

First order approx is

$$f(0) + f'(0) * (p_{ic} - 0) = p_{ic} \left(\left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \right)$$

Where

p_{ic} is the probability of a nonsense error at position i using codon c

p_j is the probability of a nonsense error at position j (using any codon)

$$f(p_{ic}) = \left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\frac{p_{ic}}{1-p_{ic}} \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right]$$

So

$$f(0) = \left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \left[\frac{0}{1-0} \right]$$

$$f(0) = 0$$

$$\begin{aligned} f'(p_{ic}) &= \frac{\delta}{\delta p_{ic}} f(p_{ic}) = \frac{\delta}{\delta p_{ic}} \left(\left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\frac{p_{ic}}{1-p_{ic}} \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \right) \\ &= \left(\left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \right) \frac{\delta}{\delta p_{ic}} \left[\frac{p_{ic}}{1-p_{ic}} \right] \\ &= \left(\left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \right) \left[\frac{(1-p_{ic})(1) - p_{ic}(-1)}{(1-p_{ic})^2} \right] \\ &= \left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \left[\frac{1}{(1-p_{ic})^2} \right] \end{aligned}$$

So

$$\begin{aligned} f'(0) &= \left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \left[\frac{1}{(1-0)^2} \right] \\ &= \left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \end{aligned}$$

$$f''(p_{ic}) = \frac{\delta}{\delta p_{ic}} \left(\left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \left[\frac{1}{(1-p_{ic})^2} \right] \right)$$

$$f''(p_{ic}) = \left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \frac{\delta}{\delta p_{ic}} \left[(1-p_{ic})^{-2} \right]$$

$$f''(p_{ic}) = \left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \left[-2(1-p_{ic})^{-3}(-1) \right]$$

$$f''(p_{ic}) = \left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \left[2(1-p_{ic})^{-3} \right]$$

So

$$f''(0) = \left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right] \left[2(1-0)^{-3} \right]$$

$$= 2 \left[\sum_{k=1}^i a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^n \left(\frac{1}{1-p_j} \right) \right]$$