
Artificial Intelligence Techniques for Mobile Robots

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Course home page:
<http://aass.oru.se/~asaffio/Teaching/AIMR/>

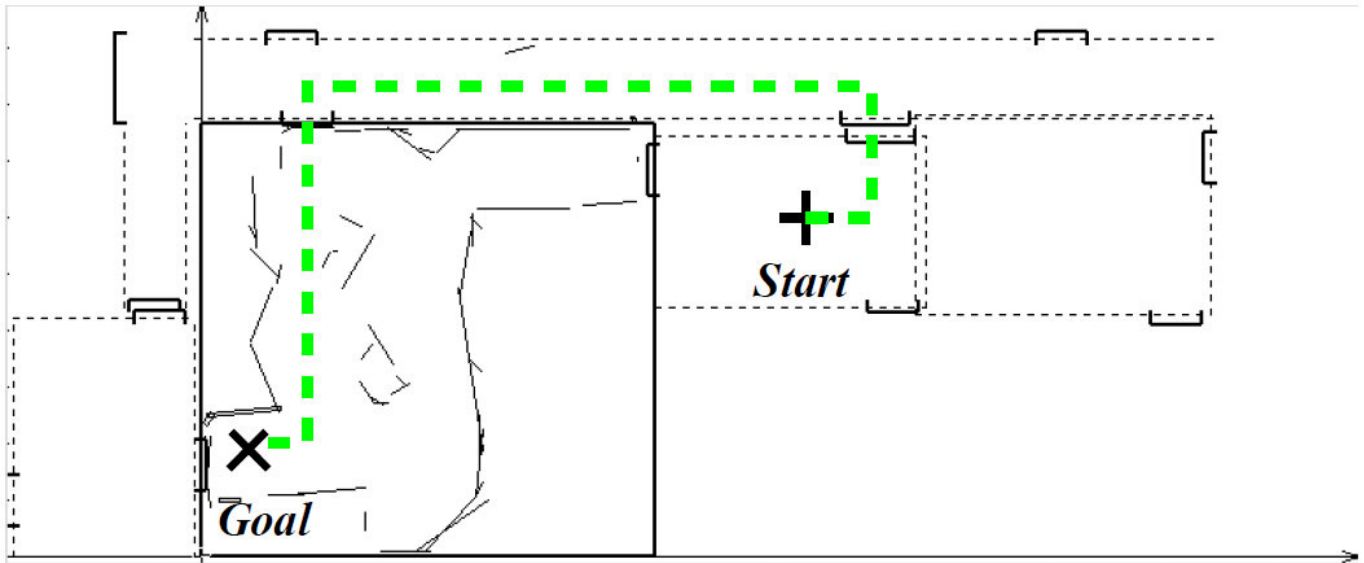
Path planning

Outline

- Searching
 - *the path planning problem*
 - *the search problem in Artificial Intelligence (AI)*
 - *path planning seen as a search problem*
- Breadth-first search
 - *intuition*
 - *the search algorithm* ⇐ Lab
 - *path generation* ⇐ Lab
- Other types of search
 - *depth-first search*
 - *heuristic search*
 - *A* search*
- Summary of types of search

Search

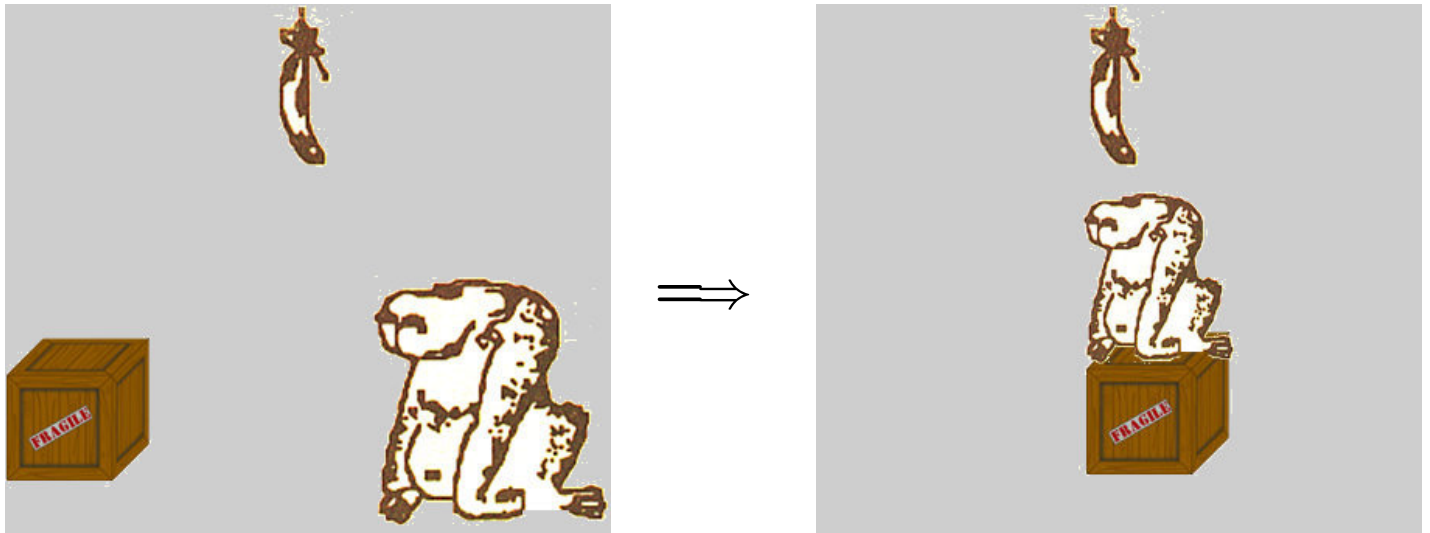
Recall: the planning problem



- Find a trajectory in the map that
 - *goes from start position to goal position*
 - *is collision free*
 - *has minimum length*
- In this lesson:
 - *assume full geometric description of the environment*
 - *ignore the kinematic and dynamic constraints*
 - *length is only quality parameter (no time, energy, ...)*
 - *ignore uncertainty*

The search problem

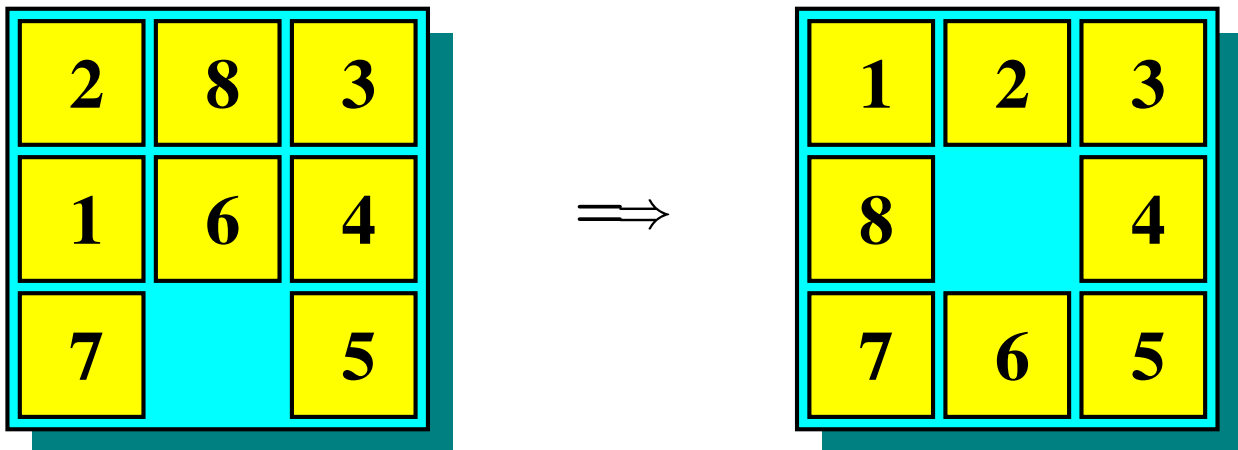
Problem solving as search in state space



- We want to reach a given state of the world...
- ...starting from an initial one...
- ...by applying state-transforming actions
 - '*push box*', '*climb box*', '*grab banana*'

The search problem

Problem solving as search in state space



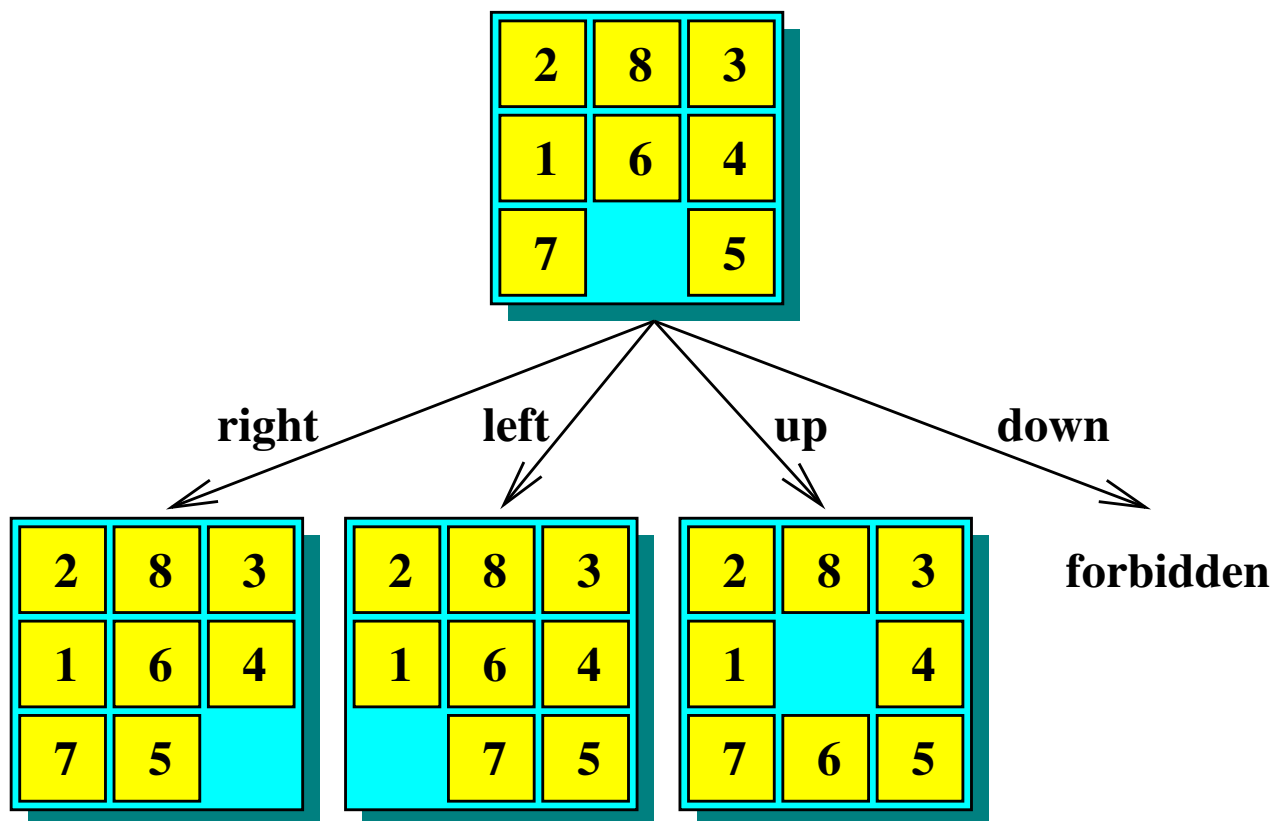
- We want to reach a given state of the world...
- ...starting from an initial one...
- ...by applying state-transforming actions
 - 'move tile 6 down' , 'move tile 4 left' ...

