Multibody simulation

Exercise 2

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Main points covered

- rotation matrix
- Euler angles
- axis/angle
- homogeneous matrix

Supplementary materials used in this exercise

All materials are available for download at the course website

1. To install the bMSd toolbox: download it, navigate to its directory ($e.g.$, `bMSd`) and type `setup_bMSd`. This will include in the Matlab's path a number of directories that contain files that you could use.
2. For this exercise, you might find the following functions useful
   - in directory `bMSd/rotation`: `rx.m`, `ry.m`, `rz.m`, `R2rpy.m`, `rpy2R.m`, `R2aa.m`, `aa2R.m`
   - in directory `bMSd/general_purpose`: `draw_frame.m`
   - in directory `bMSd/examples`: `test_rot_seq.m`
3. `points3D.m`
4. `T_matrices.m`

Task 1 (rotation matrix)

Multiplications of rotation matrices

In the previous lab you made an animation of an object following a circular path in 2D by re-computing a rotation matrix

$$\boldsymbol{R} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix},$$

for different angles θ. Perform the same task but instead of recomputing the rotation matrix from scratch using $\boldsymbol{R}(\theta)$ update the rotation matrix (in each iteration) using:

$$\boldsymbol{R}_{t_{n+1}} = \boldsymbol{R}_{t_n} \boldsymbol{R}(\Delta\theta),$$

where $\Delta\theta$ is the same angular increment you used in the previous lab (in the order of 0.01 radians).
For initialization use $\boldsymbol{R}_{t_0} = \boldsymbol{R}(0) = \boldsymbol{I}$.

- verify that it works
- explain with your own words why and how it works

Visualization (rotation matrix)

In order to see what is going on it is important that you visualize the rotation matrix. One way is to visualize the rotation matrix is to draw each matrix row as a vector. Write your own visualization function or use the provided `draw_frame.m` function.

Example usage:

- draw the the non-rotated coordinate system in black.
  `draw_frame(eye(3),k);`
- draw the provided rotation matrix $\boldsymbol{R}$ and color the different axis as follows: x - red, y - green and z - blue.
  `draw_frame(R,rgb);`

Task 2 (rotation matrix)

Problems

1. A vector ${}^{\mathcal{A}}\boldsymbol{r}$ is rotated around the z-axis $(0,0,1)$ of frame $\{\mathcal{A}\}$ by $\theta = 45$ degrees, and is subsequently rotated around the x-axis $(1,0,0)$ of frame $\{\mathcal{A}\}$ by -10 degrees. Give the rotation matrix $\boldsymbol{R}$ that accomplishes the rotations in the given order.
2. A frame $\{\mathcal{B}\}$ initially coincides with frame $\{\mathcal{A}\}$. The following transformations are performed
   - rotate frame $\{\mathcal{B}\}$ $(\pi/4)$ around the x-axis of frame $\{\mathcal{A}\}$ (we call the resulting frame $\{\mathcal{C}\}$)
   - rotate frame $\{\mathcal{C}\}$ $(\pi/4)$ around the y-axis of frame $\{\mathcal{A}\}$ (we will call the resulting frame $\{\mathcal{D}\}$)

   Give the rotation matrix ${}^{\mathcal{A}}_{\mathcal{D}}\boldsymbol{R}$
3. A frame $\{\mathcal{B}\}$ initially coincides with frame $\{\mathcal{A}\}$. The following transformations are performed
   - rotate frame $\{\mathcal{B}\}$ $(\pi/4)$ around the x-axis of frame $\{\mathcal{A}\}$ (we call the resulting frame $\{\mathcal{C}\}$)
   - rotate frame $\{\mathcal{C}\}$ $(\pi/4)$ around the y-axis of frame $\{\mathcal{C}\}$ (we will call the resulting frame $\{\mathcal{D}\}$)

   Give the rotation matrix ${}^{\mathcal{A}}_{\mathcal{D}}\boldsymbol{R}$

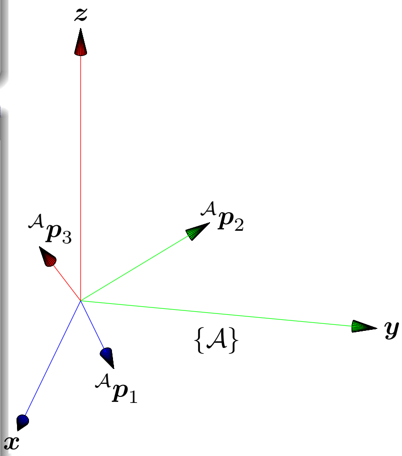
Task 3 (rotation matrix)

Given

- an orthonormal coordinate frame $\{\mathcal{A}\}$
- the positions (expressed in frame $\{\mathcal{A}\}$) of three points ${}^{\mathcal{A}}\boldsymbol{p}_1$, ${}^{\mathcal{A}}\boldsymbol{p}_2$, ${}^{\mathcal{A}}\boldsymbol{p}_3$ (see file `points3D.m`)

