Multibody simulation

Laboratory Exercise 1

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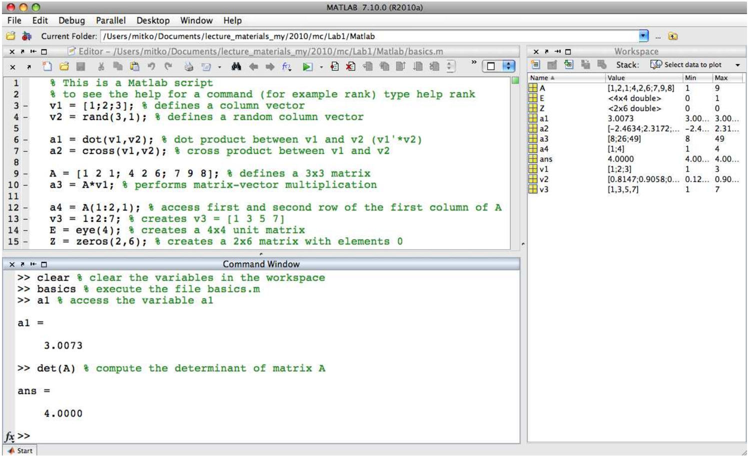
Main points covered

- basics of Matlab
- simulation of a free-falling ball
- existing physics engine



Many tutorials on-line

- http://www.cyclismo.org/tutorial/matlab/
- http://www.engin.umich.edu/class/ctms/
- many others . . .

Task 0 (warm up)

Note

In order to pass the lab you need not only to provide the answer but also show how you computed it. One quick solution is to cut and paste the commands and outputs to a text file.

1. If you are not familiar with Matlab, follow some of the on-line tutorials (for example, searching for 'Matlab introduction tutorial' will get you going). A good starting point is: http://se.mathworks.com/videos/getting-started-with-matlab-68985.html.
2. Test in Matlab the concepts we discussed during the linear algebra review. Given a = [1 2 3] and b = [2 3 1] compute the following:
   - adding the two vectors a and b
   - visualize the two vectors (a,b) and their cross product
   - describe in a sentence with your own words what a cross product does (not how it's computed but the geometrical interpretation)
   - visualize the projection of a vector on a line (through the origin), use vector 'a' and the y-axis as a line

Task 1 - rotation matrices

Linear algebra will be used to compute positions and rotations of bodies in space. A 2D rotation matrix is computed as:

$$\boldsymbol{R} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix},$$

where θ is the rotation angle. The new position is computed using $\boldsymbol{R}(\theta)p$ where p is a point in 2D space.

1. Given a point p = [1,0]', use the rotation matrix to rotate the point 45 degrees ($\theta = \pi/4$).
2. Animate/visualize a full rotation of p ($\theta = [0..2\pi]$) using a for loop. Hints: place the plot command and a delay in the loop.
3. Play around with the inverse (inv) of the rotation matrix $\boldsymbol{R}^{-1}$. Explain how it is related to $\boldsymbol{R}$? Hints: try out p3 = inv(R)*p2.
4. A rotation matrix is a orthonormal matrix - check the linear algebra lecture notes about its properties. Verify that the properties and holds by trying them on the rotation matrix R. Use $\theta = \pi/8$. Explain what each property means in your own words.

Review: basic concepts

Differentiation

$$\frac{d}{dt}(\text{position}) \rightarrow \text{velocity}$$

$$\frac{d}{dt}(\text{velocity}) \rightarrow \text{acceleration}$$

$$\frac{d}{dt}(\text{acceleration}) \rightarrow \text{jerk}$$

Integration

$$\int (\text{jerk})dt \rightarrow \text{acceleration}$$

$$\int (\text{acceleration})dt \rightarrow \text{velocity}$$

$$\int (\text{velocity})dt \rightarrow \text{position}$$

Principle of inertia

If an object is not disturbed, it continues to move with a constant (linear) velocity in a straight line if it was originally moving. Or it continues to stand still if it was standing still.

The second law of Newton

The net force ($\boldsymbol{F}$) on an object is equal to the mass (m) of the object multiplied by its acceleration ($\boldsymbol{a}$)

$$\boldsymbol{F} = m\boldsymbol{a}.$$

Linear momentum

The net force on an object is proportional to the time rate of change of its linear momentum $\boldsymbol{p}$

$$\boldsymbol{F} = m\frac{d\boldsymbol{v}}{dt} = \frac{d(m\boldsymbol{v})}{dt} = \frac{d\boldsymbol{p}}{dt} = \dot{\boldsymbol{p}},$$

where a constant mass m is assumed. The units are

$$[\mathsf{N}] = [\mathsf{kg}][\mathsf{m/s}]/\mathsf{s} = \mathsf{kgm/s}^2.$$

Facts

- on the Earth's surface, a mass of 1 kg exerts a force of approximately 9.8 N
- the linear momentum $\boldsymbol{p}$ has a magnitude and direction (identical to the direction of the velocity $\boldsymbol{v}$)
- if $\boldsymbol{F} = \mathbf{0}$ the linear momentum is conserved ($\dot{\boldsymbol{p}} = \mathbf{0}$)

Task 2 (free-falling ball)

1. How can we model a free-falling ball (what is the **equation of motion**)? You can assume zero air resistance.
2. Implement the simulation in Matlab using a for loop (make sure you can explain all lines of your code and try to divide the lines into the steps detailed in the next page).
3. What are the *parameters* that we need to set before the simulation?
4. How much time is required for the ball to fall 100 m. starting from zero initial velocity? Do not calculate this analytically but instead find it out using your simulation.
5. What is the role played by the mass m of the ball?

The core - the simulation procedure

How to do a simulation

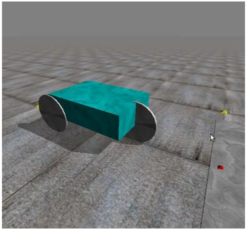
- at time t we have the position y and velocity $\dot{y}$ of the system [in this case the ball, where the y-axis is pointing upwards] (we will call $(y, \dot{y})$ the **state** of the system at time t)
- substitute the known variables in the equation of motion and find the **acceleration** of the system
- **integrate** the vector of accelerations to obtain the new state (at $t + \Delta t$)

This might be useful for “task 2”

- $\boldsymbol{r}_0$ - initial position
- $\boldsymbol{r}_f$ - position after Δt seconds
- $\boldsymbol{v}_0$ - initial velocity
- $\boldsymbol{v}_f$ - velocity after Δt seconds
- $\bar{\boldsymbol{v}} = \frac{1}{2}(\boldsymbol{v}_0 + \boldsymbol{v}_f)$ - average velocity for the period Δt
- $\boldsymbol{a}$ - constant acceleration

$$
\begin{aligned}
\boldsymbol{v}_f &= \boldsymbol{v}_0 + \boldsymbol{a}\Delta t \\
\boldsymbol{r}_f &= \boldsymbol{r}_0 + \bar{\boldsymbol{v}}\Delta t = \boldsymbol{r}_0 + \frac{1}{2}(\boldsymbol{v}_0 + \boldsymbol{v}_f)\Delta t \\
&= \boldsymbol{r}_0 + \boldsymbol{v}_0\Delta t + \frac{1}{2}\boldsymbol{a}\Delta t^2
\end{aligned}
$$

Task 3 - Using a physics engine



In the previous exercise the state consisted of two variables of the system (the ball) the position y and the velocity $\dot{y}$. This is of course highly simplified and to give an impression on how this is handled in more complex simulations you will now look into a physics engine. Note that the simulation procedure is exactly the same as in your falling ball case.

States

- start the program and drive around the buggy, create a state.dif file (an inbuilt output file of the engine)
- look in state.dif and find the corresponding state information

Integration and Δt

In the program there is the possibility to change the stepsize Δt and a delay parameter d. Increasing the delay will have the same results as running on a slower and slower computer.

- try with different settings of Δt and d. Explain with your own words what is happening and why?
- try with very large values of Δt. Explain with your own words what is happening and why.
- in the simulation menu you can select to perform a single step, try it out and explain what it means.
- if you want to have a simulation that works in “real-time” and looks good, how would you select these parameters?

Lab 1 - receipt

After completing all exercises in the lab you need to present the results to the teacher/assistant in order to pass the lab. It should be completed latest 2 weeks after the lab was given. The receipt is given to you as a proof that you passed the lab. A separate record will be kept by the teacher.

Be prepared to demonstrate all results before the presentation.

Name:

Personal number:

Approved - signature:

Date: