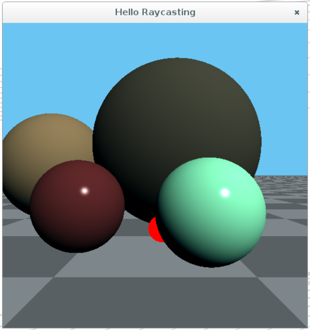
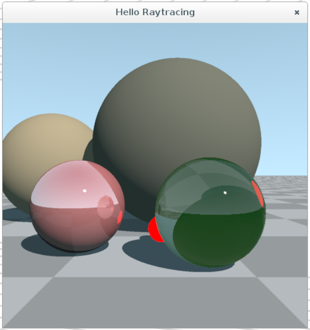
Ray-based graphics, and the rendering equation

Computer Graphics (DT3025)

Martin Magnusson
November 18, 2016





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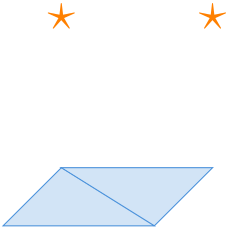
  - Tuning the bias for avoiding z-fighting.
  - Warping and partitioning for avoiding aliasing.
- (From now on, we'll deal with methods where we get proper shadows "for free"!)

Shadow volumes: how much geometry?

We have the following scene with 2 triangles and 2 omnidirectional point light sources. How many triangles have to be generated for computing the shadow volumes?



1. 10
2. 16
3. 24

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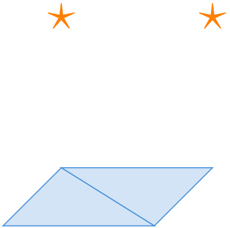
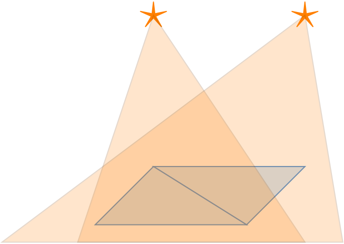


Shadow volumes: how many rendering passes?

We have the following scene with 2 triangles and 2 spot lights. How many rendering passes are required for rendering this scene with *stencil-buffer shadow volumes*?



Shadow mapping: how many rendering passes?

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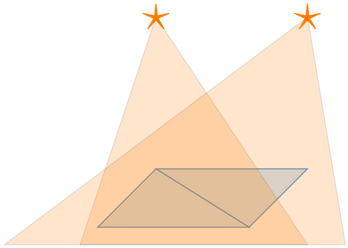

Today

- Ray casting vs rasterisation
- The rendering equation!
- Ray tracing for shadows and transparency

Reading instructions

- Hughes et al:
  - 15.1–15.2.4, 15.4–15.4.1, 15.4.3
  - 7.8
  - 29

Rasterisation outline

```
for (each triangle)
{
  for (each pixel)
  {
    if (triangle covers pixel)
    {
      update z-buffer
      keep closest hit
    }
  }
}
```

- (Remember Lecture 1.)

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- (Remember Lecture 1.)
- Rasterisation, AKA *projective rendering*.

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- (Remember Lecture 1.)
- Rasterisation, AKA *projective rendering*.
- Each object is *projected* to screen and *rasterised*.

Ray casting vs rasterisation

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Ray casting

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for (each pixel)
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  for (each object)
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- We just need to swap the for-loops!

Evolution of Lara Croft


kanasoku.info

Polygon counts

- Tom Raider I (1996): 230
- TombRaider III (1998): 300
- Angel of Darkness (2003): 4 400
- Legend (2006): 9 800
- Underworld (2008): 32 816

Speaking of Lara Croft, Schleiner (2001) is worth reading too.

Rasterisation pros and cons

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- Each pixel may need to be touched many times for complex scenes.
- Ray casting used to be hard to implement in hardware, but with general-purpose GPUs, we are getting there (Lab 4!)