Problems

1. is $\{\mathcal{A}\}$ a right-handed coordinate frame?
2. is there a rotation matrix $\boldsymbol{R}$ such that
   - $\boldsymbol{R}^T[{}^{\mathcal{A}}\boldsymbol{p}_1]$ is aligned with the $\boldsymbol{x}$ axis
   - $\boldsymbol{R}^T[{}^{\mathcal{A}}\boldsymbol{p}_2]$ is aligned with the $\boldsymbol{y}$ axis
   - $\boldsymbol{R}^T[{}^{\mathcal{A}}\boldsymbol{p}_3]$ is aligned with the $\boldsymbol{z}$ axis
3. if $\boldsymbol{R}$ in point 2 exists
   - find it
   - implement a function that returns the set of $x \to y \to z$ (current axis) Euler angles corresponding to $\boldsymbol{R}$
   - find an "axis/angle" representation of $\boldsymbol{R}$



Task 4 (axis/angle)

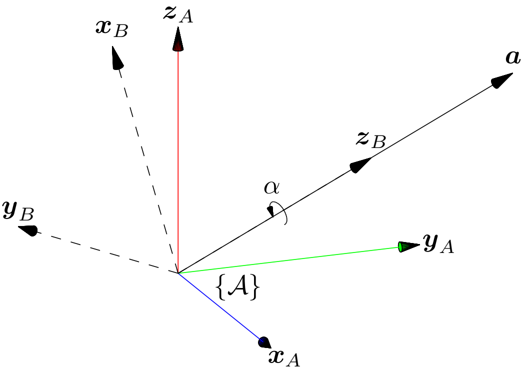
Given

axis $\boldsymbol{a}$ (expressed in frame $\{\mathcal{A}\}$) and angle α

Problem

find the rotation matrix $\boldsymbol{R}$ that rotates $\{\boldsymbol{x}_A, \boldsymbol{y}_A, \boldsymbol{z}_A\}$ around the axis $\boldsymbol{a}$ at an angle α

use `R2aa.m` to check your answer



Hints

1. construct an orthonormal frame $\{\mathcal{B}\}$ whose z-axis is co-linear with $\boldsymbol{a}$
2. find the rotation matrix ${}^{\mathcal{A}}_{\mathcal{B}}\boldsymbol{R}$
3. in order to rotate for example $\boldsymbol{x}_A$ around $\boldsymbol{a}$: (i) express $\boldsymbol{x}_A$ in frame $\{\mathcal{B}\}$; (ii) rotate ${}^{\mathcal{B}}\boldsymbol{x}_A$ around the local z-axis; (iii) take the rotated vector back to frame $\{\mathcal{A}\}$. Hence, ${}^{\mathcal{A}}_{\mathcal{B}}\boldsymbol{R}\boldsymbol{R}_z(\alpha){}^{\mathcal{B}}_{\mathcal{A}}\boldsymbol{R}[{}^{\mathcal{A}}\boldsymbol{x}_A]$.

Task 5 (homogeneous matrix)

Given

homogeneous matrix

$$ {}^{\mathcal{A}}_{\mathcal{B}}\boldsymbol{T} = \begin{bmatrix} {}^{\mathcal{A}}_{\mathcal{B}}\boldsymbol{R} & \boldsymbol{r} \\ \boldsymbol{0}^T & 1 \end{bmatrix} $$

specifying the posture of frame $\{\mathcal{B}\}$ with respect to frame $\boldsymbol{a}$, where

$$ \boldsymbol{r} = (1, 2, 3), \quad {}^{\mathcal{A}}_{\mathcal{B}}\boldsymbol{R} = \boldsymbol{R}_x(\frac{\pi}{4})\boldsymbol{R}_y(\frac{\pi}{5})\boldsymbol{R}_z(\frac{\pi}{6}) $$

Problem

express ${}^{\mathcal{A}}_{\mathcal{B}}\boldsymbol{T}$ as a product of 6 homogeneous matrices

Task 6 (homogeneous matrix)

Given

three homogeneous matrices ${}^{\mathcal{A}}_{\mathcal{B}}\boldsymbol{T}$, ${}^{\mathcal{B}}_{\mathcal{C}}\boldsymbol{T}$, ${}^{\mathcal{C}}_{\mathcal{D}}\boldsymbol{T}$ (see file T_matrices.m)

Problems

1. Plot the frames associated with each homogeneous matrix. Find the homogeneous matrices ${}^{\mathcal{A}}_{\mathcal{D}}\boldsymbol{T}$, ${}^{\mathcal{D}}_{\mathcal{B}}\boldsymbol{T}$.
2. find a set of $x \to y \to z$ (current axis) Euler angles that parametrize the rotation matrix in ${}^{\mathcal{A}}_{\mathcal{B}}\boldsymbol{T}$

Lab 2 - receipt

After completing all exercises in the lab you need to present the results to the teacher/assistant in order to pass the lab. It should be completed latest 2 weeks after the lab was given. The receipt is g iven to you as a proof that you passed the lab. A separate record will be kept by the teacher.

Be prepared to demonstrate all results before the presentation.

Name:

Personal number:

Approved - signature:

Date: