Simultaneous Transfer of Heat and Moisture in Porous Media

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Abstract—Simultaneous differential equations for the transfer of heat and moisture in porous media under the combined influence of gravity and gradients of temperature and moisture content are developed. These equations are a generalization of those derived by *Philip* and *de Vries* [1957]. In particular a consistent distinction is made between changes of moisture content in the liquid and the vapor phase.

The interaction between heat and moisture transfer in steady state heat conduction is discussed in detail using numerical values for two soils, a clay loam and a medium sand. The behavior is found to be dependent on the boundary conditions for moisture transfer, on the direction of the temperature gradient and on the ratio of two moisture diffusivities entering in the analysis.

Introduction—In a recent paper *Philip* and *de Vries* [1957] discussed the movement of liquid water and water vapor in porous media under the influence of gradients of temperature and moisture content. In the final paragraph of that paper they presented equations of simultaneous heat and moisture transfer. The present paper extends this approach.

In particular heat transfer by moisture movement is considered in greater detail. Heat of wetting and transfer of sensible heat are taken into account. Furthermore a consequent distinction is made between changes of moisture content in the liquid and the vapor phase, whereas in the final paragraph of the paper cited it was tacitly assumed that changes of moisture contained in the vapor phase are negligible. This may often be admissible, because the vapor content is usually small in comparison with the liquid content (both expressed as g/cm³). However, there are also many situations where changes in liquid and vapor content can be of the same order. This will be so in particular where these changes are due to moisture movement under the influence of temperature gradients. A number of implications of the new equations will be discussed using numerical data for a coarse textured and a medium textured soil.

Notation—The notation in the present paper is almost the same as that of the paper cited. The most important difference is that the symbol θ is used here to denote the total moisture content (liquid + vapor), whereas in the previous paper it stood for liquid moisture content. For reasons of economy the subscripts l and v are used to indicate the liquid and vapor phases respectively instead of liq and vap. Hence θ_l and θ_v are the moisture contents in the two phases, and ρ_l and ρ_v are the corresponding densities (ρw and ρ in the previous paper).

The following list contains an explanation of the symbols used and the units in which they are expressed. Numerical values of some constants are also included.

a volumetric air content, cm³ of air/cm³

c_p specific heat of water vapor at constant pressure, cal g⁻¹ °C⁻¹

c_l specific heat of liquid water, cal g⁻¹ °C⁻¹

C volumetric heat capacity of moist porous medium, cal cm⁻³ °C⁻¹

C_d volumetric heat capacity of dry porous medium, cal cm⁻³ °C⁻¹

D_{atm} molecular diffusion coefficient of water vapor in air, cm² sec⁻¹ $(=4.42 \times 10^{-4} T^{2.3}/P)$

$D_T = D_{Tl} + D_{Tv}$ thermal moisture diffusivity, cm² sec⁻¹ °C⁻¹

$D_{Tl} = K\gamma\Psi$ thermal liquid diffusivity, cm² sec⁻¹ °C⁻¹

$D_{Tv} = fD_{atm}\nu\beta h(\nabla T)_a/\rho_l\nabla T$ (with $f = S$ for $\theta_l < \theta_{lK}$, $f = a + a\theta_l/(S - \theta_{lK})$ for $\theta_l > \theta_{lK}$) thermal vapor diffusivity, cm² sec⁻¹ °C⁻¹

$D_\theta = D_{\theta l} + D_{\theta v}$ isothermal moisture diffusivity, cm² sec⁻¹

$D_{\theta l} = K\partial\Psi/\partial\theta_l$ isothermal liquid diffusivity, cm² sec⁻¹

$D_{\theta v} = \alpha a D_{atm}\nu g\rho_v(\partial\Psi/\partial\theta_l)/\rho_l RT$ isothermal vapor diffusivity, cm² sec⁻¹

E rate of evaporation, sec⁻¹

g acceleration due to gravity, cm sec⁻² $(=981)$

h relative humidity

j mechanical equivalent of heat, erg cal⁻¹ $(=4.18 \times 10^7)$

k   unit vector in vertical direction

K unsaturated hydraulic conductivity, cm sec^{-1}

L heat of vaporization, cal g^{-1} (=585 at 20°C)