Formulating the Problem

- States: descriptions of the state of the world
 - *Must mention all and only the relevant aspects*
- Operators: how we move from state to state
 - *Must specify legal moves in each state*
 - *Must use abstract actions*
 - *Actions may have a cost*
- Goal: a set of states
 - *Can be one specific state . . .*
 - *. . . or all the states satisfying a desired condition*
- Solution: a path from initial state to a goal state
 - *Encodes the sequence of actions to perform*
 - *We may want to find a minimal cost path*

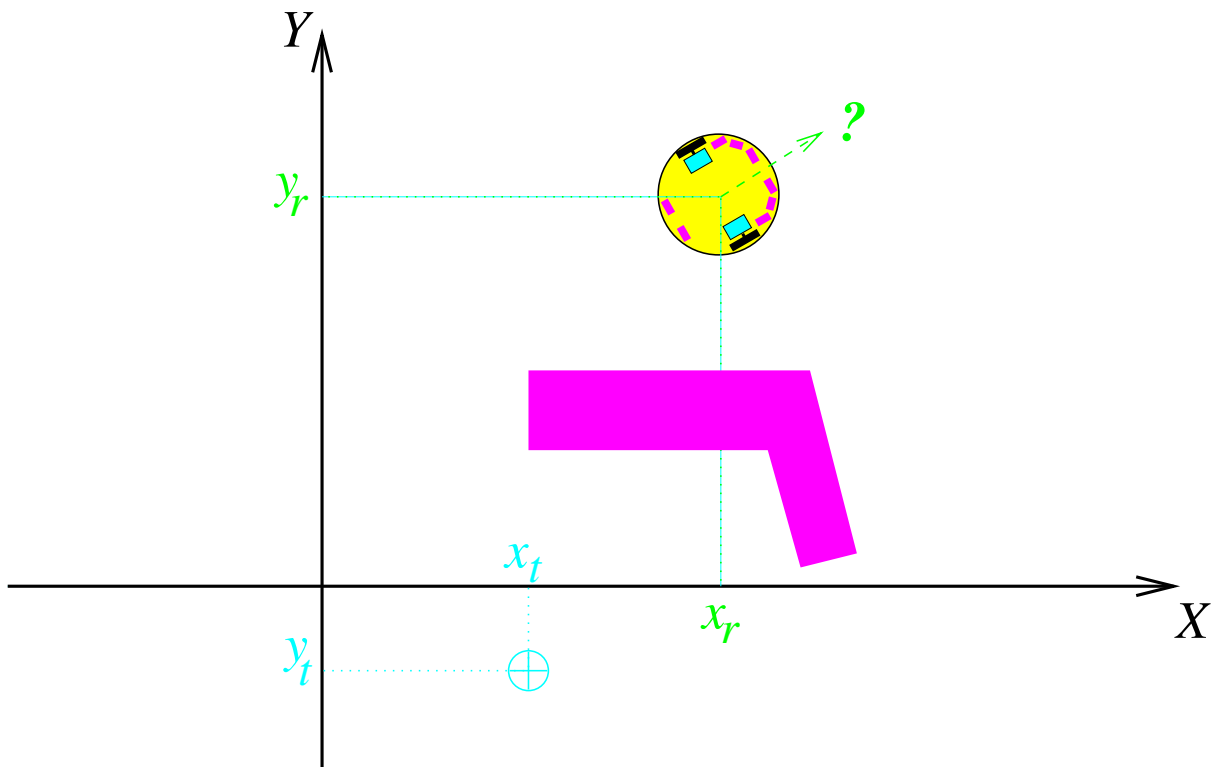
Search example

- States: 3×3 array or integers (0 for blank)
 - # of nodes: $9! = 362,880$ nodes
- Operators: 'move-1-up', 'move-1-left', ...
 - branching factor: $8 \times 4 = 32$
 - better: 'move-blank-up', 'move-blank-left', ...



- Goal: reach the given state

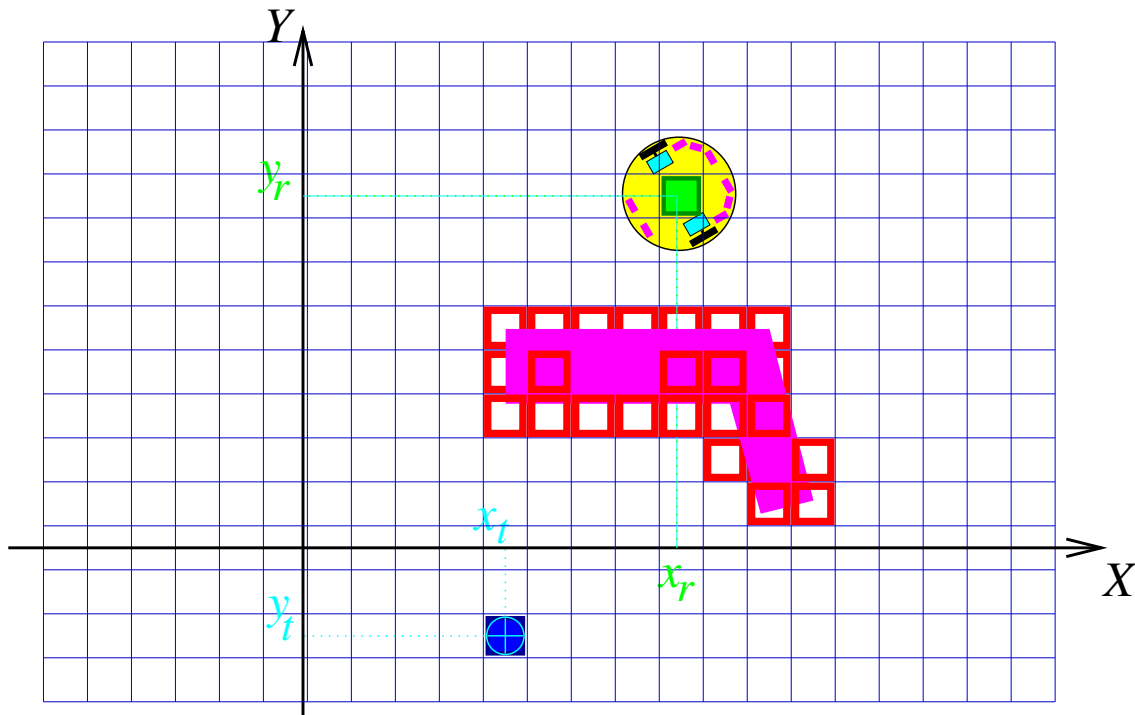
Search for robot navigation



- States:
 - *the (x_r, y_r) position of the robot*
- Operators:
 - *the possible atomic moves of the robot*
 - *“possible” = do not collide with obstacles*
- Goal:
 - *the (x_t, y_t) position of the target*

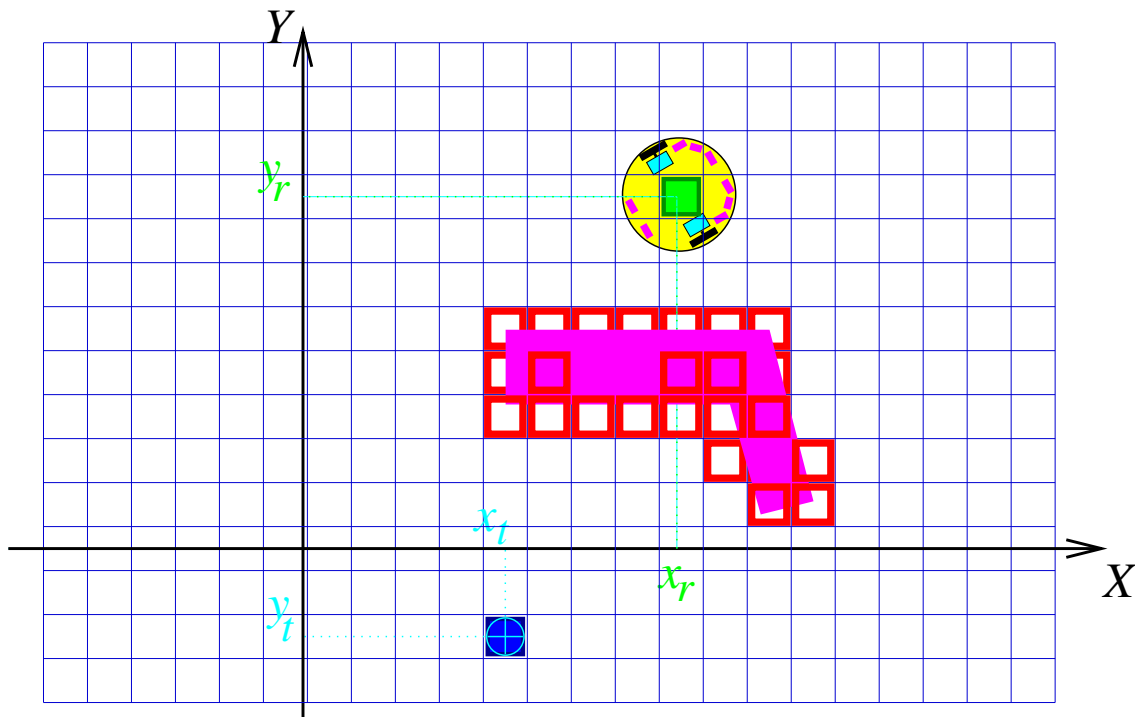
But \Rightarrow the state space is continuous!

Path planning as a search problem



- States:
 - *grid decomposition of space*
- Operators:
 - *single step between cells in the grid*
 - *move according to 4-connectivity or to 8-connectivity*
 - *cells (partially) occupied by obstacle are forbidden*
 - *note: obstacles should be “grown” by one robot radius*
- Goal:
 - *reach a given cell in the grid ...*
 - *... in a minimum number of steps*

Path planning: general strategy

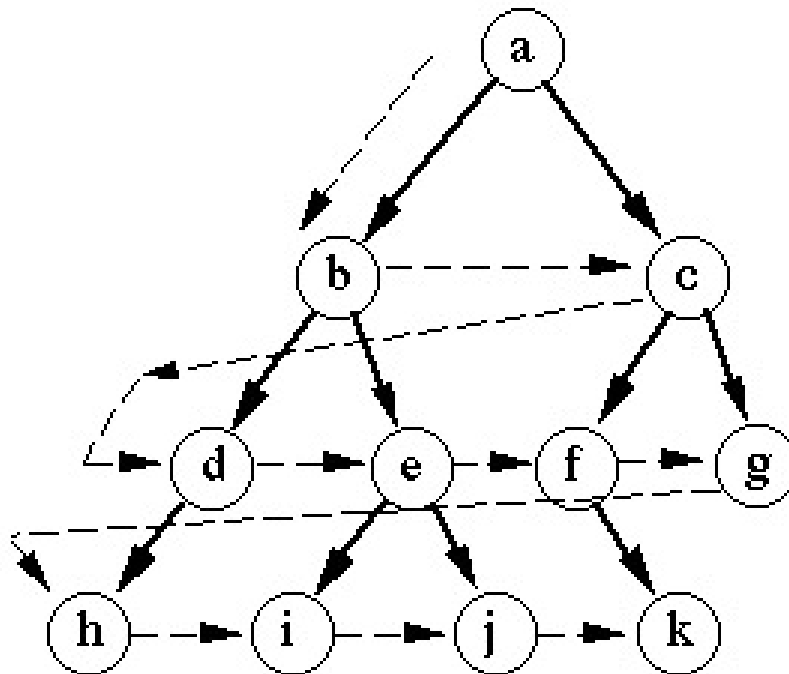


1. See what are your possible moves in current state
2. Chose and simulate execution of one move
3. If you are at the goal \Rightarrow stop
4. If you are in trouble \Rightarrow backtrack

Planning means to simulate execution in your head by trials-and-errors, before you go and execute the actions in the world.

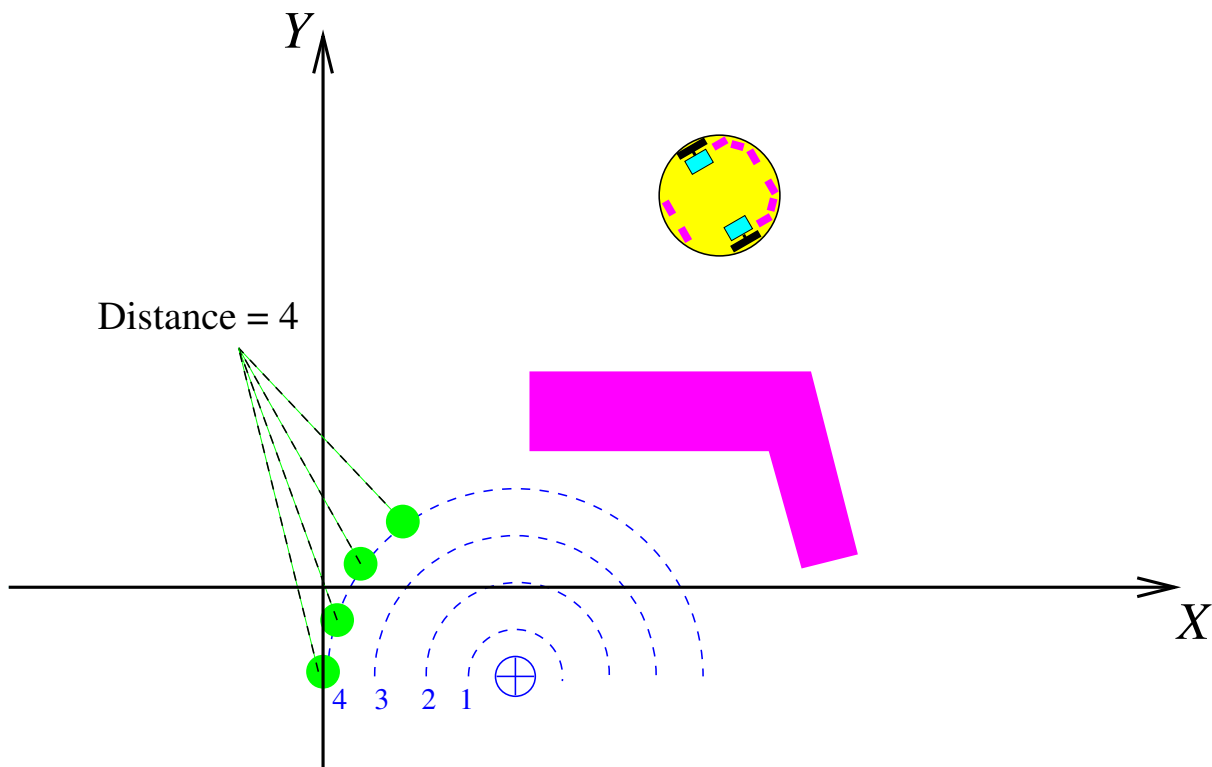
Breadth-first search

Breadth-First Search



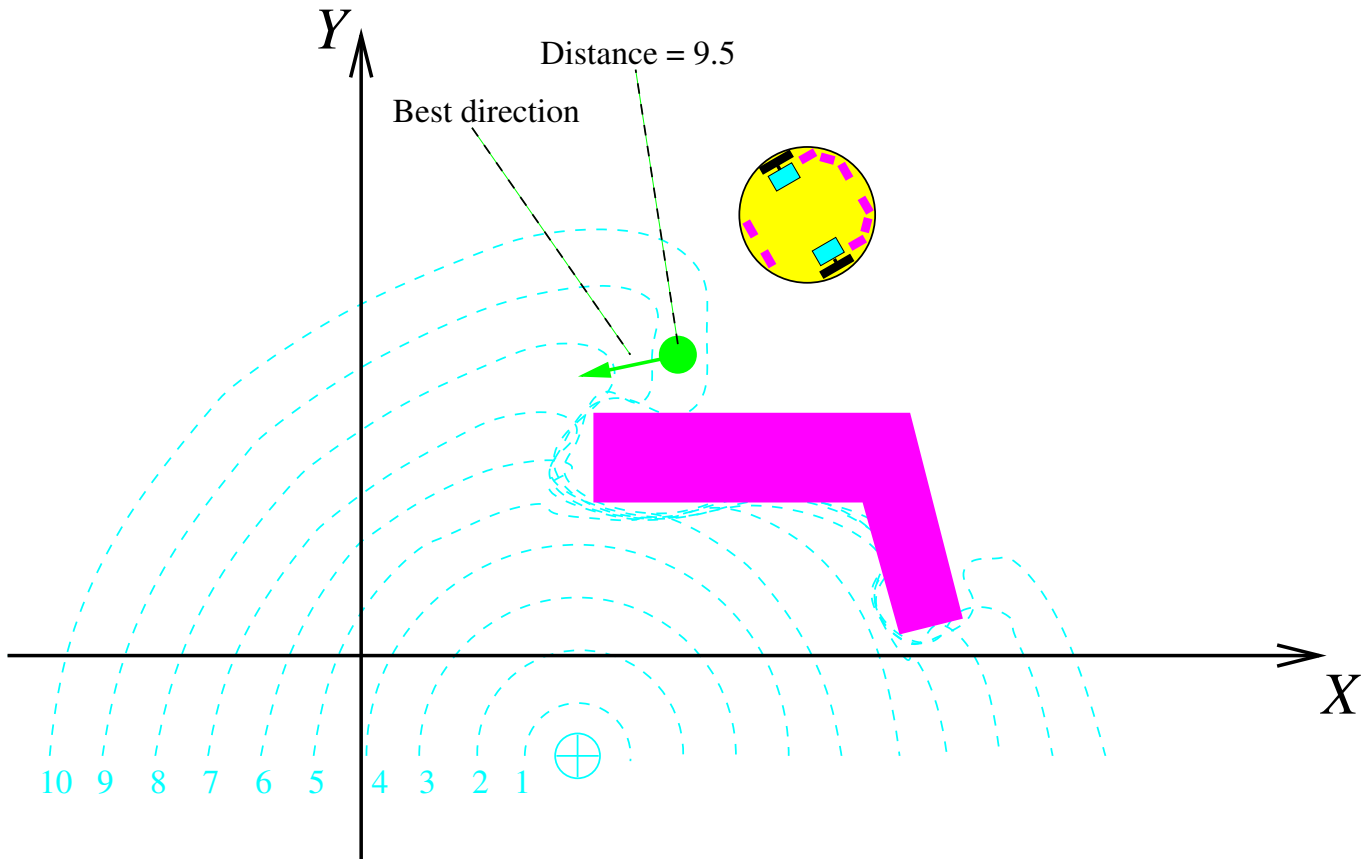
- Explore the space by layers
 - *all the nodes that can be reached in 1 move*
 - *then all the nodes that can be reached in 2 moves*
 - *then ...*
- All nodes in a layer have same distance from root
- Can be done in either direction
 - *start from initial state until you hit the goal*
 - *start from goal until you hit initial state*
 - *or both (bi-directional) until the searches meet*

In our case . . .



- Explore the space by layers
 - *all the cells that can be reached in 1 move*
 - *then all the cells that can be reached in 2 moves*
 - *then . . .*
- All cells in a layer have same distance from start
- Can be done in either direction
 - *start from robot until you hit the goal*
 - *start from goal until you hit the robot*
 - *or both (bi-directional) until the waves meet*

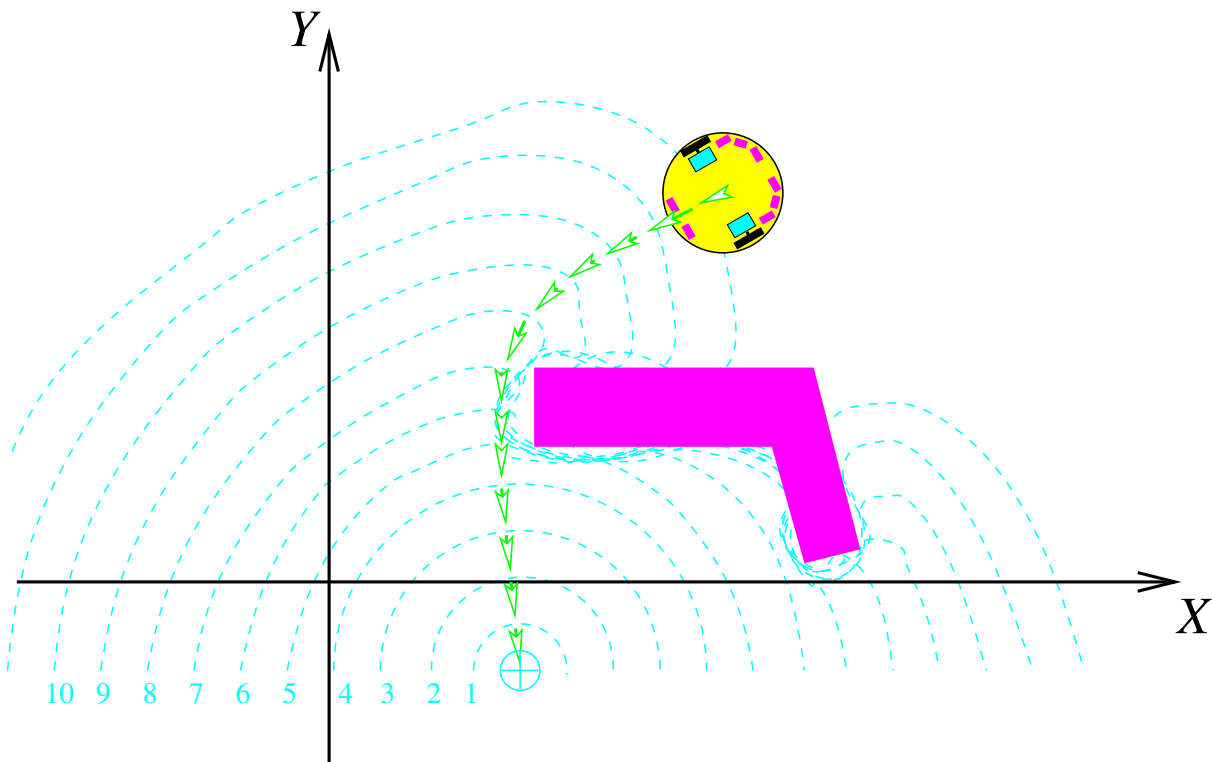
The Wavefront algorithm



- Special case of Breadth-First Search
 - propagate a “wave” from the goal
 - the wave goes around obstacles
 - the wave leaves a timestamp at each point
 - when the wave hits the robot, stop
- Now, at each point:
 - we know how far is the goal
 - we know which direction is best to go