Rasterisation example

Drive Club, Evolution Studios

Ray tracing example

Source: “Now with vitamin R”

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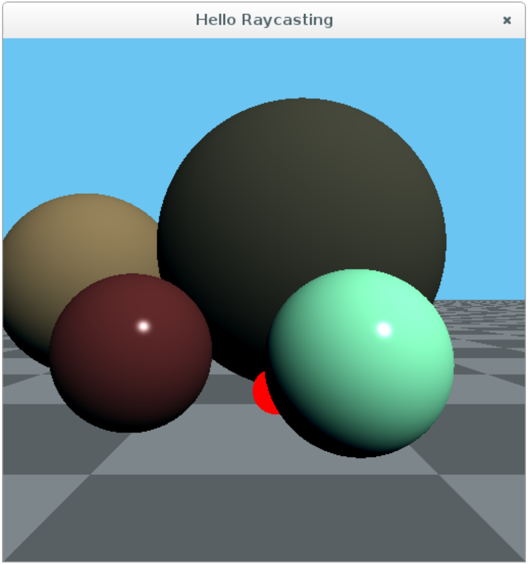
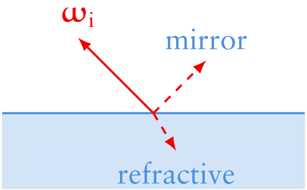
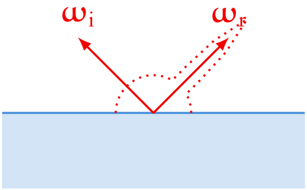
- *How do we compute the colour?*
- With a BSDF, as part of *the rendering equation*.
- Different ways of apprixmating the rendering equation leads to different rendering methods.

Ray casting example



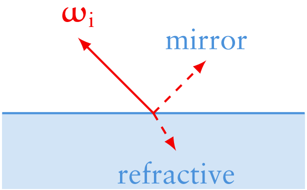
Two types of scattering

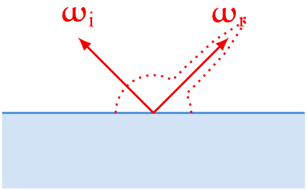




1 Mirror scattering

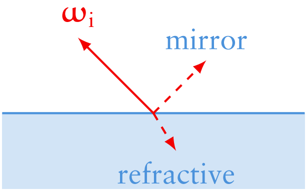
Two types of scattering

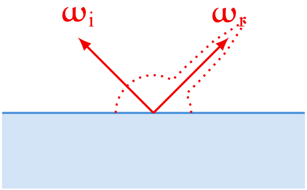




1. Mirror scattering
2. Snell-transmissive scattering (refraction)

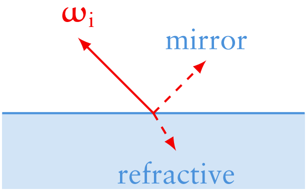
Two types of scattering

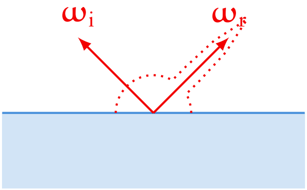




1. Mirror scattering
2. Snell-transmissive scattering (refraction)
3. Three! Three types of scattering!

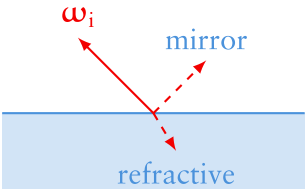
Two types of scattering

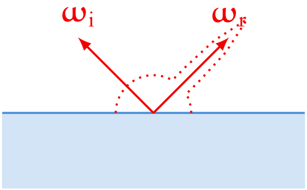




1. Mirror scattering
2. Snell-transmissive scattering (refraction)
3. Everything else

Two types of scattering





1. Impulse scattering: either mirror or refractive (i.e. Snell-transmissive).
2. Everything else (integrals).

Two types of lights

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- *Luminaire* = light source.