L_0 heat of vaporization at reference temperature T_0, cal g^{-1}

p partial pressure of water vapor, mm Hg

P total gas pressure, mm Hg

q_h heat flux, cal cm^{-2} sec^{-1}

q_l liquid flux, g cm^{-2} sec^{-1}

q_m $= q_l + q_v$ moisture flux, g cm^{-2} sec^{-1}

q_v vapor flux, g cm^{-2} sec^{-1}

R gas constant of water vapor, erg g^{-1} °C^{-1} (=4.615 × 10^6)

S porosity, cm^3/cm^3

t time, sec

T absolute temperature, °K

T_0 arbitrary reference temperature, °K

W differential heat of wetting, cal g^{-1}

x horizontal coordinate, cm

z vertical coordinate, positive upwards, cm

α tortuosity factor for diffusion of gases in soils (≈0.67)

β $= dp_0/dT$, g cm^{-3} °C^{-1} (=1.05 × 10^{-6} at 20°C)

γ temperature coefficient of surface tension of water, °C^{-1} (=−2.09 × 10^{-3} at 20°C)

θ $= \theta_l + \theta_v$ total volumetric moisture content, cm^3/cm^3

θ_l volumetric liquid content, cm^3/cm^3

θ_{lK} value of θ_l at which 'liquid continuity' fails

θ_v volumetric vapor content, cm^3 of precipitable water/cm^3

λ thermal conductivity, cal cm^{-1} sec^{-1} °C^{-1}

λ_* hypothetical thermal conductivity of moist porous medium excluding vapor movement, cal cm^{-1} sec^{-1} °C^{-1}

λ_a thermal conductivity of air, cal cm^{-1} sec^{-1} °C^{-1}

λ_v apparent thermal conductivity of an air-filled pore due to vapor diffusion, cal cm^{-1} sec^{-1} °C^{-1}

ν $= P/(P - p)$ mass flow factor

ρ_v density of water vapor, g cm^{-3}

ρ_0 density of saturated water vapor, g cm^{-3}

ρ_l density of liquid water, g cm^{-3}

Ψ water pressure, with atmospheric pressure as datum, cm

$(\nabla T)_a$ average temperature gradient in air-filled pores, °C cm^{-1}

THE EQUATIONS FOR HEAT AND MOISTURE TRANSFER

Moisture transfer—The equations for the liquid and vapor flux densities as given by *Philip* and *de Vries* [1957] are

$$q_l/\rho_l = -D_{\theta l}\nabla\theta_l - D_{Tl}\nabla T - Kk \quad (1)$$

$$q_v/\rho_l = -D_{\theta v}\nabla\theta_l - D_{Tv}\nabla T \quad (2)$$

Combination of these equations leads to the total moisture flux density

$$q_m/\rho_l = -D_\theta\nabla\theta_l - D_T\nabla T - Kk \quad (3)$$

By application of the continuity principle the following differential equations for θ_l, θ_v and θ are obtained

$$\frac{\partial\theta_l}{\partial t} = -\nabla(q_l/\rho_l) - E = \nabla(D_{\theta l}\nabla\theta_l)$$
$$+ \nabla(D_{Tl}\nabla T) + \frac{\partial K}{\partial z} - E \quad (4)$$

$$\frac{\partial\theta_v}{\partial t} = -\nabla(q_v/\rho_l) + E = \nabla(D_{\theta v}\nabla\theta_l)$$
$$+ \nabla(D_{Tv}\nabla T) + E \quad (5)$$

$$\frac{\partial\theta}{\partial t} = -\nabla(q_m/\rho_l) = \nabla(D_\theta\nabla\theta_l)$$
$$+ \nabla(D_T\nabla T) + \frac{\partial K}{\partial z} \quad (6)$$

Eq. (1), (2), (3), and (6) are identical with (12), (7), (20), and (23) respectively of the paper cited with the exception that in the latter no distinction between θ and θ_l was made. The possibility of water transfer from the liquid to the vapor phase is recognized by the introduction of an 'evaporation' term, $\pm E$, on the right hand sides of (4) and (5). In the case of condensation E is negative, of course.

Assuming that at every instant liquid water is in equilibrium with water vapor the following relation between θ_v and θ_l holds

$$\theta_v = (S - \theta_l)\rho_v/\rho_l = (S - \theta_l)\rho_0 h/\rho_l \quad (7)$$

Hence

$$\frac{\partial \theta_v}{\partial t} = \left[\frac{(S - \theta_l)\rho_0 g h}{\rho_l R T}\frac{\partial \Psi}{\partial \theta_l} - \frac{\rho_v}{\rho_l}\right]\frac{\partial \theta_l}{\partial t}$$

$$+ \frac{(S - \theta_l)h\beta}{\rho_l}\frac{\partial T}{\partial t}$$

$$= \left[\frac{D_{\theta v}}{\alpha \nu D_{\text{atm}}} - \frac{\rho_v}{\rho_l}\right]\frac{\partial \theta_l}{\partial t} + \frac{(S - \theta_l)h\beta}{\rho_l}\frac{\partial T}{\partial t} \quad (8)$$

On the right hand side of (8) a term with $\partial h/\partial T$ has been neglected. Philip and de Vries argued that this term is negligible in comparison with the second term on the right hand side of (8).

The value of E follows immediately from (5) and (8). Substitution in (4) leads to the following equation in θ_l and T

$$\left(1 + \frac{D_{\theta v}}{\alpha \nu D_{\text{atm}}} - \frac{\rho_v}{\rho_l}\right)\frac{\partial \theta_l}{\partial t} + \frac{(S - \theta_l)h\beta}{\rho_l}\frac{\partial T}{\partial t}$$

$$= \nabla(D_\theta \nabla \theta_l) + \nabla(D_T \nabla T) + \frac{\partial K}{\partial z} \quad (9)$$

Heat transfer—The heat flux density will be tentatively expressed as follows

$$q_h = -\lambda_* \nabla T + L_0 q_v + c_p(T - T_0)q_v$$

$$+ c_l(T - T_0)q_l \quad (10)$$

The first term on the right hand side represents the contribution of pure heat conduction; λ_* is the thermal conductivity of the porous medium in the hypothetical case where no moisture movement occurs. The second term represents the transfer of latent heat by vapor movement. The third and fourth terms represent the transfer of sensible heat in vapor and liquid form respectively. T_0 is an arbitrary reference temperature, L_0 is the heat of vaporization at T_0. The tentative character of (10) arises from the fact that the various terms on its right hand side have been considered as additive, which is strictly not permissible as will be discussed below. Further it has been assumed that heat transfer by convection and radiation is negligible, which is usually the case [*de Vries*, 1952a].

The total heat content per unit of volume is

$$C_d(T - T_0) + L_0\rho_l\theta_v + c_p\rho_l\theta_v(T - T_0)$$

$$+ c_l\rho_l\theta_l(T - T_0) - \rho_l \int_0^{\theta_l} W d\theta_l \quad (11)$$

where C_d is the volumetric heat capacity of the dry medium and W the differential heat of wetting. According to thermodynamic theory [*Edlefsen and Anderson*, 1943]

$$W = RT^2 \partial \ln h/\partial T = -g(\Psi - T\partial\Psi/\partial T) \quad (12)$$

Applying the principle of heat conservation we have

$$C\frac{\partial T}{\partial t} + L_0\rho_l\frac{\partial \theta_v}{\partial t} + c_p\rho_l(T - T_0)\frac{\partial \theta_v}{\partial t}$$

$$+ c_l\rho_l(T - T_0)\frac{\partial \theta_l}{\partial t} - \rho_l W\frac{\partial \theta_l}{\partial t} = -\nabla q_h$$

$$= \nabla(\lambda_* \nabla T) - L_0 \nabla q_v - c_p(q_v \nabla T) - c_l(q_l \nabla T)$$

$$- c_p(T - T_0)\nabla q_v - c_l(T - T_0)\nabla q_l \quad (13)$$

Using (4), (5) and the relation

$$L(T) = L_0 - (c_l - c_p)(T - T_0) \quad (14)$$

this reduces to

$$C\frac{\partial T}{\partial t} - \rho_l W\frac{\partial \theta_l}{\partial t} = \nabla(\lambda_* \nabla T) - L\rho_l E$$

$$- c_p(q_v \nabla T) - c_l(q_l \nabla T) \quad (15)$$

where the reference temperature no longer occurs.

Combining (1), (2), (4), (5), (8), (12) and (15) we obtain

$$[C + L(S - \theta_l)h\beta]\frac{\partial T}{\partial t} + \left[\frac{L\rho_l D_{\theta v}}{\alpha \nu D_{\text{atm}}}\right.$$

$$\left. - L\rho_v + \rho_l j^{-1}g\left(\Psi - T\frac{\partial \Psi}{\partial T}\right)\right]\frac{\partial \theta_l}{\partial t}$$

$$= \nabla[(\lambda_* + L\rho_l D_{Tv})\nabla T] + L\rho_l \nabla(D_{\theta v}\nabla \theta_l)$$

$$+ \rho_l c_p[(D_{\theta v}\nabla \theta_l + D_{Tv}\nabla T)\nabla T]$$

$$+ \rho_l c_l[(D_{\theta l}\nabla \theta_l + D_{Tl}\nabla T + K k)\nabla T] \quad (16)$$

The third term on the right hand side of (16), representing sensible heat transfer by vapor movement, is usually negligible.

In the present treatment it has been tacitly assumed that all processes of heat transfer take place uniformly throughout the porous medium, and that sources and sinks of heat (arising from evaporation-condensation and wetting-drying) are distributed uniformly as well. The former assumption is already implicit in the additive structure of the right hand side of (10). However, in reality vapor transfer takes place only in the air-filled pore space, liquid movement only in the water-filled pore space, while evaporation and condensation occur at the water-air interfaces. A complete analytical treatment of these processes would lead us to a 'microscopic' study of heat and moisture transfer in porous media. In such a complete theory the pore geometry has to be specified and proper boundary conditions must be introduced at all the solid-liquid and liquid-gas interfaces. Such
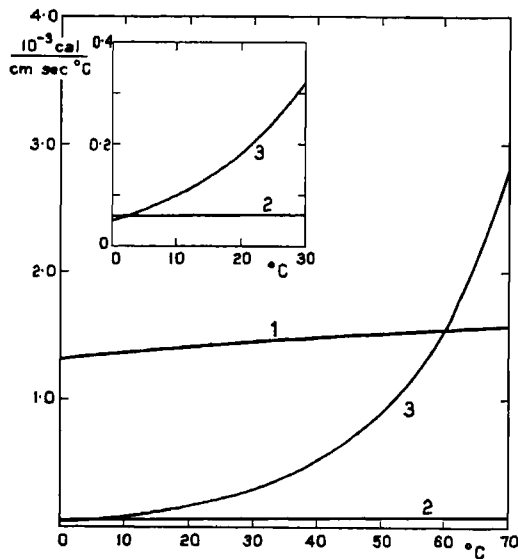
FIG. 1 – Thermal conductivities of water (curve 1) and air (curve 2) and values of $\nu\beta LD_{atm}$ (curve 3) in relation to temperature; the inset has an enlarged scale of ordinates

an analysis will be extremely difficult and it will not be attempted here.

An exception will be made for vapor distillation due to temperature gradients in the air-filled pores. *Krischer* and *Rohnalter* [1940] have shown that this leads to an apparent increase in the thermal conductivity of the air-filled pores. This theory was incorporated by *de Vries* [1952ab] in a theoretical method for calculating the thermal conductivity of porous media. In this method the conductivity of the air-filled pores is expressed as

$$\lambda_a + \lambda_v = \lambda_a + h\nu\beta LD_{atm} \qquad (17)$$

and the usual boundary conditions of continuity of temperature and (apparent) heat flux at the interfaces are used. Numerical values of $\nu\beta LD_{atm}$ are given in Figure 1 together with values of the thermal conductivities of water and air. Experimental evidence in support of this theory has also been given by *Krischer* and *Rohnalter* [1940] and *de Vries* [1952ab].

