

COMP2521 25T3

Abstract Data Types

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abstraction
abstract data types
stacks and queues
sets

Abstraction

ADTs

Stacks

Queues

Sets

Abstraction
is the process of
hiding or generalising
the **details** of an object or system
to **focus on its high-level meaning** or behaviour

Abstraction

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We drive a car by using a steering wheel and pedals

Abstraction

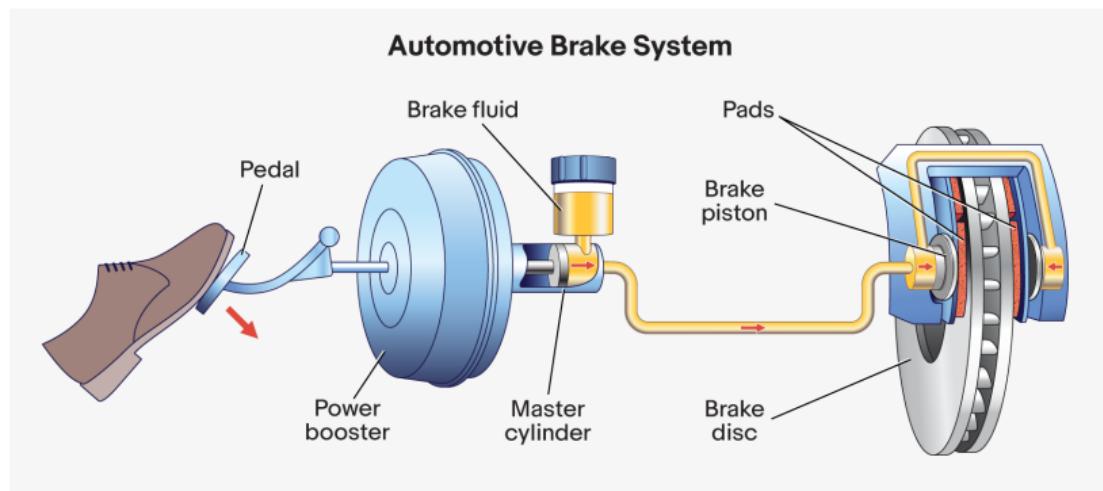
ADTs

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We drive a car by using a steering wheel and pedals



Abstraction

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We operate a television through a remote control

Abstraction

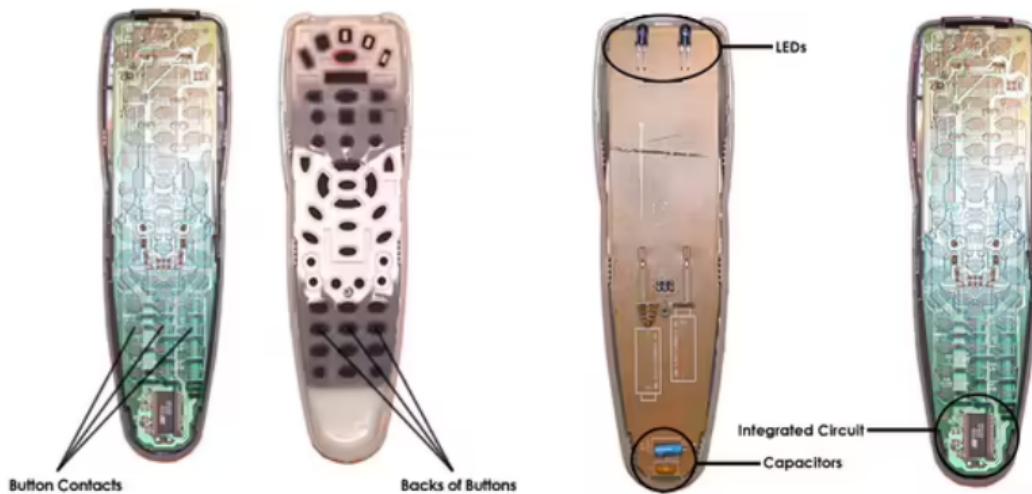
ADTs

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We operate a television through a remote control



Abstraction

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Sets

We deposit and withdraw money to/from our bank account via an ATM

Abstraction

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We deposit and withdraw money to/from our bank account via an ATM



Abstraction

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How many of these are examples of abstraction?

Using a computer

Executing a sorting program

Calling a function in C

Using an int in C

Abstraction

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How many of these are examples of abstraction?

Using a computer

Executing a sorting program

Calling a function in C

Using an int in C

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Functions abstract away the steps of a computation

```
int factorial(int n) {  
    int res = 1;  
    for (int i = 1; i <= n; i++) {  
        res *= i;  
    }  
    return res;  
}
```

int res = factorial(5);

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Modern programming languages abstract away assembly code

```
push rbp
mov rbp, rsp
mov DWORD PTR [rbp-0x14], edi
mov DWORD PTR [rbp-0x04], 0x1
mov DWORD PTR [rbp-0x08], 0x1
jmp 25 <fn+0x25>
mov eax, DWORD PTR [rbp-0x04]
imul eax, DWORD PTR [rbp-0x08]
mov DWORD PTR [rbp-0x04], eax
add DWORD PTR [rbp-0x08], 0x1
mov eax, DWORD PTR [rbp-0x08]
cmp eax, DWORD PTR [rbp-0x14]
jle 17 <fn+0x17>
mov eax, DWORD PTR [rbp-0x04]
pop rbp
ret
```

```
int factorial(int n) {
    int res = 1;
    for (int i = 1; i <= n; i++) {
        res *= i;
    }
    return res;
}
```

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Assembly languages abstract away machine code

```
0000000000000000 <fn>:  
 0: 55                      push rbp  
 1: 48 89 e5                mov rbp, rsp  
 4: 89 7d ec                mov DWORD PTR [rbp-0x14], edi  
 7: c7 45 fc 01 00 00 00    mov DWORD PTR [rbp-0x04], 0x1  
 e: c7 45 f8 01 00 00 00    mov DWORD PTR [rbp-0x08], 0x1  
 15: eb 0e                  jmp 25 <fn+0x25>  
 17: 8b 45 fc                mov eax, DWORD PTR [rbp-0x04]  
 1a: 0f af 45 f8              imul eax, DWORD PTR [rbp-0x08]  
 1e: 89 45 fc                mov DWORD PTR [rbp-0x04], eax  
 21: 83 45 f8 01              add DWORD PTR [rbp-0x08], 0x1  
 25: 8b 45 f8                mov eax, DWORD PTR [rbp-0x08]  
 28: 3b 45 ec                cmp eax, DWORD PTR [rbp-0x14]  
 2b: 7e ea                  jle 17 <fn+0x17>  
 2d: 8b 45 fc                mov eax, DWORD PTR [rbp-0x04]  
 30: 5d                      pop rbp  
 31: c3                      ret
```

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An abstract data type...

is a **conceptual model**

that defines a **set of operations** for a data structure
without specifying how these operations are implemented
or how data is stored in memory.

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Ordering from a restaurant...



The menu...



The kitchen...

Example of an ADT: Stack

A stack is a linear collection of items
with two main operations:

push

adds an item to the top of the stack

pop

removes the item at the top of the stack

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User

Stack

Operations

push

adds an item to the top of the stack

pop

removes the item at the top of the stack

User

push 8

push 3

push 7

pop

pop

push 1

Stack



Operations

push

adds an item to the top of the stack

pop

removes the item at the top of the stack

User**push 8****push 3****push 7****pop****pop****push 1****Stack****Operations****push**

adds an item to the top of the stack

pop

removes the item at the top of the stack

User

push 8

push 3

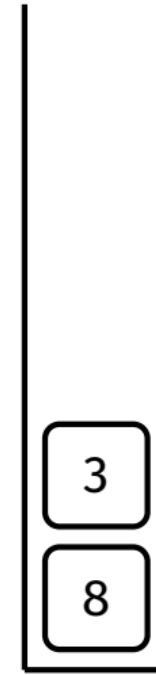
push 7

pop

pop

push 1

Stack



Operations

push

adds an item to the top of the stack

pop

removes the item at the top of the stack

User

push 8

push 3

push 7

pop

pop

push 1

Stack**Operations****push**

adds an item to the top of the stack

pop

removes the item at the top of the stack

User

push 8

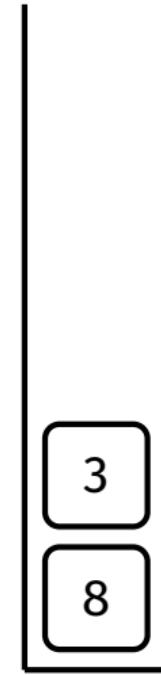
push 3

push 7

pop \Rightarrow 7

pop

push 1

Stack**Operations****push**

adds an item to the top of the stack

pop

removes the item at the top of the stack

User

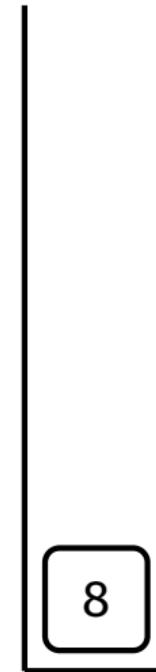
push 8

push 3

push 7

pop \Rightarrow 7pop \Rightarrow 3

push 1

Stack**Operations****push**

adds an item to the top of the stack

pop

removes the item at the top of the stack

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User

push 8

push 3

push 7

pop \Rightarrow 7pop \Rightarrow 3**push 1****Stack****Operations****push**

adds an item to the top of the stack

pop

removes the item at the top of the stack

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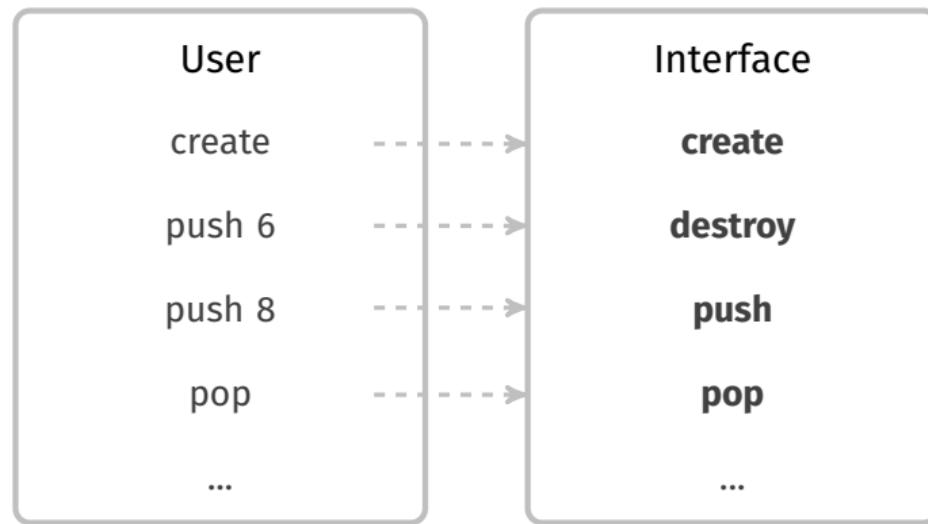
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The set of operations provided by an ADT is called the **interface**.

Users of an ADT only see and interact with the interface.



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An ADT interface must:

1. clearly describe the behaviour of each operation
2. describe the conditions under which each operation can be used

Example:

pop

removes the item at
the top of the stack

assumes that the stack is not empty

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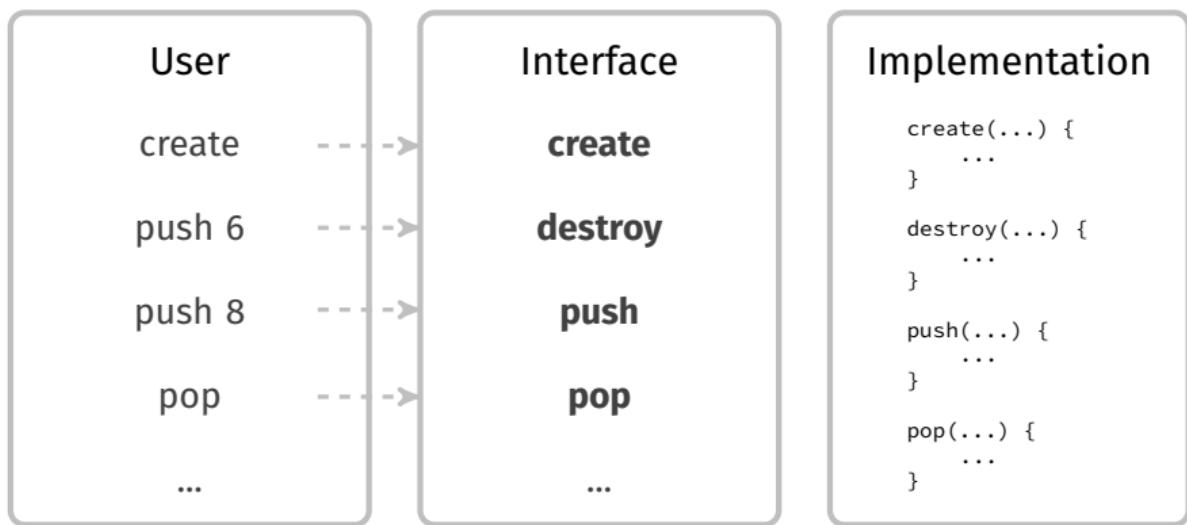
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Builders of an ADT provide an implementation of its operations.



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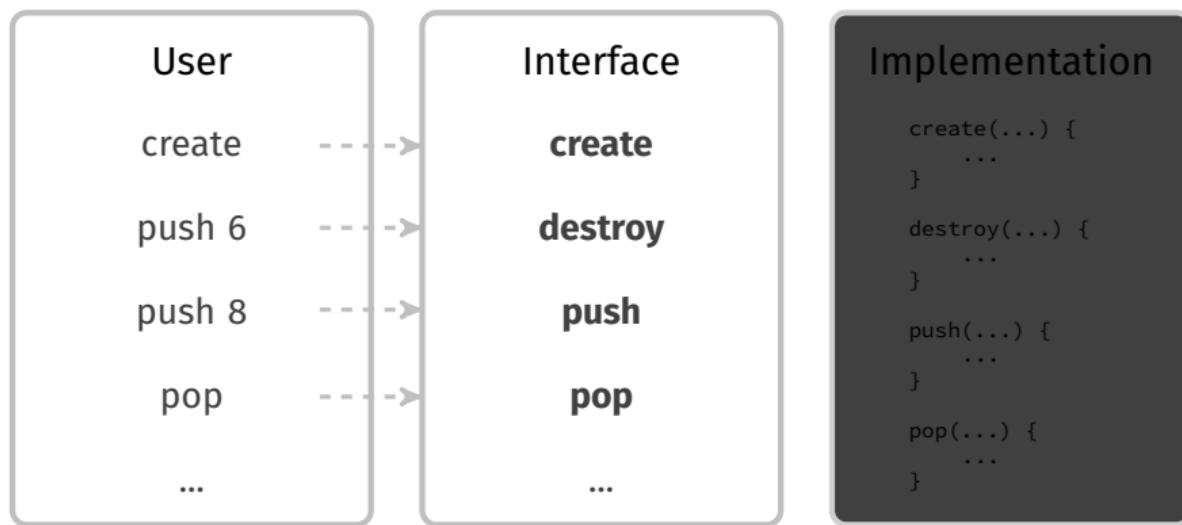
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Users of an ADT **do not** see the implementation.



