First order approximation about $p_{ic} = 0$ is

$$f(0) + f'(0) * (p_{ic} - 0) = p_{ic}\left(\left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]\right)$$

Where
p_{ic} is the probability of a nonsense error at position i using codon c
p_j is the probability of a nonsense error at position j (using any codon)

$$f(p_{ic}) = \left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\frac{p_{ic}}{1-p_{ic}}\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]$$

So...

$$f(0) = \left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]\left[\frac{0}{1-0}\right]$$

$$f(0) = 0$$

$$f'(p_{ic}) = \frac{\delta}{\delta p_{ic}} f(p_{ic}) = \frac{\delta}{\delta p_{ic}}\left(\left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\frac{p_{ic}}{1-p_{ic}}\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]\right)$$

$$= \left(\left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]\right)\frac{\delta}{\delta p_{ic}}\left[\frac{p_{ic}}{1-p_{ic}}\right]$$

$$= \left(\left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]\right)\left[\frac{(1-p_{ic})(1) - p_{ic}(-1)}{(1-p_{ic})^2}\right]$$

$$= \left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]\left[\frac{1}{(1-p_{ic})^2}\right]$$

So...

$$f'(0) = \left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]\left[\frac{1}{(1-0)^2}\right]$$

$$= \left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]$$

$$f''(p_{ic}) = \frac{\delta}{\delta p_{ic}} \left(\left[\sum_{k=1}^{i} a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^{n} (\frac{1}{1-p_j}) \right] \left[\frac{1}{(1-p_{ic})^2} \right] \right)$$

$$f''(p_{ic}) = \left[\sum_{k=1}^{i} a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^{n} (\frac{1}{1-p_j}) \right] \frac{\delta}{\delta p_{ic}} \left[(1-p_{ic})^{-2} \right]$$

$$f''(p_{ic}) = \left[\sum_{k=1}^{i} a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^{n} (\frac{1}{1-p_j}) \right] \left[-2(1-p_{ic})^{-3}(-1) \right]$$

$$f''(p_{ic}) = \left[\sum_{k=1}^{i} a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^{n} (\frac{1}{1-p_j}) \right] \left[2(1-p_{ic})^{-3} \right]$$

So...

$$f''(0) = \left[\sum_{k=1}^{i} a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^{n} (\frac{1}{1-p_j}) \right] \left[2(1-0)^{-3} \right]$$

$$= 2 \left[\sum_{k=1}^{i} a_1 + a_2(k-1) \right] \left[\prod_{j=i+1}^{n} (\frac{1}{1-p_j}) \right]$$

Based on what we've talked about with the math, it may be possible that Mike meant to assume $p_{i+1} \approx p_{i+2} \approx \cdots \approx p_n$. This feels a little questionable. They are all sufficiently close to 0, but approximating them all at once is suspect. For the first codon in a gene of length 400, p_j incorporates as many as 3^{399} different values.

Regardless, here is the approximation. To keep in the spirit of $p_j \neq p_{j+1}$, I will not simplify $\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]$ to $\frac{1}{(1-p_j)^{n-(i+1)}}$, though the calculation actually comes out the same either way.

The final result, the approximation about $p_{i+1}, \cdots, p_n \approx 0$ is

$$f(p_j) \approx \left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\frac{p_{ic}}{1-p_{ic}}\right] + ((i+1)-n)\left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\frac{p_{ic}}{1-p_{ic}}\right](p_j)$$

$$f(p_j) \approx f(0) + (f(0))(p_j)((i+1)-n)$$

I'm concerned about that last p_j term that is added to the first order term. What is that? We approximated around $p_{i+1} \approx p_{i+2} \approx \cdots \approx p_n$

$$f(p_j) = \left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\frac{p_{ic}}{1-p_{ic}}\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]$$

So...

$$f(0) = \left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\frac{p_{ic}}{1-p_{ic}}\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-0})\right]$$

$$f(0) = \left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\frac{p_{ic}}{1-p_{ic}}\right]$$

$$f'(p_j) = \frac{\delta}{\delta p_j}\left(\left[\sum_{k=1}^{i} a_1 + a_2(k-1)\right]\left[\frac{p_{ic}}{1-p_{ic}}\right]\left[\prod_{j=i+1}^{n}(\frac{1}{1-p_j})\right]\right)$$

Which I'm going to restate for simplicity as

$$f'(p_j) = (f(0))\,\frac{\delta}{\delta p_j}\left(\prod_{j=i+1}^{n}\frac{1}{1-p_j}\right)$$

$$f'(p_j) = (f(0))\left(\sum_{j=i+1}^{n}\frac{-1}{(1-p_j)^2}\right)\left(\prod_{j=i+1, j\neq i}^{n}\frac{1}{1-p_j}\right)$$

So...

$$f'(0) = (f(0)) \left(\sum_{j=i+1}^{n} \frac{-1}{(1-0)^2} \right) \left(\prod_{j=i+1, j \neq i}^{n} \frac{1}{1-0} \right)$$

$$f'(0) = (f(0)) \left(\sum_{j=i+1}^{n} \frac{-1}{(1-0)^2} \right) \left(\prod_{j=i+1, j \neq i}^{n} \frac{1}{1-0} \right)$$

$$f'(0) = (f(0)) \left((i+1) - n\right) = \left((i+1) - n\right) \left[\sum_{k=1}^{i} a_1 + a_2(k-1) \right] \left[\frac{p_{ic}}{1 - p_{ic}} \right]$$