As an alternative to the tentative formulation given above we can therefore replace the expression $\lambda_* + L\rho_l D_{Tv}$ in the first term on the right hand side of (16) by λ, this being the apparent conductivity of the medium including the effect of vapor distillation due to temperature gradients. (It will be noted that $\lambda \neq \lambda_* + L\rho_l D_{Tv}$, because the heat fluxes in different parts of the medium are not additive). This leads to the following revised

forms of Eqs. (10) and (16)

$$q_h = -(\lambda - L\rho_l D_{Tv})\nabla T + Lq_v + c_l(T - T_0)q_m \qquad (18)$$

and (with the omission of the term with c_p)

$$[C + L(S - \theta_l)h\beta]\frac{\partial T}{\partial t} + \left[\frac{L\rho_l D_{\theta v}}{a\nu D_{atm}}\right.$$

$$- L\rho_v + \rho_l j^{-1}g\left(\Psi - T\frac{\partial \Psi}{\partial T}\right)\left]\frac{\partial \theta_l}{\partial t}\right.$$

$$= \nabla(\lambda\nabla T) + L\rho_l\nabla(D_{\theta v}\nabla\theta_l) + \rho_l c_l$$

$$\cdot[(D_{\theta l}\nabla\theta_l + D_{Tl}\nabla T + Kk)\nabla T] \qquad (19)$$

In (18) use has been made of (14) to eliminate $L_\bullet$, this can of course be done in either formulation.

It might appear arbitrary to single out the particular term arising from vapor distillation due to temperature gradients. However, this term often represents the most important contribution of moisture movement to the heat transfer. Since it increases the conductivity of the poorest conducting part of the system (namely, the gaseous part) it contributes markedly to the overall conductivity of the porous medium. From (17) and Figure 1 it follows, for instance, that in the temperature range of 0° to 40°C the conductivity of the air-filled pores can be increased by a factor 2 to 9 by the vapor contribution.

The second term on the right hand side of (19) also arises from vapor diffusion, this time under the influence of moisture gradients. Since the corresponding heat flux is not proportional to ∇T it cannot be incorporated in λ in the same way as the term $\nabla(L\rho_l D_{Tv}\nabla T)$.

For the present we shall consider (10) with (16), and (18) with (19) as equivalent alternatives. We shall discuss the applicability of each of these alternatives further in the following section.

Eq. (9) and (16) or (19) govern the simultaneous transfer of moisture and heat in porous media. Eq. (16) or (19) replaces the Fourier equation of heat conduction. They are more general than (24) in the paper of Philip and de Vries; (19) reduces to the latter when heat transfer by liquid movement is negligible and $W = \partial\theta_v/\partial t = 0$. The generation of heat due to viscosity of the moving liquid has been neglected in the present treatment. It would give rise to an additional term $gq_l^2/j\rho_l K$ on the right hand sides of (16) and (19). This term is negligible under most circumstances. Eq. (9) is more general than (23) in the previous paper. Note that the coefficients occurring in these equations are functions of the dependent variables, θ_l and T. They all depend

strongly on θ_l. Their variation with T is less outspoken except for $D_{\theta v}$ and D_{Tv} which both increase rapidly with increasing temperature.

For the sake of completeness we must point out that the present theory cannot be applied in cases where hysteresis operates [Philip, 1955].

Numerical values of the various diffusivities occurring in these equations can be calculated once Ψ and K are known as functions of θ_l. The thermal properties (λ and C) of a soil can be calculated from its composition as discussed by the author [de Vries, 1952ab].

Values of the diffusivities for Yolo light clay (a clay loam) were published by Philip [1955, 1957c] and Philip and de Vries [1957]. These are based on measurements of Ψ and K by Moore [1939]. Philip [1955] extrapolated the Ψ and K

FIG. 2 – Moisture conductivity and diffusivities in relation to moisture content θ_l for a medium sand at 20°C; curve 1, K(cm sec^{-1}); curve 2, $D_{\theta v}$ (cm^2 sec^{-1}); curve 3, $D_{\theta l}$ (cm^2 sec^{-1}); curve 4, $D_\theta = D_{\theta v} + D_{\theta l}$

FIG. 3 – Water pressure and thermal moisture diffusivities in relation to moisture content, θ_l, for a medium sand at 20°C; curve 1, Ψ (cm); curve 2, D_{Tv} (cm^2 sec^{-1} °C^{-1}); curve 3, D_{Tl} (cm^2 sec^{-1} °C^{-1}); curve 4, $D_T = D_{Tv} + D_{Tl}$

curves at the low end of the moisture range. The thermal conductivity of this soil in relation to θ_l, as calculated by the author, was also published by Philip [1957c]. Its heat capacity is: $C = 0.23 + \theta_l$.

Since the numerical values will be used only for illustrative purposes we are here not so much concerned with the exact values of the various quantities as with their order of magnitude and in the way they change with θ_l and T.

To illustrate the influence of soil texture values for a sand (particle sizes 0.01 to 0.06 cm) are given in Figures 2 and 3. Values of λ and Ψ in relation to θ_l for this sand were determined by the author [de Vries, 1952a]. Its heat capacity, $C = 0.27 + \theta_l$. Values of K were calculated from the $\Psi(\theta_l)$ curve by the method of Childs and Collis-George [1950]. The calculations were formally extended to θ_l-values as low as 0.006, although it is very doubtful whether the procedure is still applicable at such low moisture contents. This was necessary because the results indicate that moisture movement in the liquid and the vapor phase are of the same order of magnitude in the moisture range $0.006 < \theta_l < 0.02$ (assuming the order of magnitude of the calculated K values to be correct). The value 0.10 was chosen for θ_{lK}. All numerical values given here are for 20°C.

It follows from the equations that true equilibrium in the soil requires that both the total potential ($= \Psi + z$; the influence of soluble salts is left out of consideration here) and the temperature are constant. In a non-equilibrium state there will always be interaction between heat and moisture transfer, even if initially the temperature gradient were zero.

It can be easily checked that for such processes as infiltration of water available in excess at the soil surface and water movement leading to evaporation from bare soil at rates of the order of 10^{-6} g cm^{-2} sec^{-1} or more the dominant term on the left hand side of (9) is $\partial\theta_l/\partial t$, while the term $\nabla(D_T\nabla T)$ on the right hand side is small in comparison with the other terms. These processes can therefore be regarded as isothermal to a good degree of approximation [Philip, 1957ab]. When q_m is of the order of 10^{-7} g cm^{-2} sec^{-1} or less moisture movement due to temperature gradients is no longer negligible [Philip, 1957c; de Vries, 1957].

Similarly in soils under natural conditions the dominant terms in (19) are often $C\partial T/\partial t$ and $\nabla(\lambda\nabla T)$.
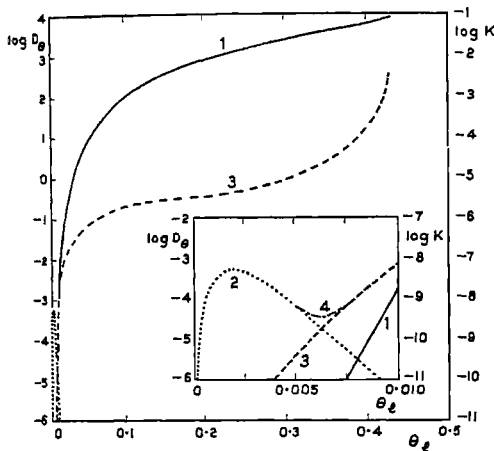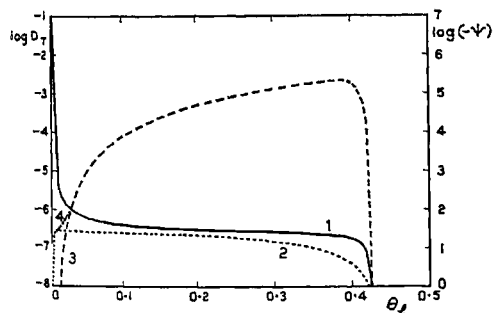
As an application of the present theory we shall

discuss steady state heat conduction in a moist soil in greater detail in the following section.

STEADY STATE HEAT CONDUCTION

We shall assume that an experiment in heat conduction is carried out in which both θ_l and T vary either in a horizontal direction or in a vertical direction and that a steady state has been reached $(\partial\theta_l/\partial t = \partial T/\partial t = 0)$. The steady state will be characterized by the magnitudes of the moisture flux q_m and the heat flux q_h in the sample.

The moisture and temperature gradient can be solved from (2), (3) and (18). For the 'vertical' case we find

$$\frac{d\theta_l}{dz} = \frac{q_h D_T - K\lambda - q_m c_l(T - T_0)D_T - q_m \lambda/\rho_l}{\lambda D_\theta - L\rho_l D_{\theta v} D_T} \quad (20)$$

$$\frac{dT}{dz} =$$
$$- \frac{q_h D_\theta - K L\rho_l D_{\theta v} - q_m c_l(T - T_0)D_\theta - q_m L D_{\theta v}}{\lambda D_\theta - L\rho_l D_{\theta v} D_T} \quad (21)$$

Similar equations for the 'horizontal' case are obtained by substituting x for z and 0 for K. In the alternative formulation (starting from (10) instead of (18)) we must substitute $\lambda_* + L\rho_l D_{Tv}$ for λ.

In thermal conductivity experiments the soil sample is usually bounded by solid surfaces [for example, *Kersten*, 1948] and in this case $q_m = 0$ in the steady state. For the horizontal case (20) and (21) then reduce to

$$\frac{d\theta_l}{dx} = \frac{q_h D_T}{\lambda D_\theta - L\rho_l D_{\theta v} D_T} \quad (22)$$

$$\frac{dT}{dx} = -\frac{q_h D_\theta}{\lambda D_\theta - L\rho_l D_{\theta v} D_T} \quad (23)$$

The apparent thermal conductivity found in an experiment of this type is therefore $\lambda - L\rho_l D_{\theta v} D_T/D_\theta$, or $\lambda_* + L\rho_l(D_{Tv}D_{\theta l} - D_{\theta v}D_{Tl})/D_\theta$ in the alternative formulation.

Values of $d\theta_l/dx$ and dT/dx calculated from (22) and (23) for Yolo light clay and the sand of Figures 2 and 3 are given in Figures 4 and 5. They hold for 20°C and $q_h = 10^{-3}$ cal cm^{-2} sec^{-1}. It will be seen that the moisture gradient in the sample is very small, except in the θ_l region where D_T is not small in comparison with D_θ. This region is narrower and at a lower average moisture content for the coarser textured soil than for the soil of finer texture. Values of $\theta_l(x)$ and $T(x)$ can be obtained by numerical integration of (22) and (23) for given boundary conditions.
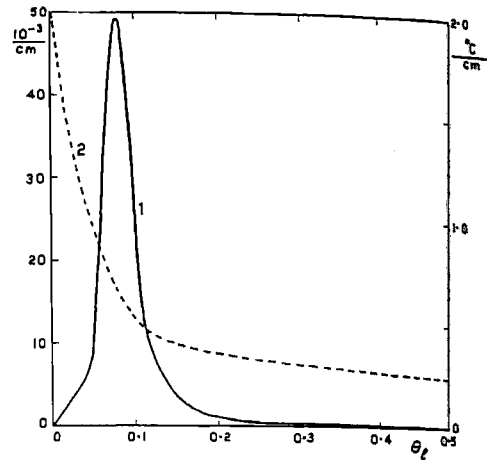

FIG. 4 – Moisture and temperature gradients in Yolo light clay for steady state heat conduction in relation to moisture content, θ_l; values computed from (22) and (23) for 20°C and $q_h = 10^{-3}$ cal cm^{-2} sec^{-1}; curve 1, $d\theta_l/dx$ (cm^{-1}); curve 2, dT/dx (°C cm^{-1})
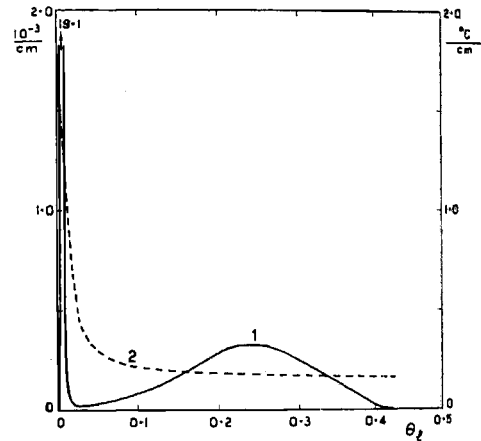

FIG. 5 – Moisture and temperature gradients in a medium sand for steady state heat conduction in relation to moisture content, θ_l; values computed from (22) and (23) for 20°C and $q_h = 10^{-3}$ cal cm^{-2} sec^{-1}; curve 1, $d\theta_l/dx$ (cm^{-1}), curve 2, dT/dx (°C cm^{-1})

We now revert to the applicability of the two alternatives, (10) and (16) or (18) and (19). We recall that temperature gradients give rise to moisture movement in the vapor and the liquid phase of which the former contributes most to the heat transfer. In steady state experiments with $q_m = 0$ there will be a return moisture flow of equal magnitude due to moisture gradients. We now distinguish the following three θ_l regions.

The region where $D_{\theta l} \gg D_{\theta v}$—In this case the

return flow is almost entirely in the liquid phase. Vapor diffusion due to temperature gradients will contribute fully to the heat transfer and equation (18) must be preferred. It will be noted that in this region $L\rho_l D_{\theta v} D_T/D_\theta \ll \lambda$, so that the apparent conductivity measured in an experiment of this type is the same as that predicted by the theory based on (17).

The region where $D_{\theta l} \ll D_{\theta v}$—Here moisture movement is almost exclusively in the vapor phase, as it can be easily checked that D_{Tl} is also small in comparison with D_{Tv} in this region. There will be no net vapor movement and (10) must therefore be used. Eq. (22) and (23) now reduce to

$$d\theta_l/dx = q_h D_{Tv}/\lambda_* D_{\theta v} \qquad (24)$$

$$dT/dx = -q_h/\lambda_* . \qquad (25)$$

The region where $D_{\theta l}$ and $D_{\theta v}$ have comparable magnitudes—In this region D_{Tl} will still be small in comparison with D_{Tv}. Return flow to the temperature induced vapor movement will take place both in the vapor and the liquid phases. Hence, vapor diffusion will contribute only partly to the transfer of heat. For steady states and $q_m = 0$ this can be taken into account by the following modification of (17)

$$\lambda_a + \lambda_v = \lambda_a + D_{\theta l} h v \beta L D_{atm}/D_\theta \qquad (26)$$

which applies to the first two regions as well.

Further we must point out that the differences between λ and $\lambda_* + L\rho_l D_{Tv}$, although not negligible, are fairly small, at least at temperatures below about 30°C. At 20°C the maximum value of $L\rho_l D_{Tv}$ is 0.15 × 10⁻³ cal cm⁻¹ sec⁻¹ °C⁻¹ for both the sand and Yolo light clay. This is small in comparison with λ (or λ_*) except at low moisture contents. The difference between λ and λ_* is generally of the same order of magnitude as $L\rho_l D_{Tv}$. For instance, for the sand at 20°C and $\theta_l = 0.09$ (calculated and measured) value of λ is 4.38 × 10⁻³, while the (calculated) values of λ_* and $L\rho_l D_{Tv}$ are 4.06 × 10⁻³ and 0.14 × 10⁻³ respectively. At 20°C and $\theta_l = 0.01$ these values are: $\lambda = 0.90 \times 10^{-3}$, $\lambda_* = 0.64 \times 10^{-3}$ and $L\rho_l D_{Tv} = 0.15 \times 10^{-3}$. However, $L\rho_l D_{Tv}$ increases rapidly with increasing temperature and at 60°C, for instance, its maximum value is about 1.0 × 10⁻³ cal cm⁻¹ sec⁻¹ °C⁻¹.

The situation is more complicated for steady state conditions in the 'vertical' case or when q_m is not zero. Then q_h is no longer strictly proportional to ∇T. The outcome of a thermal conductivity experiment, for instance, will depend on the boundary conditions for moisture flow and on the direction of the temperature gradient. However, K is practically zero in the second region and there is no difference between transfer of heat and moisture in horizontal and vertical directions in this region.

Under non-stationary conditions (16) or (19) must be used with proper initial and boundary conditions. Here also q_h will not be proportional to ∇T. An analysis of non-stationary experiments for determining the thermal conductivity using cylindrical probes will be published elsewhere (*de Vries* and *Peck* [1958ab]).

Under most circumstances it will be best to apply (18) and (19) in the first region. For the second and third regions the choice between the alternative formulations must be decided upon by a careful consideration of the experimental conditions. If, for instance, water is supplied at a sufficient rate, while q_m and q_h are in the same direction (18) and (19) can be applied over the entire range of moisture contents.

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