(the one with decreasing distance)

The result of the search



- We can keep the full distance matrix
- Or we can extract a path
 - *by looking at decreasing values*
- Discussion:
 - *what are the pros and cons of both?*

Implementation – data structures

cell: a 2-array (i,j) of indexes in the discrete grid

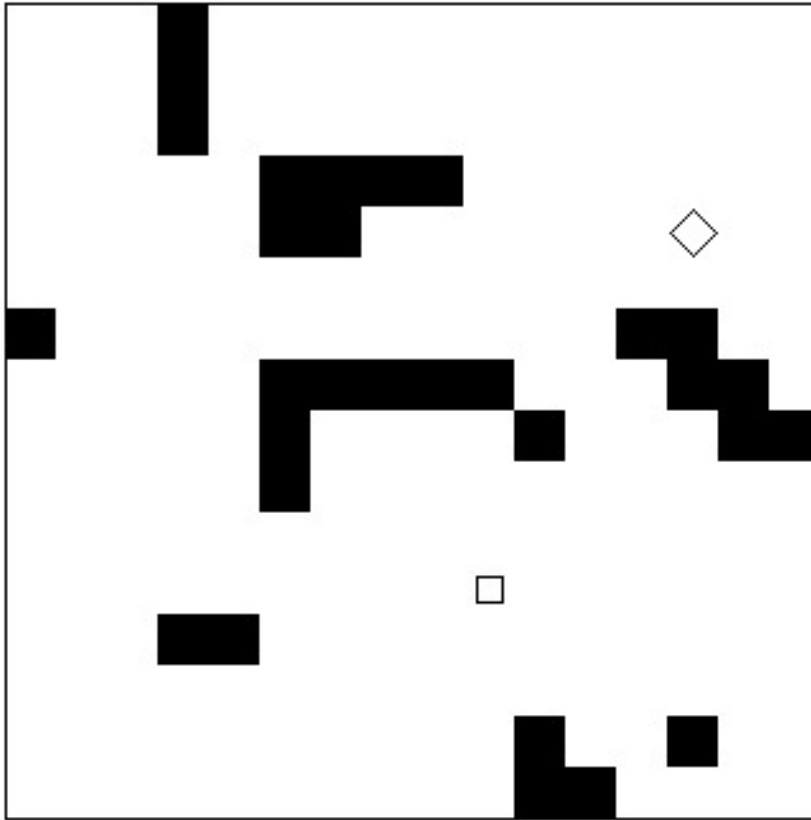
grid: $N \times N$ array of values for cells; each value in the grid is an integer with meaning:

{	$+x$	distance to goal is x steps
	0	goal cell
	-1	cell out of boundaries
	-2	cell has not been computed yet
	-3	obstacle cell
	-4	initial position

(Note: markers in the files maps.c / maps.h might be different)

queue: list of cells that must be explored next details on this in a few moments . . .

The grid at start



[0,0]->

```

-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1
-1 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -1
-1 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -1
-1 -2 -2 -2 -2 -3 -3 -3 -3 -2 -2 -2 -2 -2 -2 -1
-1 -2 -2 -2 -2 -3 -3 -2 -2 -2 -2 -2 -2 -4 -2 -1
-1 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -1
-1 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -3 -3 -2 -1
-1 -2 -2 -2 -2 -3 -3 -3 -3 -3 -2 -2 -2 -3 -3 -1
-1 -2 -2 -2 -2 -3 -2 -2 -2 -2 -3 -2 -2 -2 -3 -1
-1 -2 -2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -1
-1 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -1
-1 -2 -2 -2 -2 -2 -2 -2 -2 -2 0 -2 -2 -2 -2 -1
-1 -2 -2 -3 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -1
-1 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -1
-1 -2 -2 -2 -2 -2 -2 -2 -2 -2 -3 -2 -2 -3 -2 -1

```

<- [0,15]

[15,0]->

```

-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1

```

<- [15,15]

Implementation – main concept

procedure Search ()

put the goal cell in the queue

repeat until the queue is empty

[*] *take a cell c from the queue*

if c is the robot cell then return(success)

foreach n which is a neighbor cell of c

if n is not an obstacle and n has label -2

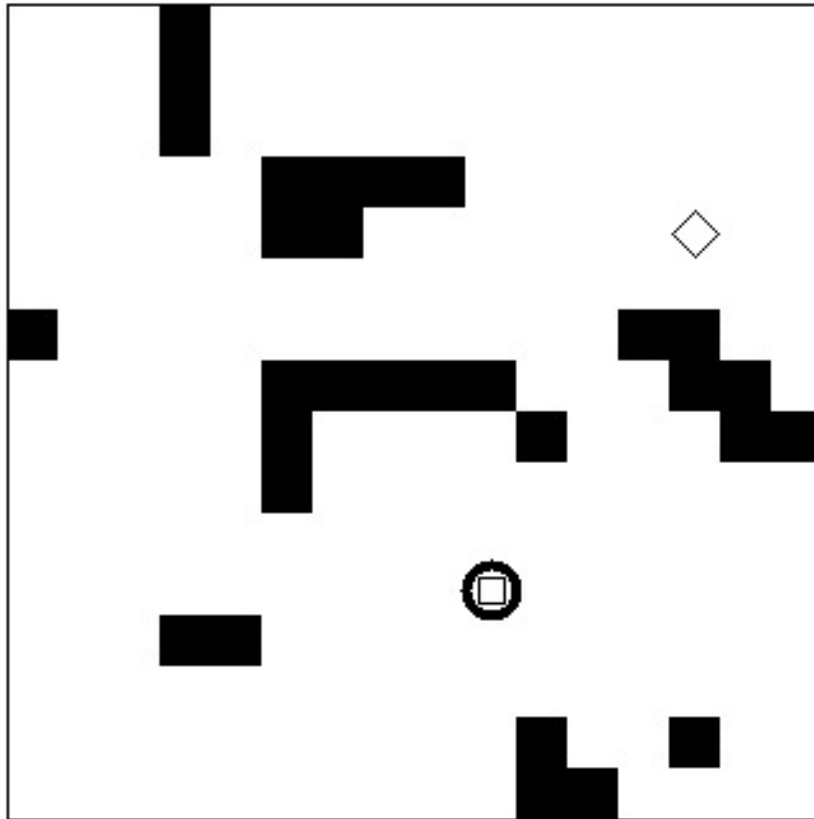
label n by $\underline{\text{grid}(c)} + 1$

[*] *insert n in the queue*

if the queue is empty then return(failure)

end

[*] depending on how we perform these operations
we obtain search strategies



Example – step 2

procedure Search ()

put the goal cell in the queue

repeat until the queue is empty

[*] *take a cell c from the queue*

if c is the robot cell then return(success)

foreach n which is a neighbor cell of c

if n is not an obstacle and n has label -2

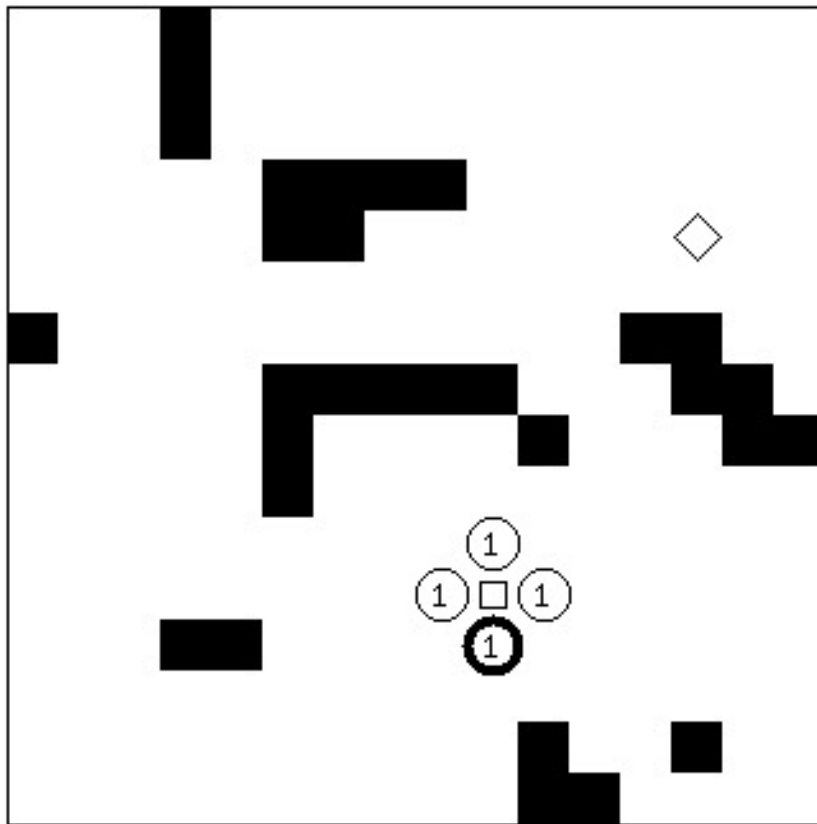
label n by $\underline{\text{grid}(c)} + 1$

[*] *insert n in the queue*

if the queue is empty then return(failure)

end

[*] depending on how we perform these operations
we obtain search strategies



Example – step 3

procedure Search ()

put the goal cell in the queue

repeat until the queue is empty

[*] *take a cell c from the queue*

if c is the robot cell then return(success)

foreach n which is a neighbor cell of c

if n is not an obstacle and n has label -2

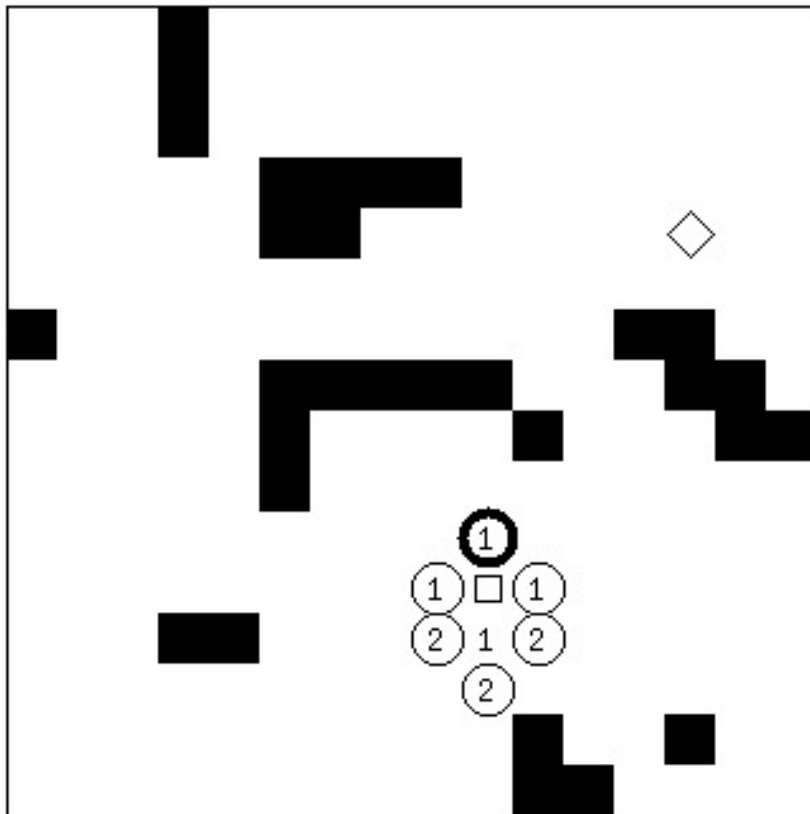
label n by $\underline{\text{grid}(c)} + 1$

[*] *insert n in the queue*

if the queue is empty then return(failure)

end

[*] depending on how we perform these operations
we obtain search strategies



Implementation – functions

```
procedure Search ([goal_i,goal_j])
  clear the queue
  push the goal cell into the queue
  while (queue is not empty)
    [i,j] := pop_queue(queue)      // take a cell from queue
    if ([i,j] is robot's start position)
      return
    // MarkCell also does insertion into the queue //
    dist := grid([i,j]) + 1
    MarkCell([i,j-1], dist)
    MarkCell([i,j+1], dist)
    MarkCell([i-1,j], dist)
    MarkCell([i+1,j], dist)
  end while
end
```

```
procedure MarkCell (Cell [i,j], int value)
  case grid([i,j]) of
    -1: // cell out of boundaries: do nothing //
      return
    -2: // unexplored cell: mark it and put it into queue //
      grid([i,j]) := value
      push [i,j] onto the queue
      return
    -3: // obstacle cell: do nothing //
      return
    -4: // initial position: put it into queue //
      push [i,j] onto the queue
      return
  else: // already explored cell, do nothing //
    return
  end case
end
```

Implementation – the queue

```
structure Q_Element
  int i, j;
  Q_Element *next;
end
```

```
structure Queue
  Q_Element *head
  Q_Element *tail
  Q_Element *hog
  Q_Element element[QMaxNum]
end
```

```
procedure clear_queue (Queue q)
  for i from 0 to (QMaxNum-1)
    q.element[i] := &(q.element[i+1])
  q.element[QMaxNum-1] := null
  q.head := null
  q.tail := null
  q.hog := &(q.element[0])
end
```

```
procedure get_el_queue (Queue q)
  p := q.hog
  q.hog := p->next
  return(p)
end
```

```
procedure free_el_queue (Q_Element el, Queue q)
  el.next := q.hog
  q.hog := &el
end
```

Implementation – the queue

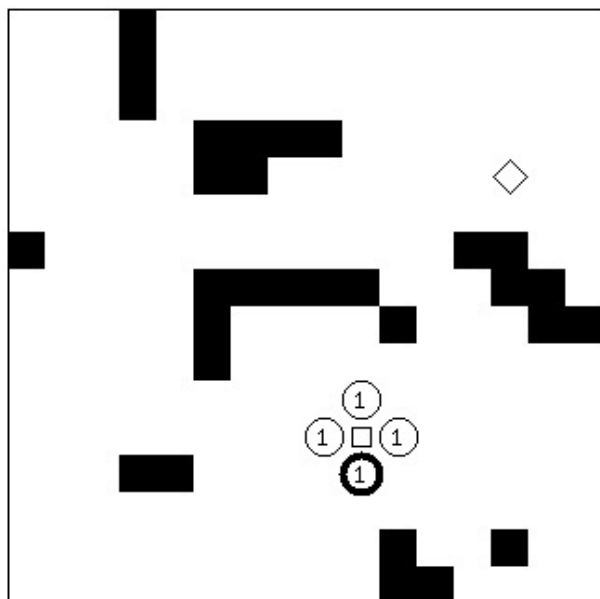
```
procedure empty_queue (Queue q)
  return(q.head = null)
end
```

```
procedure push_queue (Cell [i,j], Queue q)
  el := new_el_queue(q)
  if (el = null) return(-1)
  el.i := i
  el.j := j
  el.next := nil
  if (q.tail) q.tail->next := &el
  q.tail := &el
  if (q.head = nil) q.head := &el
end
```

```
procedure pop_queue (Queue q)
  el := q.head
  if (el = null) return(-1)
  q.head := el->next
  if (q.head = nil) q.tail := nil
  free_el_queue(el)
  return(el)
end
```

Note: First-In-First-Out (FIFO) policy.

The example again

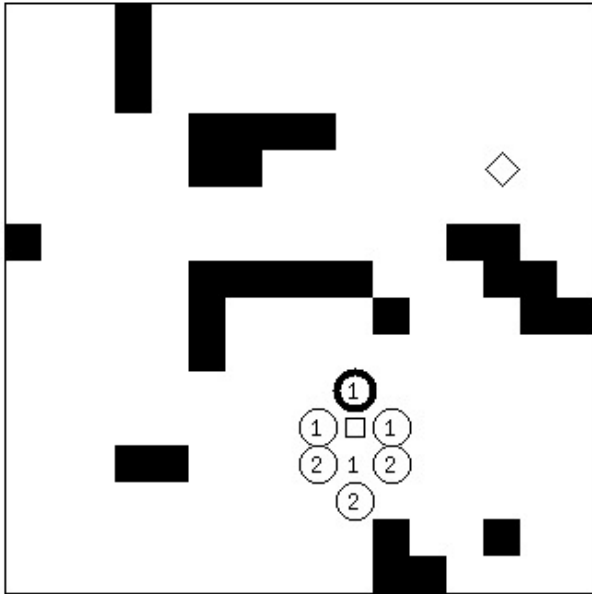


Queue: { [12,9] [10,9] [11,8] [11,10] }

```

-2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -3 -3 -3 -3 -2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -3 -3 -2 -2 -2 -2 -2 -2 -4 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -3 -3 -2 -2
-2 -2 -2 -2 -2 -3 -3 -3 -3 -3 -2 -2 -2 -3 -3 -2
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-2 -2 -2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
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```

Step 2



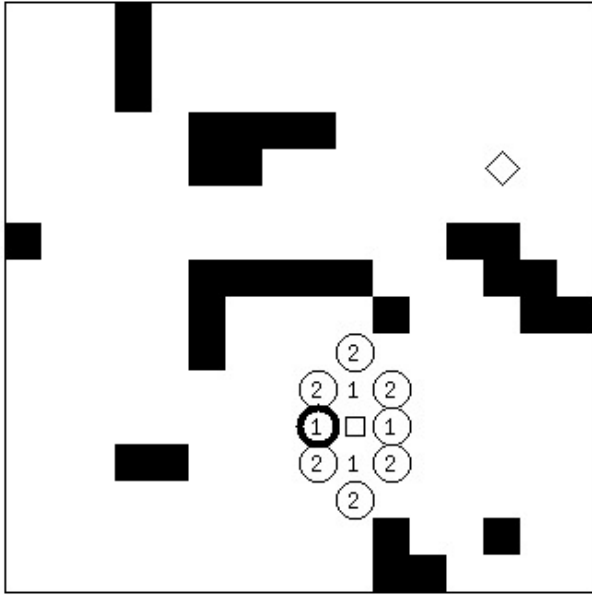
Queue: {[10,9] [11,8] [11,10] [13,9] [12,8] [12,10]}

```

-2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -3 -3 -3 -3 -2 -2 -2 -2 -2 -2
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-2 -2 -2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 -2 1 -2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 1 0 1 -2 -2 -2 -2 -2
-2 -2 -2 -3 -3 -2 -2 -2 2 1 2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 -2 2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -3 -2 -2 -3 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -3 -3 -2 -2 -2 -2

```

Step 3

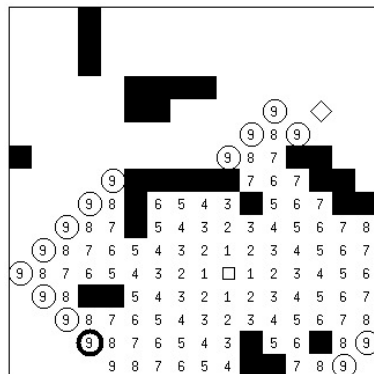
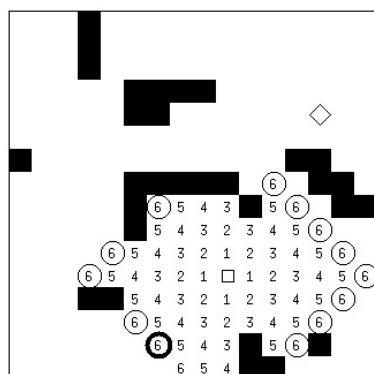
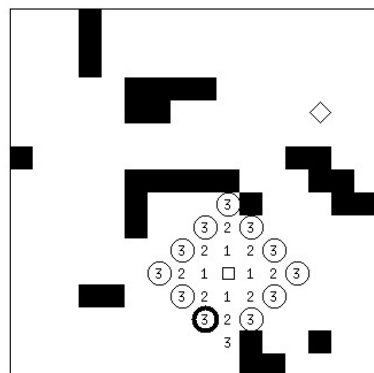


Queue: {[11,8] [11,10] [13,9] [12,8] [12,10] [9,9] [10,8] [10,10]}

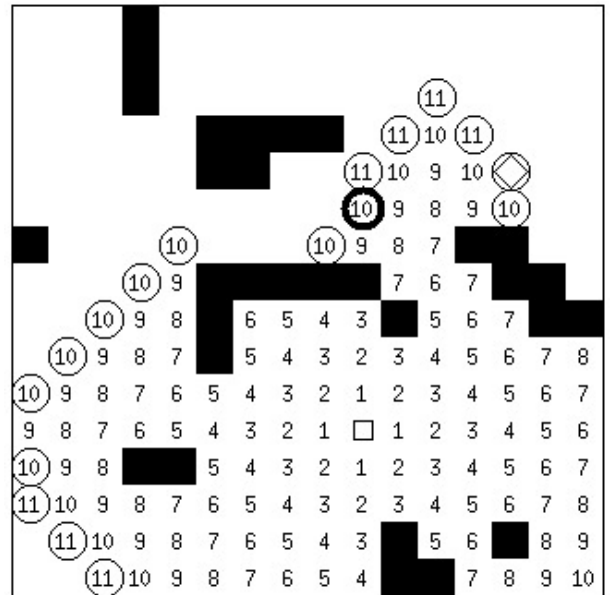
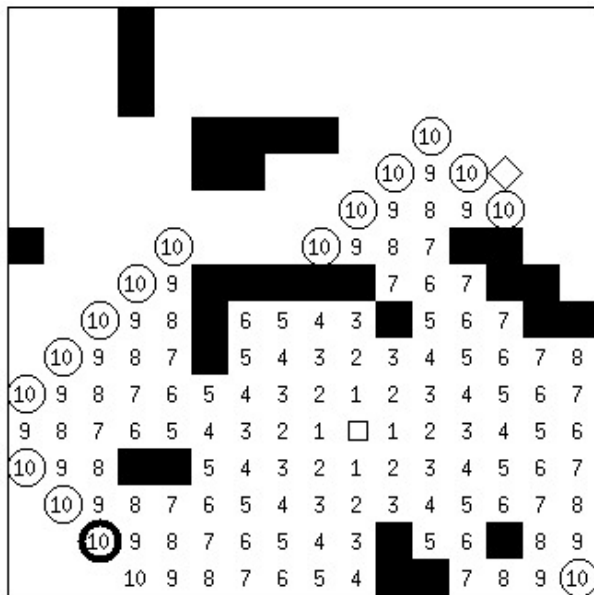
```

-2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -3 -3 -3 -3 -2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -3 -3 -2 -2 -2 -2 -2 -2 -4 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
-3 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -3 -3 -2 -2
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-2 -2 -2 -2 -2 -3 -2 -2 -2 2 -2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 2 1 2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 1 0 1 -2 -2 -2 -2 -2
-2 -2 -2 -3 -3 -2 -2 -2 2 1 2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 -2 2 -2 -2 -2 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -3 -2 -2 -3 -2 -2
-2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -3 -3 -2 -2 -2 -2

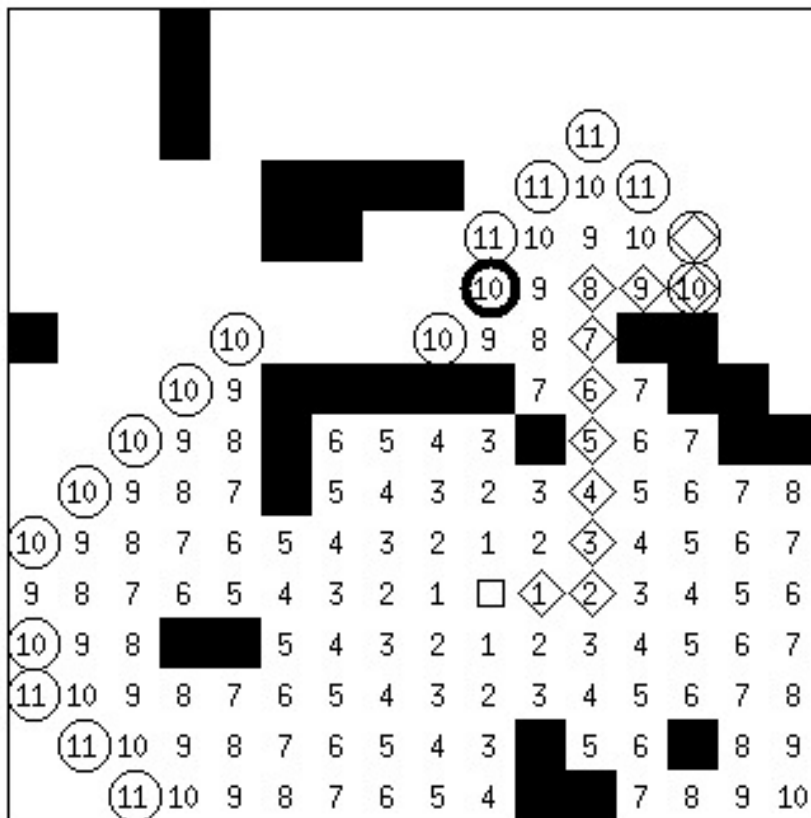
```



And the final ones



The found path



The algorithm – top level

```
procedure Plan (Cell [start_i,start_j], Cell [goal_i,goal_j])
begin

    // we first do the search //
    read_the_environment_grid();
    Search([goal_i,goal_j]);

    // we now generate the trajectory //
    // begin with the start point //
    [i,j] := [start_i,start_j];
    print([i,j]);

    repeat
        // search the neighbor with the lowest value //
        bestvalue := maxinteger;
        for [i',j'] in neighbours([i,j]) do
            if (grid([i',j']) < bestvalue) then
                bestvalue := grid([i',j']);
                [best_i,best_j] := [i',j'];
            end if
        end for

        // print it as the next step in the trajectory //
        print([best_i,best_j]);

        // move to that point //
        [i,j] := [best_i,best_j];

        // keep going until we reach the goal //
        until grid([i,j]) = 0;

    end
```

Properties of BFS

Pros in general

- *Always finds a solution (if branching factor b is finite)*
- *Finds optimal solution (if costs are all equal)*

Cons in general

- *Does not take different costs into account*
- *Time: $O(b^d)$, with $d = \text{depth}$*
- *Keeps all nodes in memory: $1 + b + b^2 + \dots + b^d$*

And in our case?

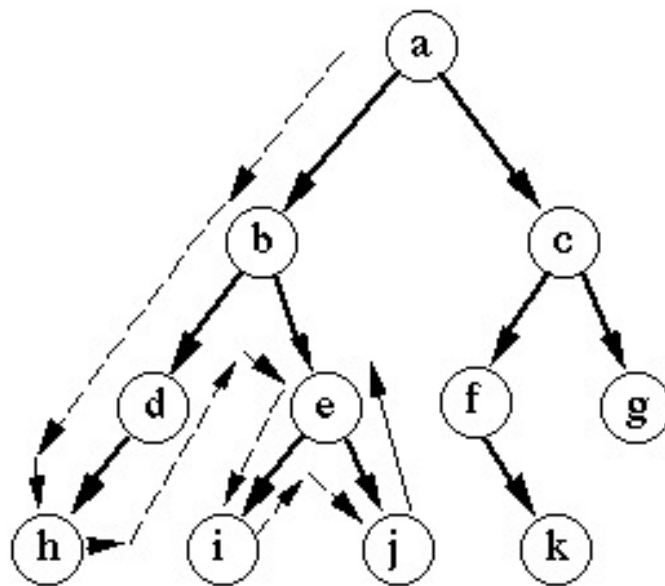
If you want to execute the path

- You need to store the path
 - *you can use the queue:*
 - *clear the queue before after the search*
 - *replace "print" by "push"*
- You need to transform $[i, j]$ into $[x, y]$
 - *decide the size of a cell, e.g., 20 mm*
 - *decide reference system (start point of robot)*
- You give the path to your robot
 - *use GoTo on all points one after the others*
 - *may need to make the way-points more sparse*
- Example:

Planned path	Way-points
[10, 13]	(0, 0)
[10, 12]	
[10, 11]	(-40, 0)
[9, 11]	
[8, 11]	(-40, 40)
[7, 11]	
[6, 11]	(-40, 80)
[5, 11]	
[5, 12]	(-20, 100)

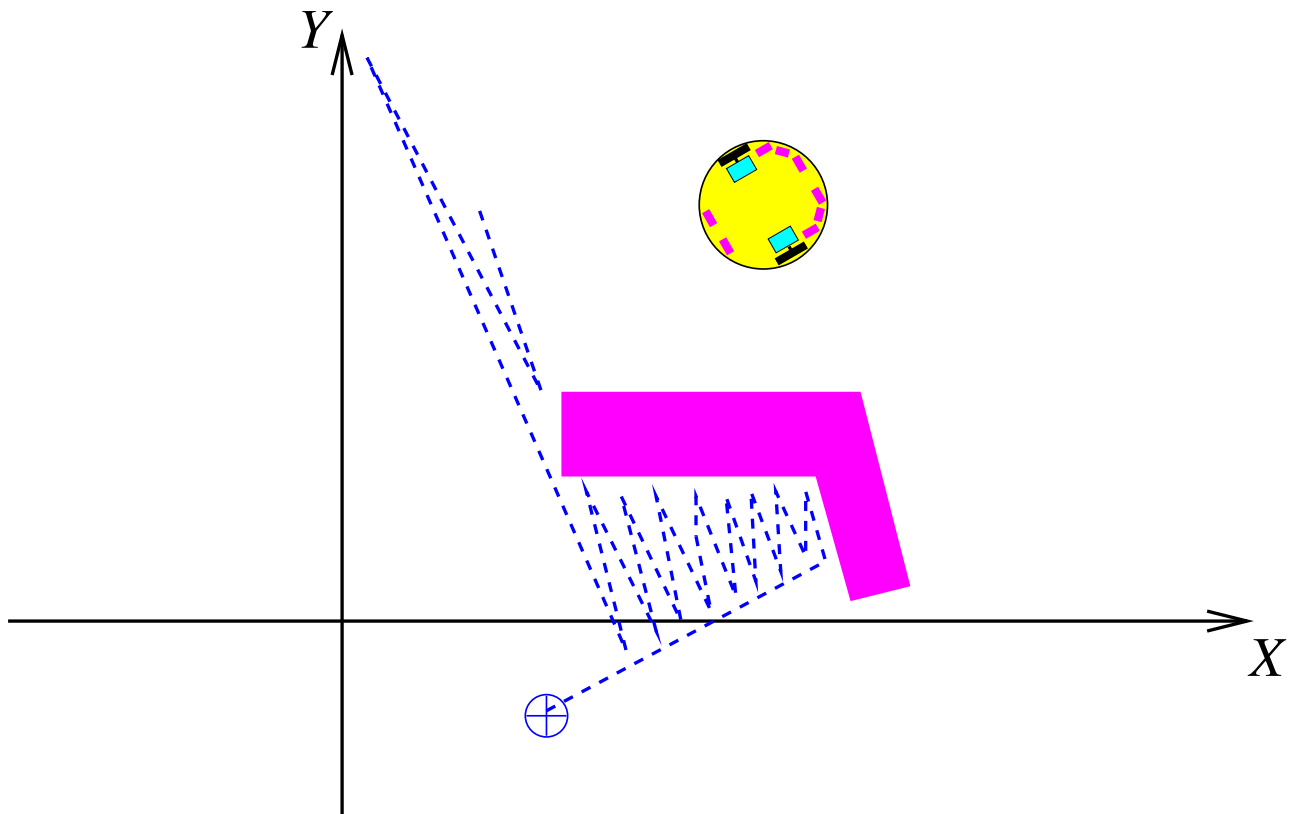
Other Types of Search

Depth-first search



- Can start from goal or from initial state
- Explore the space head-on until in trouble
 - *go all the way down a branch*
 - *when cannot go further, backtrack to most recent choice*
- “Artistic” behavior
 - *if lucky, it will go quickly to the goal*
 - *if unlucky, it will wander for ages before getting there*
- If space is infinite, it may never stop

Depth-first search



- Can start from goal or from robot
- Explore the space head-on until in trouble
 - *go all the way down in a direction*
 - *when cannot go further, return to most recent alternative*
- “Artistic” behavior
 - *if lucky, it will go quickly to the robot (or goal)*
 - *if unlucky, it will wander for ages before getting there*
- Eventually, it will hit the robot

Depth-First Search algorithm

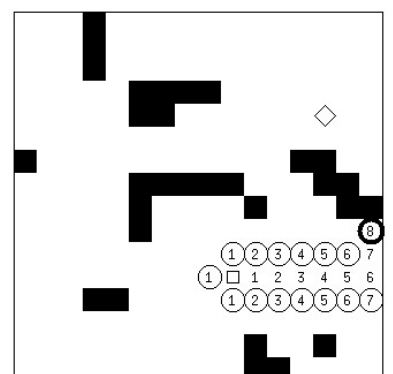
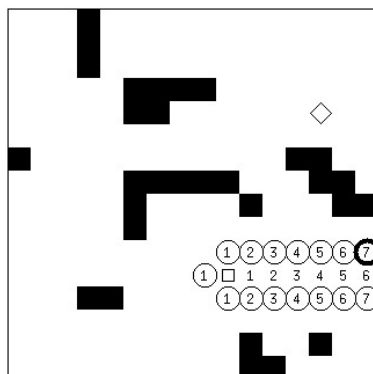
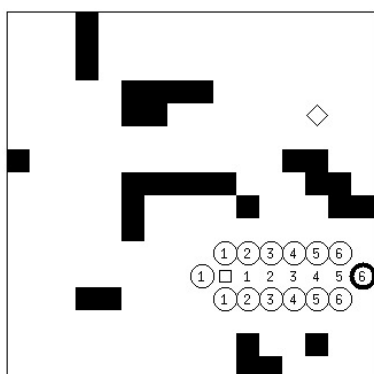
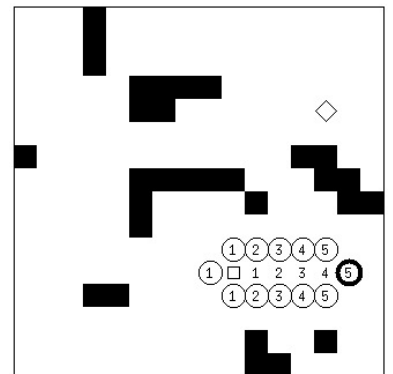
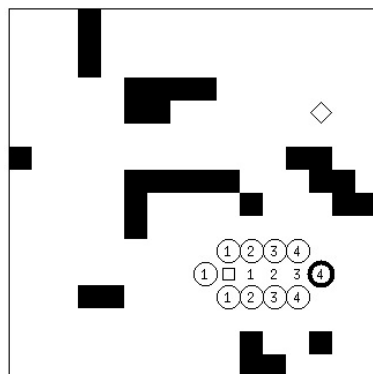
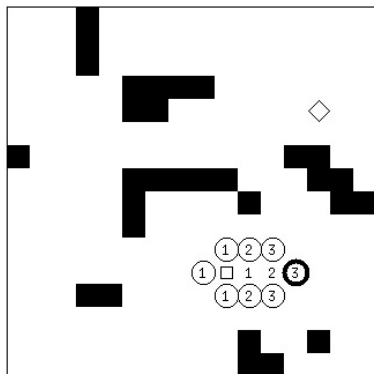
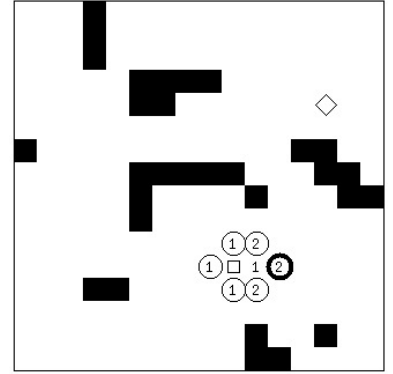
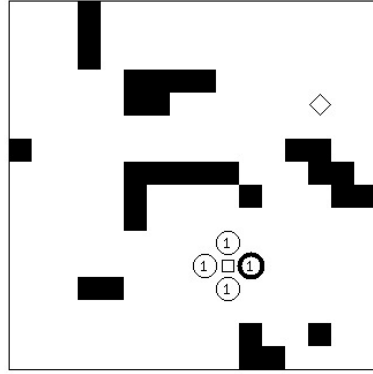
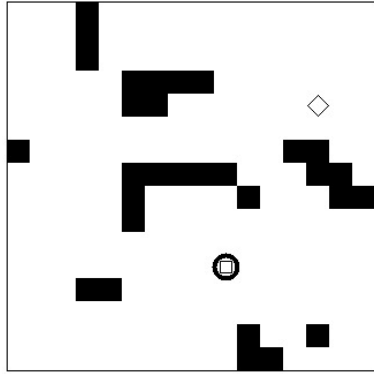
```
procedure Search ()  
  push the goal cell onto the stack  
  repeat until the stack is empty  
    pop a cell  $c$  from the stack  
    if  $c$  is the robot cell then return(success)  
    foreach  $n$  which is a neighbor cell of  $c$   
      if  $n$  is not an obstacle and  $n$  has label -2  
        label  $n$  by  $\text{grid}(c) + 1$   
        push  $n$  onto the stack  
    if the stack is empty then return(failure)  
end
```

Note: We use a Last-In-First-Out (LIFO) policy.

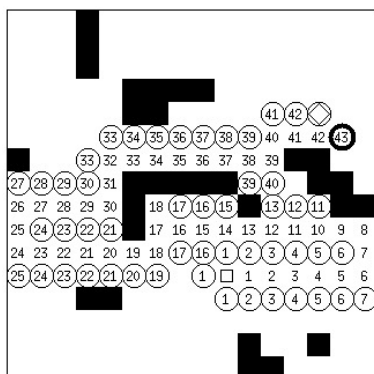
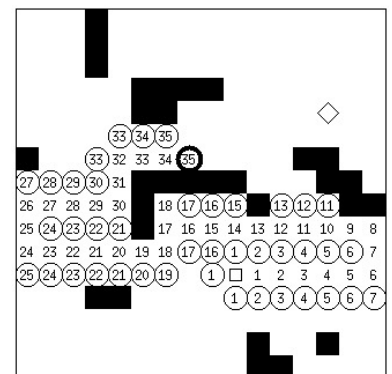
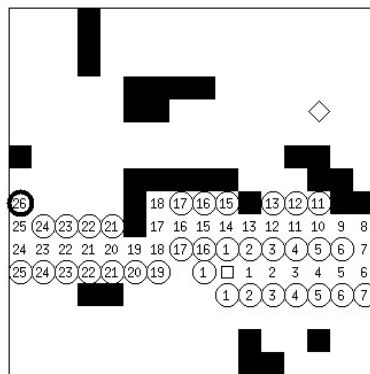
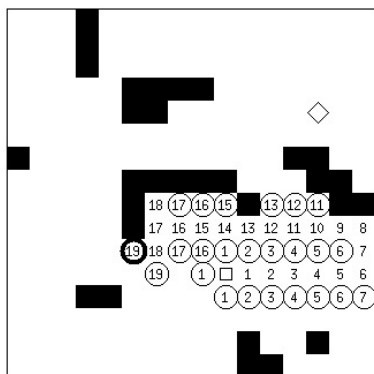
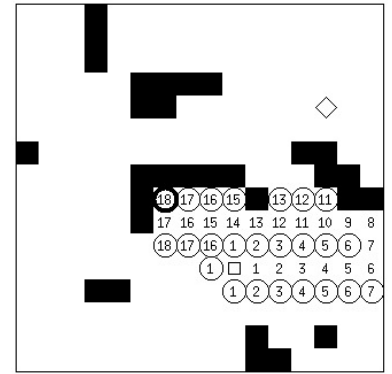
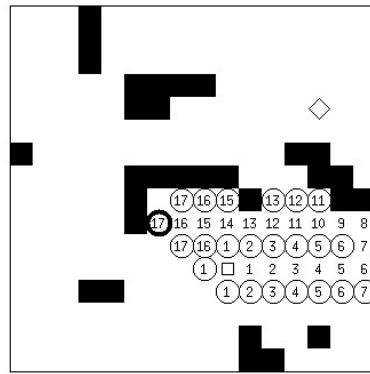
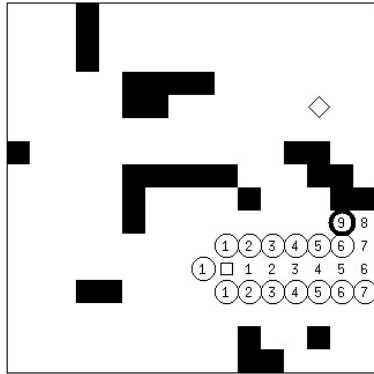
We need to make only one change to our program: use a different “push_queue”

```
procedure push_queue_lifo (Cell [i,j], Queue q)  
  el := new_el_queue(q)  
  if (el = null) return(-1)  
  el.i := i  
  el.j := j  
  el.next := q.head  
  q.head := &el  
  if (q.tail = nil) q.tail := &el  
end
```

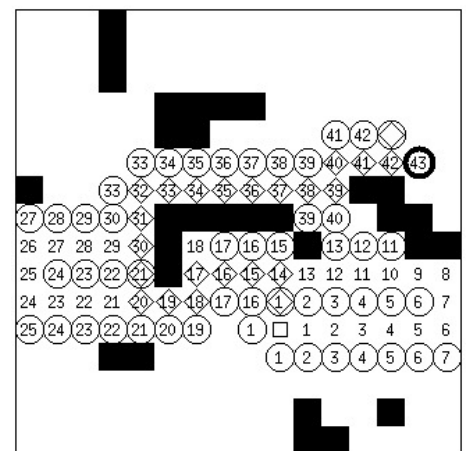
Example of depth-first search



Example (cont'd)



and the final path:



Properties of DFS

Pros in general

- *If $m = \text{max depth of search tree} \dots$*
- *Keeps only a $O(bm)$ nodes in memory (linear space)*
- *Always finds a solution (if exists) in finite graphs*

Cons in general

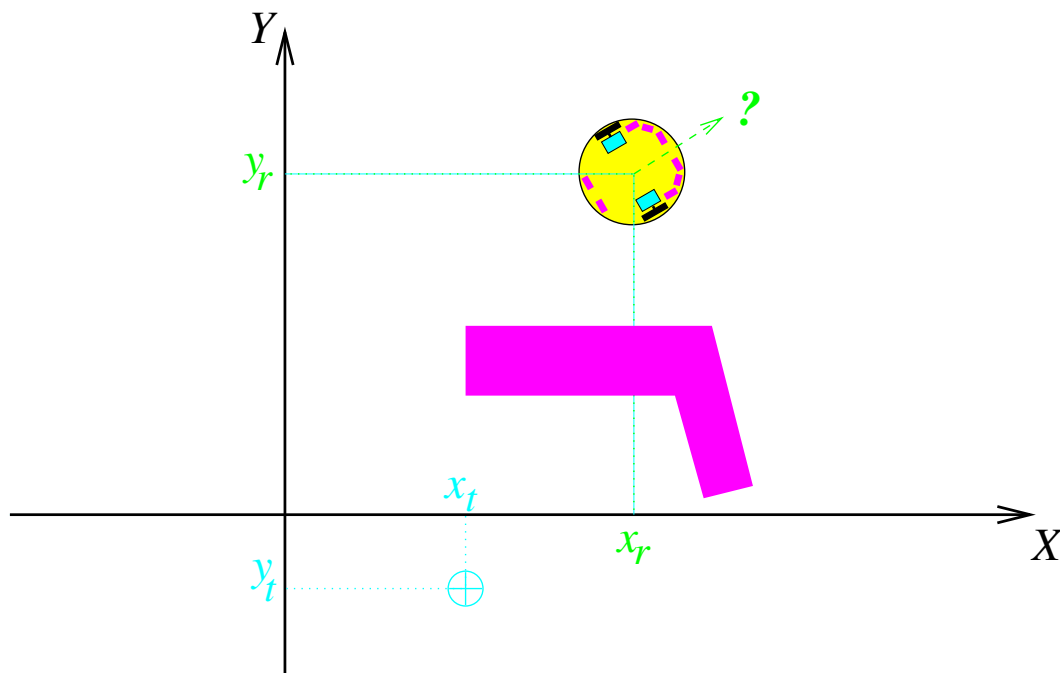
- *Time: $O(b^m)$ (very bad if $m \gg \text{depth of solution}$)*
- *Can be trapped in infinite loops*
- *Can go astray on a wrong branch in infinite graphs*

And in our case?

Improvement: Iterative-Deepening Search

- *Strategy: do not expand nodes beyond depth D*
- *Increase D when the subtree is fully explored*
- *Is complete*
- *Time: $O(b^d)$ with d depth of solution (nice)*
- *Space: $O(b^d)$ (nice)*
- *Produces optimal solution if costs are all equal*

Limits of “uninformed” search



- Both BFS and DFS explore the space “blindly”
 - *do not use any information about the goal*
 - *except recognizing when we are at the goal*
- Breadth-First Search
 - *explores all directions in parallel*
 - *tends to explore a huge number of cells*
- Depth-First Search
 - *explores one direction at the time “to the end”*
 - *needs luck in selecting a good direction*

Heuristic search

- General idea
 - *guide search towards most promising directions*
 - *use information about the domain to chose the most promising node to visit next*
 - *“promising” = value of heuristic function $h(n)$*
- The “heuristic” word
 - *something in our experience suggests that we should do it in this way, but we don’t know why*
 - *cf. Rhys Kealley’s definition of a hack:*

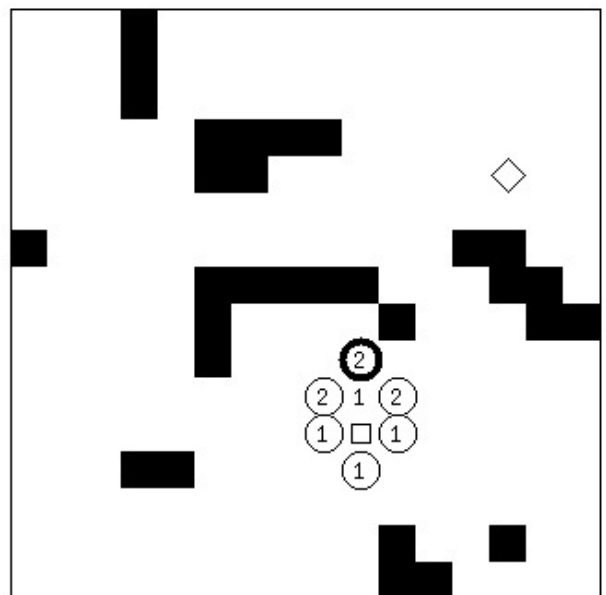
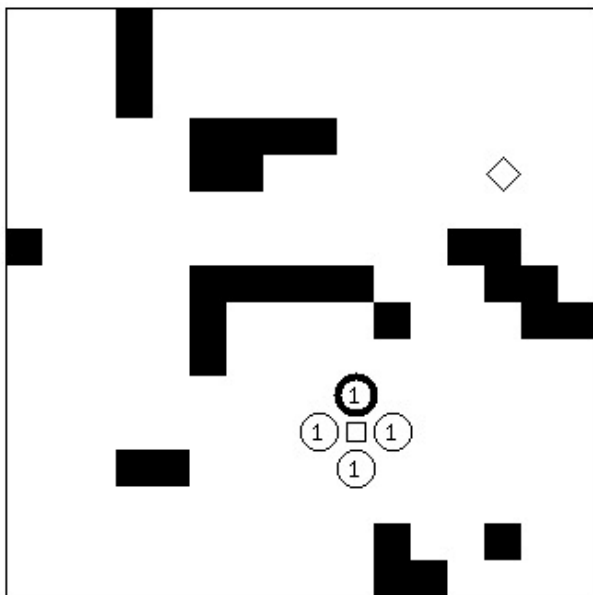
A hack is a heuristically appealing idea or approach which seems to work well in the absence of any sound background theory.
- Examples of heuristic functions in our domain:
 - *h = line-of-flight distance to goal*
 - *h = greater between x and the y distance to the goal*

Heuristic Search algorithm

```
procedure Search ()  
  put the goal cell in the queue  
  repeat until the queue is empty  
    take a cell  $c$  from the queue  
    if  $c$  is the robot cell then return(success)  
    foreach  $n$  which is a neighbor cell of  $c$   
      if  $n$  is not an obstacle and  $n$  has label -2  
        label  $n$  by  $\text{grid}(c) + 1$   
    (!) insert  $n$  in the queue ordered by  $h(n)$   
  if the queue is empty then return(failure)  
end
```

Note: Elements in the queue are ordered according to h .

Example: $h(n) = \text{Manhattan distance to goal}$



Heuristic search – Implementation

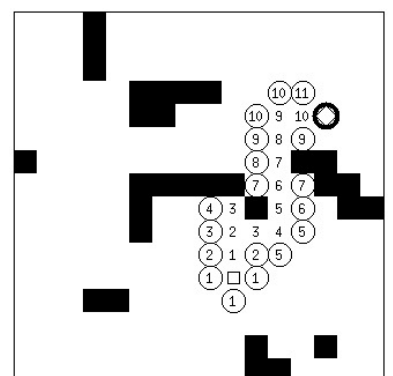
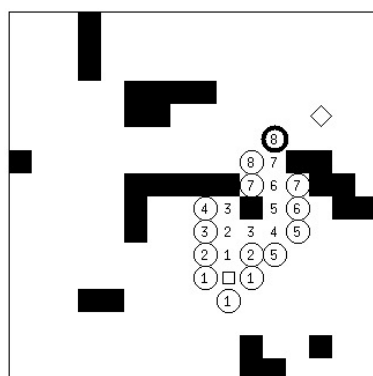
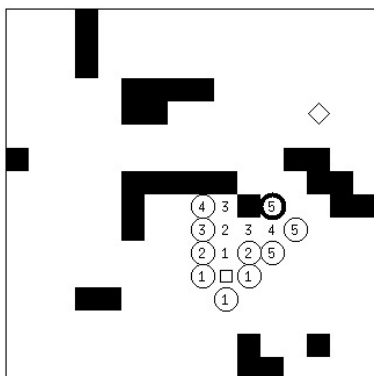
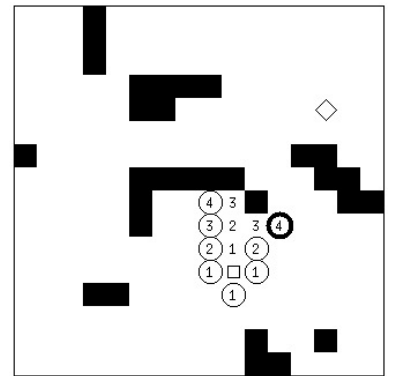
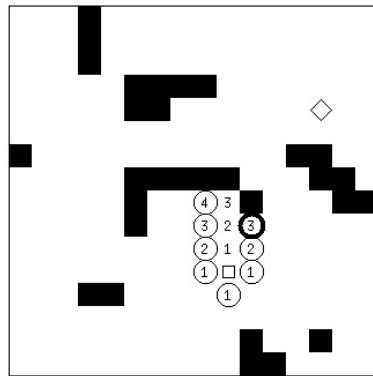
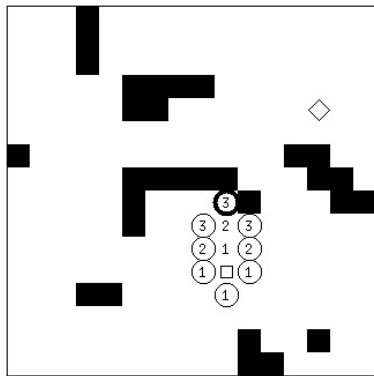
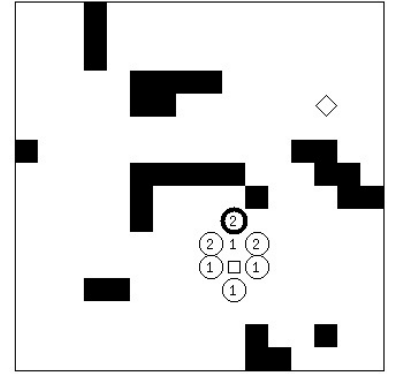
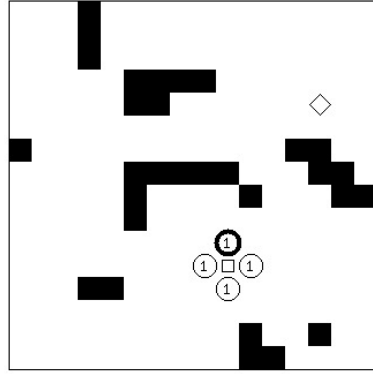
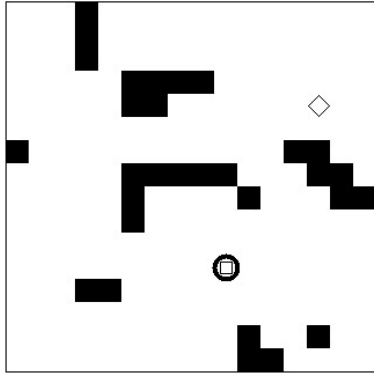
Main missing ingredient: ordered queue

```
structure Q_Element
  int i, j, key;    // queue is ordered by increasing keys
  Q_Element *next;
end

procedure push_queue_ordered (int k, Cell [i,j], Queue q)
  el := new_el_queue(q);
  if (el = null) return(-1);
  el.k := k;
  el.i := i;
  el.j := j;
  if (q.head = nil)
    el.next := nil;
    q.head := &el;
    q.tail := &el;
    return();
  else
    if (k < q.head->k)
      el.next := q.head;
      q.head := &el;
      return();
    else
      for (p := q.head; p = q.tail; p := p.next)
        if (k < p.next->k)
          el.next := p.next;
          p.next := &el;
          return();
        el.next := nil;
        p.next := &el;
        q.tail := &el;
      end
    end
  end
```

We need to replace the call `push_queue([i,j], queue)`
by `push_queue_ordered([i,j], h([i,j]), queue)`,
where $h([i,j]) = |i - \text{goal}_i| + |j - \text{goal}_j|$

Example



A* search

- Limitation of greedy search
 - $h(c)$ does not consider effort already made to get to c
 - $h(c)$ of current node and past nodes are not compared
- Idea: use heuristic function $g(h) + h(c)$:
 - $h(c)$ = distance from c to goal (still to go)
 - $g(c)$ = distance from start to c (already done)
 - Note: h is heuristic, g is known
 - Note: grounded on Bellman's principle
- Search is a mixture of depth- and breadth-first
 - priority to cells that “should” lay on a shorter path
- Pros
 - always find a solution (if exists)
 - always find the optimal solution (if h admissible)
 - can be very fast
- Cons
 - efficiency depends on the choice of h
- How would you implement it?

Summary

- If you want a simple path planner
 - *use Breadth-First Search*
 - *it always finds the best solution*
- If you want an efficient path planner
 - *use Heuristic Search*
 - *and work out your h function very carefully*
- If you want a really efficient path planner
 - *use more sophisticated forms of heuristic search*
 - *e.g.: A^**
 - *even better: IDA^* , D^* (search the web!)*
- If you have uncertainty in your domain
 - *use other A.I. techniques*
 - *e.g.: Markov decision processes, POMDP*