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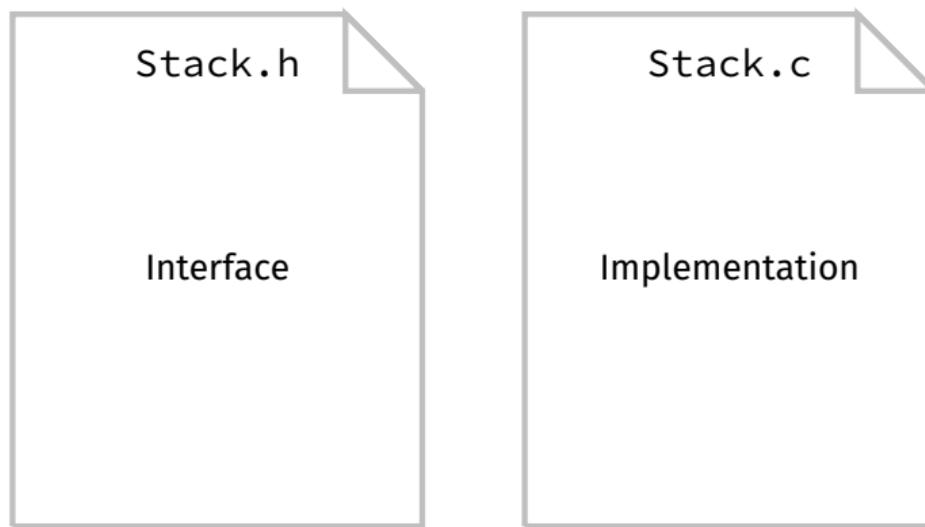
Queues

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In C, abstract data types are implemented using two files:

a **.h** file that contains the **interface**

a **.c** file that contains the **implementation**



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The interface includes:

- forward declaration of the struct for the concrete representation
 - via `typedef struct t *T`
 - **the struct is not defined in the interface**
- function prototypes for all operations
- clear description of operations
 - via comments
- a contract between the ADT and clients
 - documentation describes how an operation can be used
 - and what the expected result is *as long as the operation is used correctly*

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Stack.h

```
typedef struct stack *Stack;

/** Creates a new empty stack */
Stack StackNew(void);

/** Frees memory allocated to the stack */
void StackFree(Stack s);

/** Adds an item to the top of the stack */
void StackPush(Stack s, int item);

/** Removes the item at the top of the stack
    Assumes that the stack is not empty */
int StackPop(Stack s);
```

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The implementation includes:

- concrete definition of the data structures
 - definition of struct t
- function implementations for all operations

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Stack.c

```
struct stack {  
    ...  
};  
  
Stack StackNew(void) {  
    ...  
}  
  
void StackFree(Stack s) {  
    ...  
}  
  
void StackPush(Stack s, int item) {  
    ...  
}  
  
int StackPop(Stack s) {  
    ...  
}
```

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A user of an ADT `#includes` the interface and uses the interface functions to interact with the ADT.

user.c

```
#include "Stack.h"

int main(void) {
    Stack s = StackNew();
    StackPush(s, 6);
    StackPush(s, 8);
    int item = StackPop(s);
    ...
}
```

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Users of an ADT only see and interact with the interface –
they do not see the implementation!

user.c

```
#include "Stack.h"

int main(void) {
    Stack s = StackNew();
    StackPush(s, 6);
    StackPush(s, 8);
    int item = StackPop(s);
    ...
}
```

Stack.h

```
typedef struct stack *Stack;
...
...
```

Stack.c

```
struct stack {
    ...
};
```

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Users of an ADT only see and interact with the interface –
they do not see the implementation!

user.c

```
#include "Stack.h"

int main(void) {
    Stack s = StackNew();

    // this is not valid!
    s->...
}
```

Stack.h

```
typedef struct stack *Stack;
...
```

Stack.c

```
struct stack {
    ...
};
```

This means users cannot access the concrete representation (struct) directly.

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Naming conventions:

- ADTs are defined in files whose names start with an uppercase letter
 - For example, for a Stack ADT:
 - The interface is defined in Stack.h
 - The implementation is defined in Stack.c
 - ADT interface function names are in PascalCase and begin with the name of the ADT

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- ① Decide what operations you want to provide
 - Operations to **create, query, manipulate**
 - What are their inputs and outputs?
 - What are the conditions under which they can be used (if any)?
- ② Provide the function signatures and documentation for these operations in a .h file
- ③ The “developer” builds a concrete implementation for the ADT in a .c file
- ④ The “user” #includes the interface in their program and uses the provided functions

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What operations can you perform on a simple bank account?

- Open an account
- Check balance
- Deposit money
- Withdraw money

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```
typedef struct account *Account;

/** Opens a new account with zero balance */
Account AccountOpen(void);

/** Closes an account */
void AccountClose(Account acc);

/** Returns account balance */
int AccountBalance(Account acc);

/** Withdraws money from account
    Returns true if enough balance, false otherwise
    Assumes amount is positive */
bool AccountWithdraw(Account acc, int amount);

/** Deposits money into account
    Assumes amount is positive */
void AccountDeposit(Account acc, int amount);
```

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```
int main(void) {
    Account acc = AccountOpen();
    printf("Balance: %d\n", AccountBalance(acc));

    AccountDeposit(acc, 50);
    printf("Balance: %d\n", AccountBalance(acc));

    AccountWithdraw(acc, 20);
    printf("Balance: %d\n", AccountBalance(acc));

    AccountWithdraw(acc, 40);
    printf("Balance: %d\n", AccountBalance(acc));

    AccountClose(acc);
}
```

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Invalid usage of an ADT (breaking abstraction):

```
int main(void) {
    Account acc = AccountOpen();

    acc->balance = 1000000;

    // I'm a millionaire now, woohoo!
    printf("Balance: %d\n", AccountBalance(acc));

    AccountClose(acc);
}
```

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- Stack
- Queue
- Set
- Multiset
- Map
- Graph
- Priority Queue

A **stack** is a collection of items,
such that the **last** item to enter
is the **first** item to leave:

Last In, First Out (LIFO)

(Think stacks of books, plates, etc.)

- web browser history
- text editor undo/redo
- balanced bracket checking
- HTML tag matching
- RPN calculators
 - (...and programming languages!)
- function calls

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A stack supports the following operations:

push

add a new item to the top of the stack

pop

remove the topmost item from the stack

size

return the number of items on the stack

peek

get the topmost item on the stack without removing it

Example: Balancing Brackets

A Stack ADT can be used to check for balanced brackets.

Example of balanced brackets:

([{ }])

Examples of unbalanced brackets!

())) ((([{ })] ([]) ([

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-
[([-

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-
[([-
{	([{	-

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
		-
((-
[([-
{	([{	-
}	([}	{ = }

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-
[([-
{	([{	-
}	([{ = }	
]	([=]	

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-
[([-
{	([{	-
}	([{ = }
]	([=]
)		(=)

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-
[([-
{	([{	-
}	([{ = }
]	([=]
)		(=)
EOF		is empty

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-
[([-

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-
[([-
{	([{	-

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-
[([-
{	([{	-
}	([{ = }	

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-
[([-
{	([{	-
}	([{ = }	
)	([≠)	

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
((-
[([-
{	([{	-
}	([{ = }	
)	([≠)	fail!

```
typedef struct stack *Stack;

/** Creates a new, empty Stack */
Stack StackNew(void);

/** Frees memory allocated for a Stack */
void StackFree(Stack s);

/** Adds an item to the top of a Stack */
void StackPush(Stack s, Item it);

/** Removes an item from the top of a Stack
    Assumes that the Stack is not empty */
Item StackPop(Stack s);

/** Gets the number of items in a Stack */
int StackSize(Stack s);

/** Gets the item at the top of a Stack
    Assumes that the Stack is not empty */
Item StackPeek(Stack s);
```

How to implement a stack?

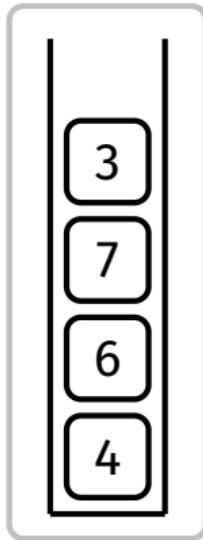
array

linked list

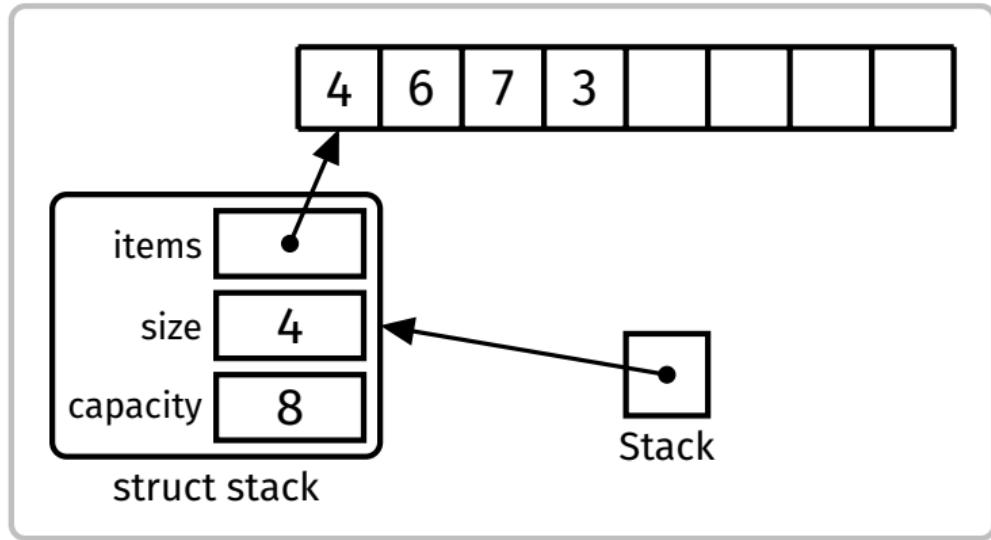
Dynamically allocate an array with an initial capacity

Fill the array sequentially – $s[0], s[1], \dots$

Maintain a counter of the number of items on the stack



Conceptual
model



Concrete representation

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Example

Perform the following operations:

PUSH(9), PUSH(2), PUSH(6), POP, POP, PUSH(8)

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PUSH(9)

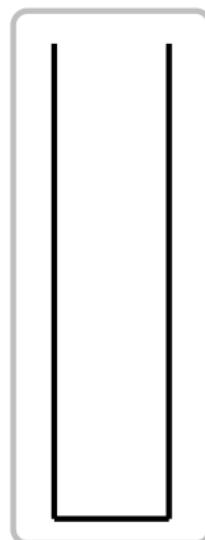
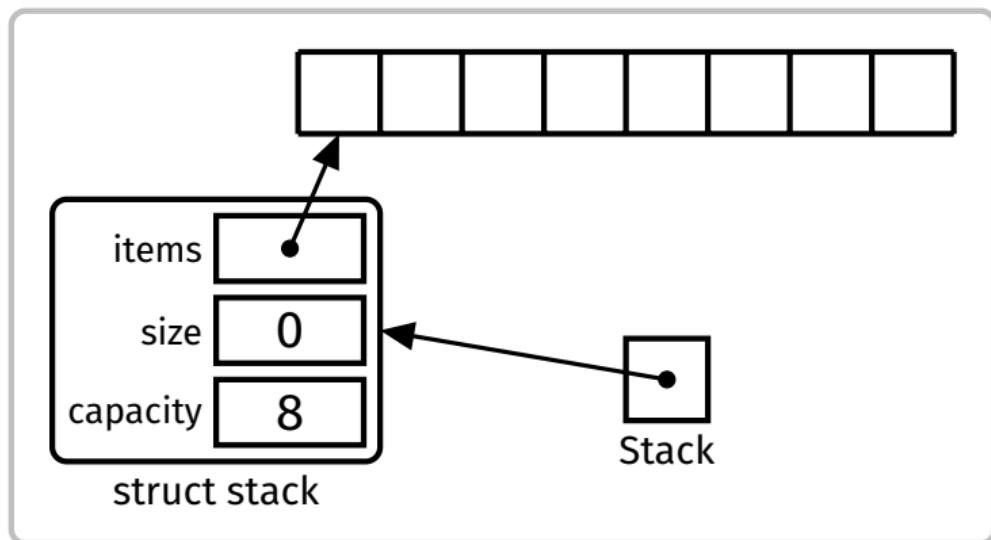
PUSH(2)

PUSH(6)

POP

POP

PUSH(8)

Conceptual
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PUSH(9)

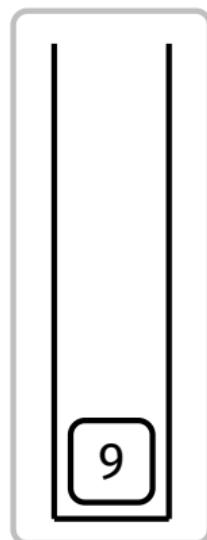
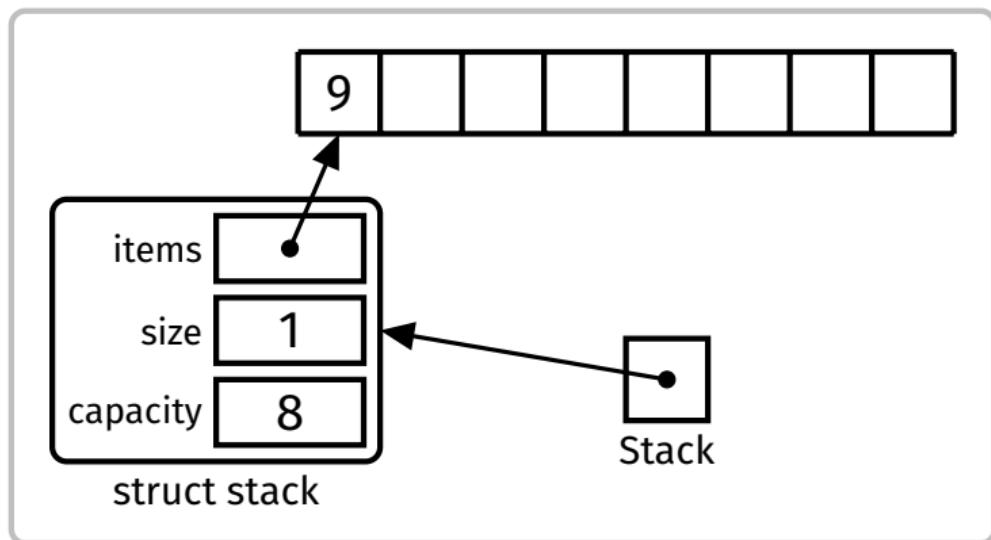
PUSH(2)

PUSH(6)

POP

POP

PUSH(8)

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PUSH(9)

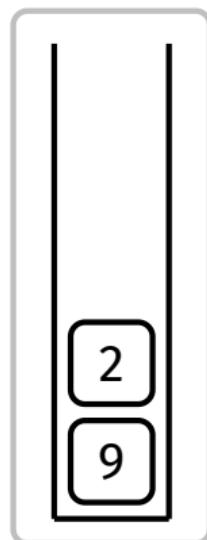
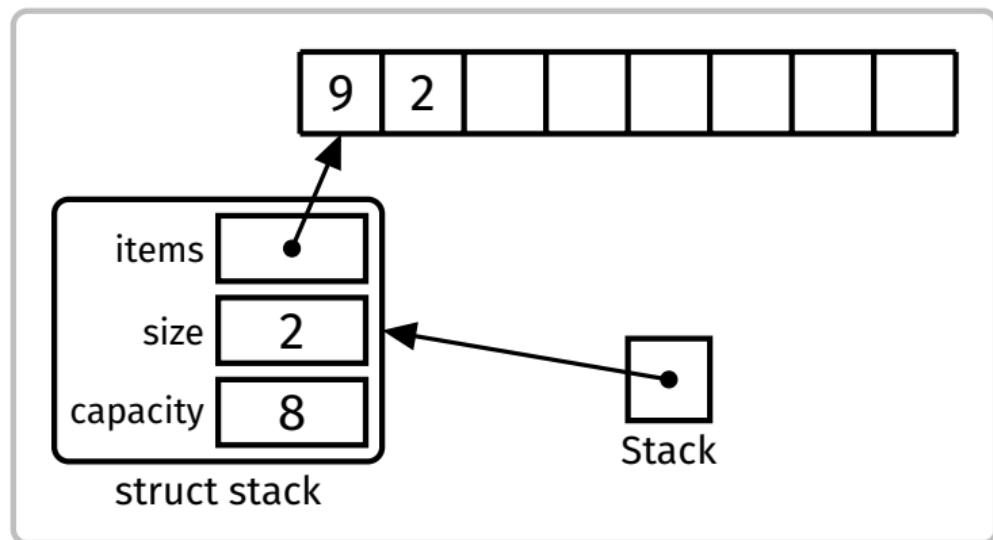
PUSH(2)

PUSH(6)

POP

POP

PUSH(8)

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PUSH(9)

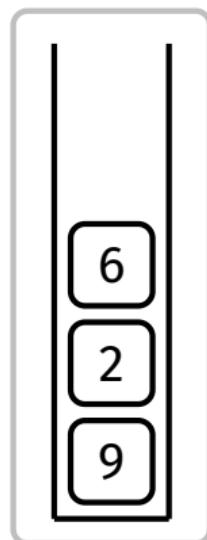
PUSH(2)

PUSH(6)

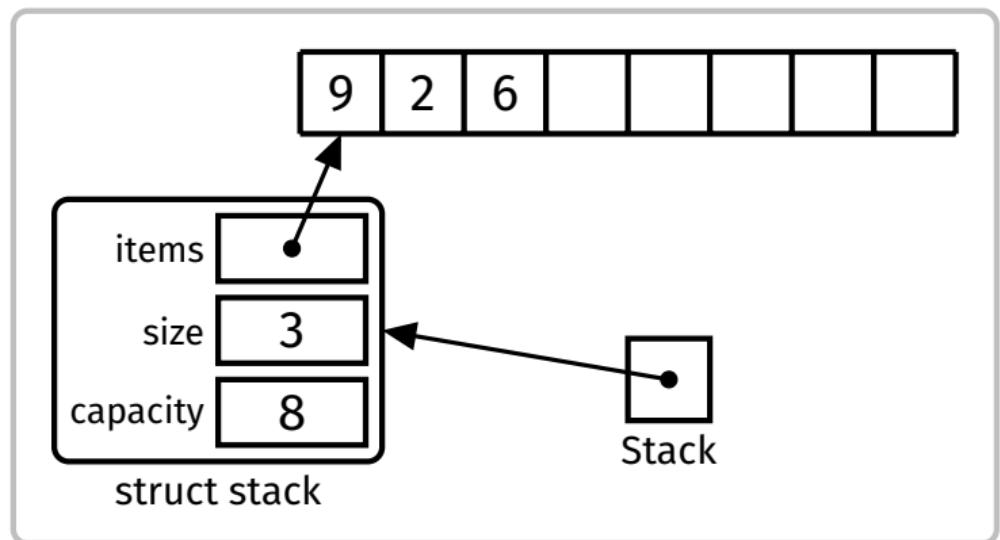
POP

POP

PUSH(8)



Conceptual model



Concrete representation

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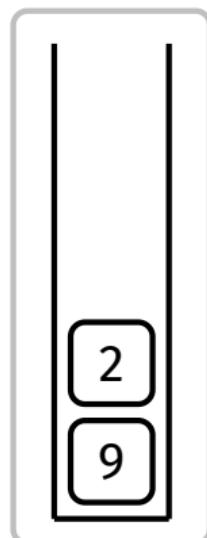
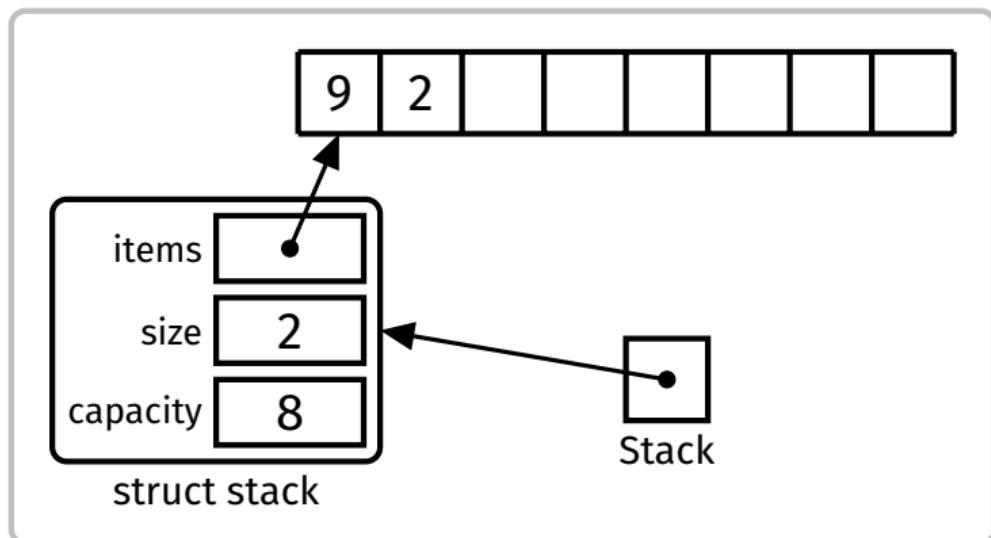
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PUSH(9) PUSH(2) PUSH(6) POP \Rightarrow 6 POP PUSH(8)Conceptual
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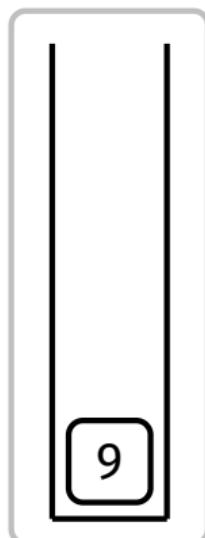
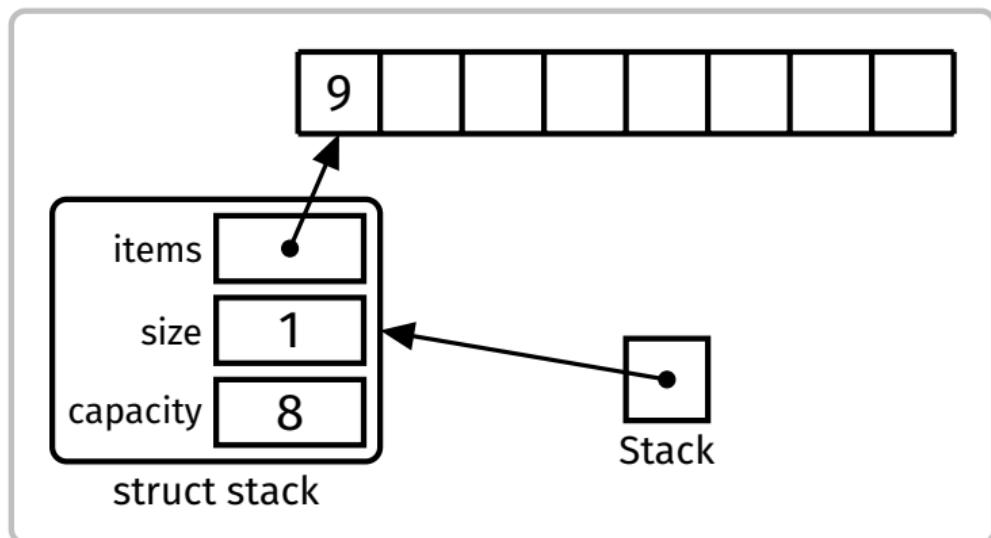
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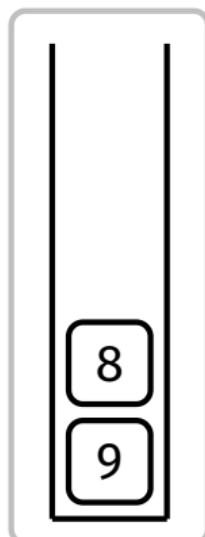
Array

Linked list

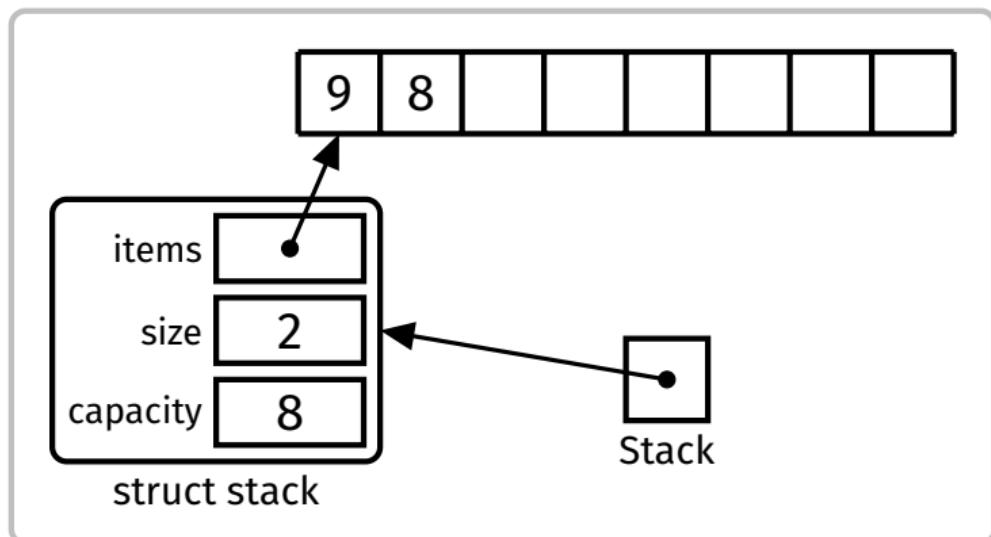
Queues

Sets

PUSH(9) PUSH(2) PUSH(6) POP \Rightarrow 6 POP \Rightarrow 2 PUSH(8)



Conceptual
model



Concrete representation

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Cost of push:

- Inserting item at index `size` is $O(1)$
- What if array is full?
 - If we double the size of the array with `realloc(3)` each time it is full, push will still be $O(1)$ on average

Cost of pop:

- Accessing item at index `(size - 1)` is $O(1)$

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Store items in a linked list

To push an item, insert it at the beginning of the list

To pop an item, remove it from the beginning of the list

Abstraction

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Interface

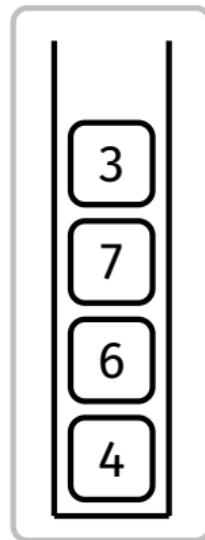
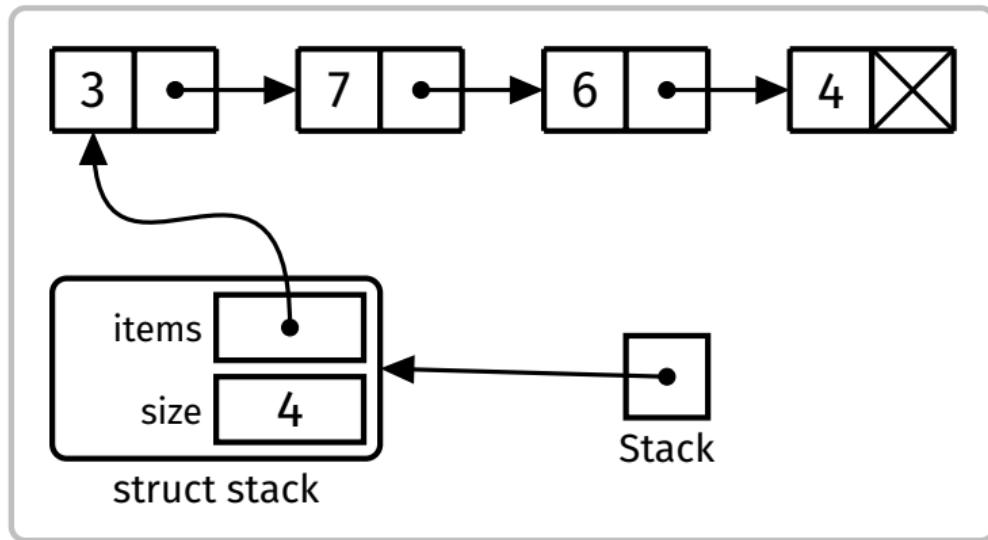
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Example

Perform the following operations:

PUSH(9), PUSH(2), PUSH(6), POP, POP, PUSH(8)

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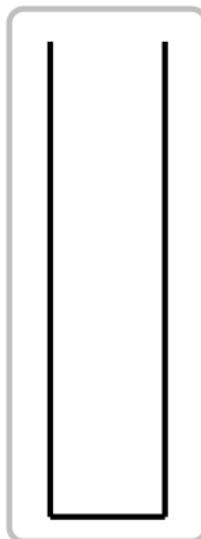
Array

Linked list

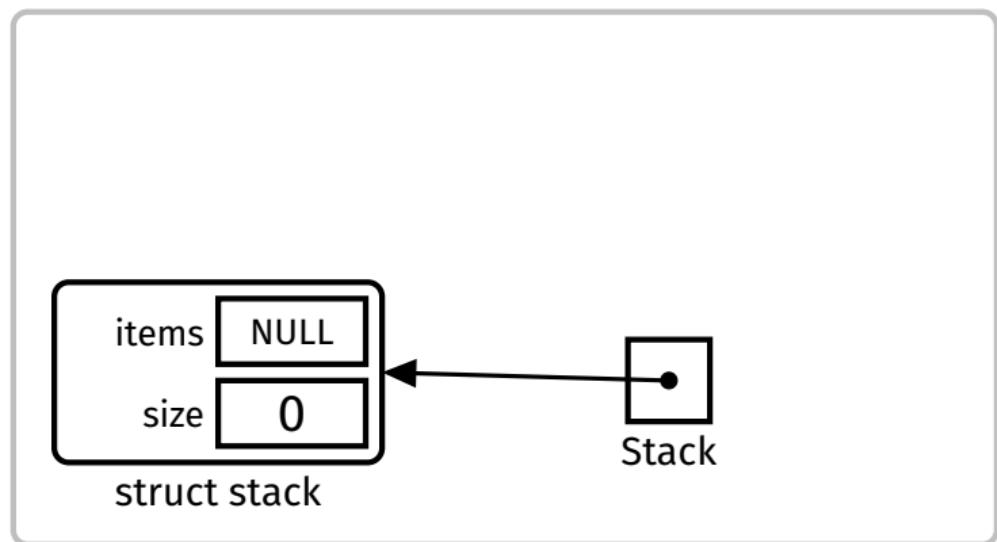
Queues

Sets

PUSH(9) PUSH(2) PUSH(6) POP POP PUSH(8)



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PUSH(9)

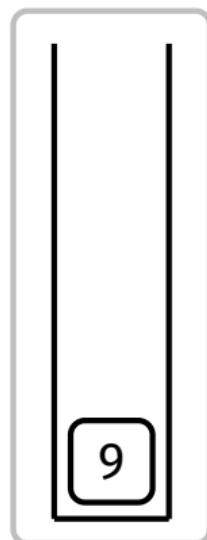
PUSH(2)

PUSH(6)

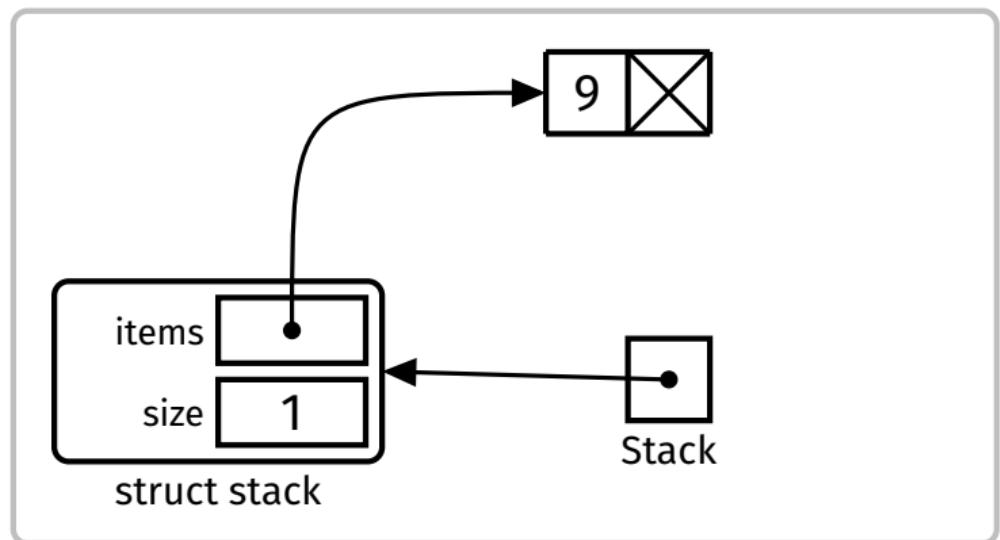
POP

POP

PUSH(8)



Conceptual model



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PUSH(9)

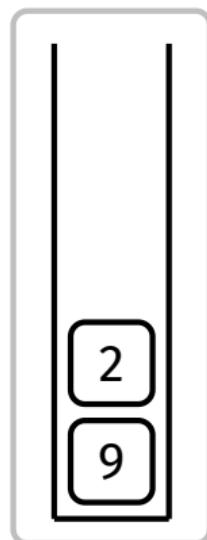
PUSH(2)

PUSH(6)

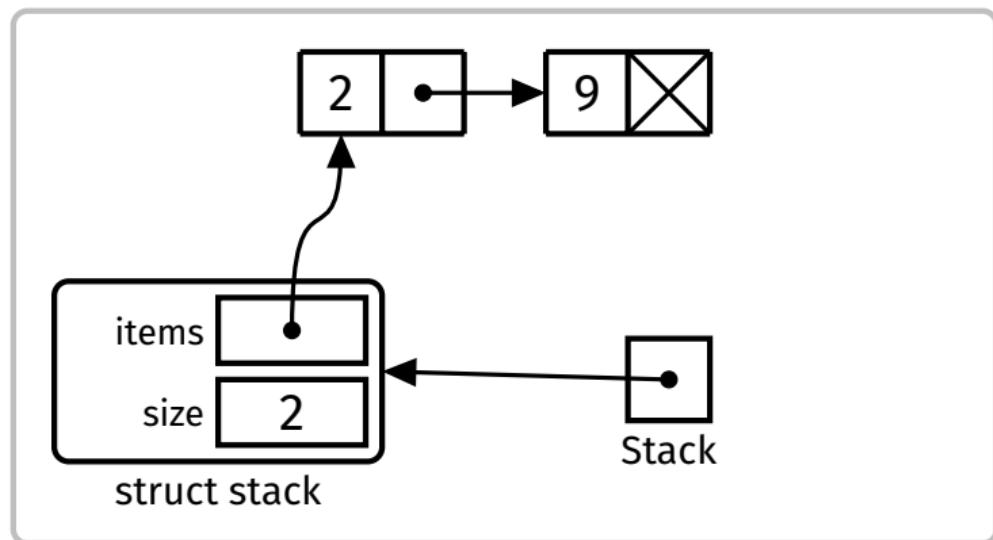
POP

POP

PUSH(8)



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PUSH(9)

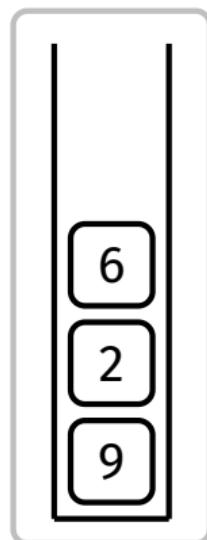
PUSH(2)

PUSH(6)

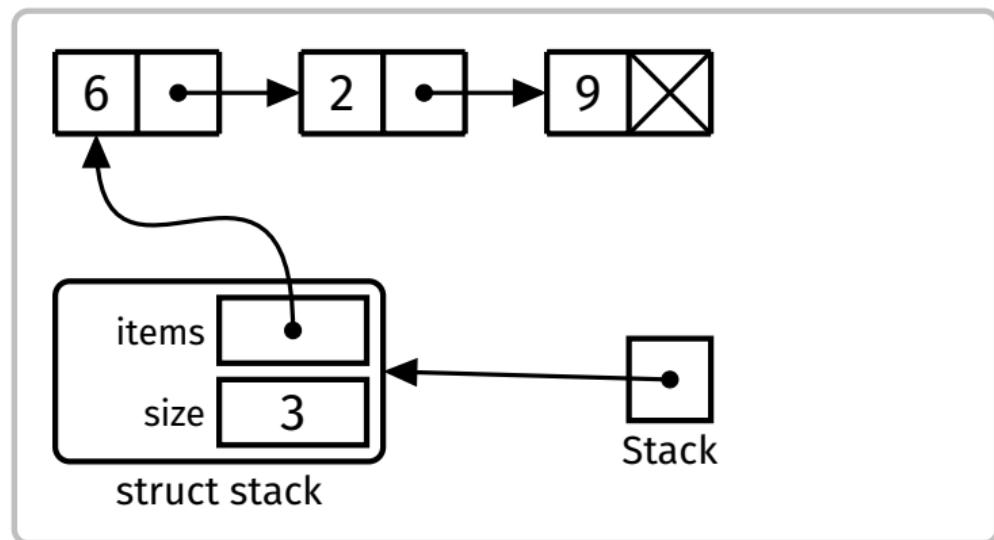
POP

POP

PUSH(8)



Conceptual model



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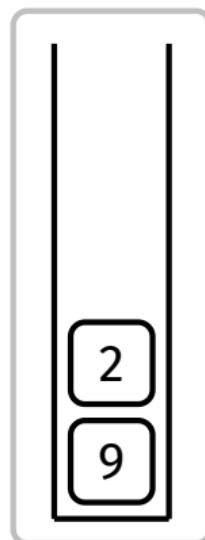
Array

Linked list

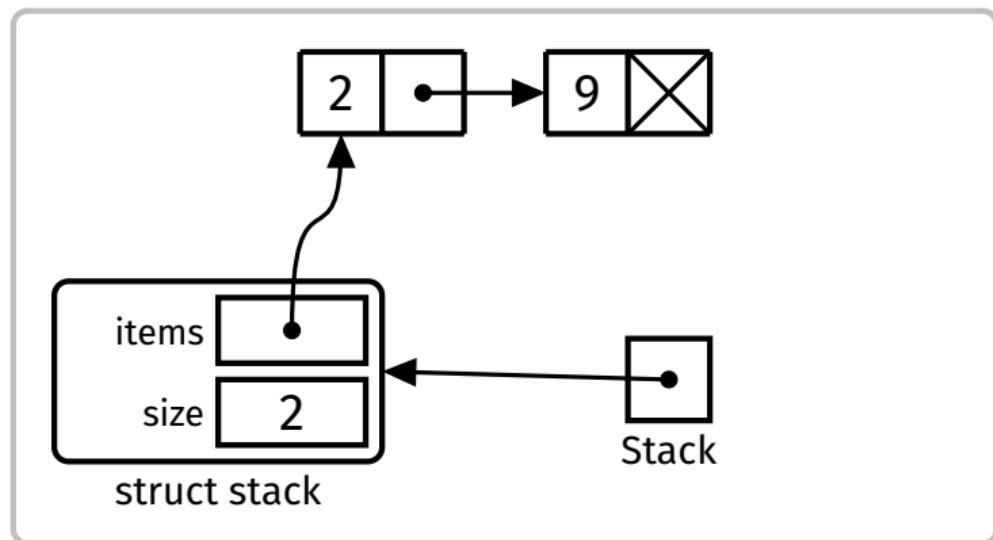
Queues

Sets

PUSH(9) PUSH(2) PUSH(6) **POP \Rightarrow 6** POP PUSH(8)



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model



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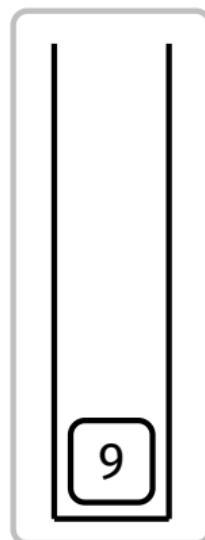
Array

Linked list

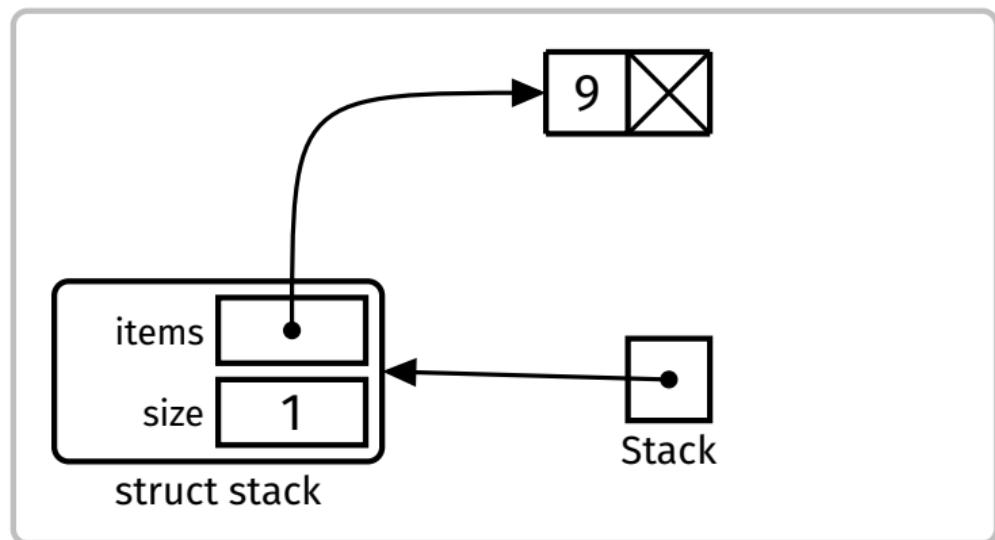
Queues

Sets

PUSH(9) PUSH(2) PUSH(6) POP \Rightarrow 6 POP \Rightarrow 2 PUSH(8)

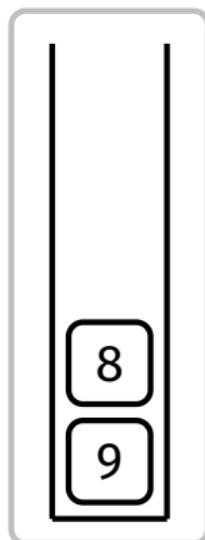


Conceptual
model

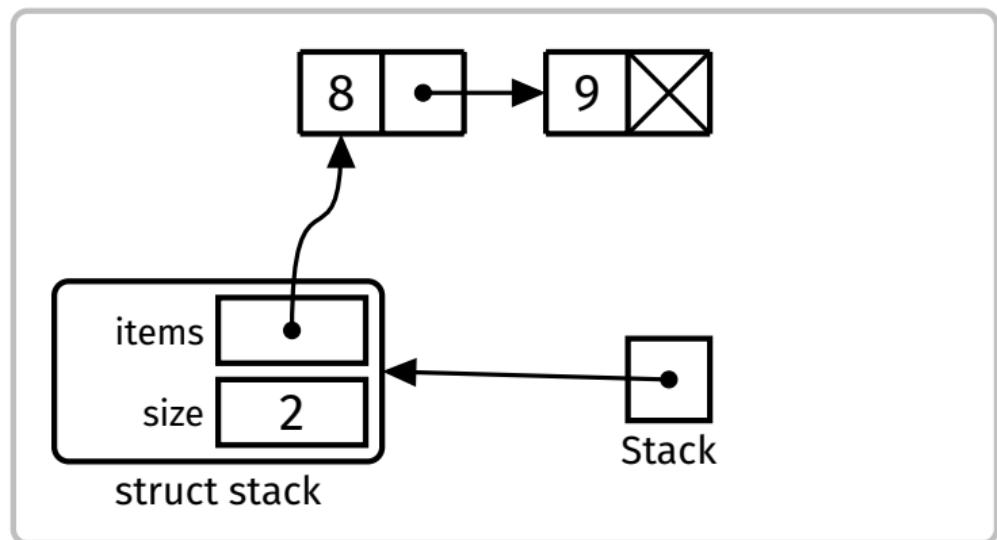


Concrete representation

PUSH(9) PUSH(2) PUSH(6) POP \Rightarrow 6 POP \Rightarrow 2 PUSH(8)



Conceptual
model



Concrete representation

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Cost of push:

- Inserting at the beginning of a linked list is $O(1)$

Cost of pop:

- Removing from the beginning of a linked list is $O(1)$

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A **queue** is a collection of items,
such that the **first** item to enter
is the **first** item to leave:

First In, First Out (FIFO)

(Think queues of people, etc.)

- waiting lists
- call centres
- access to shared resources
(e.g., printers)
- processes in a computer

Simple Queue Service (SQS)



A queue supports the following operations:

enqueue

add a new item to the end of the queue

dequeue

remove the item at the front of the queue

size

return the number of items in the queue

peek

get the frontmost item of the queue, without removing it

```
typedef struct queue *Queue;

/** Create a new, empty Queue */
Queue QueueNew(void);

/** Free memory allocated to a Queue */
void QueueFree(Queue q);

/** Add an item to the end of a Queue */
void QueueEnqueue(Queue q, Item it);

/** Remove an item from the front of a Queue
    Assumes that the Queue is not empty */
Item QueueDequeue(Queue q);

/** Get the number of items in a Queue */
int QueueSize(Queue q);

/** Get the item at the front of a Queue
    Assumes that the Queue is not empty */
Item QueuePeek(Queue q);
```

How to implement a queue?

array

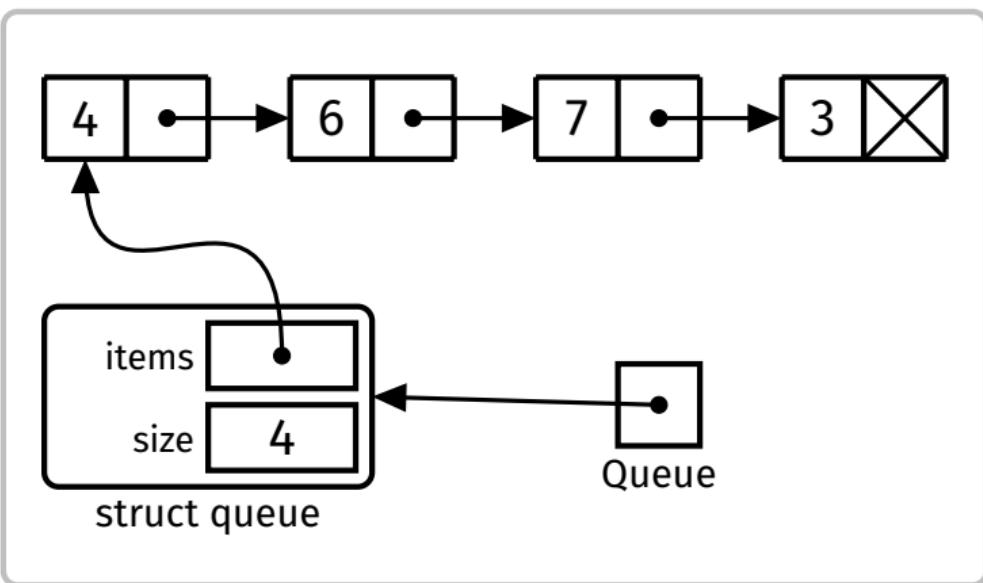
linked list (easier)

To enqueue an item, insert it at the end of the list

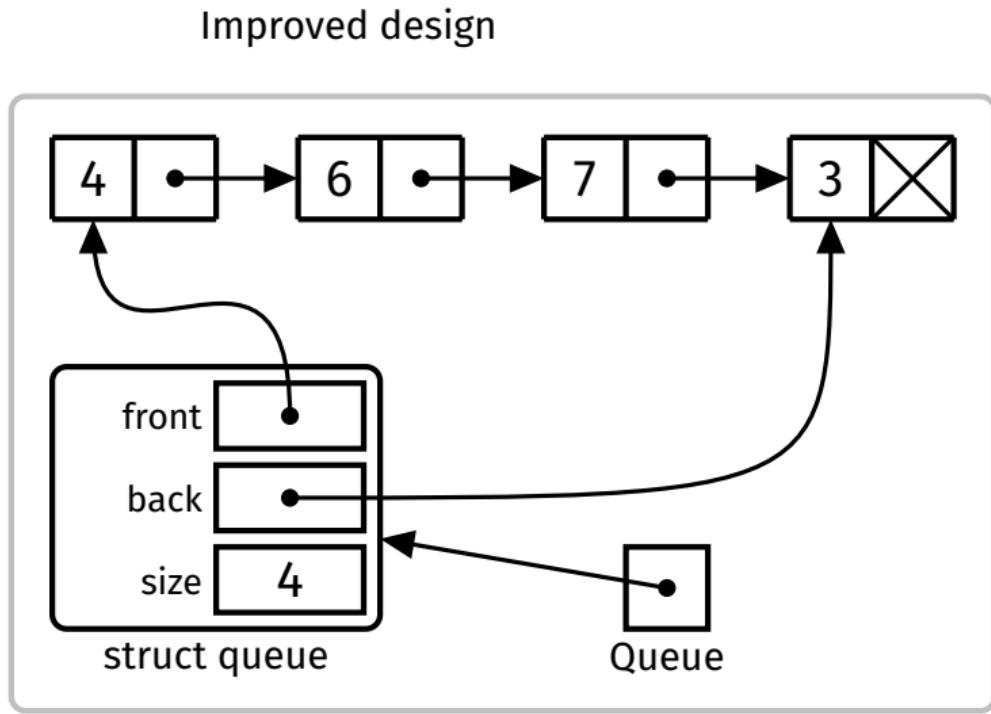
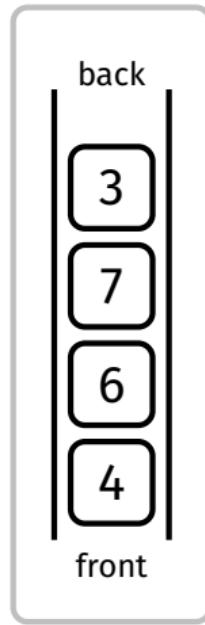
To an item, remove it from the beginning of the list

Conceptual
model

What's the problem with this design?



Concrete representation



Concrete representation

Example

Perform the following operations:

ENQ(9), ENQ(2), ENQ(6), DEQ, DEQ, ENQ(8)

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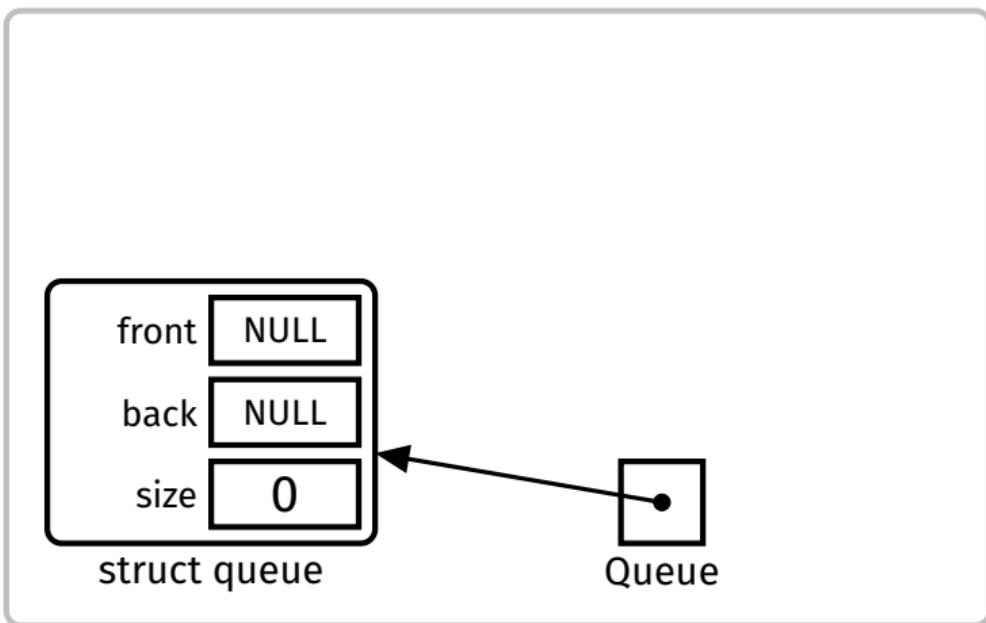
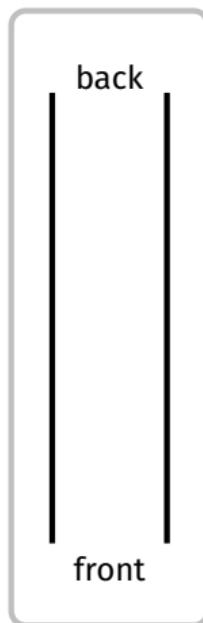
Implementation

Linked list

Array

Sets

ENQ(9) ENQ(2) ENQ(6) DEQ DEQ ENQ(8)

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ENQ(9)

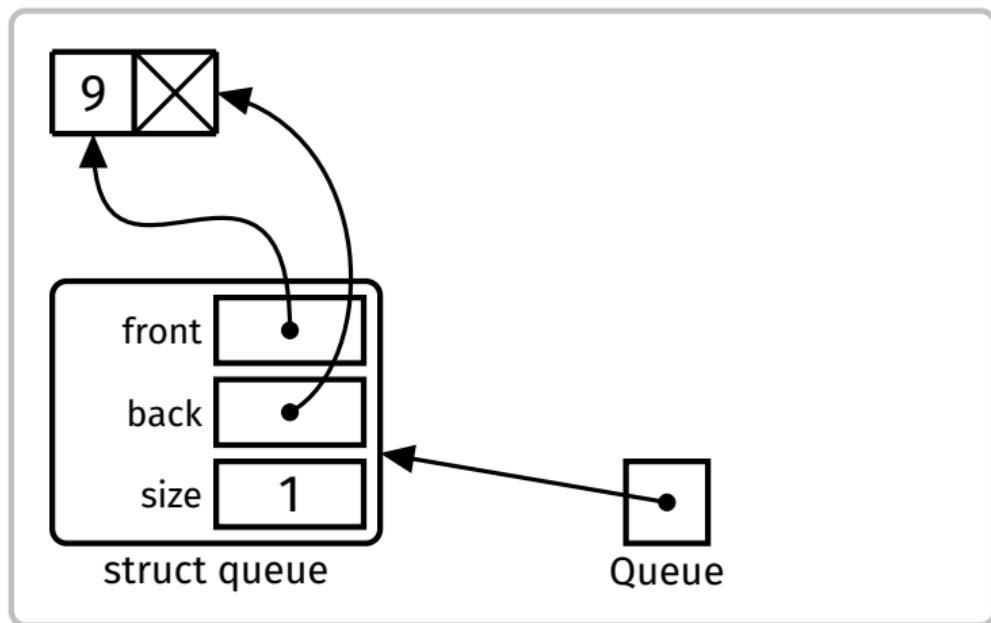
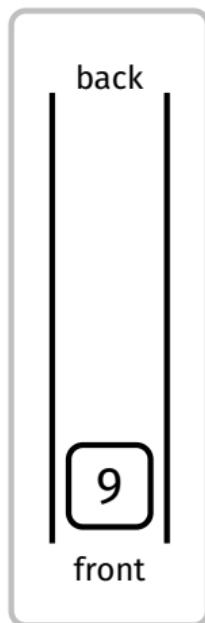
ENQ(2)

ENQ(6)

DEQ

DEQ

ENQ(8)

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ENQ(9)

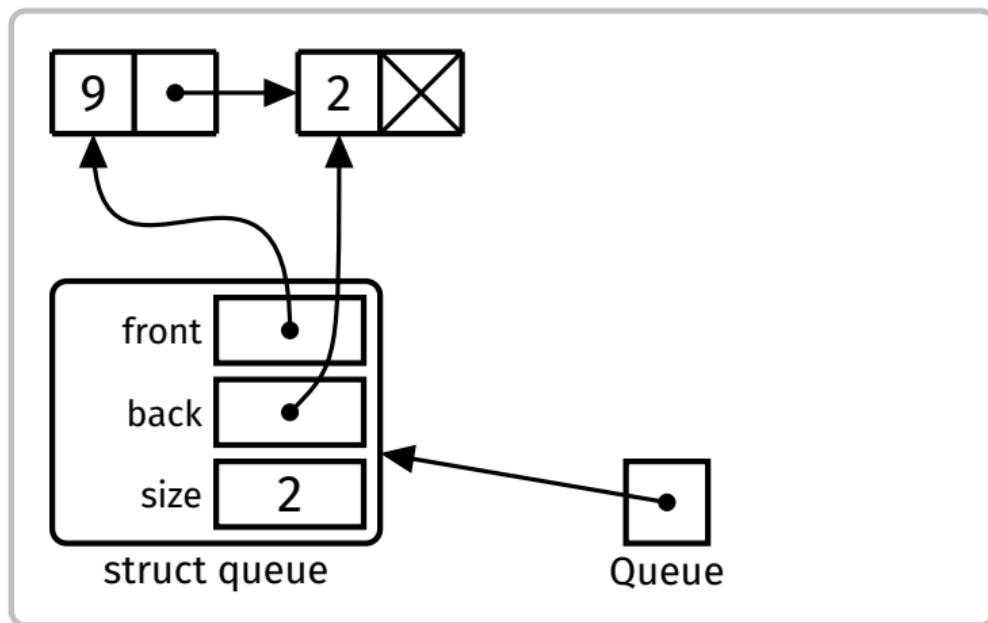
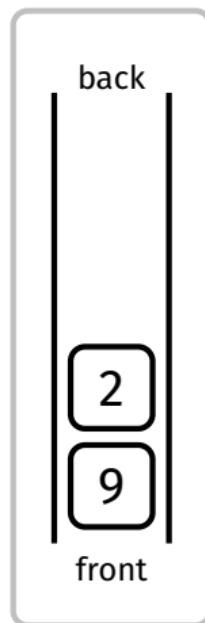
ENQ(2)

ENQ(6)

DEQ

DEQ

ENQ(8)

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ENQ(9)

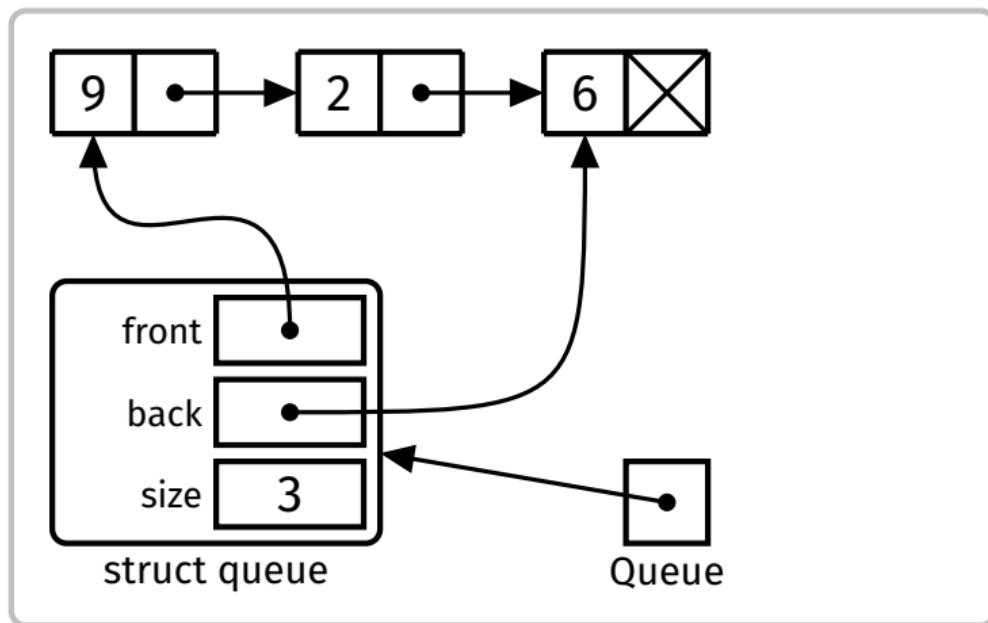
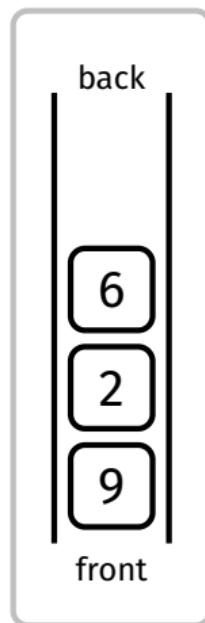
ENQ(2)

ENQ(6)

DEQ

DEQ

ENQ(8)

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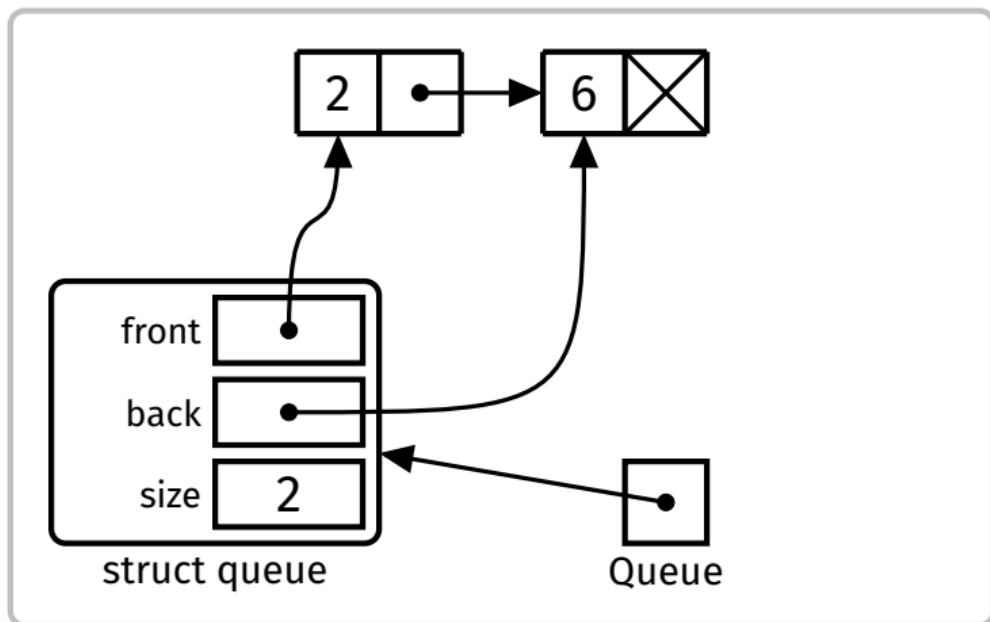
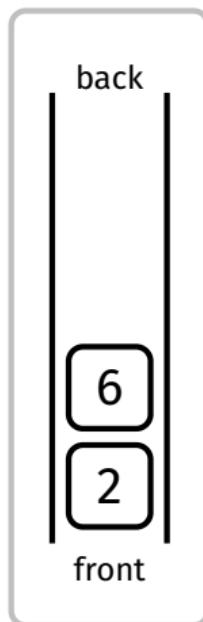
Interface

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Sets

ENQ(9) ENQ(2) ENQ(6) DEQ \Rightarrow 9 DEQ ENQ(8)Conceptual
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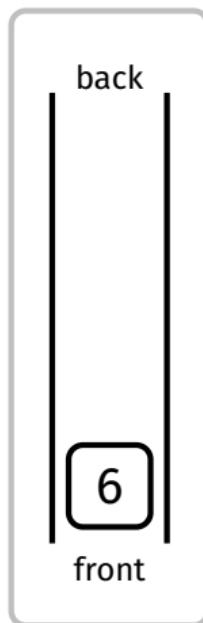
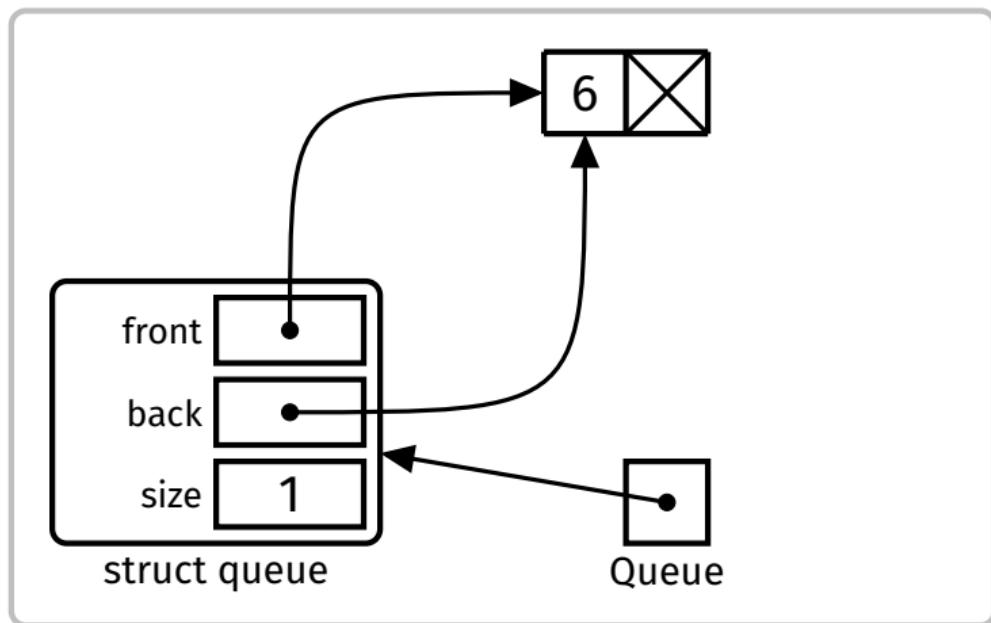
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ENQ(9) ENQ(2) ENQ(6) DEQ \Rightarrow 9 DEQ \Rightarrow 2 ENQ(8)Conceptual
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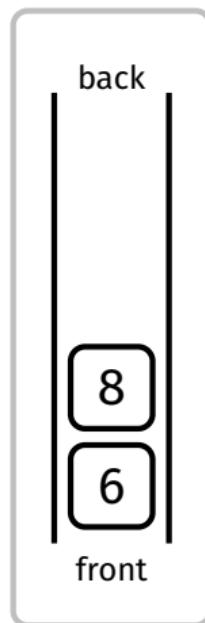
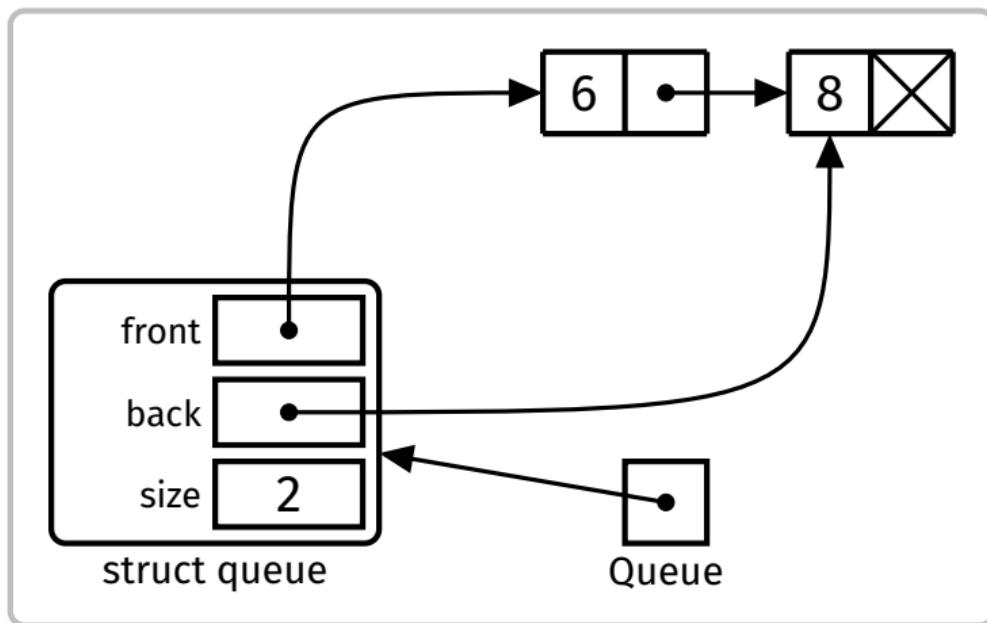
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Cost of enqueue:

- Inserting at the end of the linked list is $O(1)$

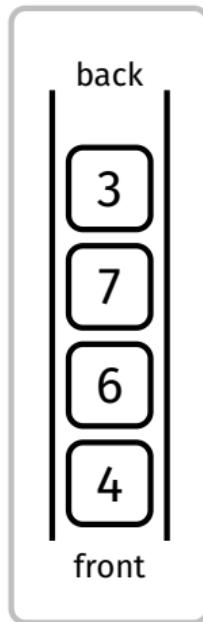
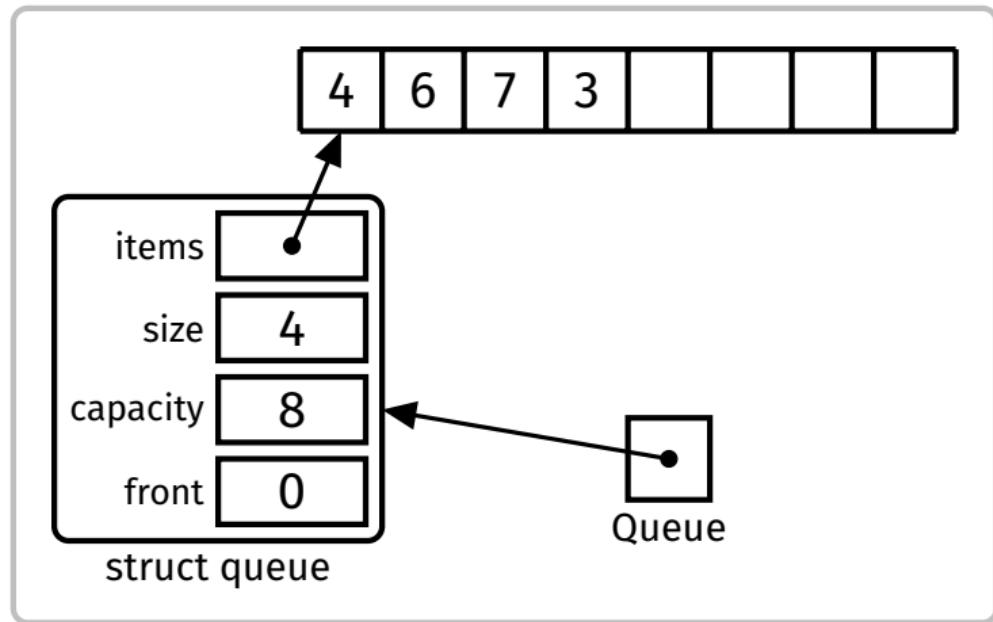
Cost of dequeue:

- Removing from the beginning of the linked list is $O(1)$

Dynamically allocate an array with an initial capacity

Maintain an index to the front of the queue

Maintain a counter of the number of items in the queue

Conceptual
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Example

Perform the following operations:

ENQ(9), ENQ(2), ENQ(6), DEQ, DEQ, ENQ(8)

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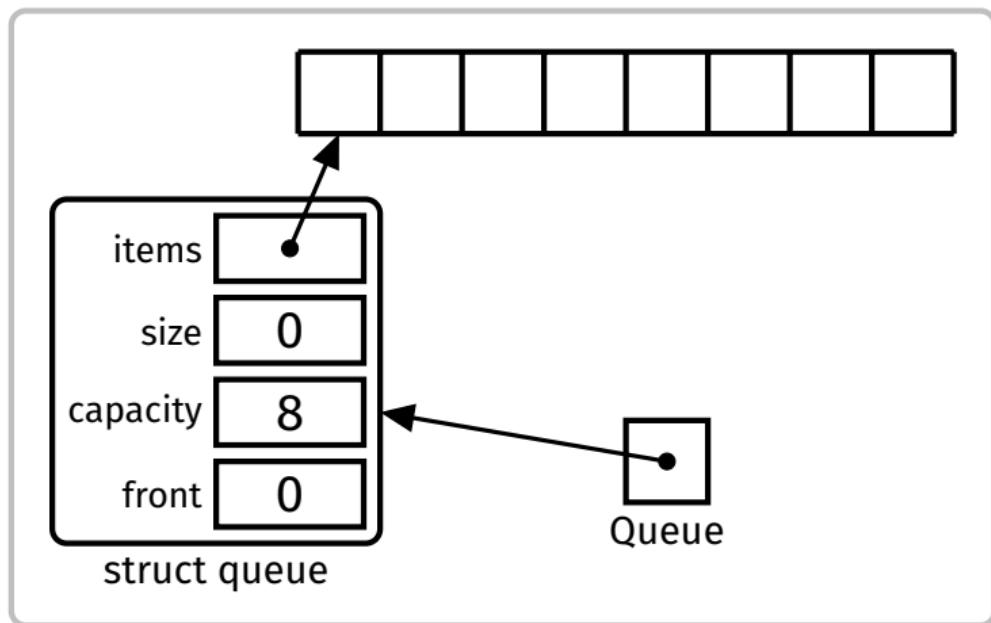
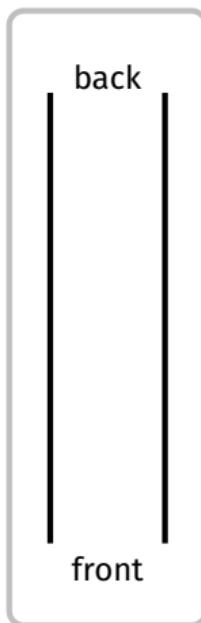
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ENQ(9)

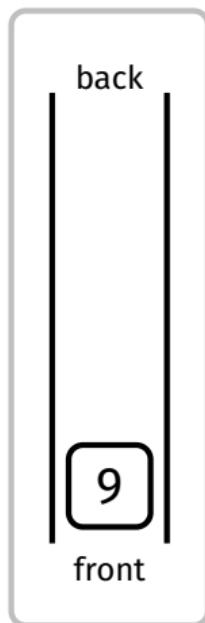
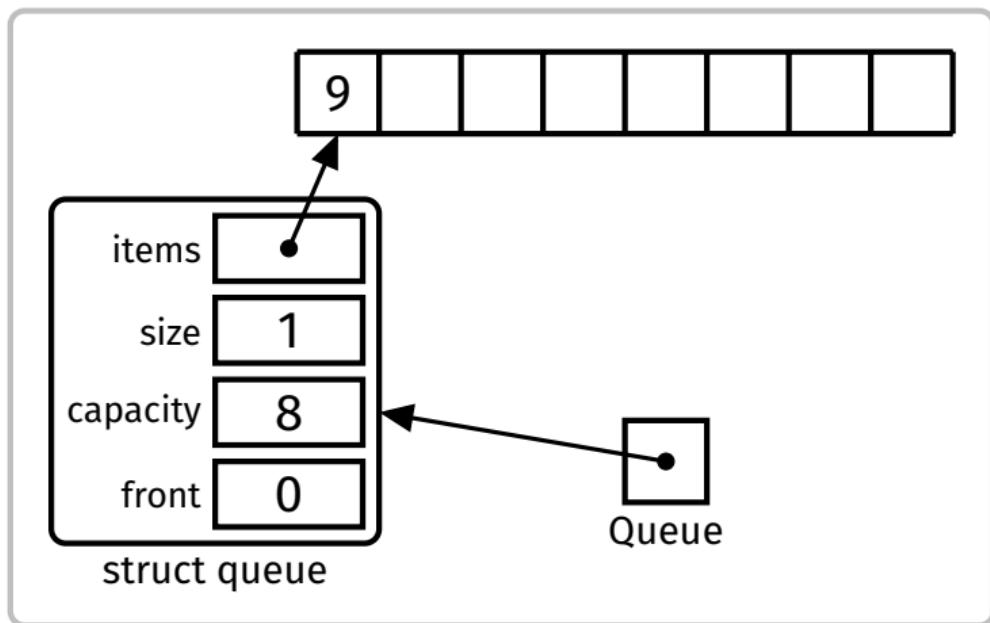
ENQ(2)

ENQ(6)

DEQ

DEQ

ENQ(8)

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ENQ(9)

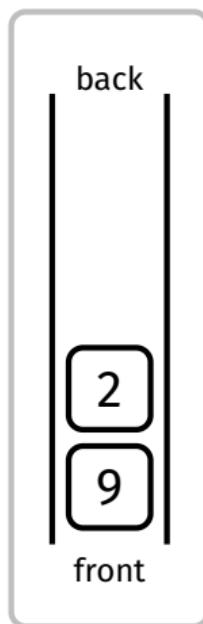
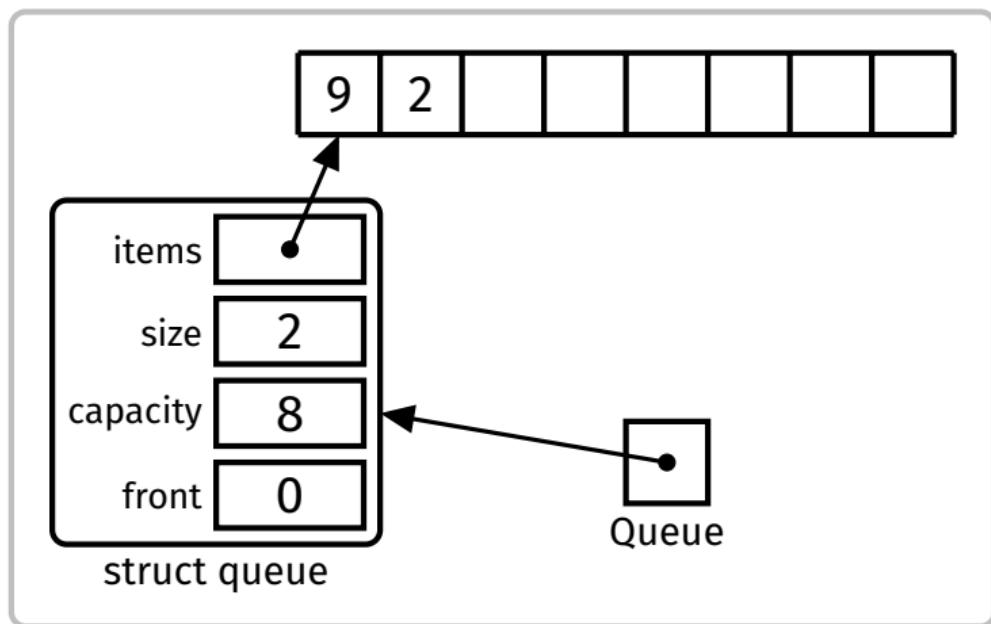
ENQ(2)

ENQ(6)

DEQ

DEQ

ENQ(8)

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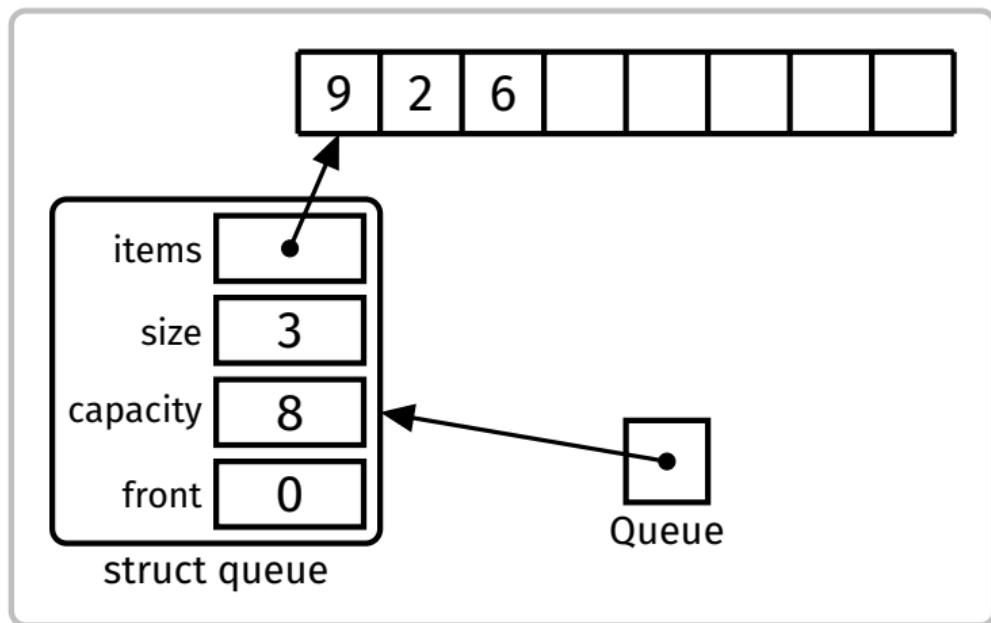
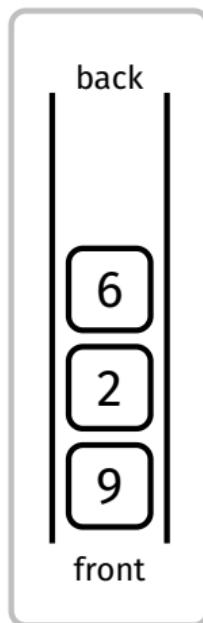
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ENQ(9) ENQ(2) ENQ(6) DEQ DEQ ENQ(8)

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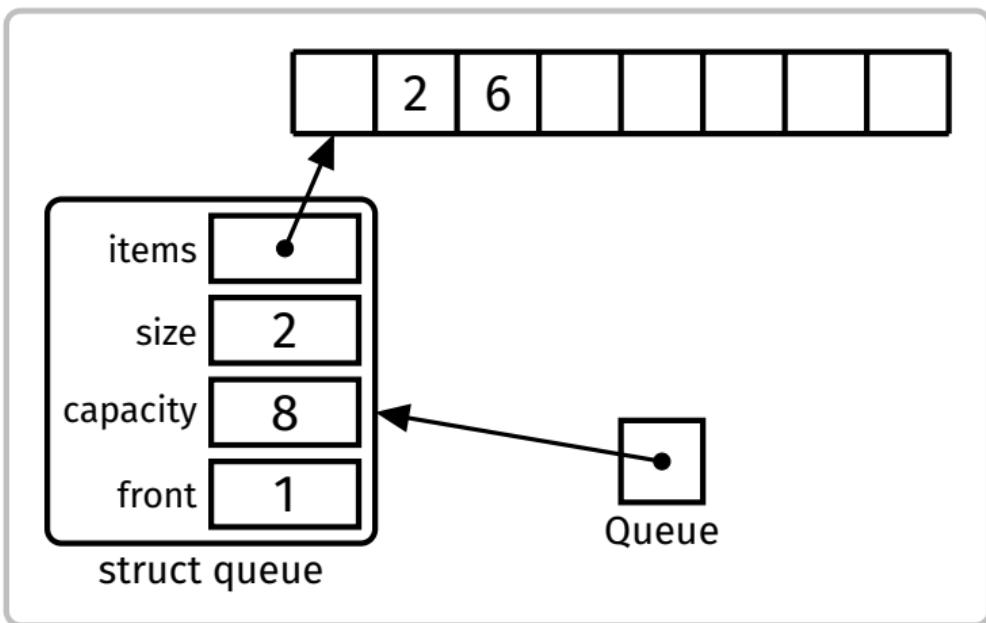
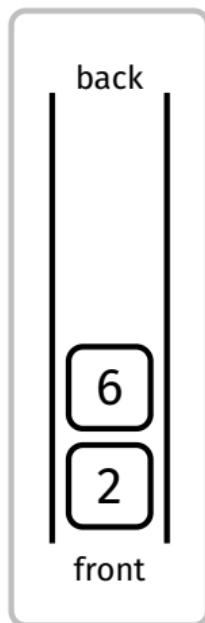
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ENQ(9) ENQ(2) ENQ(6) DEQ \Rightarrow 9 DEQ ENQ(8)

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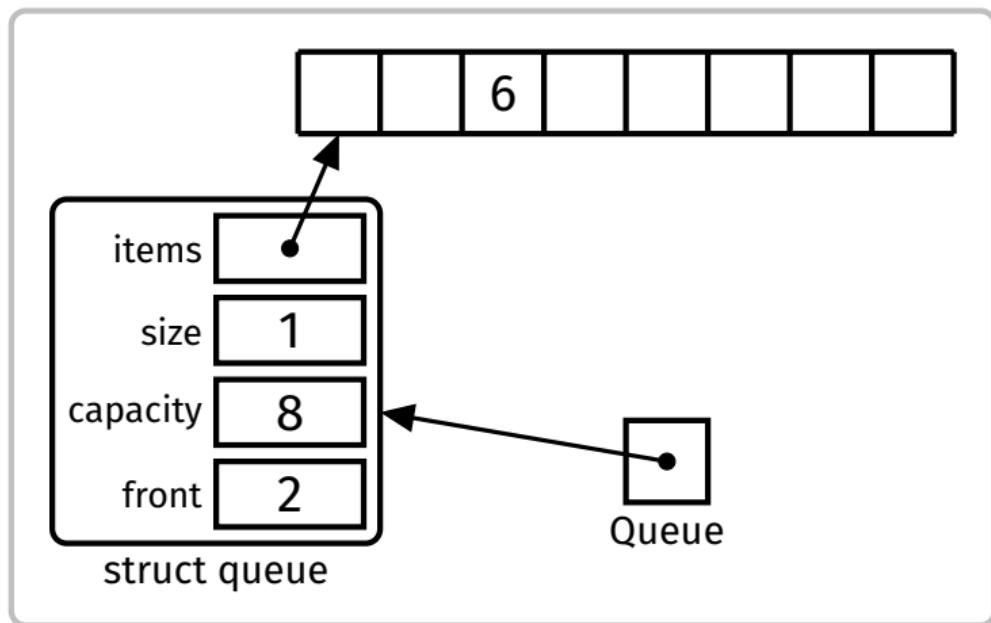
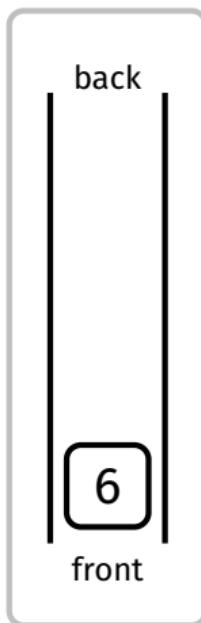
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ENQ(9) ENQ(2) ENQ(6) DEQ \Rightarrow 9 DEQ \Rightarrow 2 ENQ(8)Conceptual
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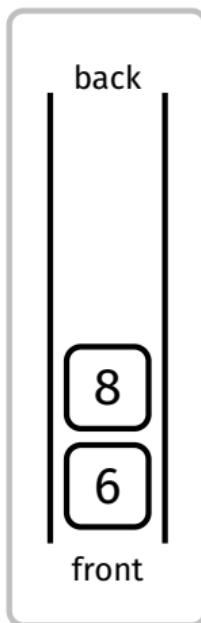
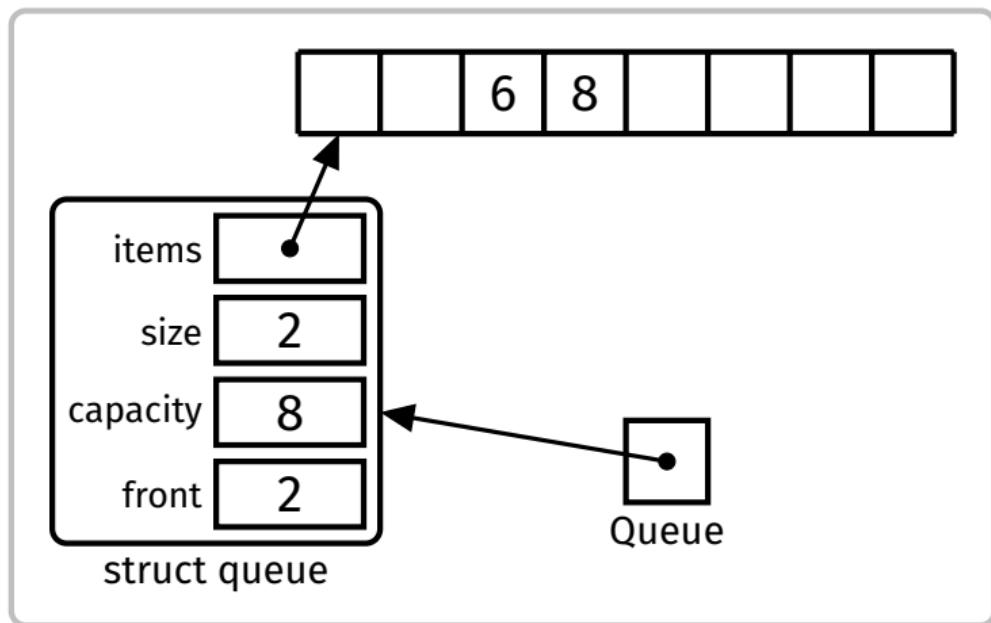
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Cost of enqueue:

- Enqueue involves calculating insertion index and inserting item at that index $\Rightarrow O(1)$

Cost of dequeue:

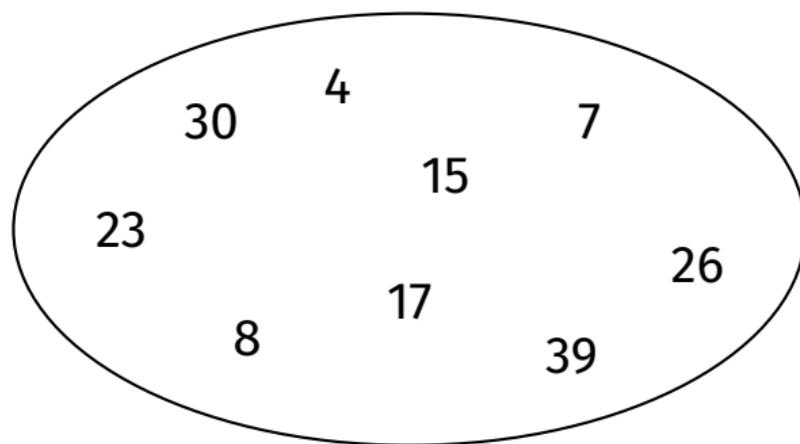
- Dequeue involves accessing item at index front $\Rightarrow O(1)$

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Summary

A set is an unordered collection of distinct elements.



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Basic set operations:

- Create an empty set
- Insert an item into the set
- Delete an item from the set
- Check if an item is in the set
- Get the size of the set
- Display the set

```
#include <stdbool.h>

typedef struct set *Set;

/** Creates a new empty set */
Set SetNew(void);

/** Free memory used by set */
void SetFree(Set set);

/** Inserts an item into the set */
void SetInsert(Set set, int item);

/** Deletes an item from the set */
void SetDelete(Set set, int item);

/** Checks if an item is in the set */
bool SetContains(Set set, int item);

/** Returns the size of the set */
int SetSize(Set set);

/** Displays the set */
void SetShow(Set set);
```

Counting and displaying distinct numbers:

```
#include <stdio.h>

#include "Set.h"

int main(void) {
    Set s = SetNew();

    int val;
    while (scanf("%d", &val) == 1) {
        SetInsert(s, val);
    }

    printf("Number of distinct values: %d\n", SetSize(s));
    printf("Values: ");
    SetShow(s);

    SetFree(s);
}
```

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Different ways to implement a set:

- Unordered array
- Ordered array
- Ordered linked list

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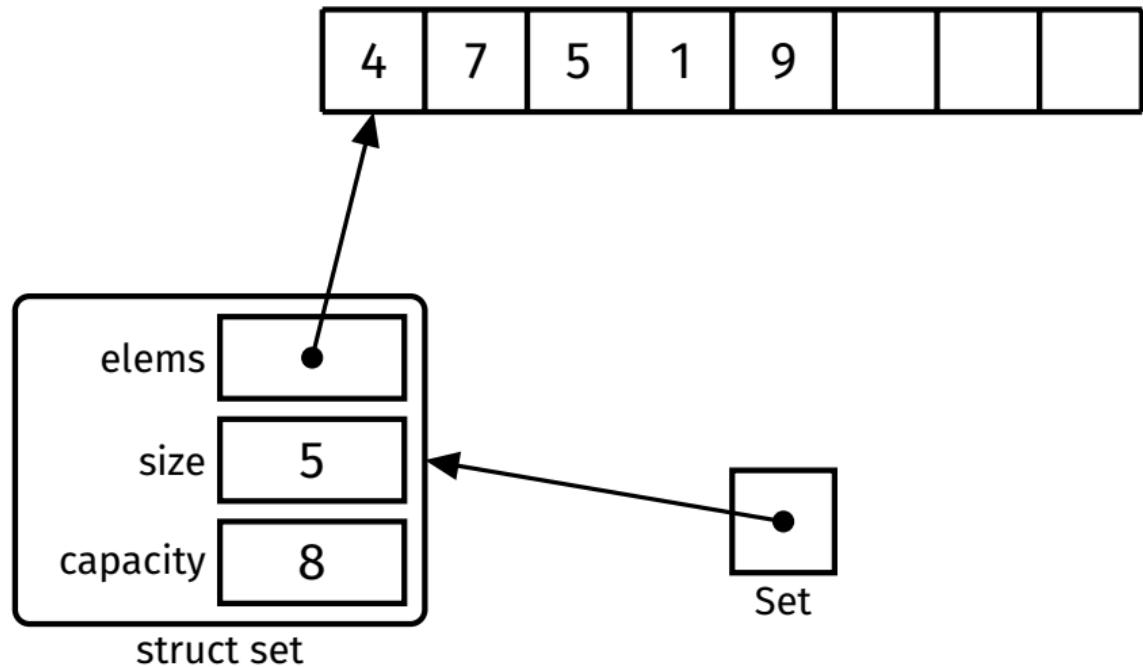
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Summary

How do we check if an element exists?

- Perform linear scan of array $\Rightarrow O(n)$

```
bool SetContains(Set s, int elem) {
    for (int i = 0; i < s->size; i++) {
        if (s->elems[i] == elem) {
            return true;
        }
    }

    return false;
}
```

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Summary

How do we insert an element?

- If the element doesn't exist, insert it after the last element

```
void SetInsert(Set s, int elem) {
    if (SetContains(s, elem)) {
        return;
    }

    if (s->size == s->capacity) {
        // error message
    }

    s->elems[s->size] = elem;
    s->size++;
}
```

Time complexity: $O(n)$

- SetContains is $O(n)$ and inserting after the last element is $O(1)$

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Summary

How do we delete an element?

- If the element exists, overwrite it with the last element

```
void SetDelete(Set s, int elem) {
    for (int i = 0; i < s->size; i++) {
        if (s->elems[i] == elem) {
            s->elems[i] = s->elems[s->size - 1];
            s->size--;
            return;
        }
    }
}
```

Time complexity: $O(n)$

- Finding the element is $O(n)$, overwriting it with the last element is $O(1)$

Abstraction

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Interface

Example Usage

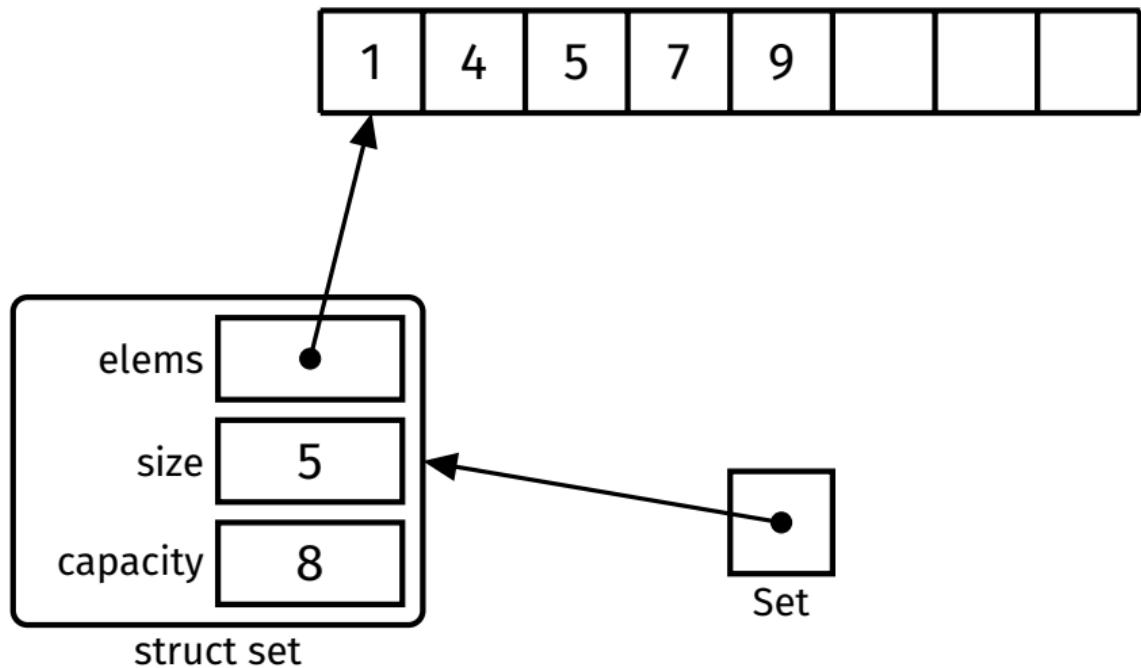
Implementation

Unordered array

Ordered array

Linked list

Summary



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Summary

How do we check if an element exists?

- Perform binary search $\Rightarrow O(\log n)$

```
bool SetContains(Set s, int elem) {
    int lo = 0;
    int hi = s->size - 1;

    while (lo <= hi) {
        int mid = (lo + hi) / 2;
        if (elem < s->elems[mid]) {
            hi = mid - 1;
        } else if (elem > s->elems[mid]) {
            lo = mid + 1;
        } else {
            return true;
        }
    }

    return false;
}
```

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Unordered array

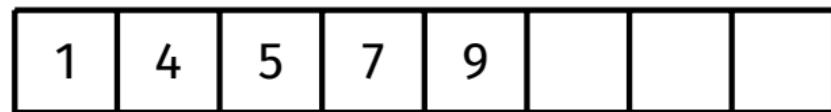
Ordered array

Linked list

Summary

How do we insert an element?

Insert 2



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Example Usage

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Ordered array

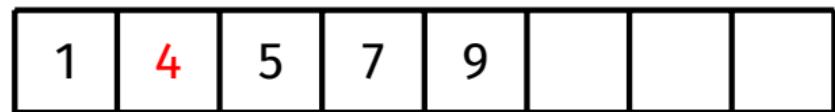
Linked list

Summary

How do we insert an element?

- Use binary search to find the index of the smallest element which is *greater than or equal to* the new element

Insert 2



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Unordered array

Ordered array

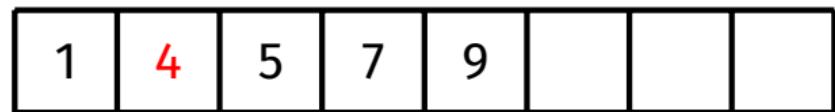
Linked list

Summary

How do we insert an element?

- Use binary search to find the index of the smallest element which is *greater than or equal to* the new element
- If this element is equal to the new element, then it already exists, so no need to do anything

Insert 2



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Example Usage

Implementation

Unordered array

Ordered array

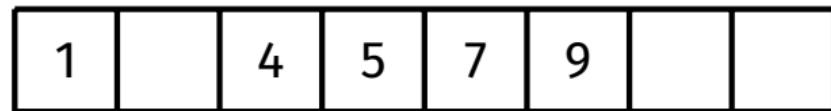
Linked list

Summary

How do we insert an element?

- Use binary search to find the index of the smallest element which is *greater than or equal to* the new element
- If this element is equal to the new element, then it already exists, so no need to do anything
- Otherwise, shift the element and everything greater than it up, and then insert the new element at that index

Insert 2



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Summary

How do we insert an element?

- Use binary search to find the index of the smallest element which is *greater than or equal to* the new element
- If this element is equal to the new element, then it already exists, so no need to do anything
- Otherwise, shift the element and everything greater than it up, and then insert the new element at that index

Insert 2

1	2	4	5	7	9		
---	---	---	---	---	---	--	--

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Summary

Time complexity of insertion?

- Binary search lets us find the insertion point in $O(\log n)$ time
- ...but we still have to potentially shift up to n elements, which is $O(n)$

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Unordered array

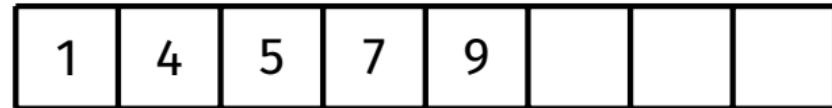
Ordered array

Linked list

Summary

How do we delete an element?

Delete 4



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Ordered array

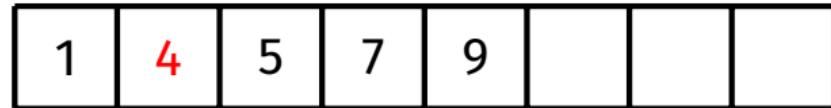
Linked list

Summary

How do we delete an element?

- Use binary search to find the element

Delete 4



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Unordered array

Ordered array

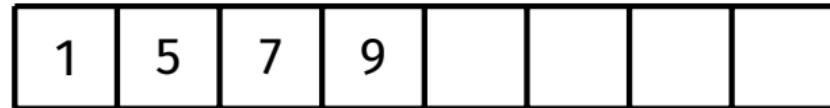
Linked list

Summary

How do we delete an element?

- Use binary search to find the element
- If the element exists, shift everything greater than it down

Delete 4



Time complexity of deletion?

- Binary search lets us find the element in $O(\log n)$ time
- ...but we still have to potentially shift up to $n - 1$ elements, which is $O(n)$

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Example Usage

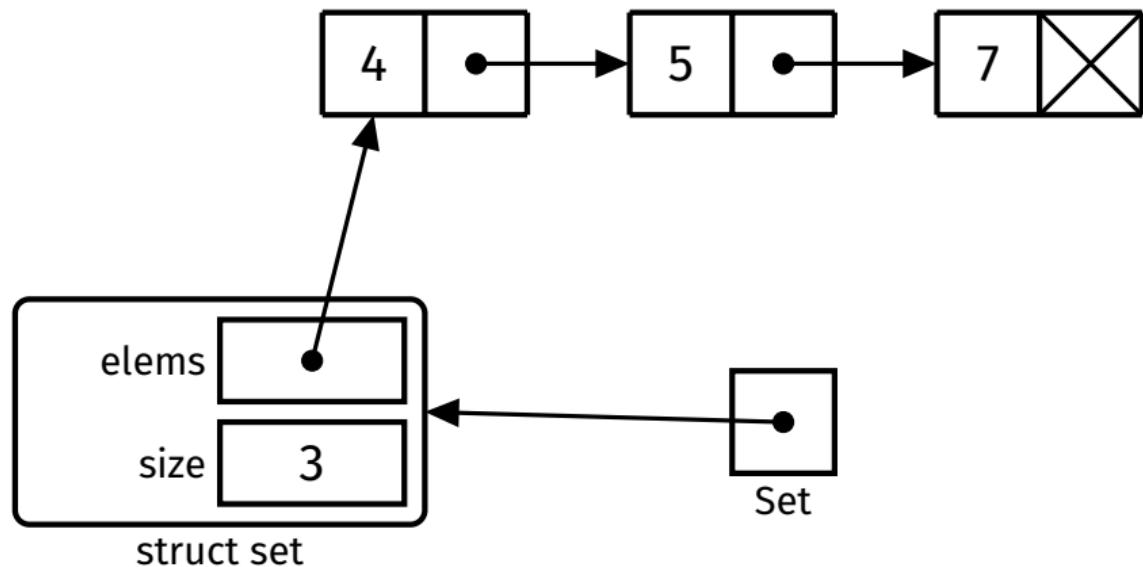
Implementation

Unordered array

Ordered array

Linked list

Summary



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Summary

How do we check if an element exists?

- Traverse the list $\Rightarrow O(n)$

```
bool SetContains(Set s, int elem) {
    for (struct node *curr = s->elems; curr != NULL; curr = curr->next) {
        if (curr->elem == elem) {
            return true;
        }
    }

    return false;
}
```

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Linked list

Summary

We always have to traverse the list from the start. Therefore...

- Insertion and deletion are also $O(n)$

However, this analysis hides a crucial advantage of linked lists:

- Finding the insertion/deletion point is $O(n)$
- But inserting/deleting a node is $O(1)$, as no shifting is required

Data Structure	Contains	Insert	Delete
Unordered array	$O(n)$	$O(n)$	$O(n)$
Ordered array	$O(\log n)$	$O(n)$	$O(n)$
Ordered linked list	$O(n)$	$O(n)$	$O(n)$