A Formal Theory of Politogenesis: Towards an Agent Simulation of Social Complexity Origins

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Abstract

Agent-based social simulation models are beginning to make significant contributions to scientific understanding of origins of human social complexity (politogenesis). However, social theory remains unclear about the prerequisites of social complexity origins; about things people must have known before the simplest societies could self-organize. In addition, there is a paucity of formal theories of politogenesis. I present a formal mathematical theory of social complexity focused on the phase of human history preceding its initial emergence in selected world regions ca. 10,000 years ago (early Holocene epoch). The formalism uses probability theory and analysis to derive a set of basic, testable results. The main prediction of the theory supports the rare nature of initial social complexity, consistent with observation. Further geospatial applications of the theory predict expected locations for politogenesis, based on prior, causal, theoretically predicted potentials.

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Contents

1 Introduction

What were the requirements of initial “politogenesis” during the early Neolithic Period? What did people already know before they generated the first, simplest polities? Under which conditions does original sociopolitical complexity become minimally viable? These and related research questions represent facets of the same core puzzle in social science regarding the origins of simple societies and eventually the genesis of first civilizations.

This paper presents a formal theory of politogenesis. The theory has intrinsic value for explaining early polity formation in regions of the world, given the paucity of formal research on politogenesis, and can be implemented as part of an agent-based social simulation. Such computational models are starting to contribute to understanding origins of social complexity in regions such as Mesopotamia (ca. 8000 BC) and South America (ca. 3000 BC). However, at present, social simulation research lacks a methodology for formalizing substantive social theories into viable agent-based models.

This paper consists of five sections. The first provides motivation and background from extant literature, with emphasis on social theories of politogenesis. In the second, the theory is presented in terms of basic concepts, assumptions, and theorems. The third section provides a formal analysis that illustrates deductions and main properties to gain a better understanding of politogenesis. The fourth provides a brief discussion of the theory in terms of initial results, broader implications, and future research. The last section presents a summary.

1.1 Motivation

An enduring and challenging research puzzle in social science is the origin of inequality and the formation of the earliest, simplest polities (NSTC, 2009: 36–38). Investigation of “origins” is as significant in social science as it is in biological and physical science. The earliest or primary stage of politogenesis consists of the formation of the simplest polities or “chiefdoms” after the Ice Age, in their simplest form, approximately 10,000 years ago (the early Holocene Period). These are “societies that break with an egalitarian ethos by featuring a centralized, hierarchical organization and ranked social order [...] that precede larger, more complex states, in many regions of the world” (Redmond, 1999: 287). Similarly, “chiefdoms are intermediate societies, neither states nor egalitarian societies. The understanding of them and their dynamics lies at the base of recent attempts to study the evolution of complex societies” (Earle, 1991: xi).

Understanding politogenesis through viable (i.e., formally modeled and empirically testable) social theories is significant for several reasons. First, the puzzle of politogenesis is the social science equivalent of “the origin of the Universe,” since social life began with the first polities about 10,000 years ago, long after humans originated. Second, a theory of politogenesis can also shed light on subsequent sociopolitical development for explaining and understanding more complex polities, such as states, empires, and civilizations. Third, understanding how and why humans formed the earliest polities can shed light on contemporary sociopolitical complexity and provide insights for the future.

However, the challenges for developing a viable theory of politogenesis are significant:

- Conceptual - Politogenesis and related ideas (e.g., inequality, chiefdom, state) are complex concepts that require careful definition. The social science literature is replete with informal definitions, mostly from anthropology, but relatively little research has applied systematic methods of concept formation (Hempel, 1952).

- Formal - There is no single mathematical formalism that seems to jointly capture static and dynamic aspects of politogensis, unlike other social phenomena where a single formalism has provided a unified formal language.[1]

- Ontological - The main entities involved in theories of politogenesis are social entities (i.e., actors, situations, perceptions, decisions, behaviors, institutions, among others) rather than variables and parameters. Object-oriented models encapsulate variables and parameters.

- Empirical - Field data on politogenic regions of the world has become available only recently and is still incomplete. Although sites for primary state formation have been identified (Uruk in Mesopotamia, Yellow River Valley in East Asia, Supe Valley in South America, and Oaxaca Valley in Mesoamerica), sites for primary chiefdom formation are far more numerous and many (most?) are not yet surveyed or known with comparable accuracy as for primary states.

- Comparative - Politogenesis occurred in several regions of the world which had both shared (nomothetic) and individual (idiosyncratic) features. Thus, politogenesis is a social phenomenon where cross-cultural comparative research is highly relevant.

- Computational - Agent-based social simulation models (ABMs) are beginning to provide viable computational alternatives to formal social theories solely based on mathematical models. However, a proper ABM where politogenesis emerges from simple, bounded-rational agents requires foundation in substantive social theory concerning realistic human and social dynamics, not merely a set of computationally effective rules (*a la* Heatbugs or Flockers). An ABM of politogenesis (*explanandum*) would provide theoretical explanation (*explanans*) in the form of a computer program.

[1]For instance, decision models and game-theoretic models have provided single formalisms for theories of bargaining and negotiations, as have dynamical systems of differential equations in a variety of social dynamics, from demography to arms races (Bittinger and Crown, 1981; Lave and March, 1993; Olinick, 1978).

Traditional approaches to formal theories of politogenesis are likely to remain wanting. A new theoretical approach is needed for overcoming these obstacles by providing formal foundations for computational theories of politogenesis. Such foundations should be both substantive and methodological.

1.2 Background Literature

The two bodies of extant literature that are most directly relevant to this research are informal (or narrative) and formal social theories of politogenesis. The focus here is on the earliest instances of politogenesis, or chiefdom formation, not subsequent state formation after one or more polities are already formed.

1.2.1 Narrative theories

Social Contract Theory (Rousseau, 1762) was arguably the first social science theory aimed at explaining politogenesis. This theory assumed that inter-personal (social) relations in hunter-gatherer society were egalitarian and lacked social institutions for collective governance.[2] However, as human population increased—today we know that the end of the Ice Age brought about warmer climate with increased food supply, in addition to plant and animal domestication (Cochran and Harpending, 2009; Fagan, 2003; Mithen, 2004; Price and Gebauer, 1995)—some groups became wealthier than others. A "contract" emerged between social groups of haves and have-nots, such that by joining in civil society all sides enjoy the rule of law against excessive oppression by the strongest or wealthiest. Social Contract Theory was followed by other philosophical theories during the Age of the Enlightenment, but it was only in the twentieth century when social theories of politogenesis became more operational and verifiable or falsifiable (as well as, eventually, amenable to mathematical formalization).

Presently the most widely accepted theories of how chiefdoms operate invoke control, tributary, or redistribution strategies (e.g., Malinowski, 1944; Service, 1962, 1972; Earle, 1989, 1991, 1997). A common mode involves control over precious goods—e.g., material goods such as water, precious stones, metals, or woods; or spiritual goods induced via altered states of consciousness (Lewis-Williams and Pearce, 2009)—that are provided by a chief or "big man" (Sahlins, 1963) as payoff in exchange for compliance or obedience. An important variation on these theories involves the manipulation of cosmic associations to induce tribute (Marcus, 1976: 43-45; Flannery et al., 1981; Flannery and Marcus, 2012).

Recent theories of social complexity based on control and cognitive components provide

[2]This assumption turned out to be mostly correct, although Rousseau and his contemporaries lacked nearly all the empirical evidence that would be eventually produced by modern archaeology in the twentieth and early twenty-first century.

effective explanations in polities already formed. However, such theories do not explain how chiefdoms formed in the first place, although they explain aspects of how they may have endured, after they formed (Wright, 1994). That is, control theories are more appropriate as explanations for synchronic sociopolitical maintenance and development, rather than for diachronic political change (emergence). Explanations of politogenesis require identification of necessary (and possibly also sufficient) conditions for primary social complexity to emerge in the first place.

1.2.2 Formal theories

The first formal approach to modeling politogenesis is arguably Nicolas Rashevsky's monograph, *Looking at History Through Mathematics* (1968), which pioneered the application of mathematics to the formation of the earliest village-based polities in Mesopotamia, China, Mexico, and similar locations during the early Neolithic period.[3] The main contribution of Rashevsky's formal "approach" (not quite a theory) was a system of equations for specifying relationships among several key quantities involved in politigenesis (e.g., size of land holdings, population and sub-population sizes, distances, areas, amounts of food produced and consumed, and agricultural yield). The result was the introduction of a new methodological approach, based on mathematical equations, rather than a specific causal theory aimed at explaining known instances of politogenesis.

Mathematical social science research conducted specifically on theories of politogenesis—i.e., formal models on origins of social complexity—has been scant since Rashevsky's pioneering contribution almost half a century ago. This is likely due to two main challenges. The first is the sheer difficulty of the subject matter, since even the formal structure of politogenesis is far from intuitive and the most promising methodological strategy is probably still unclear and fraught with difficulties.[4] The second is the paucity of data, an enduring albeit improving situation across the most relevant regions (i.e., west Asia, east Asia, northwest South America, and Mesoamerica).

Control Theory is consistent with and a special case of the more general Theory of Political Coalitions in political science (Riker, 1962), which is a formal, game-theoretic theory whereby a chief (or big man, in the sense of Sahlins) provides side-payments to confederates in exchange for their allegiance in governing a regional territory consisting of two or three levels of administrative hierarchy.

The Canonical Theory (Cioffi, 2005; Rogers and Cioffi, 2009) is among the most recent formal and interdisciplinary theories of politogenesis, building on earlier informal theories of socio-political complexity, probabilistic uncertainty, and collective action. In brief,

[3] I am grateful to the late Tom Brylawski for introducing me to Rashevsky's seminal work in mathematical social science.

[4] Rashevsky himself recognized these difficulties: "our immediate aim is not so much to develop some generally applicable theory as to develop the methodology of the new approach" (1968: 136).

this theory explains politogenesis in terms of two processes taking place on different time scales. A "fast process" consists of situational changes (in the form of opportunities and threats) that occur in the life of all societies, leading to success or failure in undertaking collective action. Societal outcomes from the fast process yield consequences (memories of leadership, experiences, payoffs, and similar lessons from collective action), which produce changes in sociopolitical complexity (beliefs, norms, expectations, social relations) when integrated over time. Politogenesis occurs as one of several possible emergent outcomes in the possibility-space (so-called "sample space") of a society.

1.2.3 Other related work

Theories of the origin of agriculture (Cowan and Watson, 1992; Watson, 1995; Smith, 1995) do not explain politogenesis, but are nonetheless insightful in illuminating ways in which early Holocene people devised adaptive strategies that enabled significant increases in the amount and reliability of food supplies. The Binford-Flannery Marginality Theory (Binford, 1968; Flannery, 1969) and the Levantine Primacy Theory (Bar-Yosef and Meadows, 1995; Bocquet-Appel and Bar-Yosef, 2010) are among the most recent and well supported theories, advancing beyond earlier theories by Pumpelly, Childe, and Braidwood, among others. According to these more recent theories, climate changes in the Levant caused increased food availability in certain areas, attracting concentrations of Natufian populations to establish the earliest communities in such areas. "Sedentarism and population growth under optimal conditions" led to "stress subsequently resulting from environmental deterioration and population movements" (Watson, 1995: 29), as well as long-term genetic changes and other consequences of agriculture (Cochran and Harpending, 2009: 85–128). The formation of such communities in selected areas is explained by the Canonical Theory as the result of endogenous demographic increases, following an opportunistic situational change provided by increased food availability in such areas.

The Circumscription Theory of polity development (Carneiro, 1970), as well as other theories of the transition from chiefdom- to state-level polities (e.g., Kirch, 2010; Grinin, 2009; Grinin and Korotayev, 2009), aim to explain primary state formation *after* primary chiefdom formation has occurred. Hence, they are not theories of primary politogenesis in a strict sense, but are nonetheless valuable for suggesting some necessary requirements. Circumscription Theory can be viewed as a special case of the Canonical Theory, with state formation occuring through endogenous (demographic pressure) or exogenous (neighboring warfare) situational changes. Canonical Theory also covers many other significant albeit special cases (e.g., climate change, technological change, disasters, and others).

Similarly, mathematical theories of primary state formation (e.g., Spencer, 1998) are formal, but not about the very earliest politogenesis. They too provide insights and conceptual development, especially on the pre-state ontology of local populations, associated local elites, and a paramount chief controlling these in a given region (i.e., the "minimal winning coalitions" predicted by Riker's earlier theory).

Politogenesis in a strict sense has not yet been modeled using agent simulations. However, several agent-based models that have focused on early societies include the following, none of which explicitly models the very earliest emergence of a primary polity with minimal governance (simple chiefdom):

1. **Kayenta Anasazi** (Dean et al., 1998). The Artificial Anasazi model is a simulation of the occupation and abandonment of the pueblo settlement in Long House Valley between 1800 BC and AD 1300.

2. **Upper Paleolithic society** (Doran et al., 1994). The EOS agent-based simulation is an abstract model of the evolution of human groups towards Neolithic society.

3. **Mesopotamian settlement patterns** (Wilkinson et al., 2007). Empirically calibrated agent-based model of Upper Mesopotamia, includes biophysics and social dynamics that generate inequality.

4. **South American early states** (Griffin and Stanish, 2007; Stanish and Levine, 2011). Agent-model of warfare and early secondary state formation in South American region of Lake Titicaca, Bolivia.

5. **Southwestern Pueblo societies** (Kohler et al., 2012). Agent-based modeling of Puebloan societies in biophysical landscapes that include social relations and strategic decision-making modeled as games.

6. **Aspero Chiefdom** (Auble, 2011; Auble, Cioffi, Magllanes, 2013). Agent-based model based on a Netlogo implementation of communities in the Supe River Valley, Peru, demonstrating the growth of Aspero and Caral (Shady Solís, 2009).

The last two of these efforts come closest to modeling politogenesis, although both still lack explicit sociopolitical dynamics in the form of governance institutions. In summary, prior research on politogenesis has begun to document the earliest polities from an archaeological perspective, but remains incomplete due to the lack of formal social theories that explain the phenomenon.

2 Theory

A set of basic concepts is needed for stating the assumptions of the theory. In turn, assumptions are used to derive core primary results (i.e., theorems and corollaries). Discussion of the theory is reserved for section 4.

2.1 Overview

Historically, human societies were simple—not yet socially complex—between the time when *Homo sapiens sapiens* originated as a subspecies of the genus *Homo* (i.e., since ca. 200,000 years ago as *terminus ad quem*, or no later than) and the time when the first chiefdoms formed (ca. 10,000 years ago, or by 8000 BC, in northern Mesopotamia and shortly after in northern China, followed by regions in the Americas). This time span lasted approximately 190,000 years, based on the known and increasingly accurate archaeological record. During this long epoch, many simple (i.e., strictly kin-based) societies expanded territorially from original homelands in Africa (Henn et al., 2011) to elsewhere around the world, while others remained in the African continent.

The dynamic phase transition from simple societies to initial social complexity that took place beginning ca. 8000 BC in northern Mesopotamia—the emergence of the first simple chiefdoms in a given region (Cioffi, 2006; Wright, 1994)—involved the *realization* of a *potential* that existed in such societies, given an ontological (causal) assumption that nothing can occur "out of the blue" (even when we do not understand the specific causes for occurrence). (1) What constituted such a potential for initial social complexity? (2) How could such a potential be characterized and specified? (3) Under which conditions would such a potential be actually realized? *These are core questions in the theory of politogenesis.*

What relevant knowledge and abilities did people have *before* the emergence of the simplest forms of sociopolitical complexity (i.e., prior to simplest chiefdoms)? The following were minimal requirements (necessary conditions, *conditio sine qua non*) for politogenesis:

1. **Kinship knowledge.** People had knowledge of their own biologically based family ties—kinship and agnation (relatives descending from a common male line; kindred relatives). Such knowledge supported extended households several times the size of a family nucleus. Kinship knowledge is relevant because it can facilitate collective action based on deontic (obligation-based) norms, such as when protecting kin or leverage kin relations to advance other goals.

2. **Communicative ability.** Behaviorally modern humans (*H. sapiens sapiens*) acquired ability to communicate through language between ca. 100,000 and 50,000 years ago, according to current estimates (Bower, 1994;). Communicative ability is necessary for coordinating collective action. It enabled large-scale hunting campaigns, where communication was essential for scouting, coordination, execution, and similar activities.

3. **Normative sociality.** A set of cooperative social norms was known to people in pre-complex societies, via biological evolution, specifically the mechanisms of kin selection and reciprocal altruism. Such base sociality supported minimally albeit sufficiently effective behavior for managing a range of contingent situations.

4. **Social identification ability.** This is the ability to classify others into in-group vs. out-group status. This ability is essential for detecting potential threats and opportunities, as well as for norm use or invocation. In-out group identification generated cognitive complexity and balancing in the sense of Heider and Abelson (1958).

5. **Environmental knowledge.** Awareness about the local and neighboring biophysical landscape. Environmental knowledge is necessary for locating resources and detecting significant change, such as in local species, "normal" climate, or other.

6. **Knowledge of normal vs. rare events.** This is a generalization at a cognitive level. Necessary for detecting situational change, such as emergent threats or opportunities, beyond the biophysical environment, and for assigning levels of urgency, significance, or priority.

7. **Food procurement ability.** Modalities for this ability include hunting, gathering, herding, farming, or preying on others (stealing). More specifically, this is the ability to maintain access to sustenance throughout the seasons of the year and longer time spans, especially in temperate regions far from the Equator, where seasonal variations determine basic food supply.

8. **Homicidal ability.** Originally derived from the hunting skill-set, homicidal ability is a necessity in some modes of collective action, such as when facing lethally aggressive adversaries. Deterrence also requires credible homicidal action.

9. **Collective action ability**. People in simple societies knew how to organize for collective action (i.e., how to lead and how to follow, and other modes of collective action), which they invented and perfected through ancient activities such as hunting large mammals (Hawkes and Bird, 2002). Collective action ability did not necessarily produce social complexity; it was merely an ability among others.

These necessary conditions for politogenesis were possessed by members of simple bands in pre-complex societies. This initial ensemble created the *potential*—albeit not the certainty—for emergence of initial social complexity. The phrase "potential for emergence of initial social complexity" denotes the "possibility," not the certainty or actual realization, of such an occurrence. Without these ideas and skills a society would have been so handicapped as to preclude the attainment of initial social complexity—as indeed happened in most of the world until about 10,000 years ago and then centuries or in some cases millennia after the first instances of social complexity in the so-called "cradles of civilization" or politogenic regions of West Asia, East Asia, South America, and Mesoamerica. While most of the world was almost completely colonized by 10,000 BC, only a few previously simple societies in northern Mesopotamia underwent the first phase transitions to complex polities.

2.2 Formalization

In this section I formalize the preceding ideas using micropolitical principles from the General Theory of Political Uncertainty (Cioffi, 1998: chs. 5–7), applied to the emergence of initial social complexity. The initial formalization is in terms of events viewed as point-sets and logic, which provides foundations for deriving probability models. Notation is introduced as needed.

Notation 1 *Events, variables, values, and other formal entities are denoted by the following symbols. An upper case hollow letter $\mathbb{X}$ denotes an **event**. Italics in upper and lower case denote **variables** X, Y, Z and **values** $\{x_1, x_2, ..., x_n\} \in X$, respectively. Boldface letters denote **vectors** or **matrices**. Historical or so-called **epochal time** is denoted by τ, whereas **analytical time**, including the random variable T, is denoted by $t \in T$. A **process** in N stages and event outcome space Ω is denoted by $\mathcal{P}_N(\Omega)$.*

2.2.1 Basic concepts

The primary concept of the theory is a *polity*, followed by its associated sociopolitical complexity or *social complexity*, for short.

Definition 1 (Polity) *A polity is a social system consisting of a society of members and a system of governance for managing public issues with policies for solving issues affecting society (Cioffi, 2008; Dahl, 1984; Easton, 1979).*

A polity is a social "artifact," or artificial system, consisting of social relations and institutions, not a natural system (Simon, 1996). The elements of Definition 1 (viz., "society," "governance," "public issues," "policies") are taken as primitives. Different types of polities have existed, as documented by political history since the emergence of the earliest polities, and many more will likely be invented in the future. All polities conform to a universal pattern consisting of the elements and relations in Definition 1. A viable universal taxonomy of polities does not currently exist. Some common dimensions for classifying polities include autonomous vs. subordinate, sedentary vs. nomadic, autocratic vs. democratic, traditional vs. modern, developed vs. undeveloped, and simple vs. complex, among others.

As all social object-entities, a polity has a set of attributes and operations that define its state $S(\tau)$ at any given point in epochal time τ (Cioffi, 2008, 2011). Attributes include common features such as territory, population, social identities, political culture, form of government, and—importantly—*social complexity*, along with other geographic, social, economic, and governmental features.

Definition 2 (Social complexity) *Social complexity is the attribute of a polity that defines the scope and depth of its institutional organization, measured by an ordinal scale with values labeled as "chiefdom" (simplest social complexity), "state," or "empire" (Cioffi, 2006; Feinman and Marcus, 1998; Flannery, 1972; Guidi, 2000; Spencer, 2012).*

Definition 3 (Chiefdom) *A chiefdom or chiefly polity is an autonomous polity with a ranked society governed by a paramount leader (chief) with putative authority over regional territory, supported by a coalition of lesser confederate leaders, where each rules a local community of households (Carneiro, 1981; Earle, 1989, 1991, 1997; Flannery, 1972; Marcus, 1989; Spencer, 1998).*[5]

In a chiefdom, the paramount leader or chief is supported by a coalition of confederates, who provide support to the chief (tribute, respect, obedience) in return for chiefly side-payments (prestige goods, blessings, local authority, among others) used for their households and local village-level subordinates (Riker, 1962)—as in a two-way, up-and-down, mutual win-win, exchange network of benefits. The structure of power relations in a chiefdom is a star-hierarchy centralized by the chief (root), where confederates occupy intermediate nodes and branches that control subordinates (leaves). Formally,

$$Chiefdom \supset \{Communities\} \supset \{Households\}, \tag{1}$$

where each community of households is ruled by a local chief that is subordinate to the paramount ruler of the regional chiefdom.

A chiefdom is a relatively unstable polity due to multiple constitutive reasons, among which are the following:

1. **Resources.** A chief has constant need to procure, control, and distribute valuable resources needed to maintain a system of coalition side-payments (bribes) for satisfying confederates (Riker, 1962);
2. **Confederates.** Aspiring subordinate confederate leaders and others can develop capacity and willingness to usurp paramount chiefly power (Cioffi, Rogers, Honeychurch, 2013);

[5] A *state* polity has greater complexity than a chiefdom, due to attributes such as (but not limited to) specialized institutions (military, judiciary, religious), exclusive legitimate use of force, reliable government revenue (through taxation, not just tribute), territorial control within state boundaries, and more reliable sovereignty. An *empire* has even greater complexity than a state, particularly in terms of territorial extent and demographic diversity, including multiple political cultures coordinated through collective action under a unified overall constitutional regime (societal-governmental relation). Horizontal and vertical polities (Ferguson and Mansbach, 1996) are also characteristic of empires and other highly complex polities (Cioffi, 2011).

3. **Religion.** Chiefly authority often depends on religious or mythical association to the supranatural world of ancestors, deities, or magic, requiring ritual and psychologically effective and efficient strategies vis-à-vis the populace (Lewis-Williams and Pearce, 2009); and

4. **Territoriality.** Territorial control is at best putative, not reliable, due to lacking standing military capability to enforce boundary security among neighbors.

These circumstances result in political instability as a characteristic property of chiefdom polities—especially when the polity is stressed by issues (endogenous or exogenous). In spite of fundamental instability, a chiefdom represents a significant base or ground level of social complexity; a quantum level above pre-chiefly polities that are not complex, in a socio-political sense, because they are governed by strictly kin-based institutions that lack the scope and depth of collective problem-solving attainable by a chiefdom on a societal scale. A chiefdom is the simplest "public" polity, in a political sense (Sartori, 1973), whereas simple, less-than-chiefly polities are said to be "private" and consist mostly of households related by kinship.

So far the concepts of polity, social complexity, and chiefdom have been introduced from a static perspective. We now consider the dynamic concept of *emergence* in initial or original social complexity. Formally,

Definition 4 (Original social complexity: Politogenesis) *The earliest or initial occurrence of social complexity in a given polity is an emergent phenomenon generated by a contingent process $\mathcal{P}_N(\Omega)$ of dynamic phase transition from a pre-chiefdom polity (lacking social complexity) to a chiefdom polity (basic social complexity), where N and Ω denote the number of contingencies and the outcome space of the process, respectively.*

Specifically, social complexity occurs or "emerges" when a society undergoes a phase transition in the organization of its constitutive social and governance relations, undergoing sociopolitical change from a state of socially unranked, decentralized, relative equality, primarily based on kin-relations among households, to a state of ranked, centralized authority typically (albeit not exclusively) based on non-kin relations.

2.2.2 Assumptions

Assumption 1 (Society as simple prior to complex) *Prior to developing initial sociopolitical complexity (event $\mathbb{C}$), a polity is in a ground-state $s_0 \in S$ with simple kin-based organization (event $\mathbb{S}$).*

Assumption 2 (Potential as antecedent condition) *Initial social complexity occurs (event $\mathbb{C}$) if and only if (i.f.f.) an a prior potential $\mathbb{P}$ emerges or forms in the state space S of a previously simple polity, such that $\mathbb{C}$ occurs when $\mathbb{P}$ is realized. Conversely, complexity cannot occur without prior formation and subsequent realization of an associated potential.*

Figure 1 shows the forward-sequential causal logic tree for occurrence of initial complexity $\mathbb{C}$ within the social outcomes space Ω, according to Assumption 2. Given a polity in a simple state with only kin-based organization (event $\mathbb{S}$), the potential for sociopolitical complexity may or may not occur (events $\mathbb{P}$ and $\sim \mathbb{P}$, respectively). If $\sim \mathbb{P}$, then the polity does not change. If $\mathbb{P}$ occurs, then the potential may or may not be realized (events $\mathbb{R}$ and $\sim \mathbb{R}$, respectively). If $\sim \mathbb{R}$, then the polity becomes metastable (event $\mathbb{S}$*). If $\mathbb{R}$, then the outcome is the occurrence of initial sociopolitical complexity (event $\mathbb{C}$). A metastable polity becomes complex upon realization of its politogenic potential; but the complexity outcome $\mathbb{C}$ is not realizable without a prior potential in terms of knowledge and ability conditions 1–9.

Thus, $\Omega = \{\mathbb{S}, \mathbb{S}^*, \mathbb{C}\}$, where $\mathbb{C}$ is an outcome in the contingent process $\mathcal{P}_3(\Omega)$ of politogenesis with three antecedents.

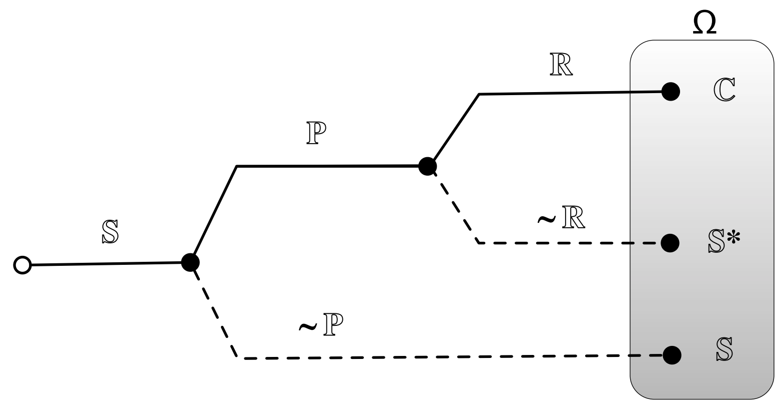

Figure 1: Forward sequential causal logic tree for initial sociopolitical complexity, denoted as the contingent process $\mathcal{P}_3(\Omega)$ of politogenesis with three antecedents. *Source:* Drawn by the author.

In causal logic form, initial complexity $\mathbb{C}$ at time τ implies an associated prior potential $\mathbb{P}$ at some prior time $\tau - \delta$ as necessary condition, $\mathbb{C}(\tau) \rightarrow \mathbb{P}(\tau - \delta)$, but not conversely, for some $\delta < \tau$.

In turn, $\mathbb{P}$ has the backward-conditional causal logic structure of a compound event.

Assumption 3 (Potential as compound event) *The emergence of potential for initial social complexity $\mathbb{P}$ is a compound event, not a singleton or elementary event, as specified* *by an event function $\Psi(\cdot)$ in terms of a set $\{\mathbb{X}_1, \mathbb{X}_2, \mathbb{X}_3, ..., \mathbb{X}_n\}$ of more elementary events causally linked to the occurrence of $\mathbb{P}$. Formally,*

$$\Psi : \mathbb{P} \Leftarrow \{\mathbb{X}_1, \mathbb{X}_2, \mathbb{X}_3, ..., \mathbb{X}_n\}, \tag{2}$$

so

$$\mathbb{P} = \Psi(\mathbb{X}_1, \mathbb{X}_2, \mathbb{X}_3, ..., \mathbb{X}_n), \tag{3}$$

where $\mathbb{X}_i$ denotes the ith causal event for $i = 1, 2, 3, ..., n$.

Assumption 4 (Specific requirements) *The event function Ψ for the compound event $\mathbb{P}$ includes the following minimally necessary causal events $\mathbb{X}_i$ on required knowledge and abilities (conditions 1–9 detailed above):*

1. *$\mathbb{X}_{kin}$ = Kinship knowledge,*
2. *$\mathbb{X}_{com}$ = Communicative ability,*
3. *$\mathbb{X}_{norm}$ = Normative knowledge,*
4. *$\mathbb{X}_{id}$ = Social identity knowledge,*
5. *$\mathbb{X}_{env}$ = Environmental knowledge,*
6. *$\mathbb{X}_{rare}$ = Knowledge of normal vs. rare events,*
7. *$\mathbb{X}_{food}$ = Food procurement ability,*
8. *$\mathbb{X}_{kill}$ = Homicidal ability, and*
9. *$\mathbb{X}_{ca}$ = Collective action ability.*

Based on these requirements (conditions 1–9), the potential for initial complexity $\mathbb{P}$ can be specified by the conjunctive event equation

$$\begin{aligned} \mathbb{P} &= \Psi(\mathbb{X}_{kin}, \mathbb{X}_{com}, \mathbb{X}_{norm}, ..., \mathbb{X}_{ca}), &(4)\\ &\Leftarrow \langle \mathbb{X}_{kin} \wedge \mathbb{X}_{com} \wedge \mathbb{X}_{norm} \wedge ... \wedge \mathbb{X}_{ca} \rangle, &(5)\end{aligned}$$

which specifies the sequential conjunction $\langle \bigwedge_i \mathbb{X}_i \rangle$ of causal events that generate $\mathbb{P}$. Equation 20 can be used to generate Figure 2, which extends Figure 1 by specifying necessary conditions 1–9 for $\mathbb{P}$.

Similarly, event $\mathbb{R}$, which consists of the actual realization of $\mathbb{P}$, (see Figure1), can be specified by the conjunctive event equation

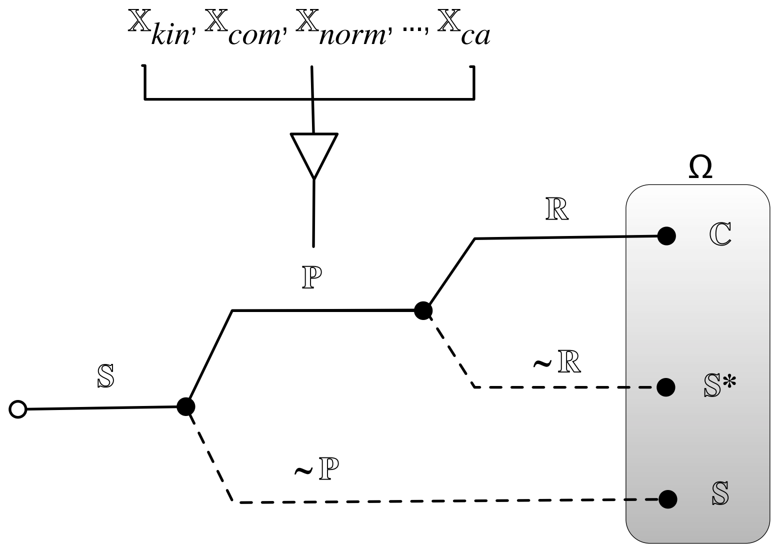

Figure 2: Forward sequential causal logic tree for initial politogenesis $\mathbb{C}$ grafted with a first-order backward conditional causal tree for complexity potential $\mathbb{P}$ (conditions 1-9; section 2.1). *Source:* Drawn by the author.

$$\mathbb{R} = \mathbb{O} \wedge \mathbb{W}, \tag{6}$$

where $\mathbb{O}$ and $\mathbb{W}$ denote the occurrence of willingness and opportunity, respectively (Cioffi and Starr, 1995).

2.2.3 Theorems

The preceding concepts and assumptions are used to derive two sets of principles (theorems). The first are first-order principles, based on immediate causal events that are once-removed (proximate causes).

Notation 2 (Probability of an event) *Let* $X = \Pr(\mathbb{X})$*. For instance,* $S = \Pr(\mathbb{S})$*,* $P = \Pr(\mathbb{P})$*,* $R = \Pr(\mathbb{R})$*,* $C = \Pr(\mathbb{C})$*, and so forth.*

Theorem 1 (Probability of politogenesis) *The probability of initial social complexity (event* $\mathbb{C} \in \Omega$ *in Figure 1–2) is given by*

$$C = S \cdot P \cdot R = \prod_{i=S}^{R} X_i \tag{7}$$

$$= c^3, \tag{8}$$

where c denotes some uniform probability on the closed interval $[0, 1]$ taken across causal events $\mathbb{S}$, $\mathbb{P}$, and $\mathbb{R}$.

Proof. Theorem 1 follows from Assumption 2 and application of the Fundamental Theorem for the Probability of a Compound Event. Theorems 2 – 4 below follow *simili modo*.□

Remark 1 (Multiplication of probabilities) *If the causal event probabilities are conditional or dependent, then they still multiply.*

Theorem 2 (Probability of metastability) *The probability of metastability is give by $S^* = S \cdot P \cdot (\neg R)$.*

Remark 2 (Closed unit-interval) *Probability c is strictly defined on the closed unit-interval $[0, 1]$, such that, $0 \nless c \ngtr 1$ so $0 \leq C \leq 1$, $\forall c$.*

Theorem 3 (Probability of potential for politogenesis) *The probability of potential for politogenesis $\mathbb{P} \in \mathcal{P}_3(\Omega)$ as a function of first-order causal events $\mathbb{X}_i$ (Assumption 4) is given by*

$$P = X_{kin} \cdot X_{com} \cdot X_{norm} \cdot ... \cdot X_{ca} = \prod_{i=kin}^{ca} X_i \tag{9}$$

$$= x^{\Theta}, \tag{10}$$

where x denotes some uniform probability taken across Θ causal events, which are nine assuming $\mathbb{X}_{kin}$ to $\mathbb{X}_{ca}$ (causal necessary conditions 1-9, Assumption 4).

Remark 3 (Unit interval support of x) *Remark 2 w.r.t. Theorem 2 above applies similarly to Theorem 3, such that $0 \geq P \leq 1$, $\forall \Theta$.*

Theorem 4 (Probability of realization) *The probability of realizing a politogenic potential $\mathbb{R} \in \mathcal{P}_3(\Omega)$ as a function of first-order causal conditions for opportunity $\mathbb{O}$ and willingness $\mathbb{W}$ is given by*

$$R = O \cdot W \tag{11}$$

$$= r^2, \tag{12}$$

where r denotes some uniform probability taken across $\mathbb{O}$ and willingness $\mathbb{W}$ causal events.

The following are second-order principles that extend the preceding first-order principles. These are stated in terms of more distant, at least twice-removed, causal event. These results are useful because second-order conditions are closer to observation and operational events than first-order conditions that are commonly more abstract.

Theorem 5 (Second-order probability of politogenesis) *The second-order probability of initial social complexity is given by*

$$\begin{aligned} \mathbb{C} &= \Psi(\mathbb{S}; \mathbb{X}_{kin}, \mathbb{X}_{com}, \mathbb{X}_{norm}, ..., \mathbb{X}_{ca}; \mathbb{O}, \mathbb{W}), & (13)\\ &\Leftarrow \langle \mathbb{S} \wedge \mathbb{X}_{kin} \wedge \mathbb{X}_{com} \wedge \mathbb{X}_{norm} \wedge ... \wedge \mathbb{X}_{ca} \wedge \mathbb{O} \wedge \mathbb{W} \rangle, & (14) \end{aligned}$$

and

$$\begin{aligned} C &= S(\prod_{i=kin}^{ca} X_i) OW & (15)\\ &= y^{\Gamma}, & (16) \end{aligned}$$

where y is some uniform probability taken across the set of Γ second-order causal events for $\mathbb{C}$ and $\Gamma > \Theta$.

Note that $\Gamma = 12$ in equation 14. Theorem 5 explains why politogenesis was such a rare occurrence in history (early Holocene). Since $\Gamma = \Theta + 3$ (by eq. 14) and $\Theta = 9$, it follows that $C(y; \Gamma) = y^{12}$, which yields a relatively minuscule probability of politogenesis for arbitrary values of y. If Assumption 4 is incomplete ($\Theta > 9$), then politogenesis as well as its potential are even rarer.

Theorem 6 (Second-order probability of metastability) *The second-order probability of metastability in a pre-complex society is given by*

$$\begin{aligned} \mathbb{S}^* &= \Psi(\mathbb{S}; \mathbb{X}_{kin}, \mathbb{X}_{com}, \mathbb{X}_{norm}, ..., \mathbb{X}_{ca}; \sim \mathbb{R}) & (17)\\ &= \Psi(\mathbb{S}; \mathbb{X}_{kin}, \mathbb{X}_{com}, \mathbb{X}_{norm}, ..., \mathbb{X}_{ca}; \sim \mathbb{O}, \sim \mathbb{W}) & (18)\\ &\Leftarrow \langle \mathbb{S} \wedge \{\mathbb{X}_i\} \wedge \sim \mathbb{R} \rangle & (19)\\ &\Leftarrow \langle \mathbb{S} \wedge \{\mathbb{X}_i\} \wedge [(\sim \mathbb{O}) \vee (\sim \mathbb{W})] \rangle & (20) \end{aligned}$$

and

$$\begin{aligned} S^* &= S^*(S, X_i, O, W) & (21)\\ &= S(\prod_{i=kin}^{ca} X_i)(1 - OW), & (22)\\ & & (23) \end{aligned}$$

so

$$\begin{aligned} S^*(z;\Theta) &= z^{\Theta+1}(1-z^2) & (24)\\ &\approx z^{\Theta+1}, & (25)\end{aligned}$$

where z is some uniform probability taken across each of the second-order causal events in equations 4–5 and Θ is the number of necessary conditions for $\mathbb{P}$ (conditions 1–9).

For $\Theta = 9$, $S^* = z^{10}(1-z^2) = z^{10} - z^{12} \approx z^{10}$, so also metastability is rare. However, relative to politogenesis, we get $S^* > C^*$, because $z^{10} > y^{12}$. Therefore, metastability is not as rare as politogenesis, which is consistent with logic and validated by the known early Holocenic record.

3 Analysis

In this section I present an initial analysis of the theory to derive additional results in the form of other basic theorems extending beyond the core.

Let the nabla ∇ symbol denote the common del differential operator used to define the multivariate gradient and associated vector derivatives (Laplacian $\nabla^2 f$, divergence $\nabla \cdot f$, curl $\nabla \times f$) in standard multivariate analysis,

$$\nabla f \equiv \sum_{i=1}^{N} \frac{\partial f}{\partial X_i}. \qquad (26)$$

Three extensions of multivariate analysis are relevant for better understanding core theorems of politogenesis: sensitivity, gradient, and normalization.

3.1 Sensitivity

The first extension is to analyze variation with respect to normalized dimensions, since standard comparative statics yield incomparable results.

Definition 5 (Sensitivity; Cioffi, 1998: 302–303) *Let $Y = f(X_1, X_2, X_3, ..., X_N)$ be a differentiable function in N independent continuous variables that assume real values. Then the sensitivity of Y with respect to $X_i \in X$, denoted by $S_{X_i}(Y)$, is defined as the normalized variation operator of percentage change in Y with respect to percentage change in X_i:*

$$S_{X_i}(Y) \equiv \frac{\frac{\Delta Y}{Y} \times 100}{\frac{\Delta X_i}{X_i} \times 100} = \frac{\partial Y}{\partial X_i}\frac{X_i}{Y}. \qquad (27)$$

Alternatively, when Y is a function of discrete variables, the first-order difference replaces the first-order derivative:

$$S_{X_i}(Y) \equiv (Y_{X_i+1} - Y_{X_i})\frac{X_i}{Y}. \tag{28}$$

Normalized variation (sensitivity) is called elasticity in mathematical economics. This is a dimensionless value that is directly comparable across the N dimensions of Y, whereas derivatives are not always comparable because they are expressed in different units (e.g, probabilities and cardinalities in the core theorems).

3.2 Gradient

A second extension of the standard gradient is with respect to cases where Y is defined by a hybrid of continuous and discrete variables, as is common in politogenetic systems and processes.

Definition 6 (The "nabladot" concrete operator; Cioffi, 2013) *Let $f(X_1, X_2, X_3, ..., X_N; Y_1, Y_2, Y_3, ..., Y_M)$ denote a multivariate function, such that some independent variables are continuous with real values (X) and others are discrete with integer values (Y). The "nabladot" operator ∇ is a vector defined as*

$$\nabla F \equiv \frac{\partial f}{\partial X_1}\hat{\mathbf{x}}_1 + \frac{\partial f}{\partial X_2}\hat{\mathbf{x}}_2 + \cdots + \frac{\partial f}{\partial X_N}\hat{\mathbf{x}}_N + \Delta_{y_1} f \hat{\mathbf{y}}_1 + + \Delta_{y_2} f \hat{\mathbf{y}}_2 + \cdots + \Delta_{y_M} f \hat{\mathbf{y}}_M \tag{29}$$

$$= \sum_{i=1}^{N} \frac{\partial f}{\partial X_i}\hat{\mathbf{x}}_i + \sum_{j=1}^{M} \Delta_{y_j} f \hat{\mathbf{y}}_j, \tag{30}$$

where $\partial/\partial X_i$ and Δ_{y_j} denote the first-order derivative and first-order difference w.r.t. X_i continuous variables and Y_j discrete variables, respectively, and $\hat{\mathbf{x}}_i$ and $\hat{\mathbf{y}}_j$ are versors (unit vectors) in their respective dimensions.[6]

Nabladot is a "concrete" operator in the sense of Graham, Knuth, and Patashnik (1994), because it is a hybrid operator defined in terms of both continuous *and* discrete variables. Nabladot is a useful operator in mathematical social science, where the concurrence of discrete and continuous variables is common, significant, and requires analysis. Note that, in general, $\nabla f \neq \nabla f$, as seen in the following instance.

[6]Note that the first-order difference is defined as $\Delta_y f = \Delta f/\Delta y = f(y+1) - f(y)$.

Theorem 7 (Gradient of the probability of potential for politogenesis) *The gradient of the probability P of potential for politogenesis is given by*

$$\nabla P = \Theta x^{\Theta-1}\hat{\mathbf{x}} + (x^{\Theta+1} - x^{\Theta})\hat{\vartheta}, \tag{31}$$

so P is increasing in x and decreasing in Θ; and

$$\mid \nabla P \mid \approx \Theta x^{\Theta-1}, \tag{32}$$

so ∇P points mainly in the direction of x.

Probability x matters more than cardinality Θ, although both affect politogenetic potential $\mathbb{P}$. This result is consistent with the Dominance Principle for Political Conjunction (Cioffi, 1998: 189): $S_x(P) > S_\Theta$, because

$$\frac{\partial P}{\partial x}\left(\frac{x}{P}\right) > (P_{\Theta+1} - P_\Theta)\left(\frac{\Theta}{P}\right). \tag{33}$$

This means that, for instance, the onset of metastability for politogenesis is more sensitive to the individual probability of required conditions (1–9) than to their total number. This is important, because the cardinality of the potential may be in question, but what matters most is not that, but rather the individual probability of each requirement.

3.3 Normalization

Definition 7 (Normalized nabladot ∇^*) *The normalized nabladot operator ∇^*, or normalized deldot, is defined in terms of partial sensitivies, as*

$$\nabla^* F \equiv \sum_{i=1}^{N} \frac{\partial f}{\partial X_i}\frac{X_i}{f}\hat{\mathbf{x}}_i + \sum_{j=1}^{M} \Delta_{Y_j} f \frac{Y_j}{f}\hat{\mathbf{y}}_j, \tag{34}$$

where $\hat{\mathbf{x}}_i$ and $\hat{\mathbf{y}}_j$ are versors (unit vectors).

4 Discussion

This paper began with a set of core and enduring research questions on politogenesis: What were the requirements of initial "politogenesis" during the early Neolithic Period? What did people need to know before they could organize the first, simplest polities? Under which conditions does original sociopolitical complexity become minimally viable? The theory presented in this paper provides answers to these questions. The necessary requirements for politogenesis are explicit (conditions 1–9), as well as providing a set of initially derived implications that serve as explanations for politogenesis.

The main assumption of the theory concerns the nine core requirements, formalized as Assumption 4. Are these requirements empirically and theoretically justifiable? Are they consistent as an axiomatic basis? The claim is that set of requirements is empirically grounded, not hypothetical. For example, the normative sociality of people in simple, pre-complex societies (bands) is known to biologists in terms of kin selection and reciprocal altruism (Hamilton, 1964; Sherman, 1977). This fact is known by relatively few social scientists (Boyd and Richerson, 2005; Fukuyama, 1999: ch. 10; 2010: ch. 2; McElreath and Boyd, 2007; Palmer, 1991). Another example is the communicative ability acquired by humans through the emergence of language by 50,000 years ago (Bower, 1994). Members of simple societies spoke among themselves to accomplish levels of collective action eventually required for politogenesis—writing was not yet an option. Similarly, people in simple societies already knew how to kill each other (homicidal ability) and how to organize for collective action (as when they hunted large animals). This core set constituted various types of knowledge and abilities possessed by members of pre-complex societies; it was an impressive ensemble of human knowledge and abilities that supported the initial potential for politogenesis, event before such potential would be realized in a few places. The status of these assumptions is empirically supported, not hypothetical, and the basic argument is that they represent a core of necessary and sufficient conditions for politigenesis—i.e., all of them are required and no other assumption is needed.

The main result is arguably explaining the extreme rareness of politogenesis, due to its "hypoprobability" (Cioffi, 1998:183)—i.e., the exponential decrease of compound probability caused by conjunction. This theoretical expectation is confirmed by observation, given that politogenesis is known to have occurred during the early Neolithic Period in the Levant, China, Peru, and Mesoamerica. The theory explains this observed rareness based on Theorems 1–6, especially the first (Probability of Politogenesis).

The theory also addresses several of the main challenges in politogenesis research (Section 1.1). Concepts are formally defined using primitive terms, using only those that are strictly necessary. The theory is formalized using logic and probability theory, because politogenesis is a causal albeit not a deterministic outcome (Suppes, 1970, 1984). The event-based causal structure of the theory captures the ontology of politogenesis in a way that is closer to agent simulation than the systems dynamics of differential equation-based systems. The theory has empirical ties linking assumptions and principles to "the plain of observation" (Hempel, 1965), including the prediction of extreme rareness in politogenic events in the Holocene. Various regional instances of politogenesis (in so-called "cradles of civilization") can be explained by different levels of politogenic potential and their realization. As a result, the theory can be used to implement an agent-based social simulation model of politogenesis and early sociopolitical development. The environment would consist of a biophysical landscape inhabited by a kin-based society. The initial agent rules would be based exclusively on knowledge and skills such as those specified by conditions 1–9. Situational changes would activate agent decision-making and produce decisions, behaviors, and emergent patterns that would generate politogenesis. Implementing an agent-based model is not trivial; but is increasingly viable through the kind of event-based formal theory proposed in this paper (see, e.g., Cioffi et al., 2008; Cioffi and Harrison, 2011).

The role of collective action in politogenesis should be emphasized and clarified, especially since it is a political concept that remains largely absent in the anthropological literature on origins of sociopolitical complexity. Collective action was a social strategy invented by members of bands in pre-complex societies—i.e., before politogenisis took place. Hunting large animals requires collective action for success. Leadership is ubiquitous in collective action, since leaders can induce, motivate, execute, and engage in other essential activities. Collective action resolves social dilemmas by well-known mechanisms (Lichbach, 1996). However, collective action per se is insufficient for politigenesis; it is a necessary condition among others defined in Assumption 4 (Specific Requirements). For example, the occurrence of a food surplus (a hypothesis with increasing empirical evidence among anthropological archaeologists), either resulting from discovery and exploitation of a rich ecological niche or by inventing agriculture, requires collective action as a specific mechanism that is necessary and sufficient for exploiting the food surplus. Food surplus without collective action does not lead to politogenesis, because food is simply consumed or wasted. A food storage facility must be designed, built, and maintained/managed, otherwise the surplus is worthless. All this requires collective action, leading to significant changes in social relations and the emergence of ranked societies.

The ancient pre-human and paleolithic origins of reciprocal altruism, kin selection, some forms of collective action (both peaceful and violent), technological adaptations, and other elementary social institutions raise interesting puzzles regarding current theory and research on "emergence" using agent-based modeling. The emergence of norms, cooperation, and other institutions is often motivated by issues and policies from a contemporary societal content. However, such phenomena began to occur prior to politogenesis, so agent models of such primordial institutions should be referenced in the context of pre-human and paleolithic societies, which provide a richer and more insightful context on contemporary instances.

Further geospatial development and application of this theory could be used to predict precise locations for politogenesis, based on prior causal potentials. For instance, given a global grid of regional location and estimated politogenetic potential at various times, the theory can be used to predict where and when initial sociopolitical complexity would occur. Politogenetic potentials for each region of space-time could be estimated by rating the occurrence of each of the requirements, since they did not have uniform distribution across regions or time. The theory is testable: Locations with highest potentials should coincide with the four politogenic regions known from archaeology (the Levant, China, Peru, Mesoamerica) as well as related areas that may not have generated states until relatively recent times but did generate chiefdoms (e.g., North America, Amazonia, subsaharan Africa, south Asia, Europe).

5 Summary

Agent-based social simulation models are beginning to make significant contributions to scientific understanding of origins of human social complexity (politogenesis). However, social theory across the individual social sciences and Social Science as a whole remains unclear about the prerequisites of social complexity origins; i.e., about things people must have known before the simplest societies could be organized. I present a formal mathematical theory of social complexity focused on the phase of human history immediately preceding its initial emergence in selected world regions ca. 10,000 years ago (early Holocene epoch). The formalism uses logic, probability theory, and analysis to derive a set of basic testable results. The main derivation from the theory is supportive of the relatively rare nature of initial social complexity, consistent with observation. Further geospatial application of the theory could predict locations for politogenesis, based on prior causal potentials.

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