#6: p ∈ Real [0, ∞)

production costs (safer and riskier)

#7: cs ∈ Real (0, ∞)

#8: cr ∈ Real (0, ∞)

penalty on product failure

#9: n ∈ Real [0, ∞)

prob whistleblower emerges

#10: β ∈ Real (0, 1)

*** Section 3

eq (1) Utility

operational product (nondefective)

#11: v − p

defective

#12: −p − δ

** Subsection 3.1: Production and profit

eq (2): profits

#13: profits = p − cs

#14: profitr = p − cr

** Subsection 3.2: Optimal production and safety w/o WB

eq (3) exp total surplus

#15: ets = (1 − ϕs)·v − cs − ϕs·δ

#16: etr = (1 − ϕr)·v − cr − ϕr·δ

eq (4) δs

#17: (1 − ϕs)·v − cs − ϕs·δ ≥ (1 − ϕr)·v − cr − ϕr·δ

#18: SOLVE((1 − ϕs)·v − cs − ϕs·δ ≥ (1 − ϕr)·v − cr − ϕr·δ, δ)

#19: $$\text{IF}\left(\phi r - \phi s < 0,\ \delta \le \frac{cr - cs + v\cdot(\phi r - \phi s)}{\phi s - \phi r}\right) \vee \text{IF}\left(\phi r - \phi s > 0,\ \delta \ge \frac{cr - cs + v\cdot(\phi r - \phi s)}{\phi s - \phi r}\right)$$

#20: $$\delta \ge \frac{cr - cs + v\cdot(\phi r - \phi s)}{\phi s - \phi r}$$

#21: $$\delta s = \frac{cr - cs + v\cdot(\phi r - \phi s)}{\phi s - \phi r}$$

#22: $$\delta s = \frac{cr - cs}{\phi s - \phi r} - v$$

eq (5): δmax

#23: $(1 - \phi s)\cdot v - cs - \phi s\cdot\delta \geq 0$

#24: SOLVE$((1 - \phi s)\cdot v - cs - \phi s\cdot\delta \geq 0, \delta)$

#25: $$\delta \leq -\frac{cs + v\cdot(\phi s - 1)}{\phi s}$$

#26: $$\delta max = -\frac{cs + v\cdot(\phi s - 1)}{\phi s}$$

#27: $$\delta max = \frac{v\cdot(1 - \phi s)}{\phi s} - \frac{cs}{\phi s}$$

*** Section 4: Whistleblowers

** Subsection 4.2: Social value of WB

eq (6) total surplus after WB reproduction

#28: $etrsw = (1 - \phi s)\cdot v - cr - cs - \phi s\cdot\delta$

eq (7) deriving δw

#29: $(1 - \phi s)\cdot v - cr - cs - \phi s\cdot\delta \geq (1 - \phi r)\cdot v - cr - \phi r\cdot\delta$

#30: SOLVE$((1 - \phi s)\cdot v - cr - cs - \phi s\cdot\delta \geq (1 - \phi r)\cdot v - cr - \phi r\cdot\delta, \delta)$

#31: $$\text{IF}\left(\phi r - \phi s < 0, \delta \leq \frac{cs + v\cdot(\phi s - \phi r)}{\phi r - \phi s}\right) \vee \text{IF}\left(\phi r - \phi s > 0, \delta \geq \frac{cs + v\cdot(\phi s - \phi r)}{\phi r - \phi s}\right)$$

$$\#32:\quad \delta \geq \frac{cs + v\cdot(\phi s - \phi r)}{\phi r - \phi s}$$

$$\#33:\quad \delta w = \frac{cs + v\cdot(\phi s - \phi r)}{\phi r - \phi s}$$

$$\#34:\quad \delta w = \frac{cs}{\phi r - \phi s} - v$$

eq (8): Social value of WB: W

$$\#35:\quad w = (1 - \phi s)\cdot v - cr - cs - \phi s\cdot\delta - ((1 - \phi r)\cdot v - cr - \phi r\cdot\delta)$$

$$\#36:\quad w = -cs + v\cdot(\phi r - \phi s) + \delta\cdot(\phi r - \phi s)$$

$$\#37:\quad w = v\cdot(\phi r - \phi s) + \delta\cdot(\phi r - \phi s) - cs$$

** Subsection 4.3: Effects of WB on product satefy

eq (9): profit with whistleblowers (2 possibilities).

$$\#38:\quad profitrw = profitr = p - cr$$

$$\#39:\quad profitrw = profitrsw = p - cr - cs$$

eq (10) deriving βs

$$\#40:\quad p - cs \geq \beta\cdot(p - cr - cs) + (1 - \beta)\cdot(p - cr)$$

$$\#41:\quad \text{SOLVE}(p - cs \geq \beta\cdot(p - cr - cs) + (1 - \beta)\cdot(p - cr), \beta)$$

$$\#42:\quad \beta \geq \frac{cs - cr}{cs}$$

$$\#43:\quad \beta s = \frac{cs - cr}{cs}$$

$$\#44:\quad \beta s = 1 - \frac{cr}{cs}$$

char Figure 3

$$\#45:\quad \frac{d}{d\,cs}\left(\beta s = \frac{cs - cr}{cs}\right)$$

$$\#46:\quad 0 < \frac{cr}{2\,cs}$$

$$\#47:\quad \frac{d}{d\,cs}\,\frac{d}{d\,cs}\left(\beta s = \frac{cs - cr}{cs}\right)$$

$$\#48:\quad 0 > -\frac{2\cdot cr}{3\,cs}$$

** Subsection 4.4: WB effects on profit, utility, and total surplus

Figure 4

$$\#49:\quad eprofitrw = \beta\cdot(p - cr - cs) + (1 - \beta)\cdot(p - cr)$$

$$\#50:\quad \frac{d}{d\beta}\,(eprofitrw = \beta\cdot(p - cr - cs) + (1 - \beta)\cdot(p - cr))$$

$$\#51:\quad 0 > -cs$$

eq (11): exp utility under low β => risky product is initially produced

#52: $\mathrm{eurw} = \beta\cdot((1-\phi s)\cdot v - p - \phi s\cdot\delta) + (1-\beta)\cdot((1-\phi r)\cdot v - p - \phi r\cdot\delta)$

#53: $\frac{d}{d\beta}(\mathrm{eurw} = \beta\cdot((1-\phi s)\cdot v - p - \phi s\cdot\delta) + (1-\beta)\cdot((1-\phi r)\cdot v - p - \phi r\cdot\delta))$

#54: $v\cdot(\phi r - \phi s) + \delta\cdot(\phi r - \phi s)$

#55: $0 < (v+\delta)\cdot(\phi r - \phi s)$

eq (12) eus (large β => safer product)

#56: $\mathrm{eus} = (1-\phi s)\cdot v - p - \phi s\cdot\delta$

eq (13): exp utility for low β

#57: $\mathrm{etrw} = \beta\cdot(p - cr - cs) + (1-\beta)\cdot(p - cr) + \beta\cdot((1-\phi s)\cdot v - p - \phi s\cdot\delta) + (1-\beta)\cdot((1-\phi r)\cdot v - p - \phi r\cdot\delta)$

#58: $\mathrm{etrw} = -cr - cs\cdot\beta + v\cdot(\beta\cdot(\phi r - \phi s) - \phi r + 1) + \delta\cdot(\beta\cdot(\phi r - \phi s) - \phi r)$

#59: $\mathrm{etrw} = v\cdot(\beta\cdot(\phi r - \phi s) - \phi r + 1) + \delta\cdot(\beta\cdot(\phi r - \phi s) - \phi r) - cr - cs\cdot\beta$

In the paper, (13) is typed as

#60: $\mathrm{etrw} = (1 - \beta\cdot\phi s - (1-\beta)\cdot\phi r)\cdot v - (cr + \beta\cdot cs) - (\beta\cdot\phi s + (1-\beta)\cdot\phi r)\cdot\delta$

are these the same? [Yes]

#61: $v\cdot(\beta\cdot(\phi r - \phi s) - \phi r + 1) + \delta\cdot(\beta\cdot(\phi r - \phi s) - \phi r) - cr - cs\cdot\beta - ((1 - \beta\cdot\phi s - (1-\beta)\cdot\phi r)\cdot v - (cr + \beta\cdot cs) - (\beta\cdot\phi s + (1-\beta)\cdot\phi r)\cdot\delta)$

#62: 0

$$\#63: \quad \frac{d}{d\beta}\,(etrw = (1 - \beta\cdot\phi s - (1 - \beta)\cdot\phi r)\cdot v - (cr + \beta\cdot cs) - (\beta\cdot\phi s + (1 - \beta)\cdot\phi r)\cdot\delta)$$

$$\#64: \quad -cs + v\cdot(\phi r - \phi s) + \delta\cdot(\phi r - \phi s)$$

$$\#65: \quad v\cdot(\phi r - \phi s) + \delta\cdot(\phi r - \phi s) - cs$$

> 0 by Assumption 4.

*** Section 5: WB combined with damage compensation to buyers

eq (14) utility with penalty

Nondefective product

#66: v – p

defective with compensation

#67: n – p – δ

Don't buy at all

#68: 0

eq (15) Profit functions including compensation

profit s: no WB and no damage/compensation

#69: profitsn = p – cs

profit s: no WB with damage compensation

#70: profitsn = p – cs – n

profir r: no WB and no damage/compensation

#71: profitrn = p – cr

profit r with compensation but no WB

#72: profitrn = p − cr − n

profit r: WB reproduction but no compensation

#73: profitrn = p − cr − cs

profit r: WB reproduction and compensation

#74: profitrn = p − cr − cs − n

** Subsection 5.1: Compensation and profit incentives to produce safer product

eq (16) expected profit with compnesation

#75: eprofitsn = p − cs − ϕs·n

#76: eprofitrn = β·(p − cr − cs − ϕs·n) + (1 − β)·(p − cr − ϕr·n)

#77: p − cs − ϕs·n ≥ β·(p − cr − cs − ϕs·n) + (1 − β)·(p − cr − ϕr·n)

#78: SOLVE(p − cs − ϕs·n ≥ β·(p − cr − cs − ϕs·n) + (1 − β)·(p − cr − ϕr·n), n)

#79: $$\text{IF}\left(\beta\cdot(\phi r-\phi s)-\phi r+\phi s<0,\ n\geq\frac{cr+cs\cdot(\beta-1)}{(\beta-1)\cdot(\phi r-\phi s)}\right)\vee\text{IF}\left(\beta\cdot(\phi r-\phi s)-\phi r+\phi s>0,\ n\leq\frac{cr+cs\cdot(\beta-1)}{(\beta-1)\cdot(\phi r-\phi s)}\right)$$

#80: $$n\geq\frac{cr+cs\cdot(\beta-1)}{(\beta-1)\cdot(\phi r-\phi s)}$$

#81: $$ns=\frac{cr+cs\cdot(\beta-1)}{(\beta-1)\cdot(\phi r-\phi s)}$$

Figure 5 behavior

#82: $$\frac{d}{d\beta}\left(ns = \frac{cr + cs\cdot(\beta - 1)}{(\beta - 1)\cdot(\phi r - \phi s)}\right)$$

#83: $$0 > \frac{cr}{(\beta - 1)^2\cdot(\phi s - \phi r)}$$

#84: $$\frac{d}{d\beta}\frac{d}{d\beta}\left(ns = \frac{cr + cs\cdot(\beta - 1)}{(\beta - 1)\cdot(\phi r - \phi s)}\right)$$

#85: $$0 > \frac{2\cdot cr}{(\beta - 1)^3\cdot(\phi r - \phi s)}$$

ns evaluated at β=0

#86: $$ns = \frac{cr + cs\cdot(0 - 1)}{(0 - 1)\cdot(\phi r - \phi s)}$$

#87: $$ns = \frac{cr - cs}{\phi s - \phi r}$$

ns=0 when βs = ?

#88: $$0 = \frac{cr + cs\cdot(\beta - 1)}{(\beta - 1)\cdot(\phi r - \phi s)}$$

#89: $$\text{SOLVE}\left(0 = \frac{cr + cs\cdot(\beta - 1)}{(\beta - 1)\cdot(\phi r - \phi s)}, \beta\right)$$

#90: $$\beta = \frac{cs - cr}{cs}$$

#91: $$\beta = 1 - \frac{cr}{cs}$$

** Subsection 5.2: Effects of compensation on profit, utility, and total surplus

slope of eprofitsn and eprofitrn in Figure 6

#92: $$\frac{d}{dn}(eprofitrn = \beta\cdot(p - cr - cs - \phi s\cdot n) + (1 - \beta)\cdot(p - cr - \phi r\cdot n))$$

#93: $$\beta\cdot(\phi r - \phi s) - \phi r$$

#94: $$\phi r\cdot(\beta - 1) - \beta\cdot\phi s$$

#95: $$\frac{d}{dn}(eprofitsn = p - cs - \phi s\cdot n)$$

#96: $$0 > -\phi s$$

eq (18): Buyer utility when n < ns

#97: $$eurn = \beta\cdot((1 - \phi s)\cdot v - p + \phi s\cdot(n - \delta)) + (1 - \beta)\cdot((1 - \phi r)\cdot v - p + \phi r\cdot(n - \delta))$$

eq (19): Buyer utility when n ≥ ns

#98: $$eusn = (1 - \phi s)\cdot v - p + \phi s\cdot(n - \delta)$$

#99: $$\frac{d}{dn}(eurn = \beta\cdot((1 - \phi s)\cdot v - p + \phi s\cdot(n - \delta)) + (1 - \beta)\cdot((1 - \phi r)\cdot v - p + \phi r\cdot(n - \delta)))$$

#100: $$\beta\cdot(\phi s - \phi r) + \phi r$$

#101: $\phi r \cdot (1 - \beta) + \beta \cdot \phi s > 0$

#102: $\frac{d}{dn}(eusn = (1 - \phi s) \cdot v - p + \phi s \cdot (n - \delta))$

#103: $0 < \phi s$

Total surplus n ≥ ns

#104: etsn = p − cs − φs·n + ((1 − φs)·v − p + φs·(n − δ))

#105: $etsn = -cs + v \cdot (1 - \phi s) - \delta \cdot \phi s$

Total surplus n < ns

#106: etrn = (β·(p − cr − cs − φs·n) + (1 − β)·(p − cr − φr·n)) + (β·((1 − φs)·v − p + φs·(n − δ)) + (1 − β)·((1 − φr)·v − p + φr·(n − δ)))

#107: $etrn = -cr - cs \cdot \beta + v \cdot (\beta \cdot (\phi r - \phi s) - \phi r + 1) + \delta \cdot (\beta \cdot (\phi r - \phi s) - \phi r)$

show that etrn < etsn as plotted in Figure 6

#108: $-cs + v \cdot (1 - \phi s) - \delta \cdot \phi s - (-cr - cs \cdot \beta + v \cdot (\beta \cdot (\phi r - \phi s) - \phi r + 1) + \delta \cdot (\beta \cdot (\phi r - \phi s) - \phi r))$

#109: $cr + cs \cdot (\beta - 1) - v \cdot (\beta \cdot (\phi r - \phi s) - \phi r + \phi s) + \delta \cdot (\beta - 1) \cdot (\phi s - \phi r)$

This should be > 0 (except for the cost part, all other terms are >0).