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- $R(P, \omega) =$ the first point hit when going from P toward ω

The reflectance equation

$$L^{\mathrm{ref}}(P, \omega_{\mathrm{o}}) = \int_{\omega_{\mathrm{i}} \in \mathbf{S}^2_+(P)} L(P, -\omega_{\mathrm{i}}) f_{\mathrm{r}}(P, \omega_{\mathrm{i}}, \omega_{\mathrm{o}}) (\omega_{\mathrm{i}} \cdot \mathbf{n}_P) \, \mathrm{d}\omega_{\mathrm{i}}$$

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Impulses (point lights and mirror reflections) can't really be integrated like this, but let's pretend we can stuff them inside the integral too for now (just make a sum instead of an integral).

The rendering equation (take 1)

$$
\begin{aligned}
L(P, \omega_\mathrm{o}) &= L^\mathrm{e}(P, \omega_\mathrm{o}) + L^\mathrm{ref}(P, \omega_\mathrm{o}) \\
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- But L appears on both sides of the equation.
- This is an *integral equation* and is difficult to solve exactly.
- We can only hope to approximate the solution.

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  - and the radiance from another point. (Just a recursive function.)

What about $R(P, \omega)$?

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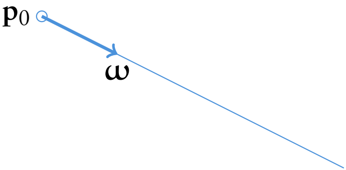
- (…and that integral).
- For a ray with starting point P and direction $\boldsymbol{\omega}$, how to compute which surface point it hits first?

Intersection testing

- How to represent a ray?
- Given start point $\mathbf{p}_0$ and (normalised) direction $\boldsymbol{\omega}$,

$$R(t) = \mathbf{p}_0 + t\boldsymbol{\omega}.$$



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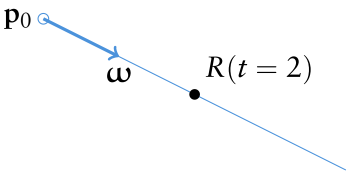

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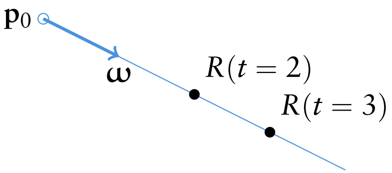

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- or as the normal $\mathbf{n} = (a, b, c)$ and a distance d from the origin.
- Implicit plane equation

$$\begin{aligned} H(\mathbf{p}) &= ax + by + cz + d = 0 \\ &= \mathbf{n} \cdot \mathbf{p} + d = 0 \end{aligned}$$

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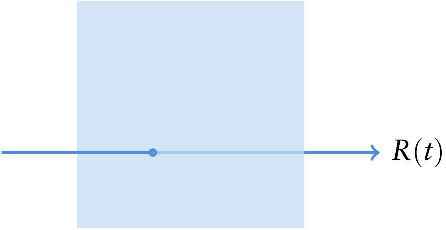
  - Does not generate points.
- Now: how to check if ray intersects plane?

Ray–plane intersection

- Intersection iff both equations are satisifed.



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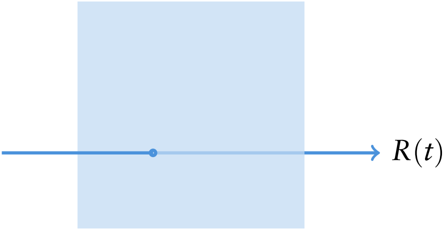

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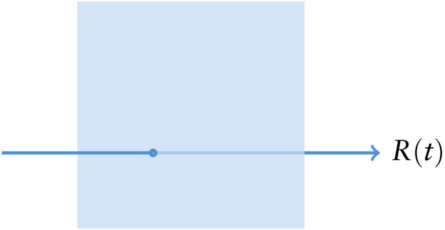

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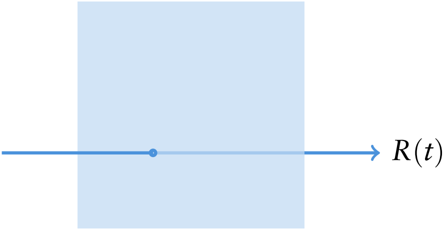

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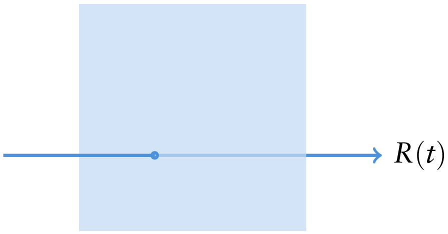

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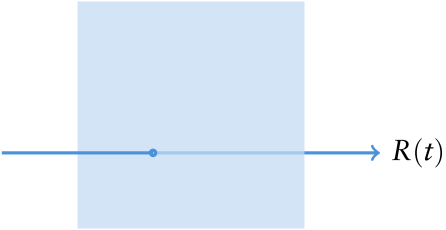

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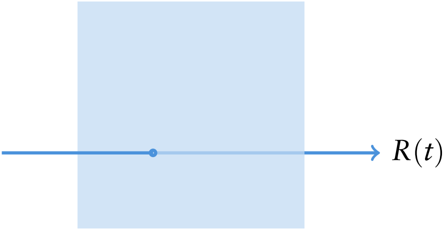

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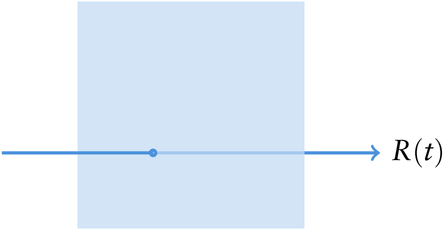

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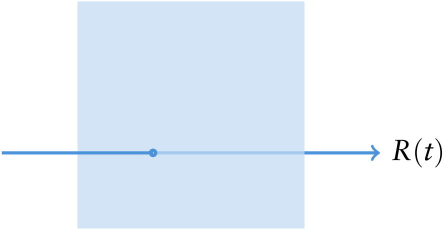

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voilà!

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- Check that t is not behind us:

$$R(t) > t_{\text{min}}.$$

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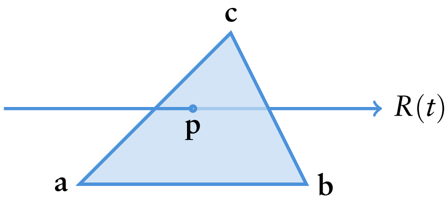
- Finally, we also need to compute the normal at the point $R(t)$, for computing *lighting and secondary rays*.
- For plane, normal is constant for all points.

Ray–triangle intersection

Two options:

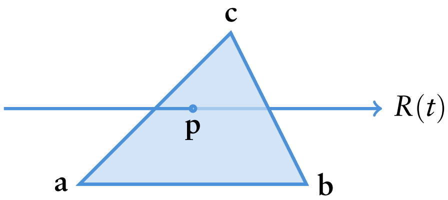
1. First ray–plane intersection, then test if $R(t)$ is inside triangle (as with rasterisation).



Ray–triangle intersection

Two options:

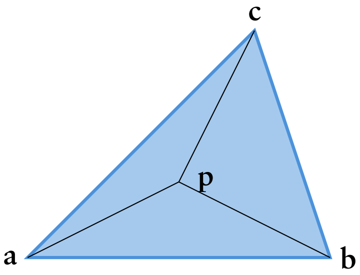
1. First ray–plane intersection, then test if $R(t)$ is inside triangle (as with rasterisation).
2. Better: use barycentric coordinates.



Triangle representation

- Implicit triangle-patch equation

$$
\begin{aligned}
T(\alpha, \beta, \gamma) &= \alpha \mathbf{a} + \beta \mathbf{b} + \gamma \mathbf{c} \\
\alpha + \beta + \gamma &= 1 \\
0 \leq \alpha, \beta, \gamma &\leq 1
\end{aligned}
$$



Just the same as when doing texture interpolation, etc.

Simplified triangle representation

- Since $\alpha + \beta + \gamma = 1$ we can use that $\alpha = 1 - \beta - \gamma$ and get rid of α.

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Ray–triangle intersection

- So, does our ray intersect the triangle?

$$T(\beta, \gamma) = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a}) \tag{1}$$

I think this version is conceptually simpler than the book's.

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I think this version is conceptually simpler than the book's.

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- Intersection if $\beta + \gamma < 1$ and $\beta, \gamma > 0$ and $t > 0$.

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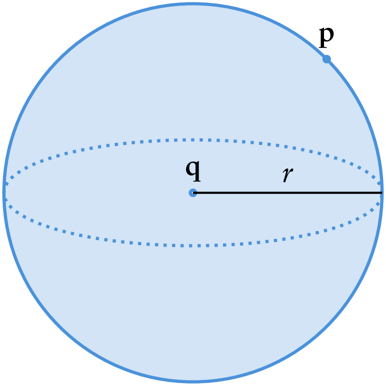
Sphere representation

- Implicit sphere equation:
- Sphere centred at $\mathbf{q}$ with radius r. Point $\mathbf{p}$ is on sphere iff

$$(\mathbf{p} - \mathbf{q}) \cdot (\mathbf{p} - \mathbf{q}) = r^2.$$



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  - Two solutions: ray pierces sphere. Choose smallest positive t.

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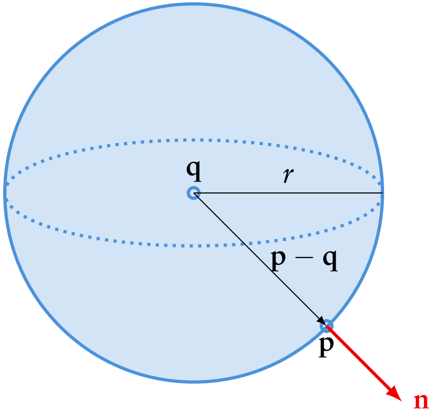
- (Useful in Lab 4.)

Sphere normal

- ... and then we need the normal at $\mathbf{p}$ too.



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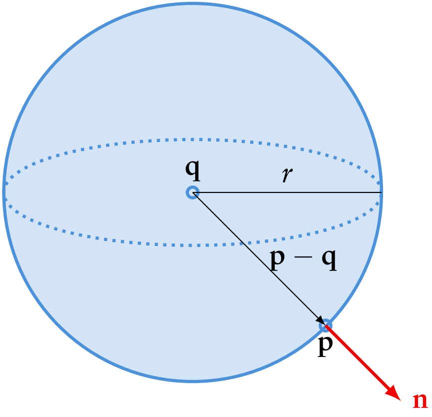
- $\mathbf{n} = \mathbf{p} - \mathbf{q} / \left\|\mathbf{p} - \mathbf{q}\right\|$



Light transport

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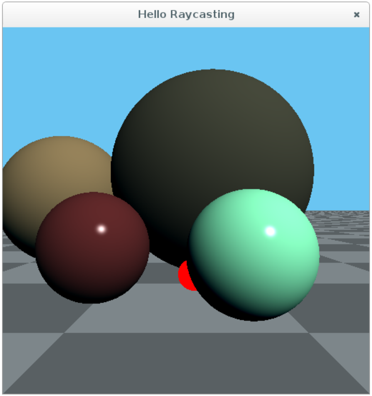
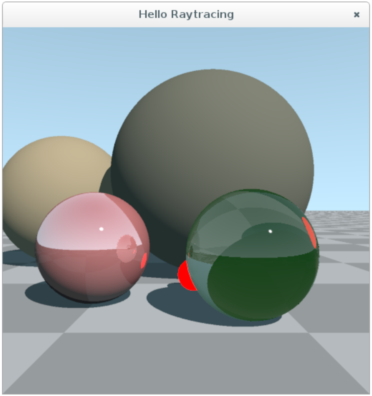
- Sooner or later, light reaches eye.

Ray casting vs ray tracing

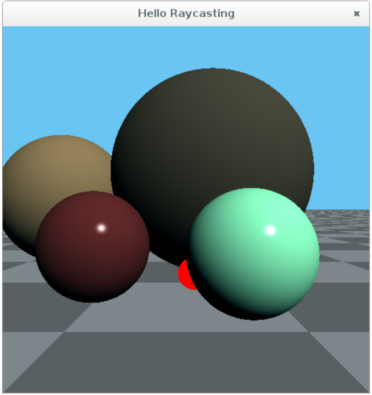
- Ray casting: stop at first intersection.

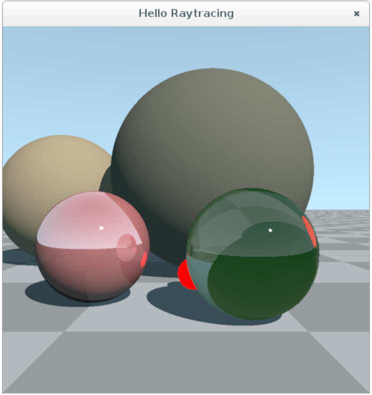




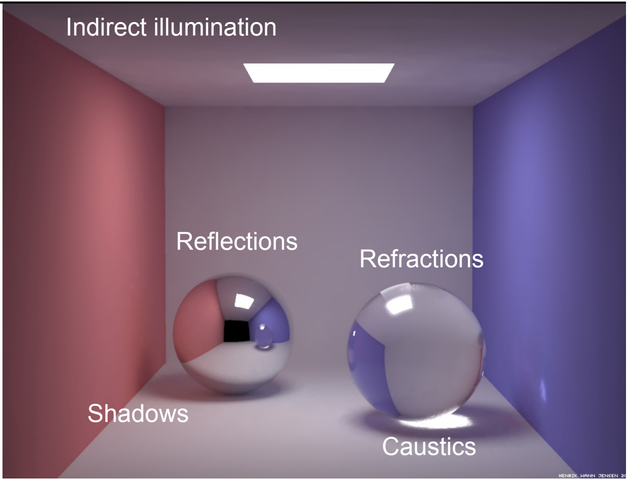
Ray casting vs ray tracing

- Ray casting: stop at first intersection.
- Ray tracing: secondary rays for reflections, shadows, etc.





Physically based rendering



Henrik Wann Jensen

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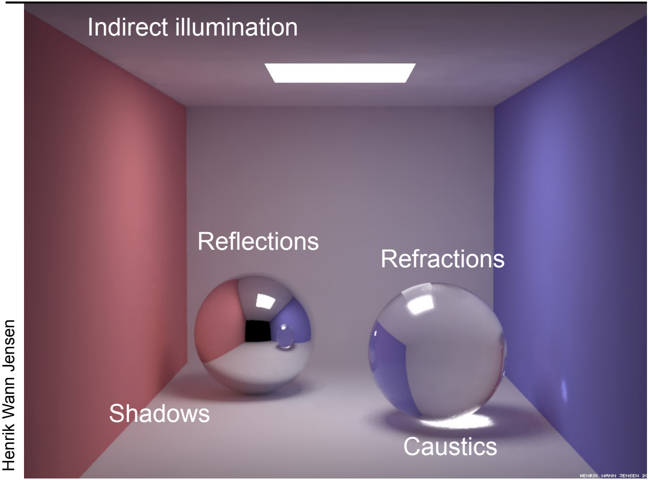

With ray tracing, we get (hard) shadows, reflections, refractions, but *not* indirect illumination and caustics.

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- *We just took one more sample from the integral.*

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    - That's how light *actually travels*.
- In ray casting, we go backwards: *EDL*.

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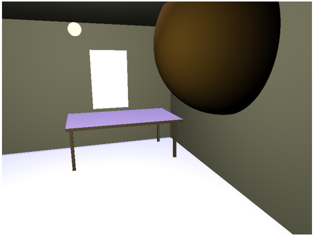
A taxonomy of tracing

- Ray casting $E(D|G)L$



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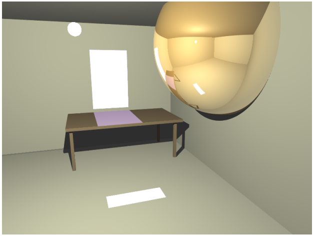

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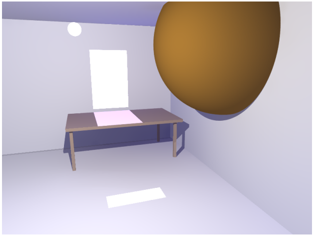

Radiosity

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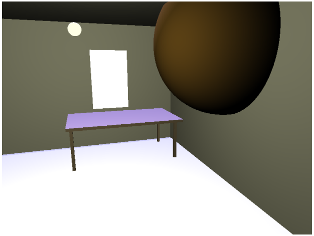

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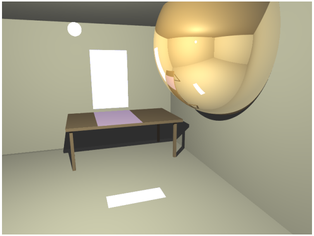

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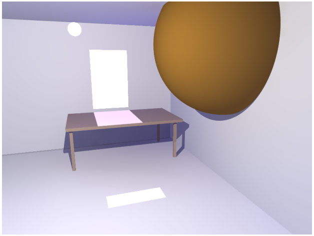

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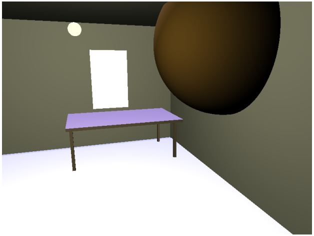

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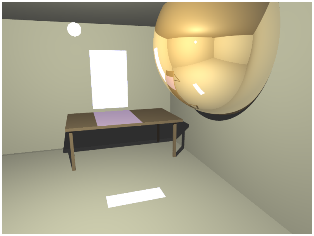

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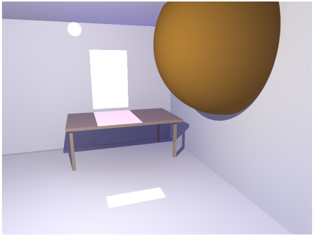

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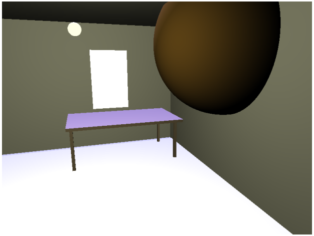

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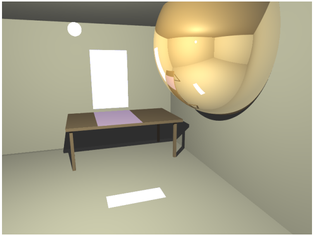

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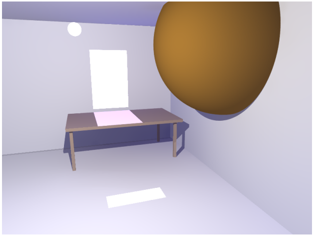

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  - Different rendering techniques approximate it in different ways
- Ray–object intersections: planes, triangles, spheres.

Summary

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- The rendering equation: all-purpose solution for computing what light comes from a point
  - Different rendering techniques approximate it in different ways
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- Ray–object intersections: planes, triangles, spheres.
- Ray tracing: emit secondary rays to render shadows and transparency (incl refraction)
- Classification of light-transport paths to describe different types of rendering methods

Next lecture: more reflections, transmission, materials

Time and place

- Mon 21 Nov, 13.15–15.00
- T-141

Reading material

- We'll stick to Chapter 29, mostly.

References

Anne-Marie Schleiner (2001). “Does Lara Croft Wear Fake Polygons? Gender and Gender-Role Subversion in Computer Adventure Games”. In: *Leonardo* 34.3, pp. 221–226.