

COMP2521 25T3

Graphs (II)

Graph Traversal

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bfs and dfs
path checking
path finding

Common problems on graphs:

- Is there a path between two vertices?
- What is the shortest path between two vertices?
- Which vertices are reachable from a particular vertex?
- Is the graph connected?
- Is there a cycle?
- How many connected components are there?
- Is there a simple path/cycle that passes through all vertices?

All of the above problems can be solved by
a **systematic exploration** of a graph via its **edges**.

This systematic exploration is called **traversal** or **search**.

Two primary methods for graph traversal/search:

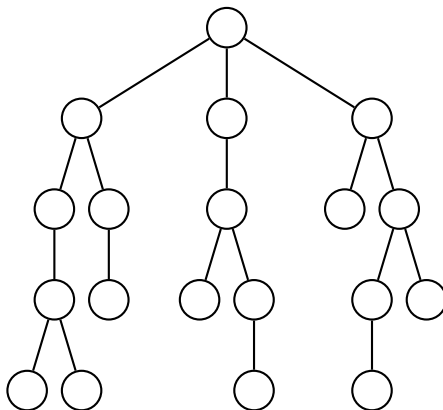
Breadth-first search (BFS)

- Prioritises exploring widely over exploring deeply
 - “Go wide”
- Implemented iteratively (using a queue)

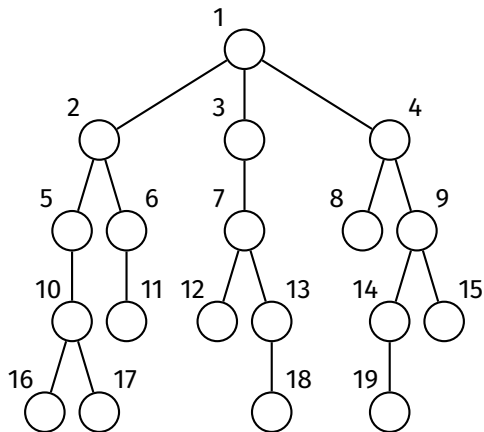
Depth-first search (DFS)

- Prioritises exploring deeply over exploring widely
 - “Go deep”
- Implemented recursively or iteratively (using a stack)

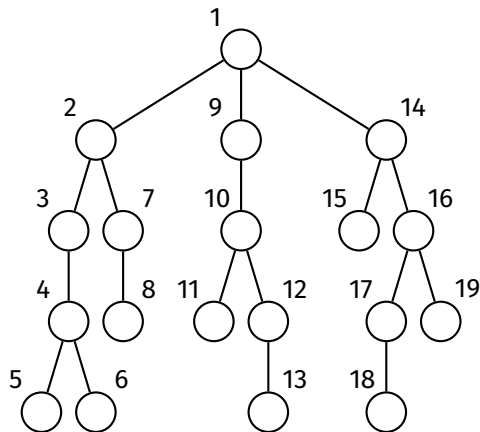
In what order would BFS and DFS visit the nodes of this tree?
(Assume that nodes towards the left have higher priority)



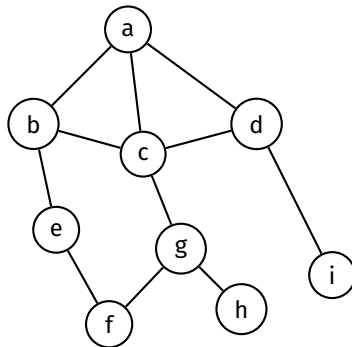
Breadth-first search



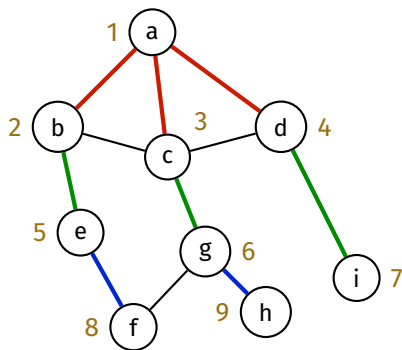
Depth-first search



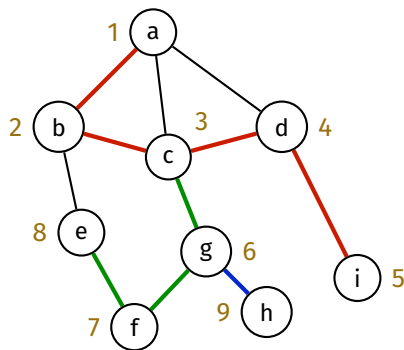
In what order would BFS and DFS visit the vertices of this graph?
(Assume that nodes containing smaller letters have higher priority)



Breadth-first search



Depth-first search



Breadth-first search visits vertices
in order of distance from the starting vertex.

BFS is implemented iteratively using a queue.

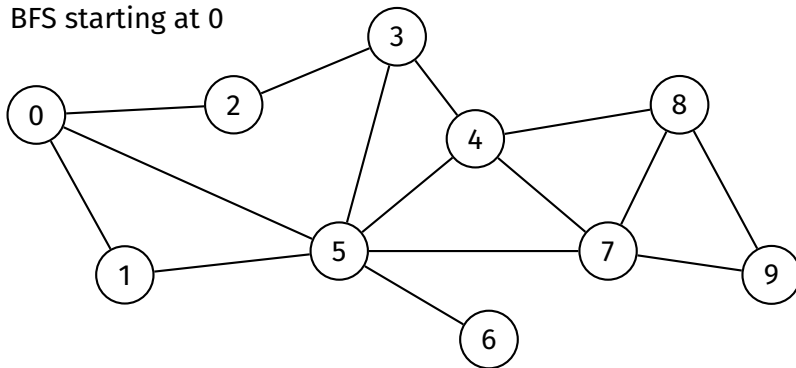
Data structures used in BFS:

- Visited array
 - To keep track of which vertices have been visited
- Predecessor array
 - To keep track of the predecessor of each vertex
 - The predecessor of v is the vertex from which we reached v
 - i.e., the vertex before v on the path to v
- Queue
 - First-in-first-out data structure
 - Stores unvisited vertices in the order that they should be visited

Algorithm:

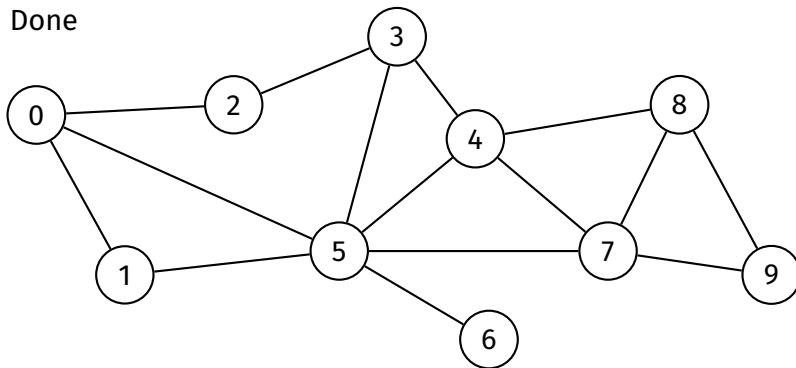
- ➊ Create/initialise data structures:
 - Create visited array, initialised to false
 - Create predecessor array, initialised to -1
 - Create empty queue
- ➋ Mark starting vertex as visited and enqueue it
- ➌ While the queue is not empty:
 - ➊ Dequeue a vertex
 - Let this vertex be v
 - ➋ **Explore** v - that is, for each of v 's unvisited neighbours:
 - ➊ Mark it as visited
 - ➋ Set its predecessor to v
 - ➌ Enqueue it

BFS starting at 0



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
pred	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
queue	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Done



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7
queue	0	1	2	5	3	4	6	7	8	9

```
bfs( $G$ ,  $src$ ):
```

```
    Input: graph  $G$ , starting vertex  $src$ 
```

```
    create visited array, initialised to false
```

```
    create predecessor array, initialised to -1
```

```
    create queue  $Q$ 
```

```
    visited[ $src$ ] = true
```

```
    enqueue  $src$  into  $Q$ 
```

```
    while  $Q$  is not empty:
```

```
         $v$  = dequeue from  $Q$ 
```

```
        for each neighbour  $w$  of  $v$  in  $G$  where visited[ $w$ ] = false:
```

```
            visited[ $w$ ] = true
```

```
            predecessor[ $w$ ] =  $v$ 
```

```
            enqueue  $w$  into  $Q$ 
```

When using a predecessor array in BFS,
the predecessor array can double as a visited array

$\text{predecessor}[v] = -1$ means v is not visited

```
bfs( $G$ ,  $src$ ):
```

```
    Input: graph  $G$ , starting vertex  $src$ 
```

```
    create predecessor array, initialised to -1
```

```
    create queue  $Q$ 
```

```
    predecessor[ $src$ ] =  $src$  // <- mark  $src$  as visited
```

```
    enqueue  $src$  into  $Q$ 
```

```
    while  $Q$  is not empty:
```

```
         $v$  = dequeue from  $Q$ 
```

```
        for each neighbour  $w$  of  $v$  in  $G$  where predecessor[ $w$ ] = -1:
```

```
            predecessor[ $w$ ] =  $v$ 
```

```
            enqueue  $w$  into  $Q$ 
```


BFS is $O(V + E)$ when using the adjacency list representation:

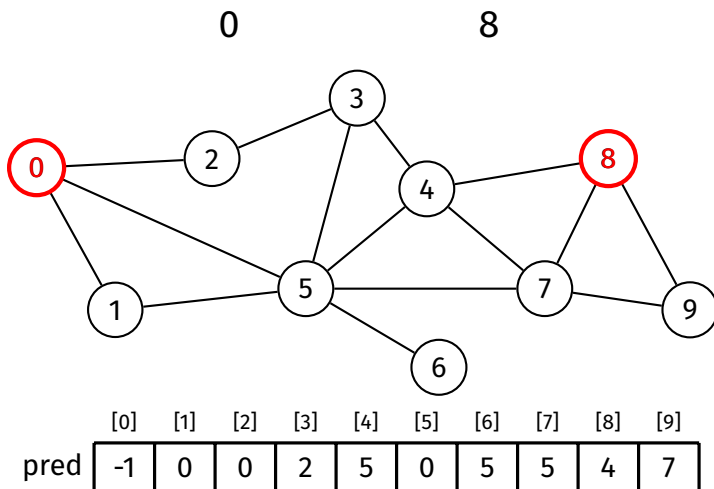
- Typical queue implementation has $O(1)$ enqueue and dequeue
- Each vertex is visited at most once $\Rightarrow O(V)$
- For each vertex, all of its edges are considered once $\Rightarrow O(E)$

A BFS finds the shortest path between the starting vertex and all other vertices.

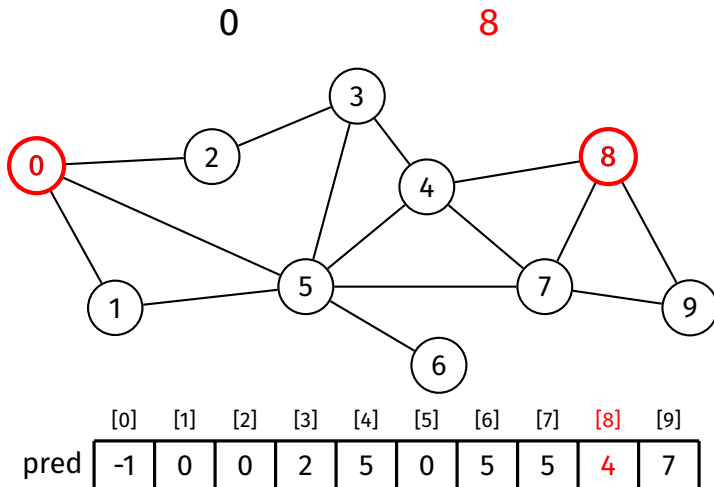
- Shortest path in terms of the number of edges

The shortest path between *src* and *dest* can be found by tracing backwards through the predecessor array (from *dest* to *src*).

Example: Shortest path from 0 to 8



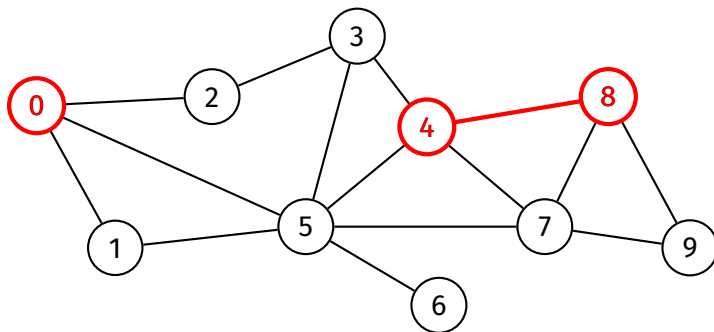
Example: Shortest path from 0 to 8



Example: Shortest path from 0 to 8

0

4 → 8

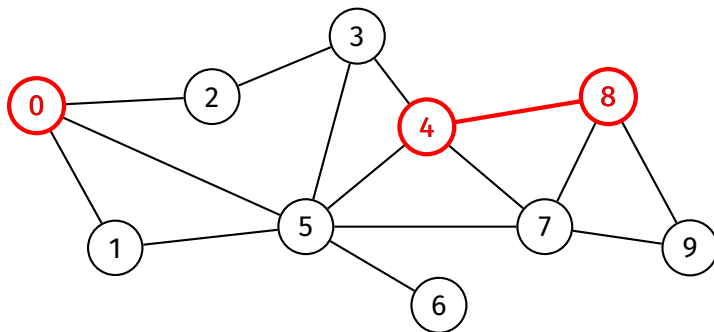


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

Example: Shortest path from 0 to 8

0

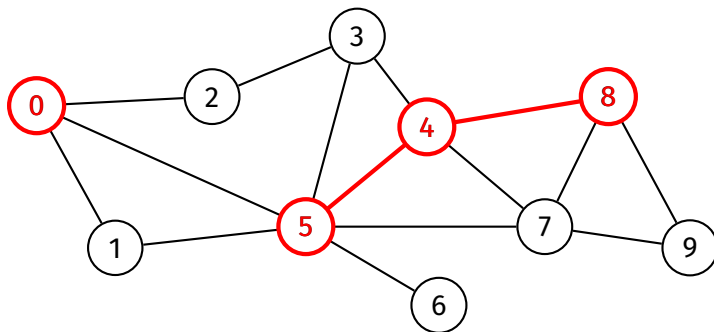
4 → 8



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

Example: Shortest path from 0 to 8

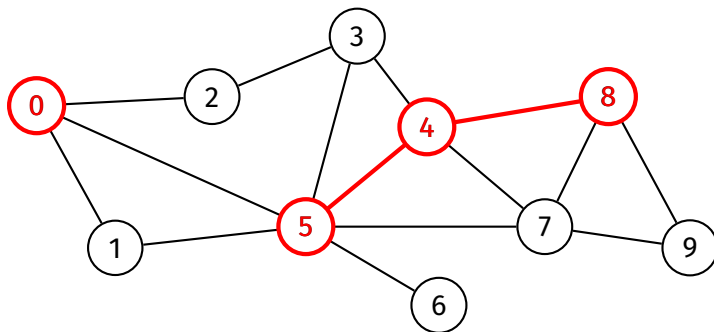
0 5 → 4 → 8



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

Example: Shortest path from 0 to 8

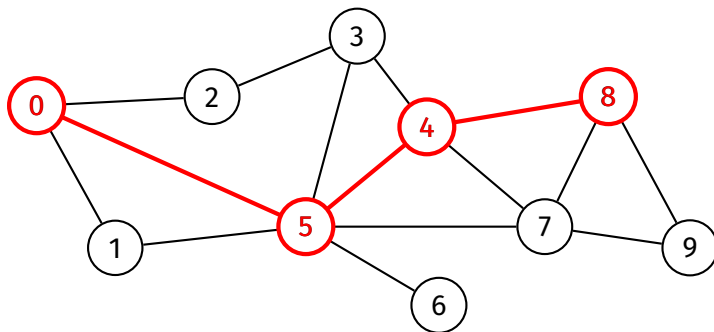
0 5 → 4 → 8



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

Example: Shortest path from 0 to 8

$0 \rightarrow 5 \rightarrow 4 \rightarrow 8$



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

```
bfsFindPath( $G$ ,  $src$ ,  $dest$ ):
```

```
    Input: graph  $G$ , vertices  $src$  and  $dest$ 
```

```
    ... BFS starting from  $src$  ...
```

```
    if predecessor[ $dest$ ]  $\neq$  -1:
```

```
         $v = dest$ 
```

```
        while  $v \neq src$ :
```

```
            print  $v$ , "<-"
```

```
             $v = predecessor[v]$ 
```

```
    print  $src$ 
```

Graph
Traversal

BFS

DFS

Recursive
Iterative

Ideas/Issues

Appendix

Depth-first search goes as far down one path
as possible until it reaches a dead end,
then backtracks until it finds a new path to take,
then repeats

DFS can be implemented recursively or iteratively.

Depth-first search is described recursively as:

- ① Mark current vertex as visited
 - The first time, this is the starting vertex
- ② For each neighbour of the current vertex:
 - If it has not been visited:
 - Recursively traverse starting from that vertex

The recursion naturally induces backtracking.

```
dfs( $G$ ,  $src$ ):
```

```
    Input: graph  $G$ , starting vertex  $src$ 
```

```
    create visited array, initialised to false
```

```
    dfsRec( $G$ ,  $src$ , visited)
```

```
dfsRec( $G$ ,  $v$ , visited):
```

```
    Input: graph  $G$ , vertex  $v$ , visited array
```

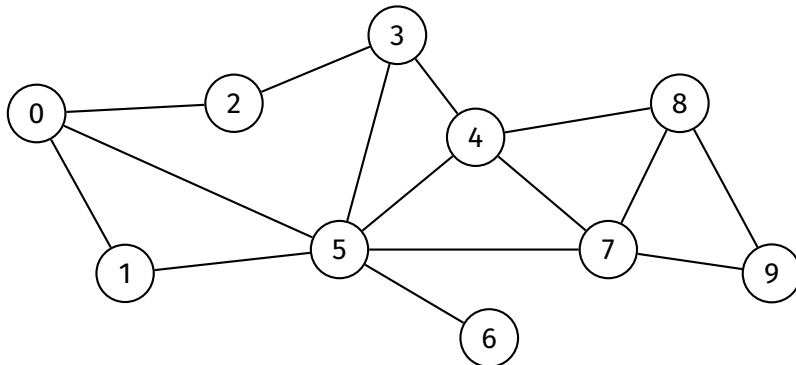
```
    visited[ $v$ ] = true // "visit"  $v$ 
```

```
    for each neighbour  $w$  of  $v$  in  $G$ :
```

```
        if visited[ $w$ ] = false:
```

```
            dfsRec( $G$ ,  $w$ , visited)
```

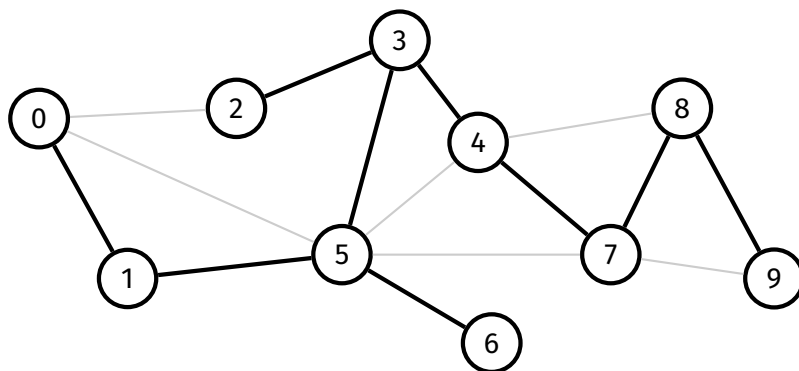
DFS starting at 0



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0

visit order

Done



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

visit order 0 1 5 3 2 4 7 8 9 6

Recursive DFS is $O(V + E)$ when using the adjacency list representation:

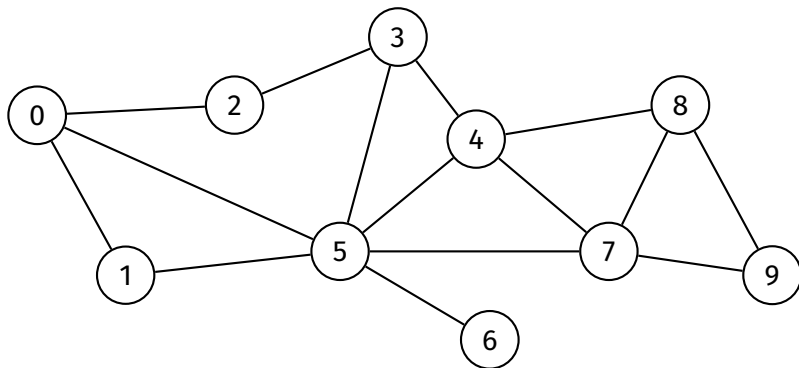
- Each vertex is visited at most once $\Rightarrow O(V)$
 - Function is called on each vertex at most once
- For each vertex, all of its edges are considered once $\Rightarrow O(E)$

Recursive DFS can be adapted to check if a path exists between two vertices.

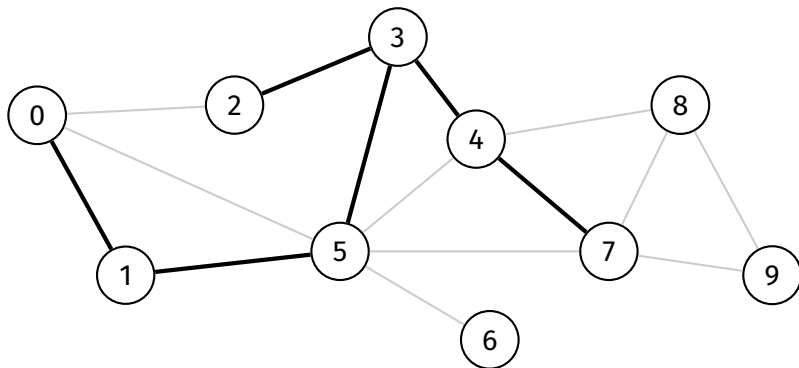
Idea:

- To check if a path exists between src and $dest$:
 - If $src = dest$, then there is a path (the empty path)
 - Otherwise, for each neighbour of src , recursively check if there is a path from that neighbour to $dest$

Does there exist a path between 0 and 7 in this graph?



Answer: Yes



```
dfsHasPath( $G$ ,  $src$ ,  $dest$ ):
```

```
    Input: graph  $G$ , vertices  $src$  and  $dest$ 
```

```
    Output: true if there is a path from  $src$  to  $dest$   
             false otherwise
```

```
    create visited array, initialised to false
```

```
    return dfsHasPathRec( $G$ ,  $src$ ,  $dest$ , visited)
```

```
dfsHasPathRec( $G$ ,  $v$ ,  $dest$ , visited):
```

```
    Input: graph  $G$ , vertices  $v$  and  $dest$ , visited array
```

```
    visited[ $v$ ] = true
```

```
    if  $v = dest$ :
```

```
        return true
```

```
    for each neighbour  $w$  of  $v$  in  $G$ :
```

```
        if visited[ $w$ ] = false:
```

```
            if dfsHasPathRec( $G$ ,  $w$ ,  $dest$ , visited):
```

```
                return true
```

```
    return false
```

Graph
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BFS

DFS

Recursive
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Analysis**Path checking**

Path finding

Iterative

Ideas/Issues

Appendix

$O(V + E)$ when using the adjacency list representation:

- Algorithm is just a modified recursive DFS with return statements

How to get the path?

Idea:

- Record the predecessor of each vertex during the DFS
- Trace backwards through the path after the DFS

```
dfsFindPath( $G$ ,  $src$ ,  $dest$ ):
```

Input: graph G , vertices src and $dest$

create predecessor array, initialised to -1

```
predecessor[ $src$ ] =  $src$ 
```

```
if dfsFindPathRec( $G$ ,  $src$ ,  $dest$ , predecessor):
```

```
     $v$  =  $dest$ 
```

```
    while  $v \neq src$ :
```

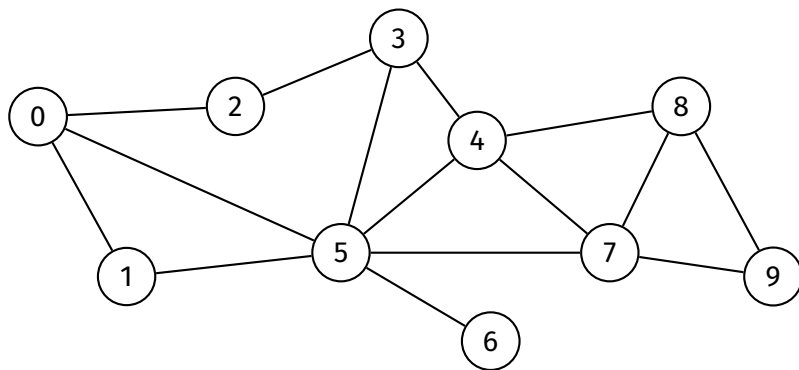
```
        print  $v$ , "<-"
```

```
         $v$  = predecessor[ $v$ ]
```

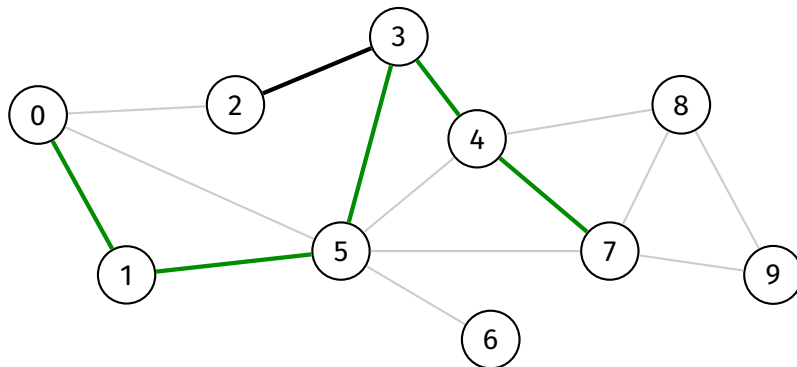
```
    print  $src$ 
```

```
dfsFindPathRec( $G$ ,  $v$ ,  $dest$ , predecessor):  
    if  $v = dest$ :  
        return true  
  
    for each neighbour  $w$  of  $v$  in  $G$ :  
        if predecessor[ $w$ ] = -1:  
            predecessor[ $w$ ] =  $v$   
            if dfsFindPathRec( $G$ ,  $w$ ,  $dest$ , predecessor):  
                return true  
  
    return false
```


Find a path from 0 to 7



Path found:



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	0	0	3	5	3	1	-1	4	-1	-1

Clearly, DFS is not guaranteed to find the shortest path.

DFS can be implemented iteratively.

Iterative DFS is similar to BFS, but there are a few crucial differences:

- DFS uses a stack instead of a queue
- DFS marks a vertex as visited after removing it from the stack, not when adding it (which is what BFS does, but with a queue)

```
dfs( $G$ ,  $src$ ):
```

```
    Input: graph  $G$ , vertex  $src$ 
```

```
    create visited array, initialised to false
```

```
    create predecessor array, initialised to -1
```

```
    create stack  $S$ 
```

```
    push  $src$  onto  $S$ 
```

```
    while  $S$  is not empty:
```

```
         $v$  = pop from  $S$ 
```

```
        if visited[ $v$ ] = true:
```

```
            continue // i.e., return to start of loop
```

```
        visited[ $v$ ] = true
```

```
        for each neighbour  $w$  of  $v$  in  $G$  where visited[ $w$ ] = false:
```

```
            predecessor[ $w$ ] =  $v$ 
```

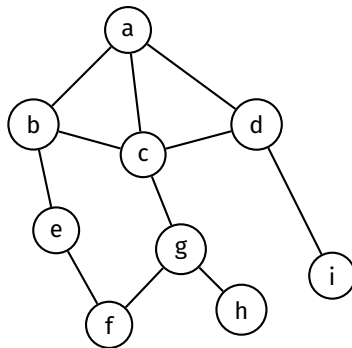
```
            push  $w$  onto  $S$ 
```

Iterative DFS is $O(V + E)$ when using the adjacency list representation.

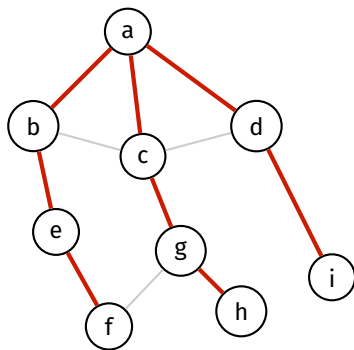
- Typical stack implementation has $O(1)$ push and pop
- Each vertex visited at most once $\Rightarrow O(V)$
- For each vertex, all of its edges are considered $\Rightarrow O(E)$

The edges traversed in a graph traversal form a **spanning tree**.

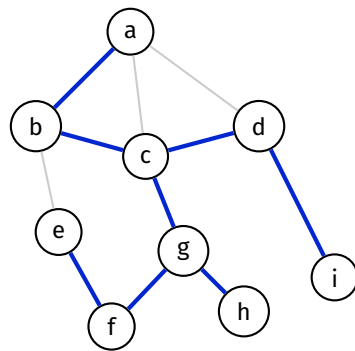
Consider the following graph:



A traversal starting at vertex 'a' forms the following spanning trees:

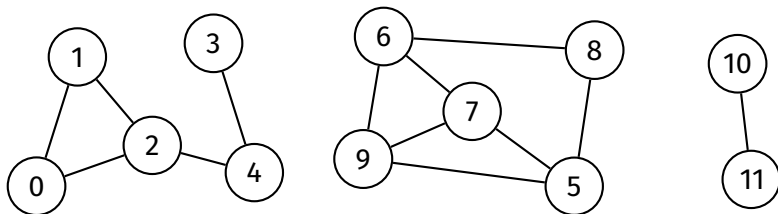


Breadth-first search



Depth-first search

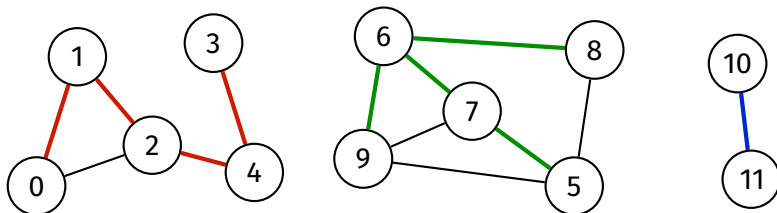
If a graph is not connected,
a graph traversal starting from a given vertex
will not traverse the entire graph



Solution

After initial traversal is complete,
perform traversal again on an unvisited vertex,
repeat until all vertices are visited

This produces a **spanning forest**



```
dfs( $G$ ):  
    Input: graph  $G$   
  
    create predecessor array, initialised to -1  
  
    for each vertex  $v$  in  $G$ :  
        if predecessor[ $v$ ] = -1:  
            dfsRec( $G$ ,  $v$ , predecessor)  
  
    ...
```

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BFS

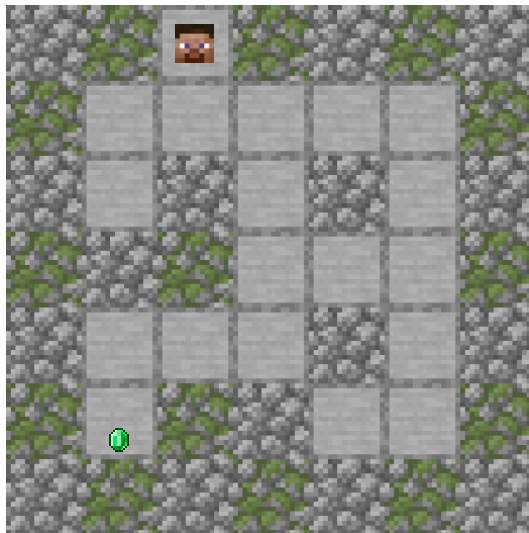
BFS Example

DFS

DFS Example

Path-Checking

Example



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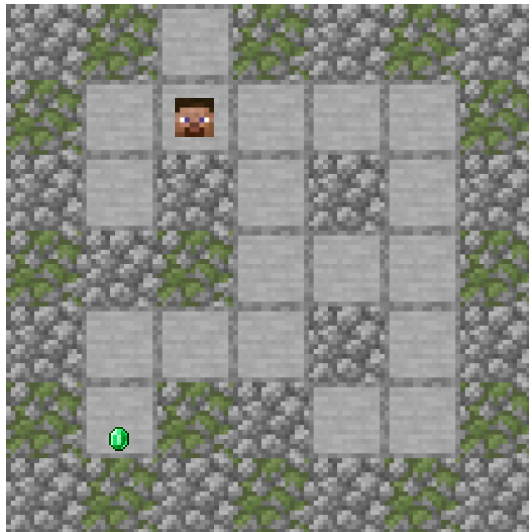
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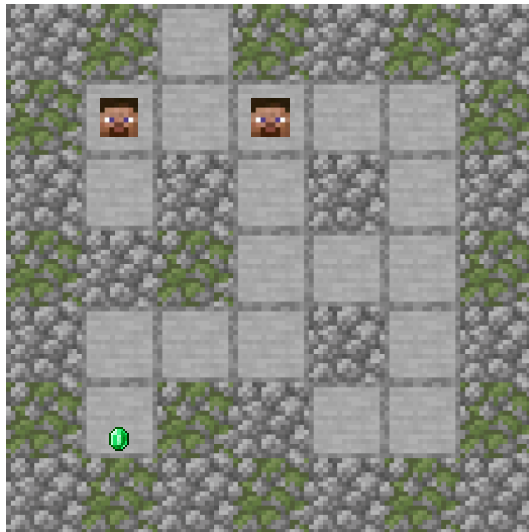
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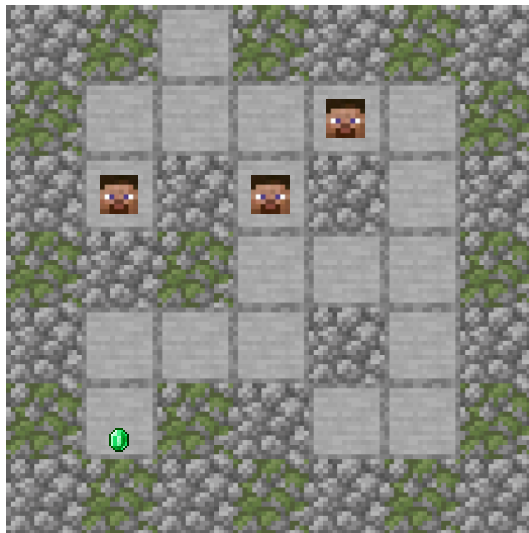
BFS Example

DFS

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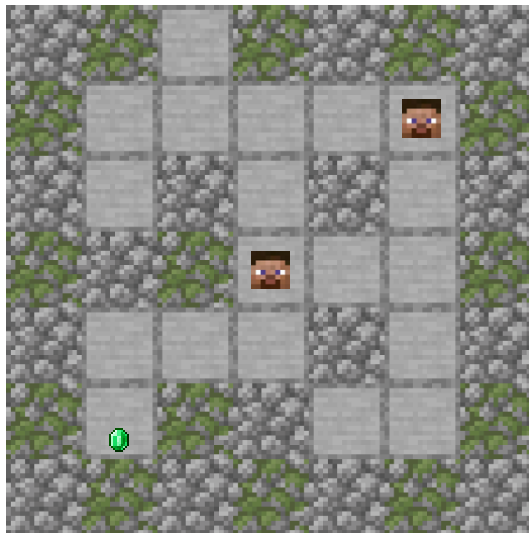
BFS Example

DFS

DFS Example

Path-Checking

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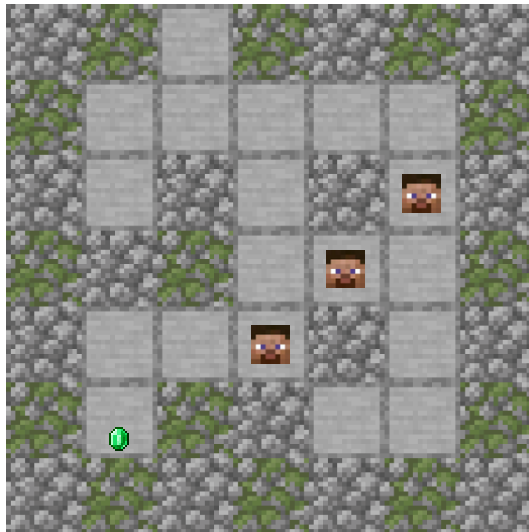
BFS Example

DFS

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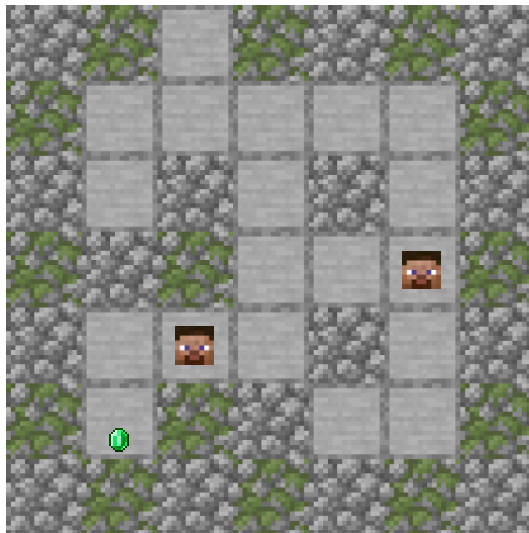
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DFS Example

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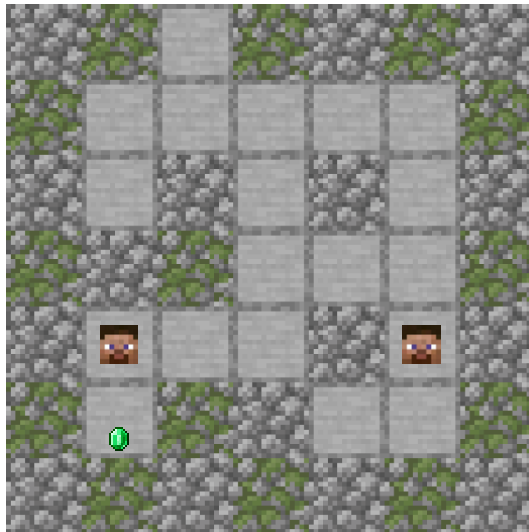
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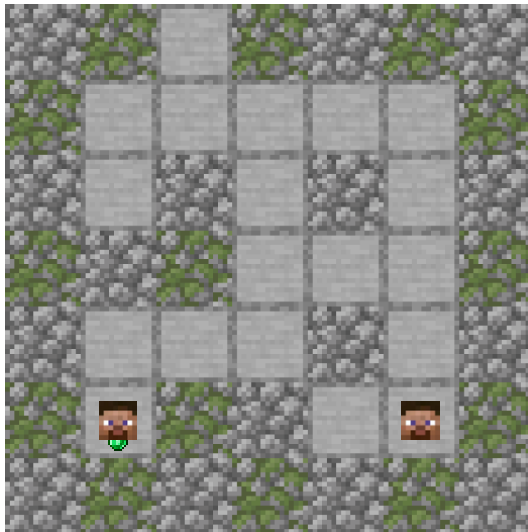
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How do we avoid revisiting the same tiles?

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How do we avoid revisiting the same tiles?

Mark tiles as they are visited!

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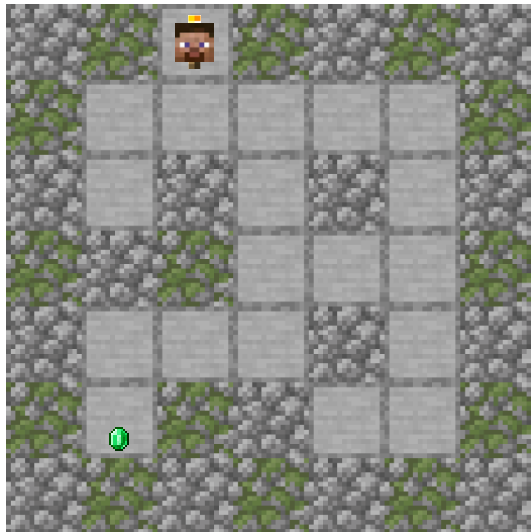
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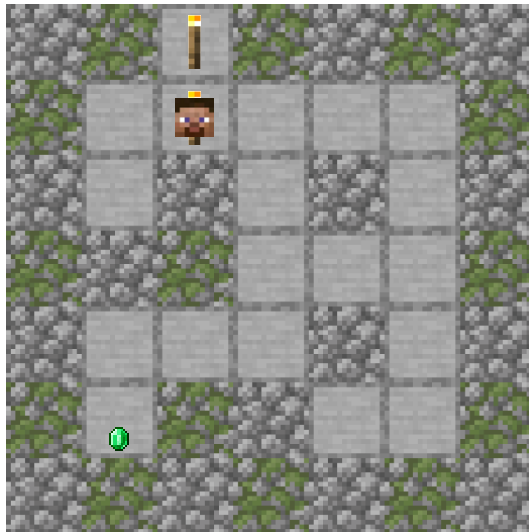
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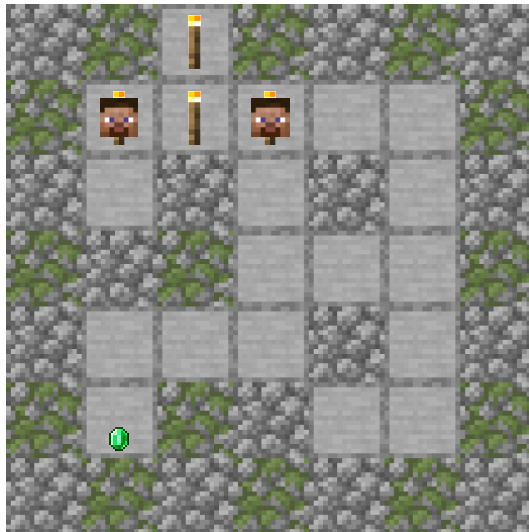
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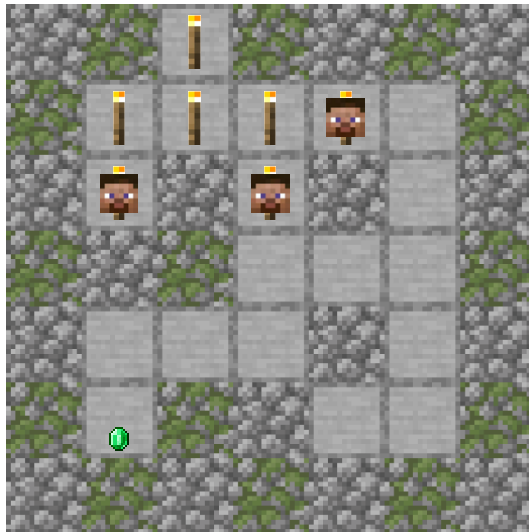
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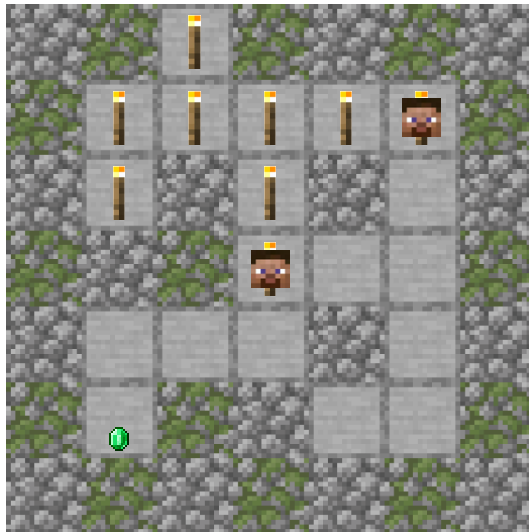
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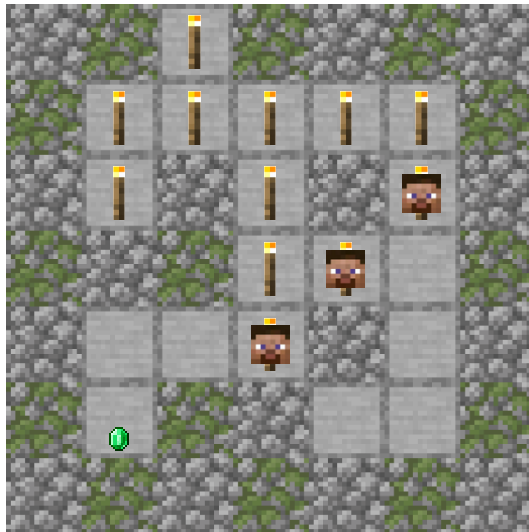
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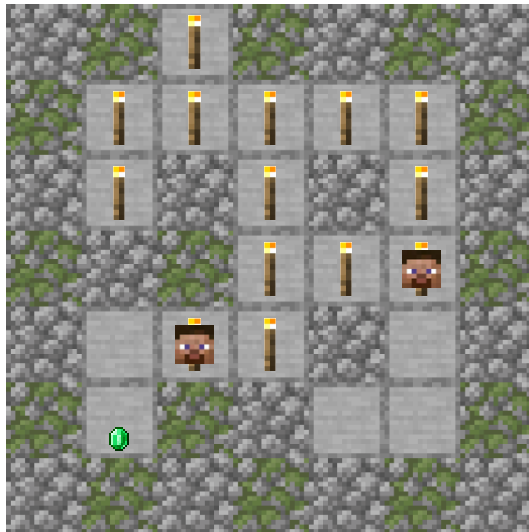
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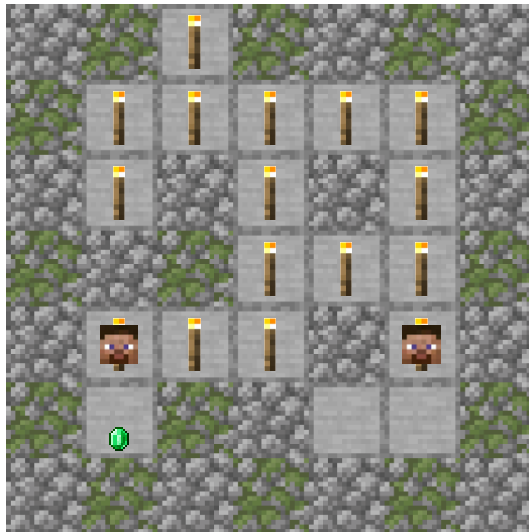
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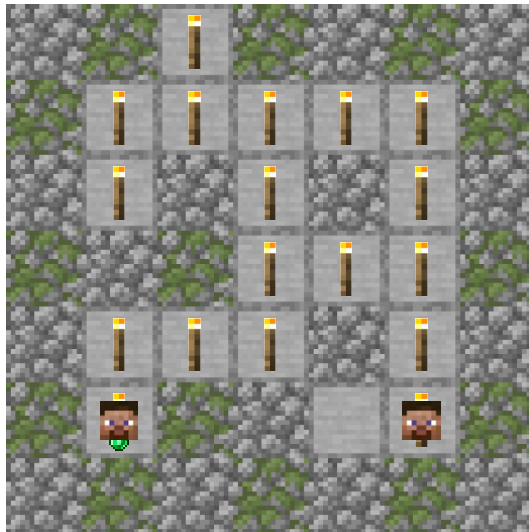
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How do we find our way back to the entrance?

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How do we find our way back to the entrance?

For each tile that we visit,
keep note of the tile we were on directly before it!
This is called the *predecessor*.

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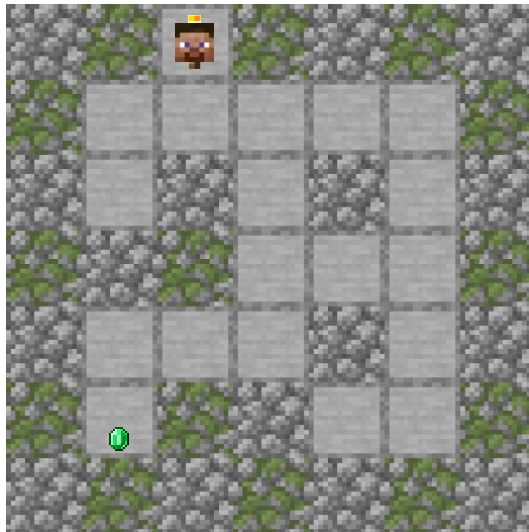
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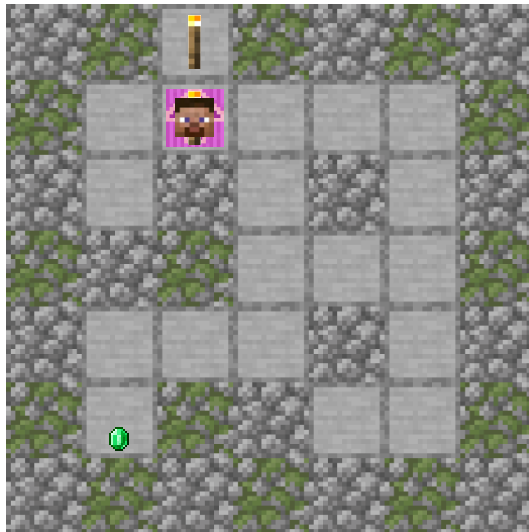
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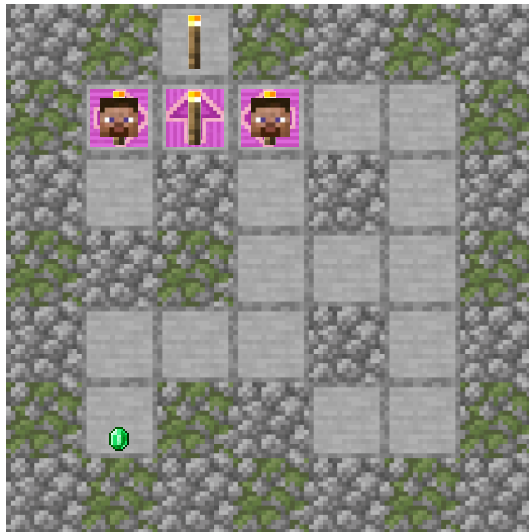
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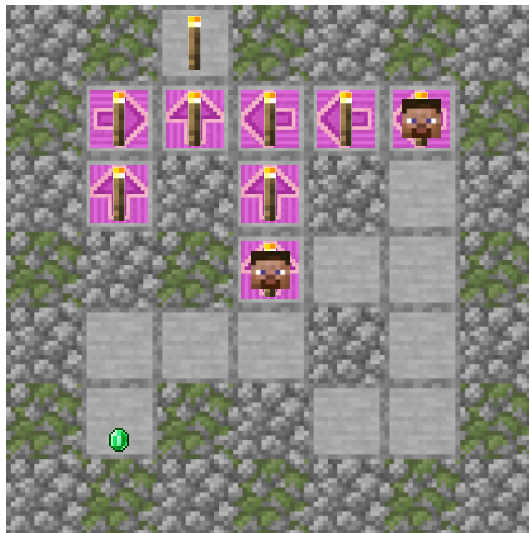
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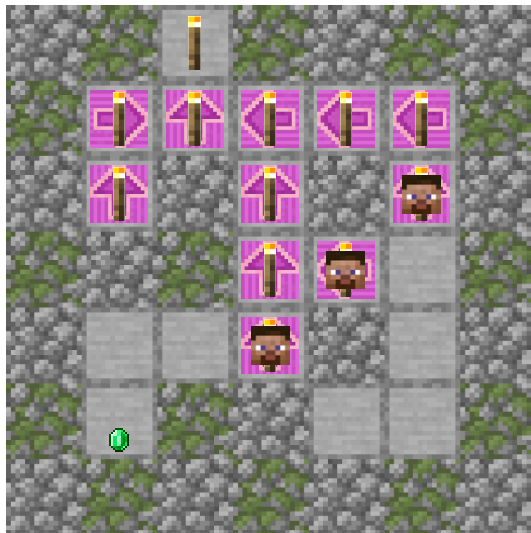
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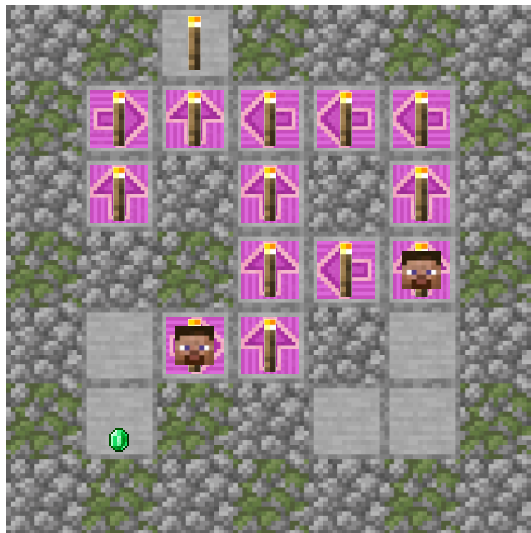
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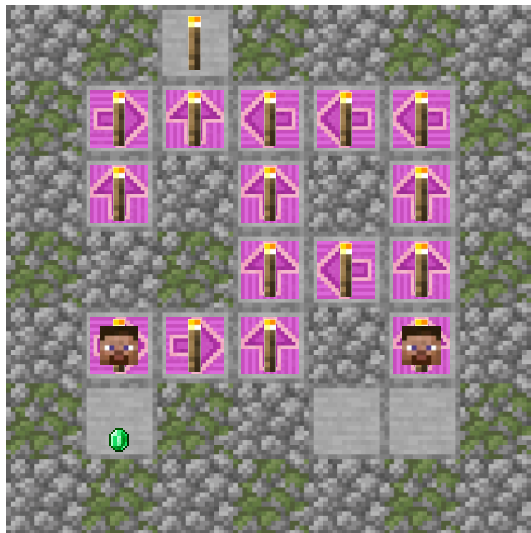
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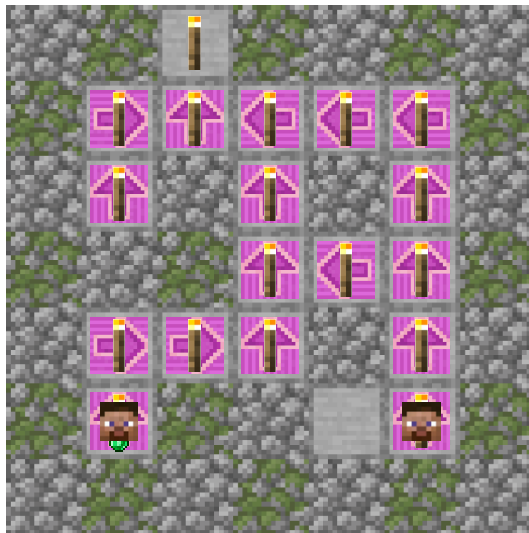
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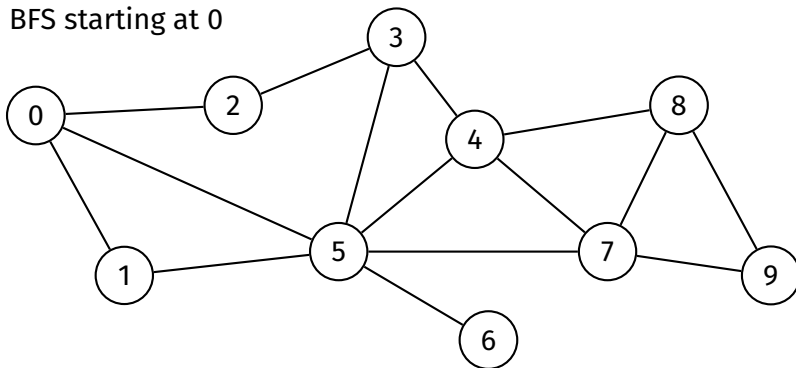
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BFS starting at 0

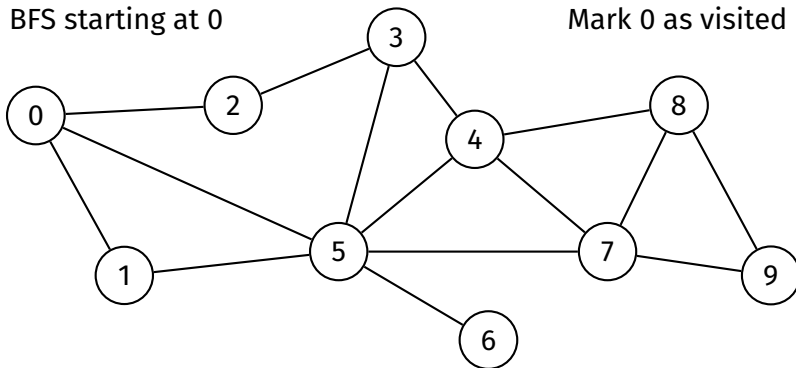


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0
pred	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

queue

BFS starting at 0

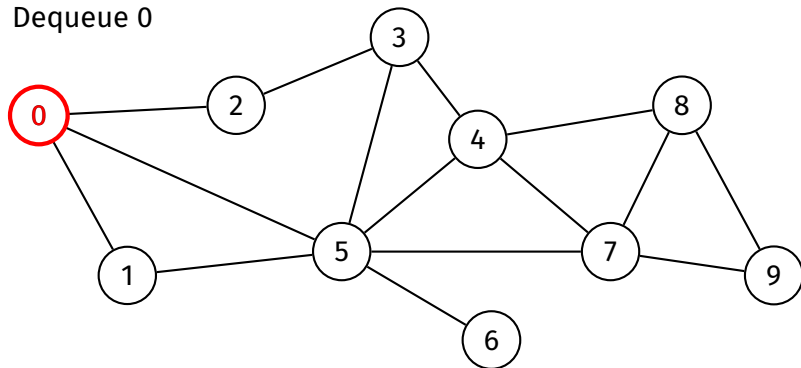
Mark 0 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0
pred	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

queue 0

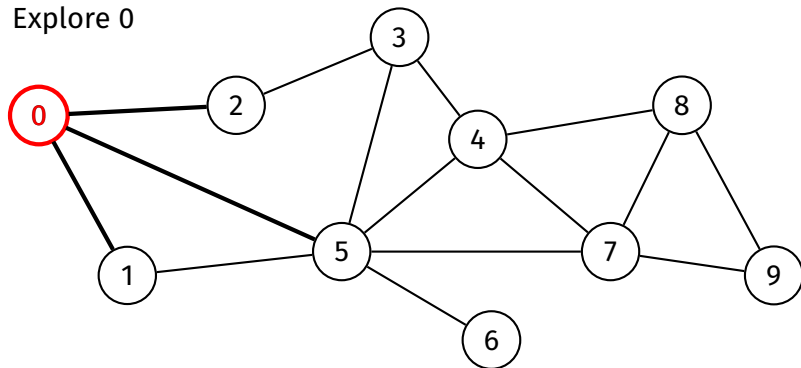
Deque 0



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0
pred	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

queue 0

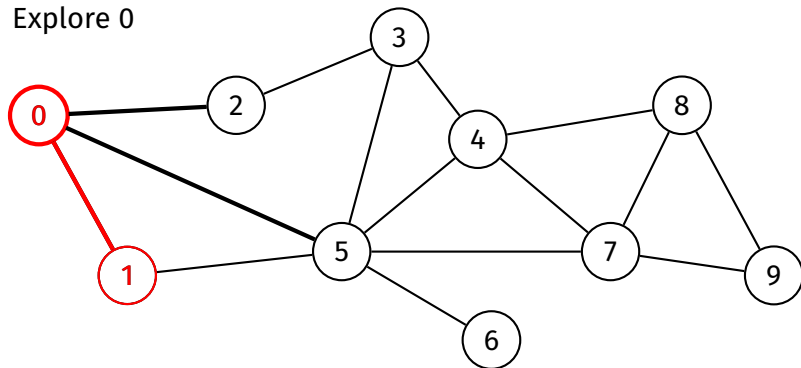
Explore 0



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0
pred	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

queue 0

Explore 0

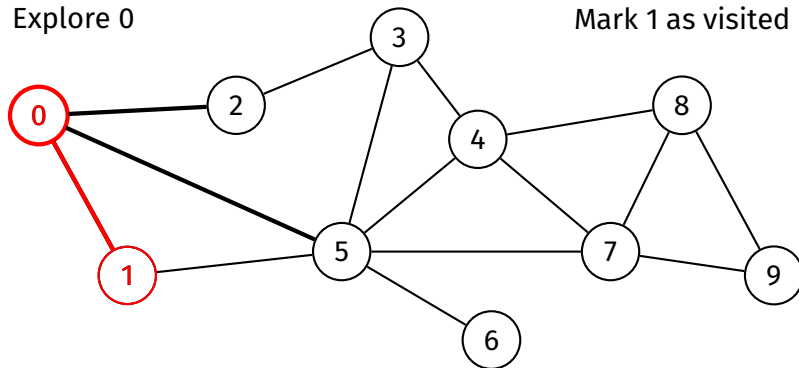


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0
pred	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

queue 0

Explore 0

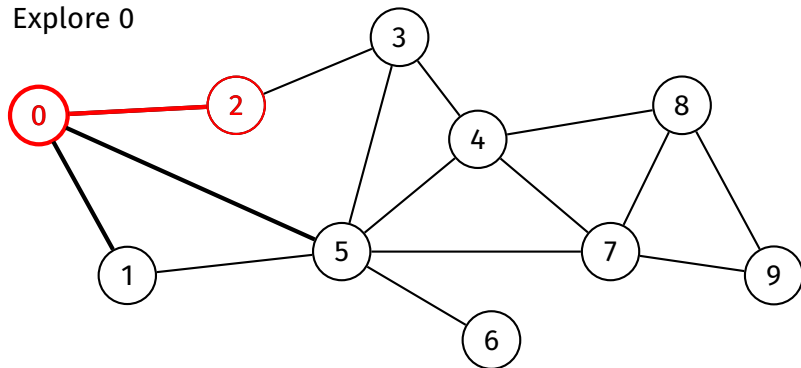
Mark 1 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0
pred	-1	0	-1	-1	-1	-1	-1	-1	-1	-1

queue 0 1

Explore 0

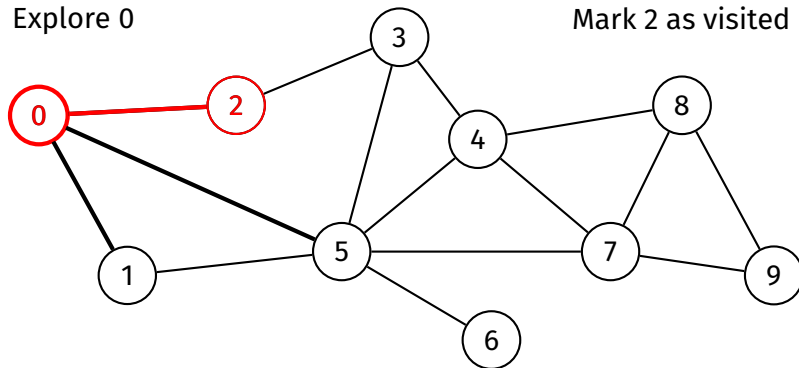


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0
pred	-1	0	-1	-1	-1	-1	-1	-1	-1	-1

queue 0 1

Explore 0

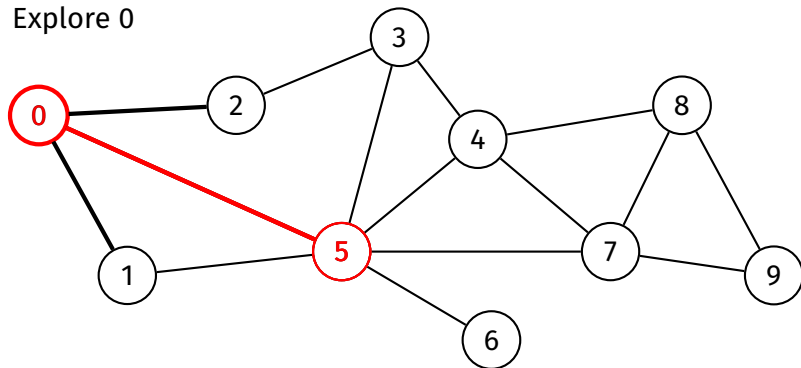
Mark 2 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	0	0	0	0	0
pred	-1	0	0	-1	-1	-1	-1	-1	-1	-1

queue 0 1 2

Explore 0

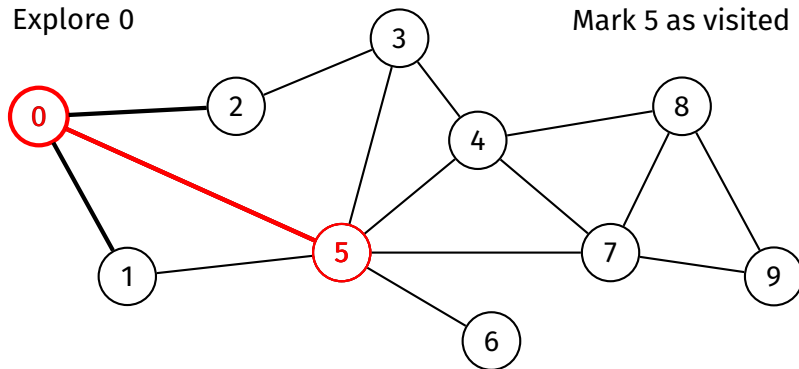


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	0	0	0	0	0
pred	-1	0	0	-1	-1	-1	-1	-1	-1	-1

queue 0 1 2

Explore 0

Mark 5 as visited

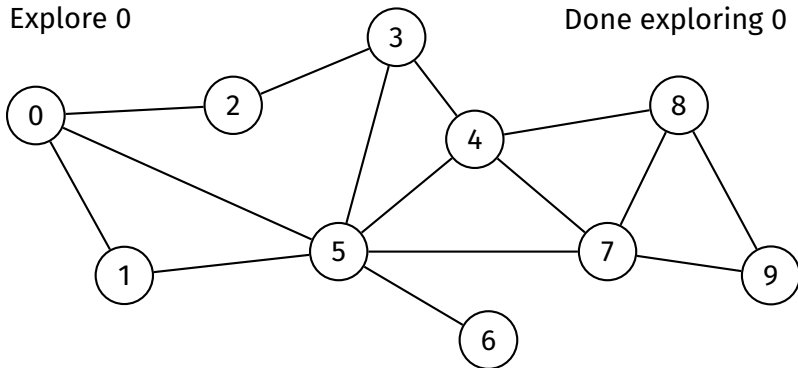


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

Explore 0

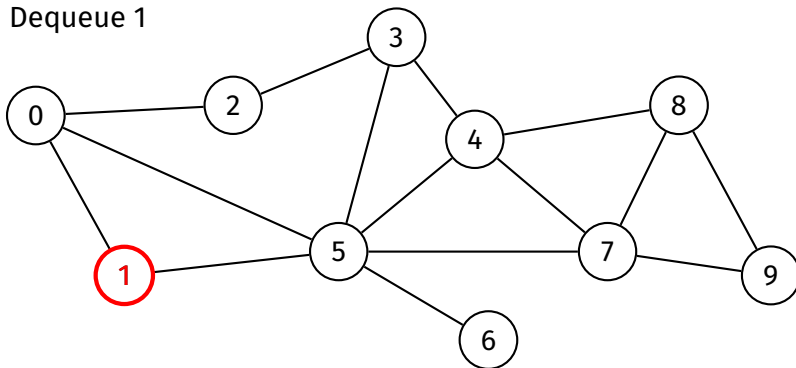
Done exploring 0



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

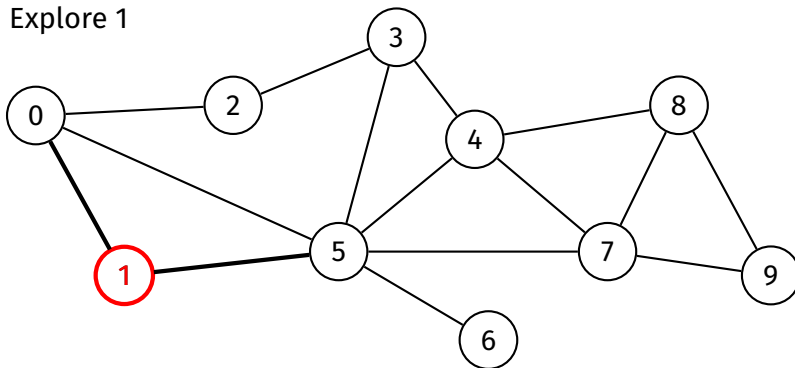
Deque 1



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

Explore 1

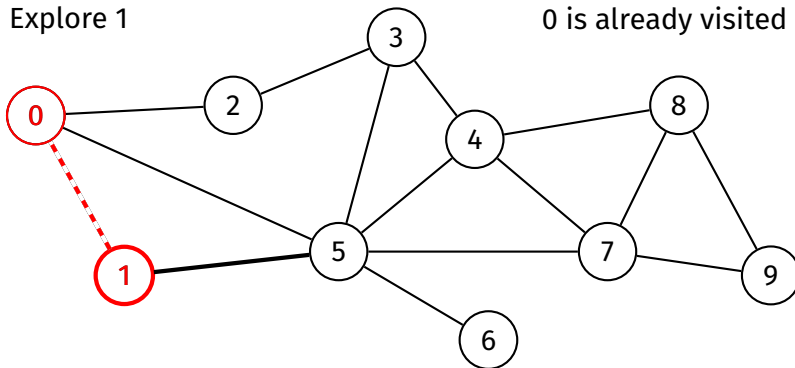


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

Explore 1

0 is already visited

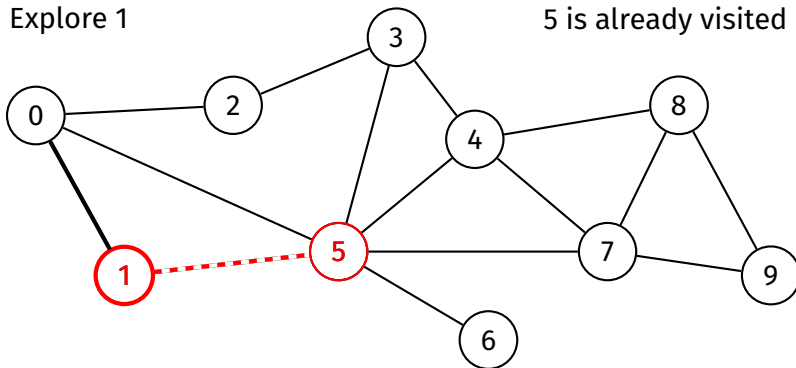


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

Explore 1

5 is already visited

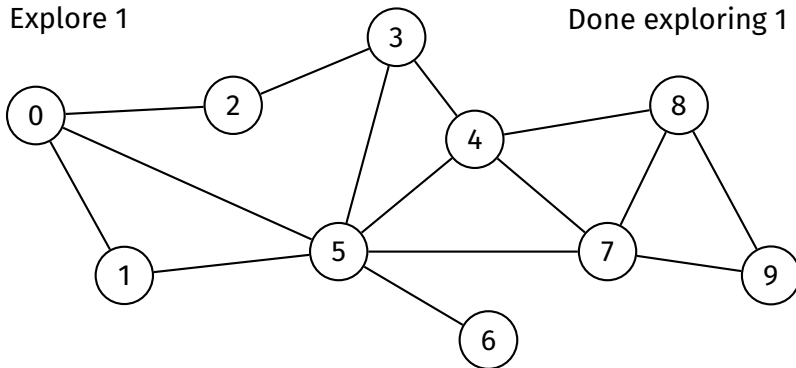


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

Explore 1

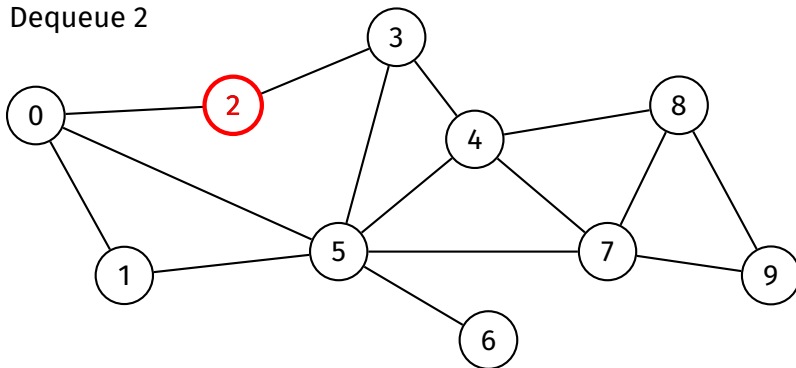
Done exploring 1



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

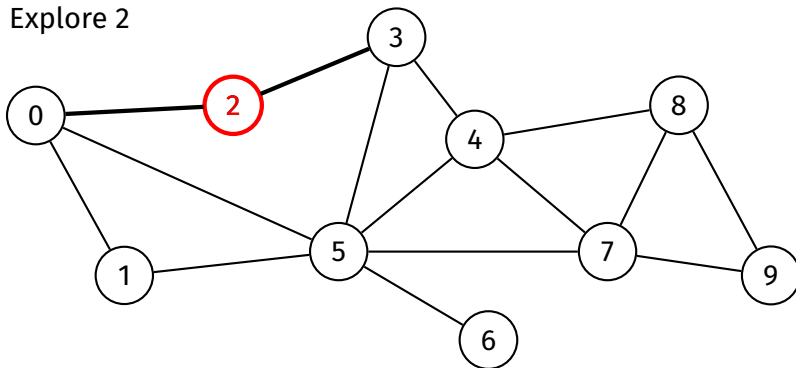
Deque 2



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

Explore 2

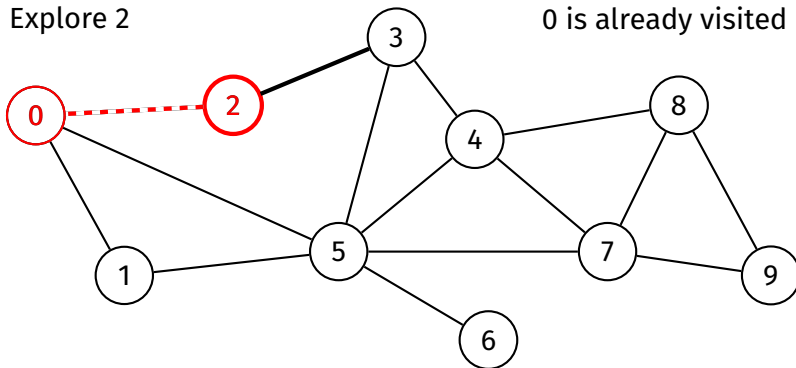


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

Explore 2

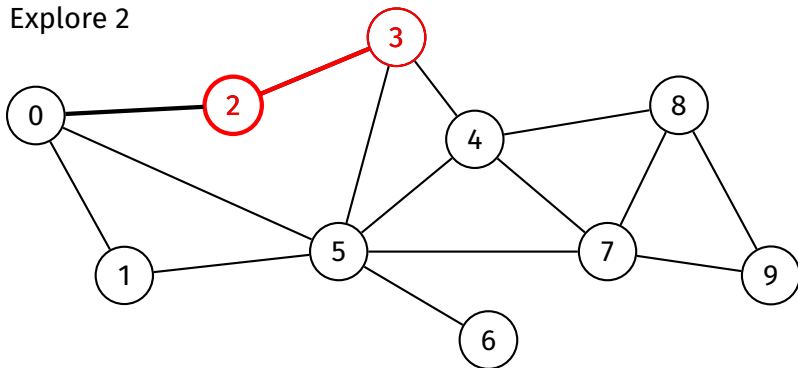
0 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

Explore 2

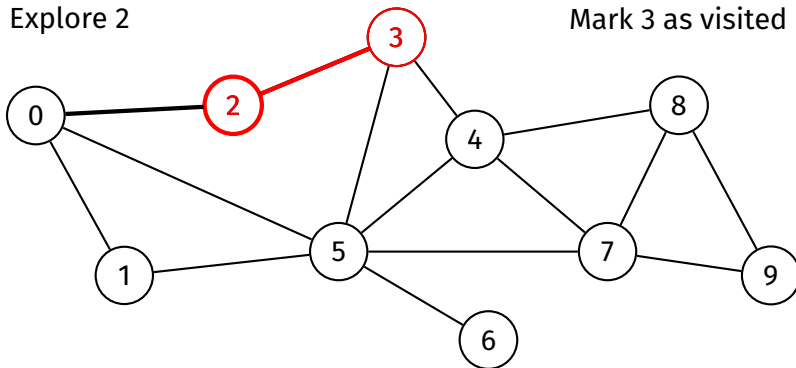


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

Explore 2

Mark 3 as visited

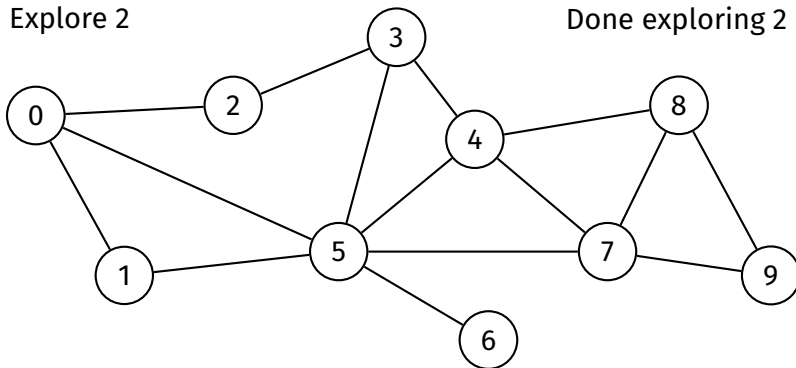


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

Explore 2

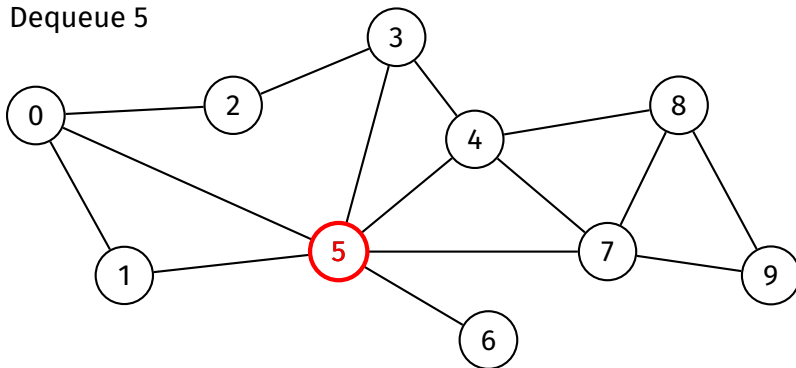
Done exploring 2



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

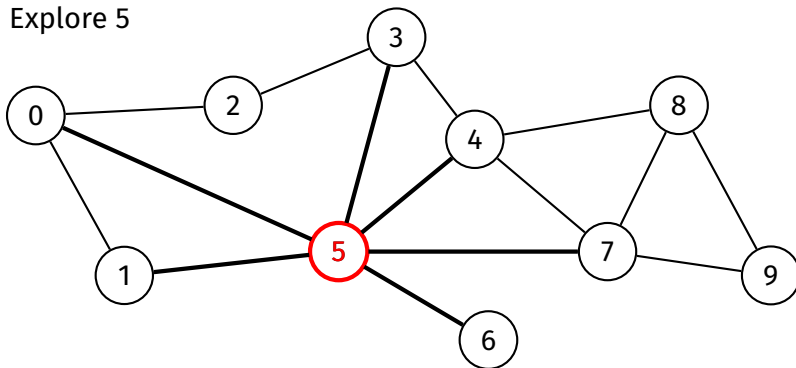
Deque 5



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

Explore 5

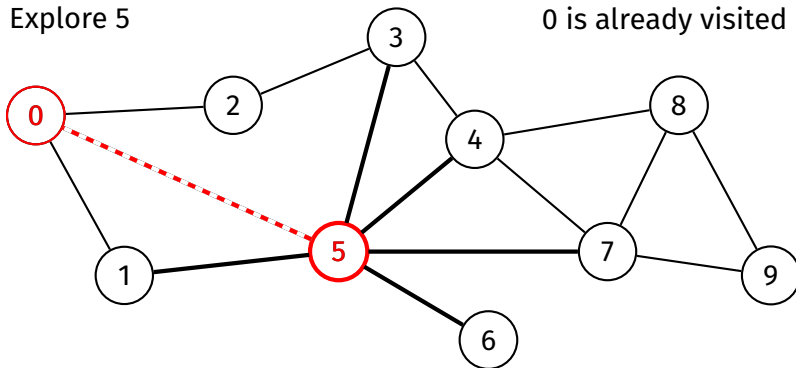


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

Explore 5

0 is already visited

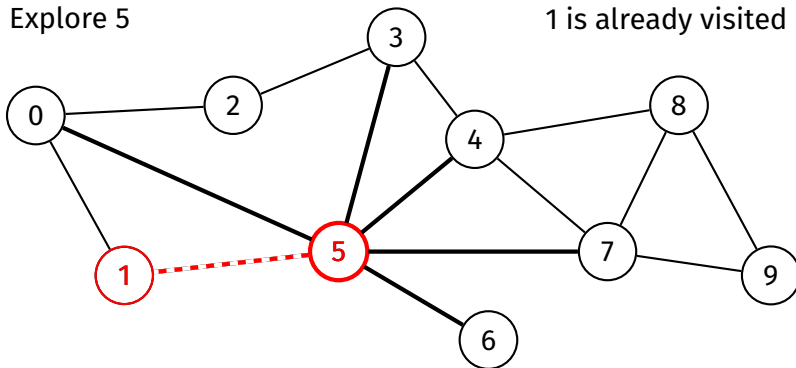


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

Explore 5

1 is already visited

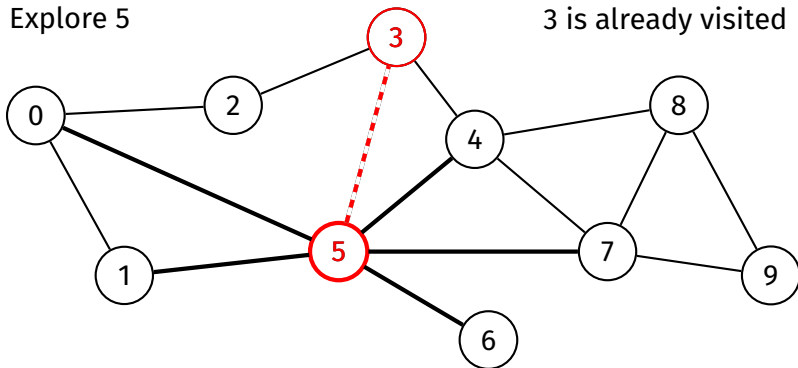


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

Explore 5

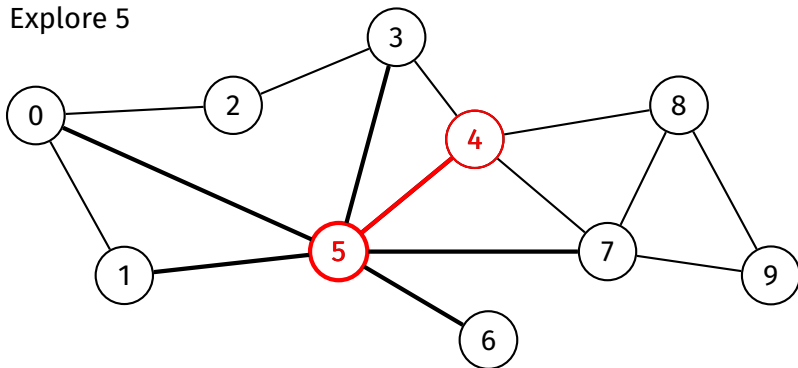
3 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

Explore 5

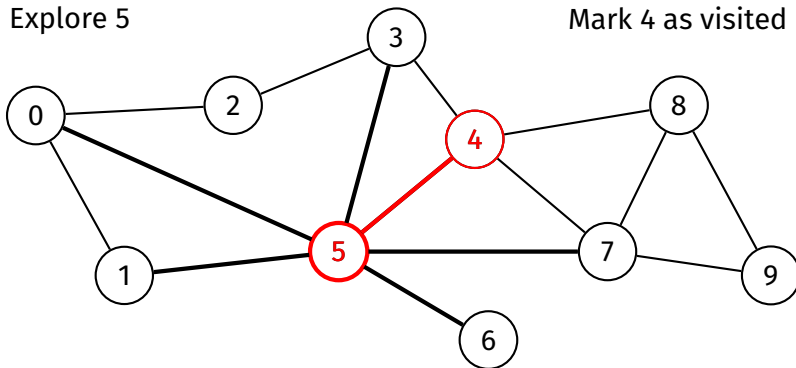


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

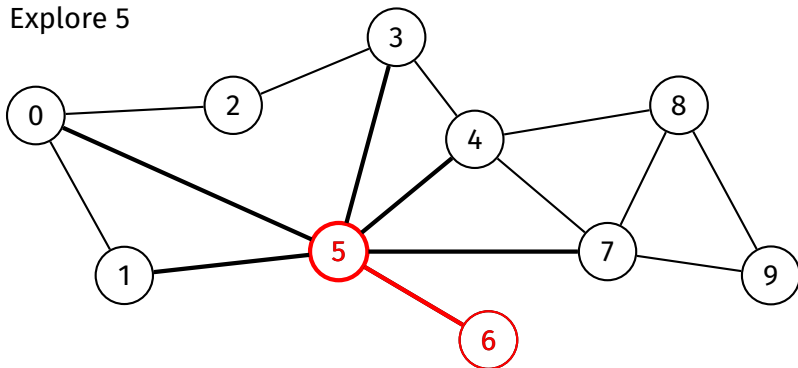
Explore 5

Mark 4 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0
pred	-1	0	0	2	5	0	-1	-1	-1	-1
queue	0	1	2	5	3	4				

Explore 5

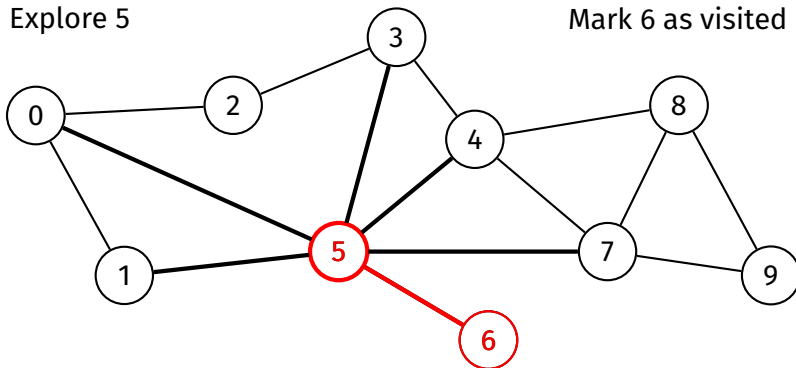


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0
pred	-1	0	0	2	5	0	-1	-1	-1	-1

queue 0 1 2 5 3 4

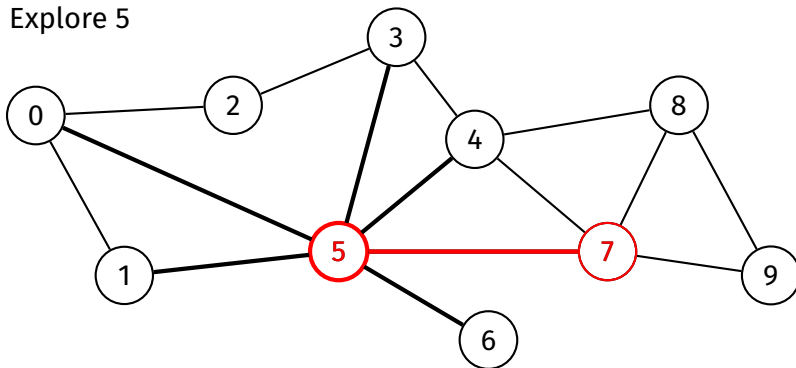
Explore 5

Mark 6 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	0	0	0
pred	-1	0	0	2	5	0	5	-1	-1	-1
queue	0	1	2	5	3	4	6			

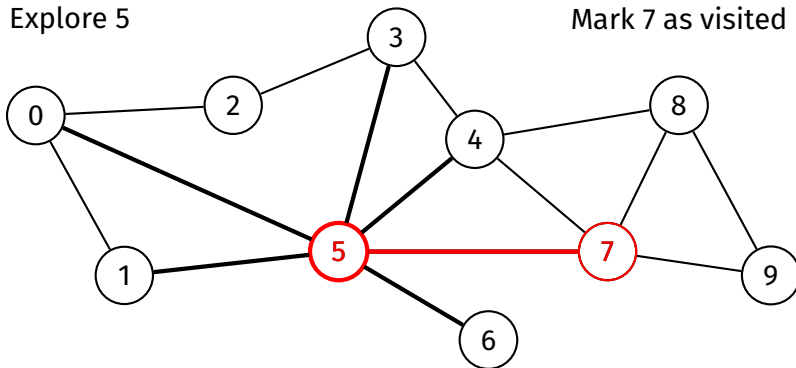
Explore 5



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	0	0	0
pred	-1	0	0	2	5	0	5	-1	-1	-1
queue	0	1	2	5	3	4	6			

Explore 5

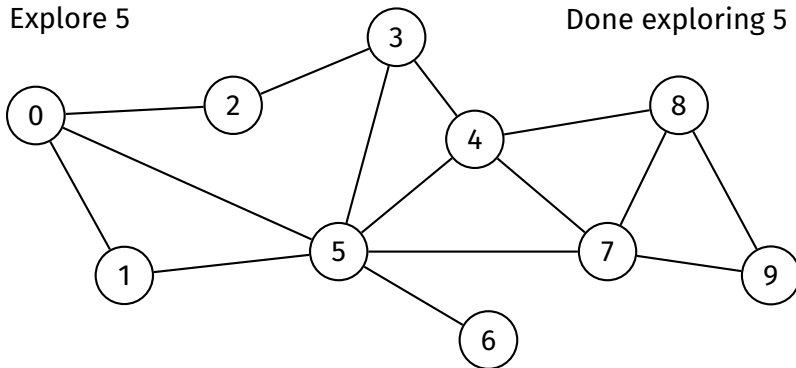
Mark 7 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1
queue	0	1	2	5	3	4	6	7		

Explore 5

Done exploring 5



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

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DFS

Ideas/Issues

Appendix

BFS

BFS Example

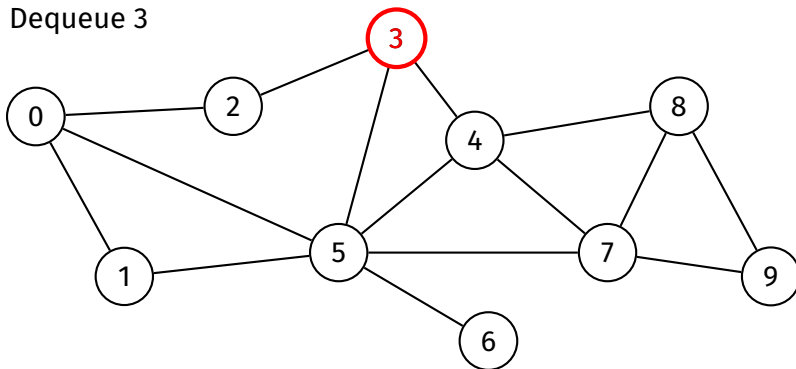
DFS

DFS Example

Path-Checking

Example

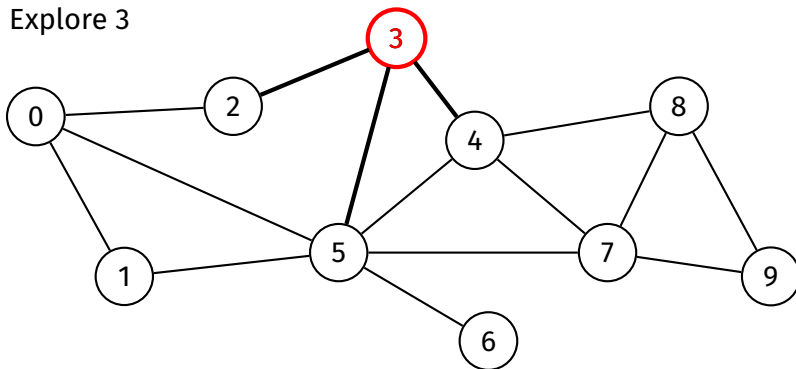
Deque 3



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

Explore 3

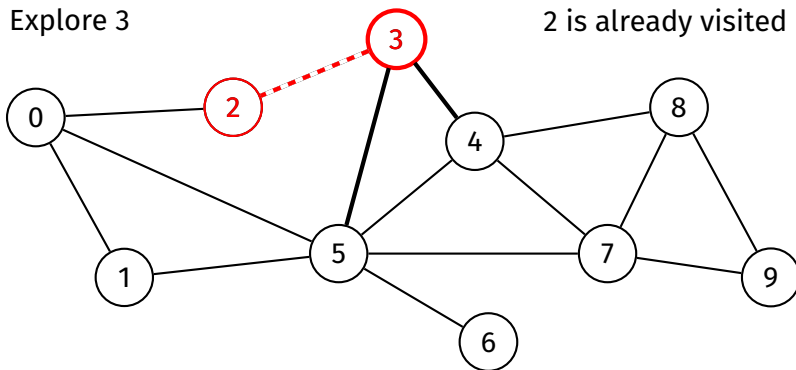


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

Explore 3

2 is already visited

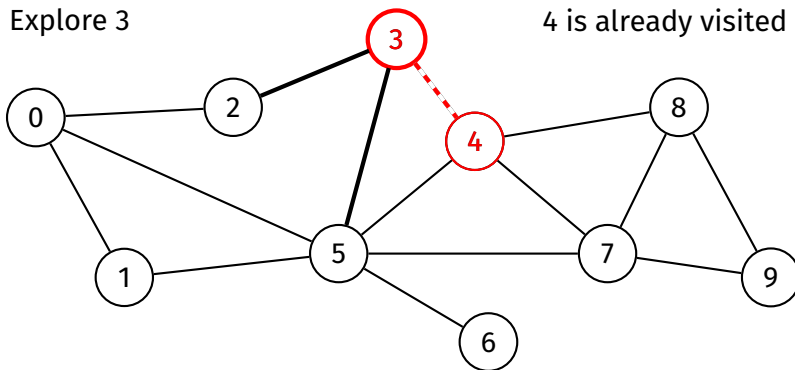


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

Explore 3

4 is already visited

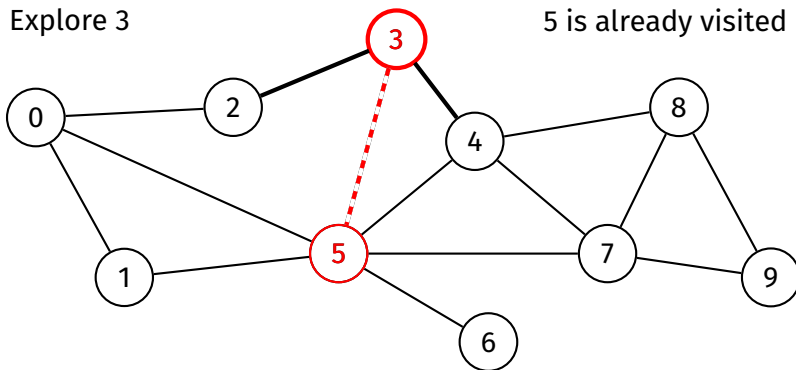


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

Explore 3

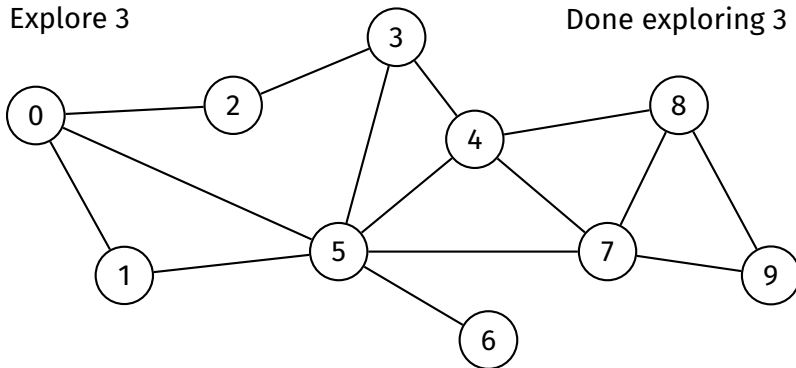
5 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1
queue	0	1	2	5	3	4	6	7		

Explore 3

Done exploring 3



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

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BFS

BFS Example

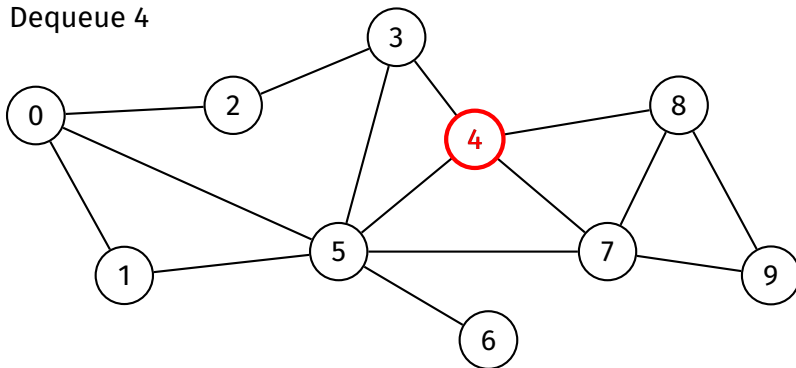
DFS

DFS Example

Path-Checking

Example

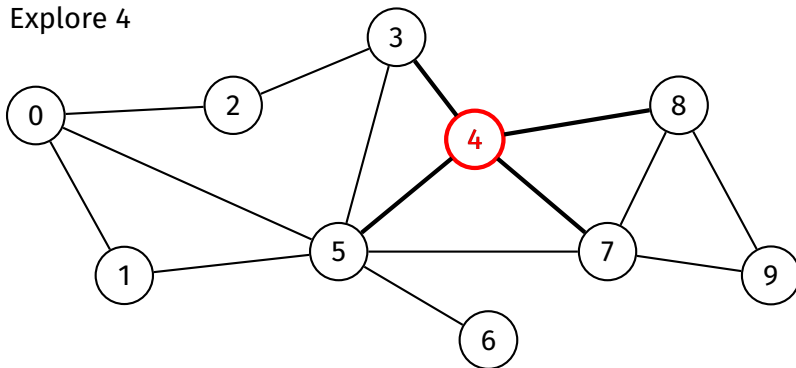
Deque 4



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

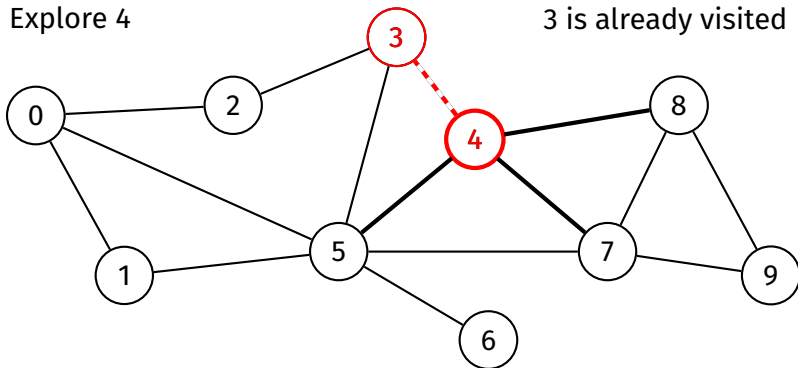
Explore 4



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

3 is already visited

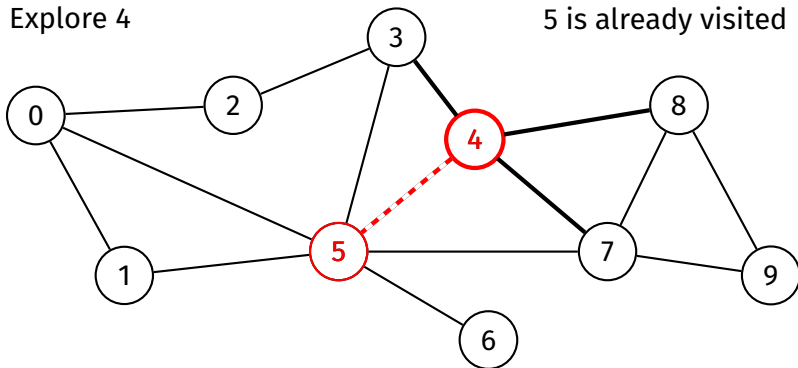


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

Explore 4

5 is already visited

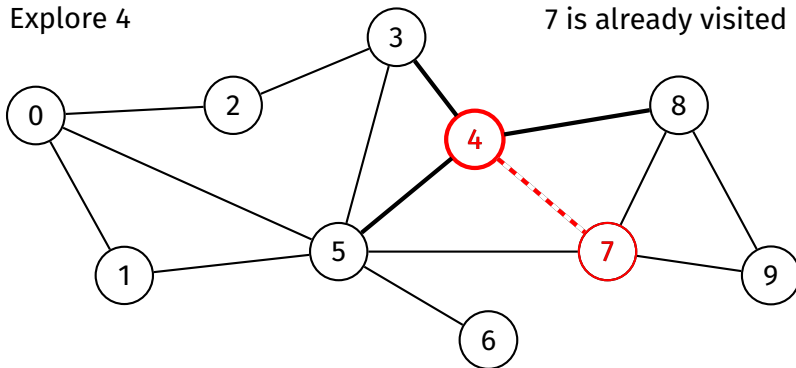


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

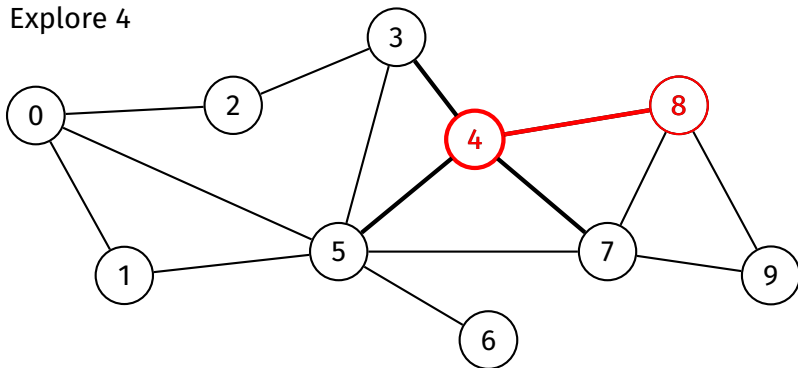
Explore 4

7 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1
queue	0	1	2	5	3	4	6	7		

Explore 4

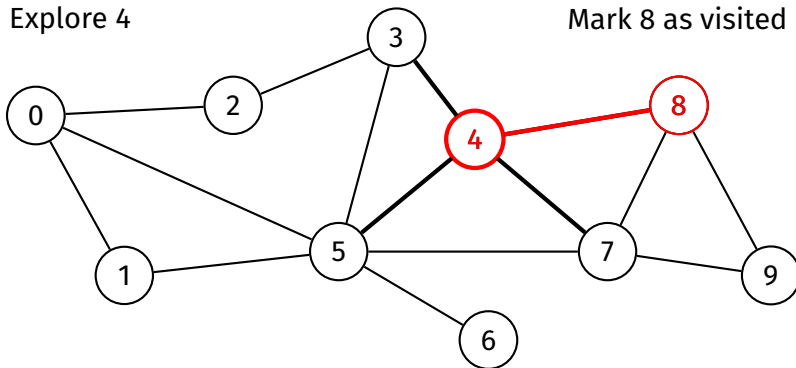


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

Explore 4

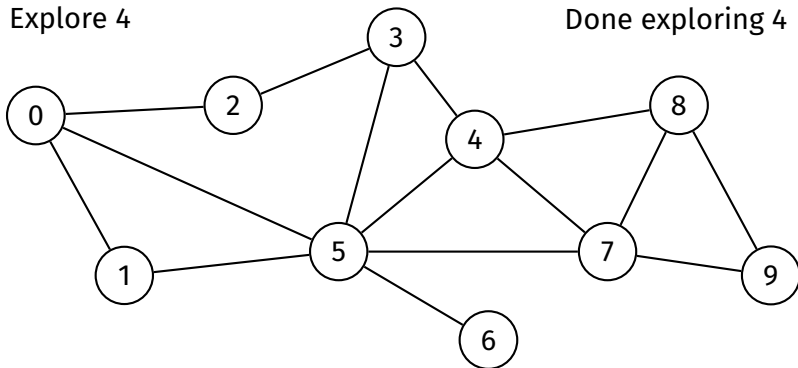
Mark 8 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1
queue	0	1	2	5	3	4	6	7	8	

Explore 4

Done exploring 4



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1
queue	0	1	2	5	3	4	6	7	8	

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BFS

BFS Example

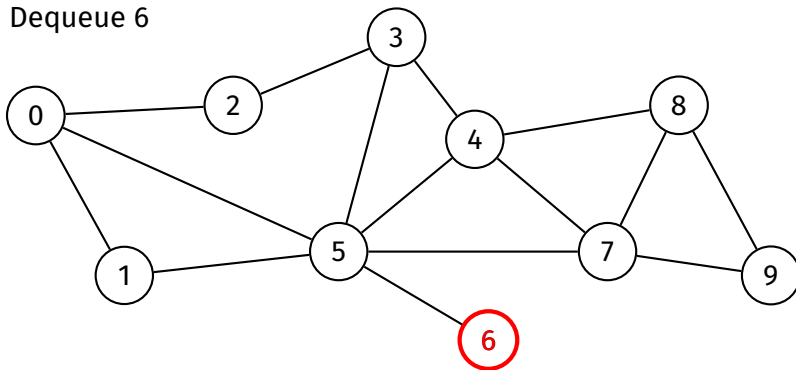
DFS

DFS Example

Path-Checking

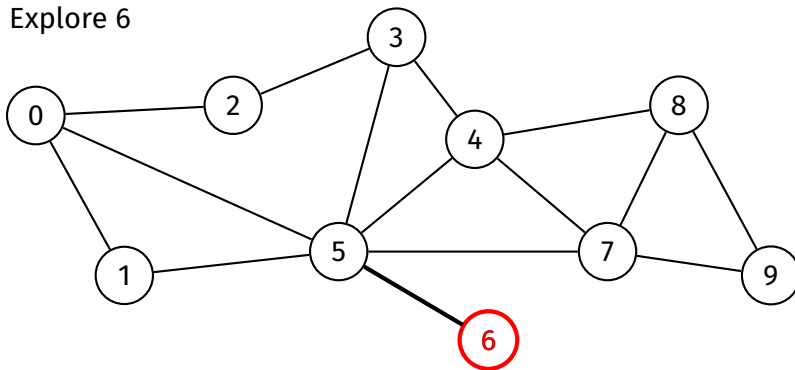
Example

Deque 6



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1
queue	0	1	2	5	3	4	6	7	8	

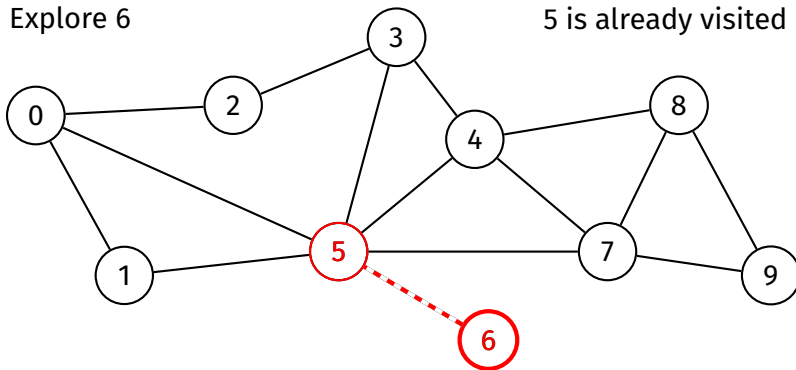
Explore 6



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1
queue	0	1	2	5	3	4	6	7	8	

Explore 6

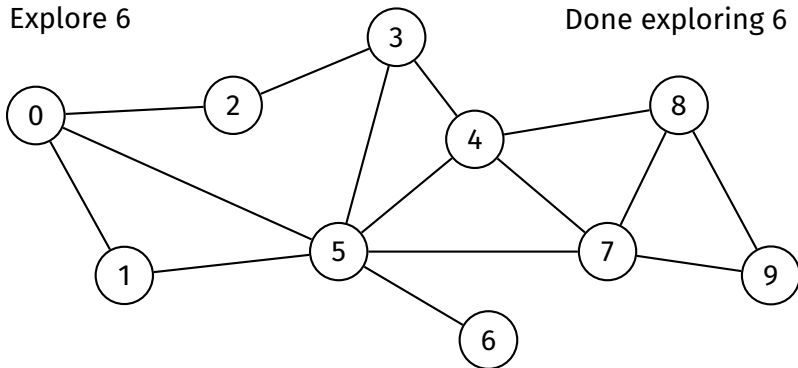
5 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1
queue	0	1	2	5	3	4	6	7	8	

Explore 6

Done exploring 6



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

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BFS

BFS Example

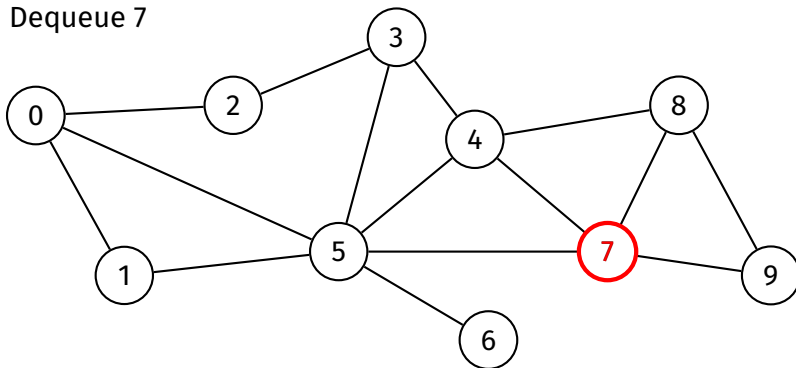
DFS

DFS Example

Path-Checking

Example

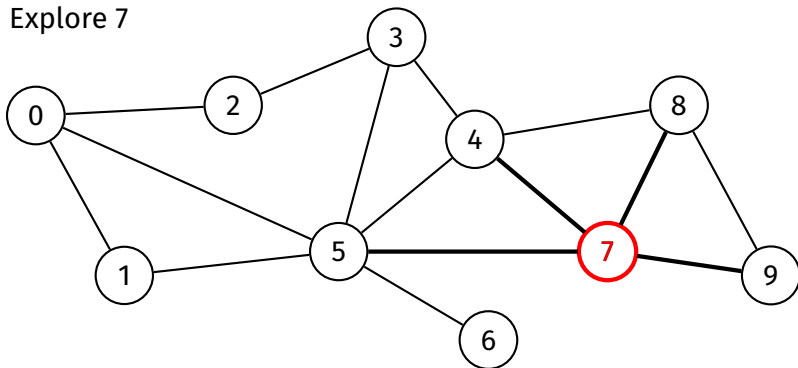
Deque 7



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

Explore 7

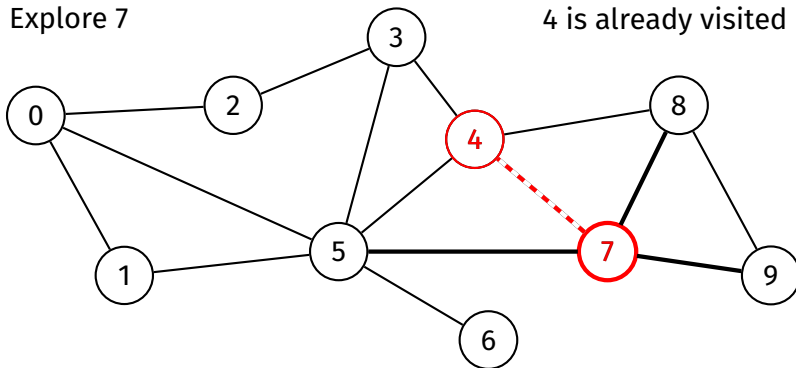


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

Explore 7

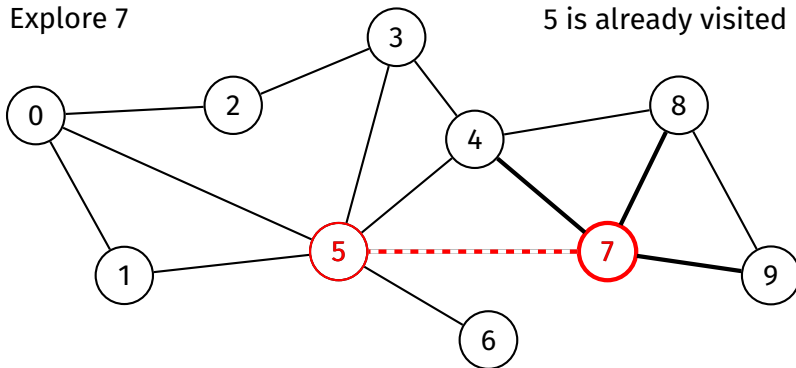
4 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1
queue	0	1	2	5	3	4	6	7	8	

Explore 7

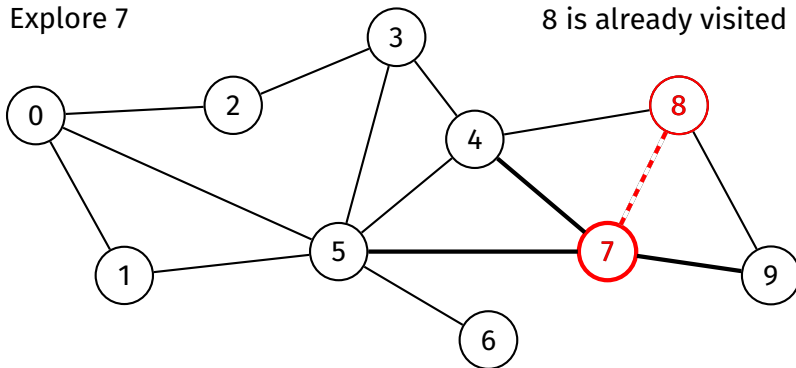
5 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1
queue	0	1	2	5	3	4	6	7	8	

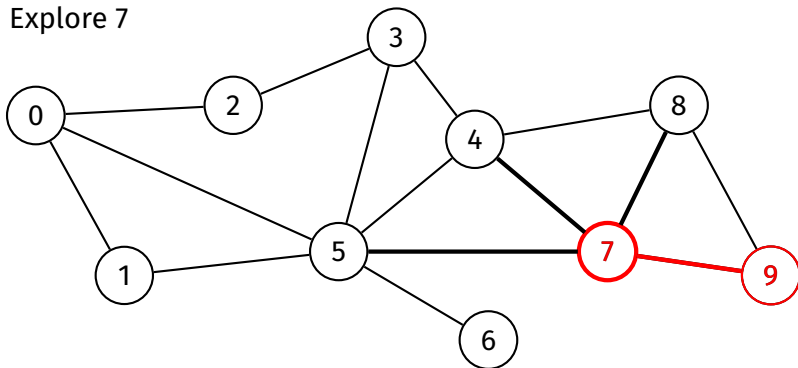
Explore 7

8 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1
queue	0	1	2	5	3	4	6	7	8	

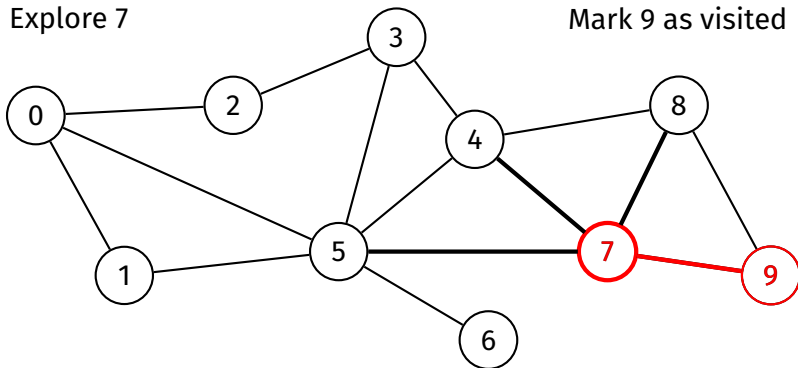
Explore 7



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1
queue	0	1	2	5	3	4	6	7	8	

Explore 7

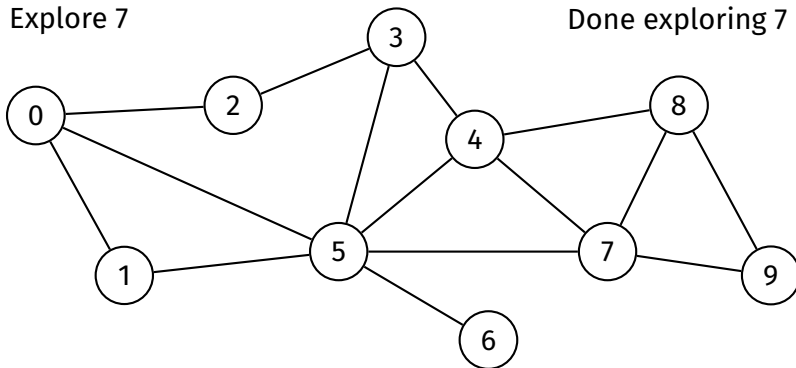
Mark 9 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7
queue	0	1	2	5	3	4	6	7	8	9

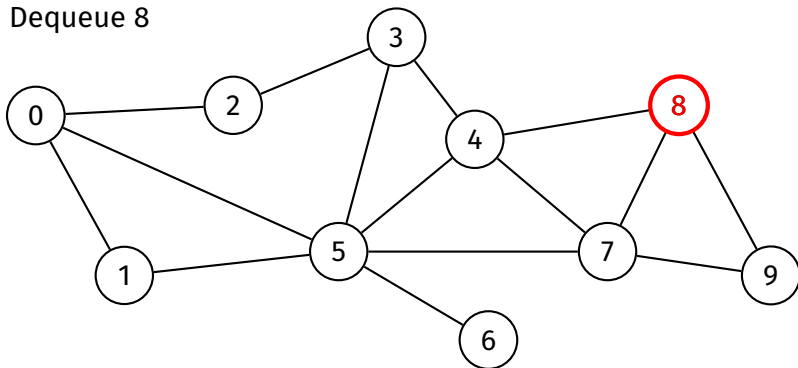
Explore 7

Done exploring 7



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7
queue	0	1	2	5	3	4	6	7	8	9

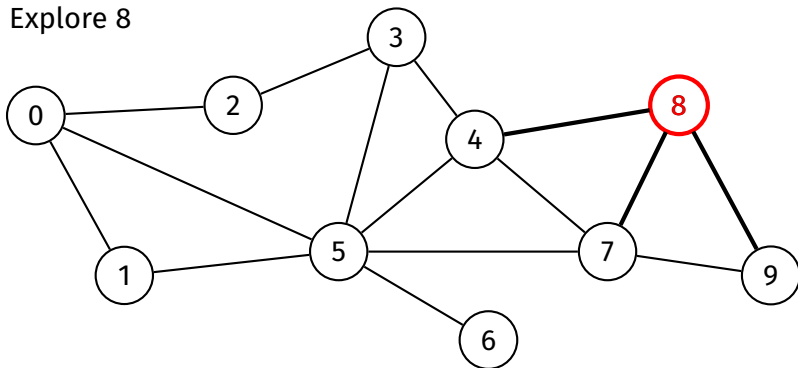
Deque 8



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

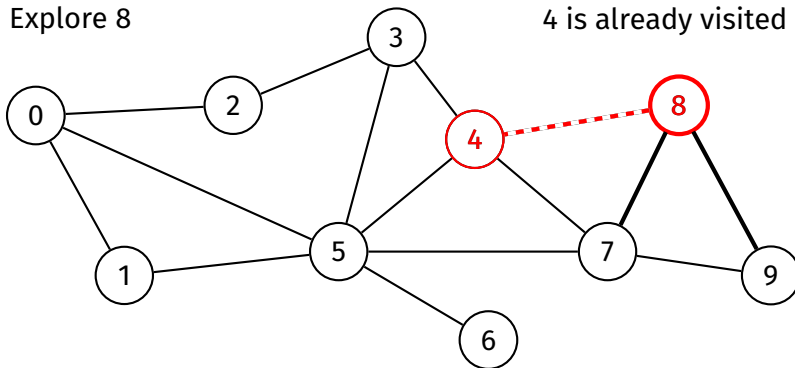
Explore 8



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7
queue	0	1	2	5	3	4	6	7	8	9

Explore 8

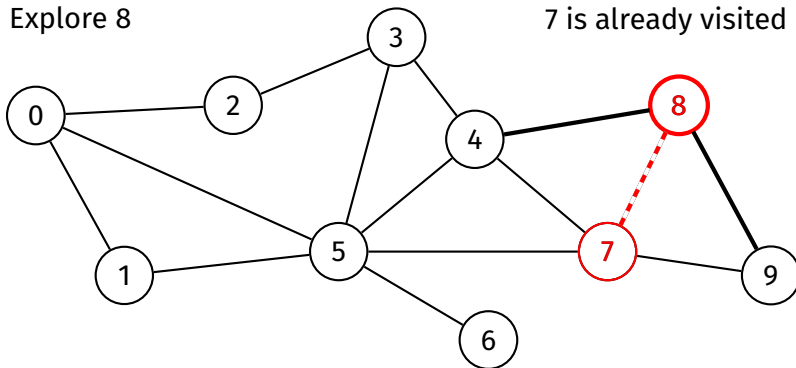
4 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7
queue	0	1	2	5	3	4	6	7	8	9

Explore 8

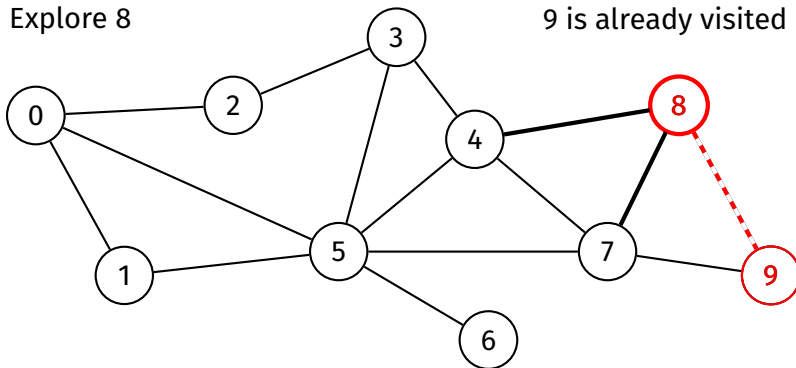
7 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7
queue	0	1	2	5	3	4	6	7	8	9

Explore 8

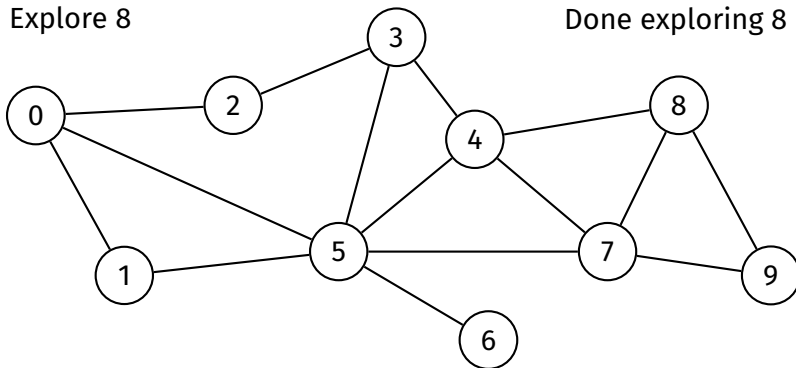
9 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7
queue	0	1	2	5	3	4	6	7	8	9

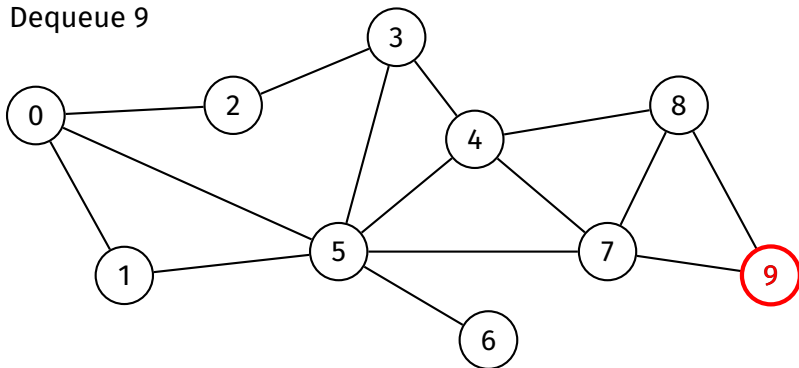
Explore 8

Done exploring 8



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7
queue	0	1	2	5	3	4	6	7	8	9

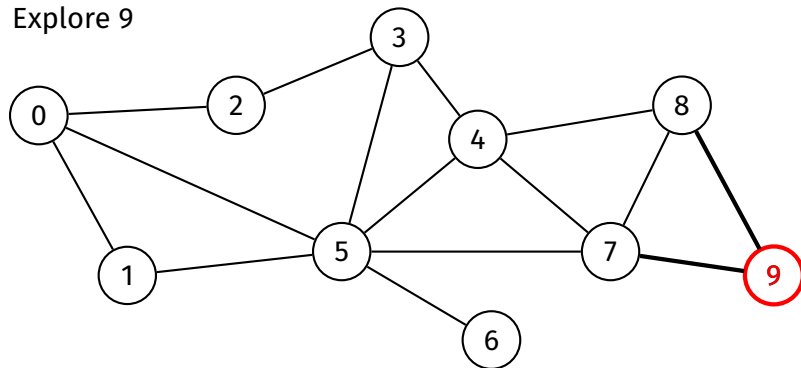
Deque 9



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

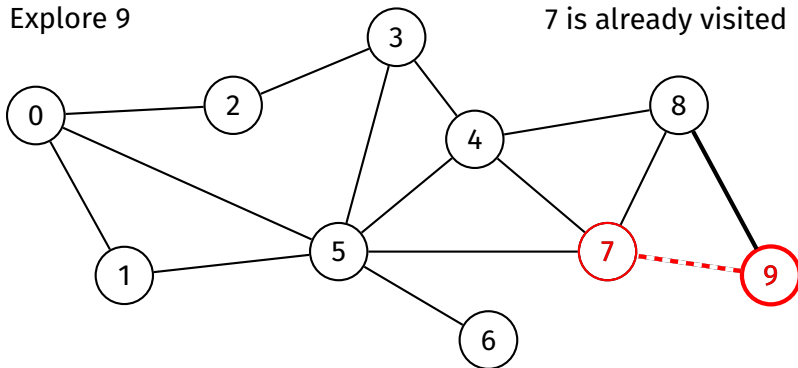
Explore 9



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7
queue	0	1	2	5	3	4	6	7	8	9

Explore 9

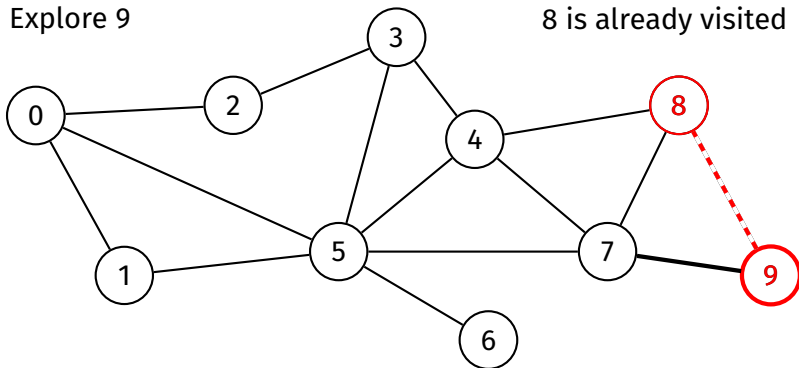
7 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7
queue	0	1	2	5	3	4	6	7	8	9

Explore 9

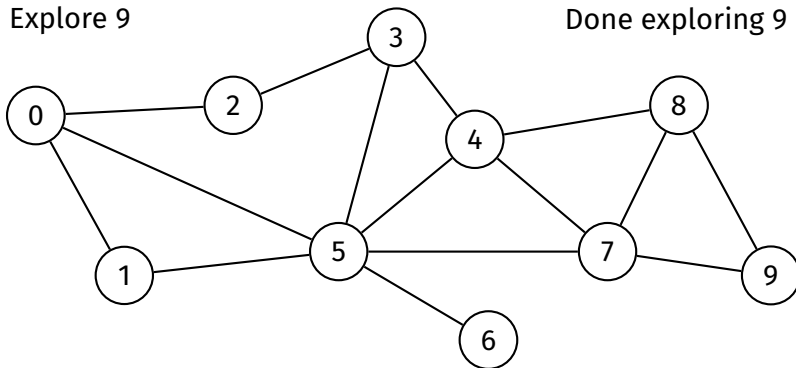
8 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7
queue	0	1	2	5	3	4	6	7	8	9

Explore 9

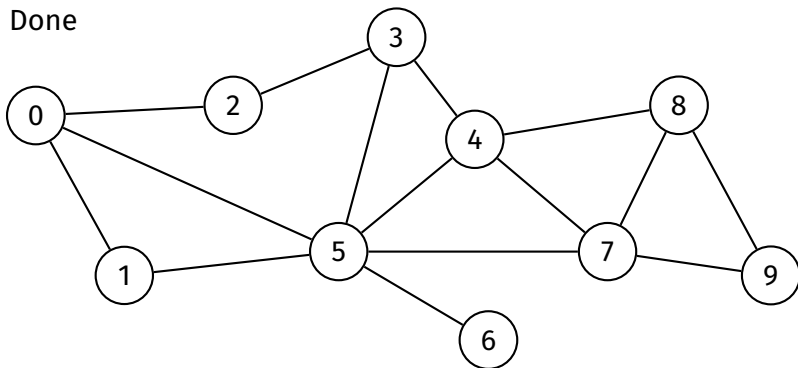
Done exploring 9



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

Done



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

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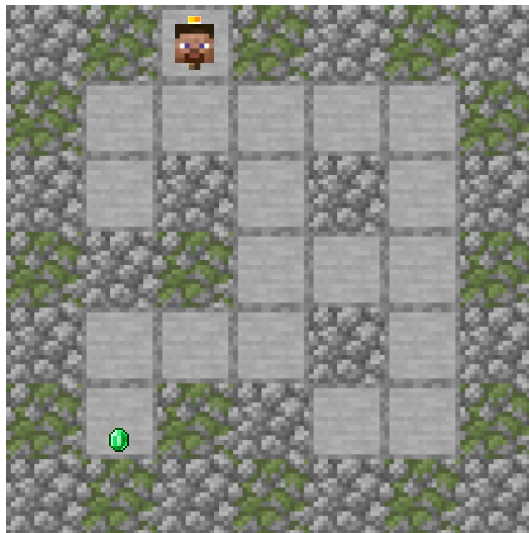
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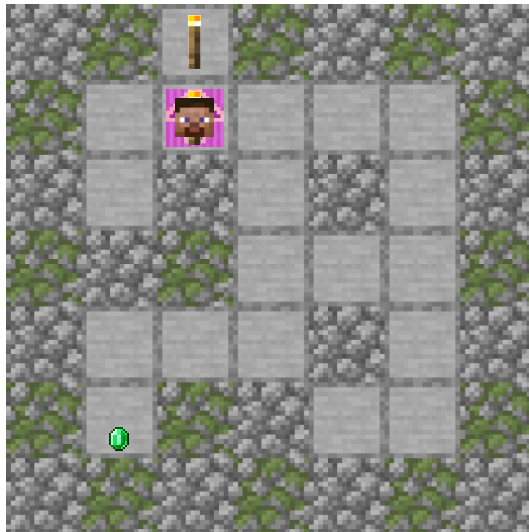
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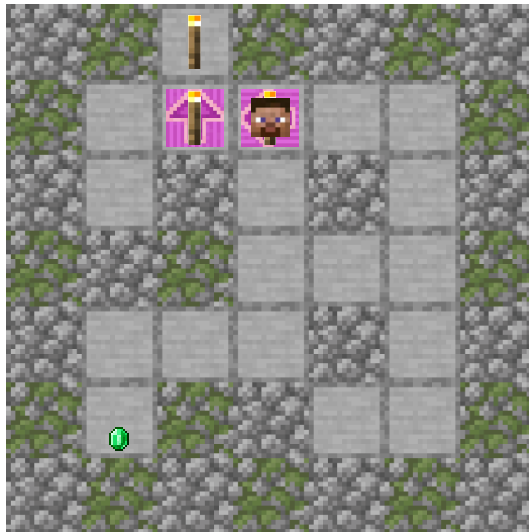
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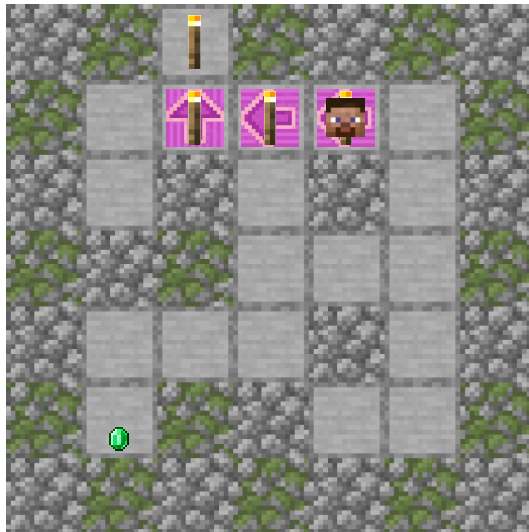
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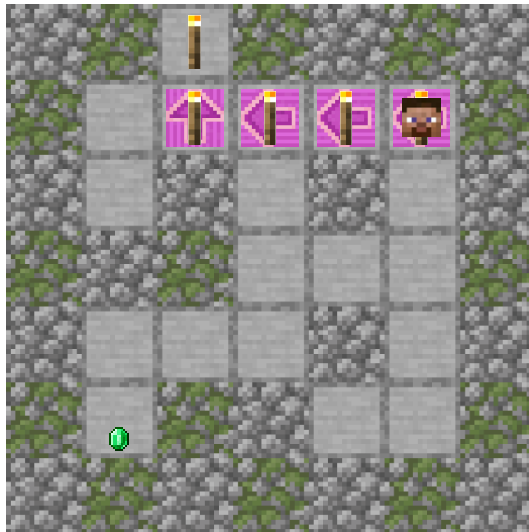
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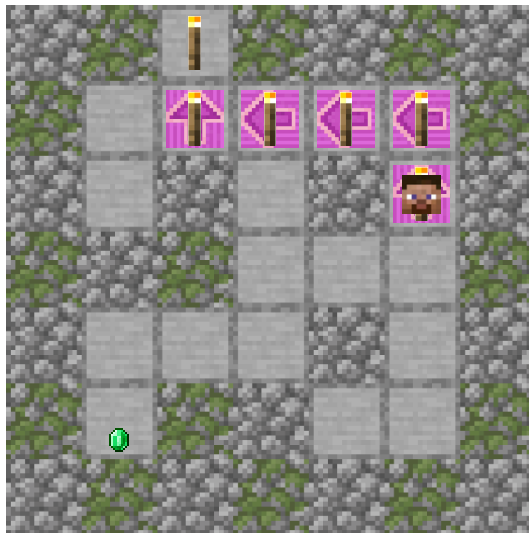
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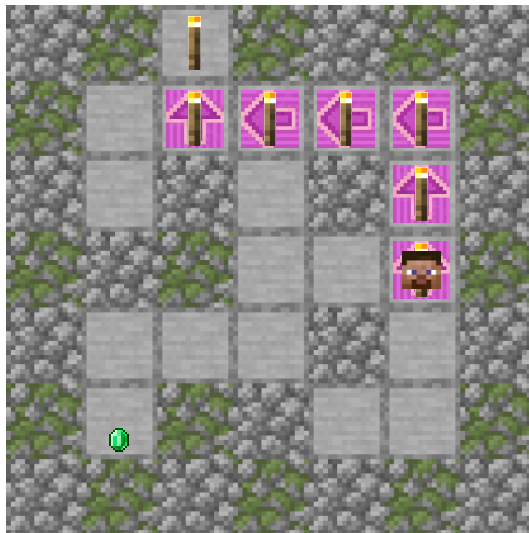
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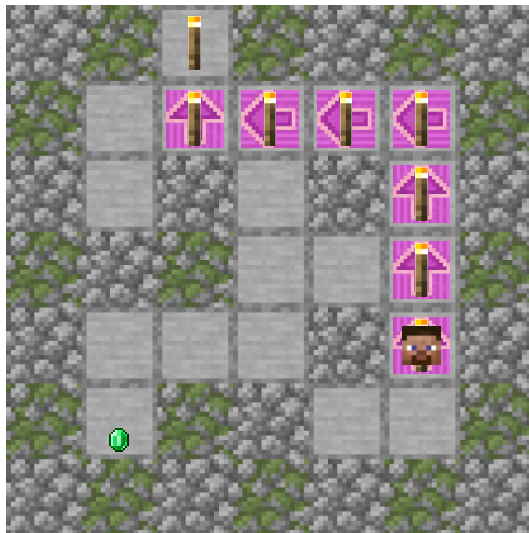
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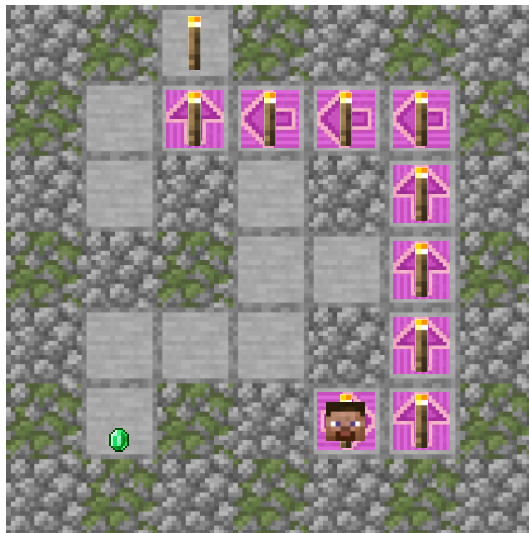
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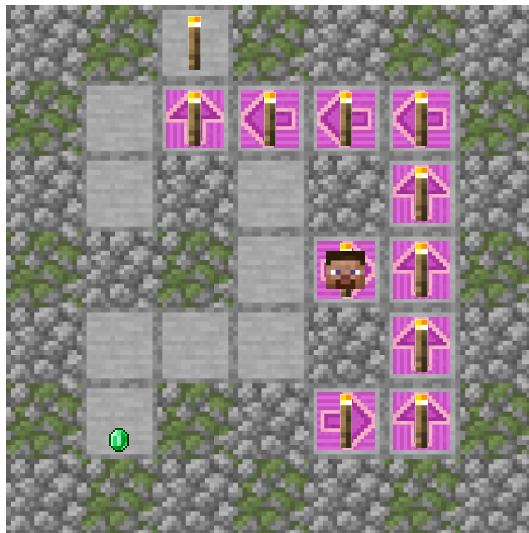
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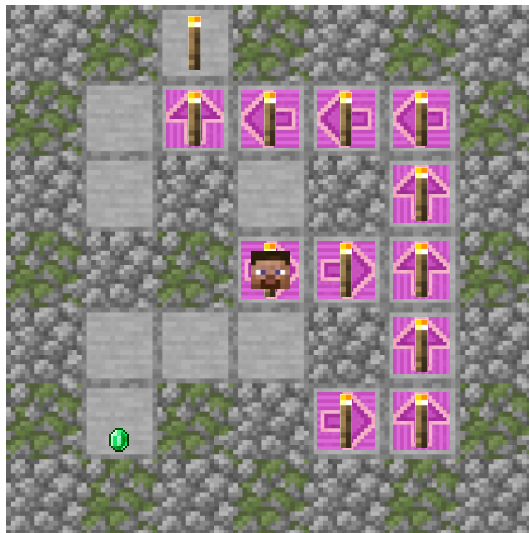
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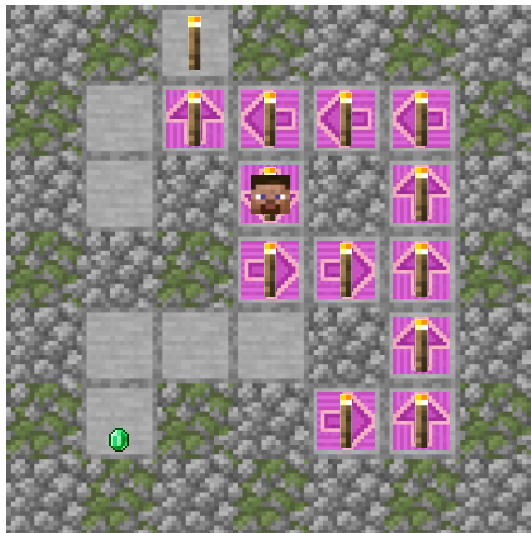
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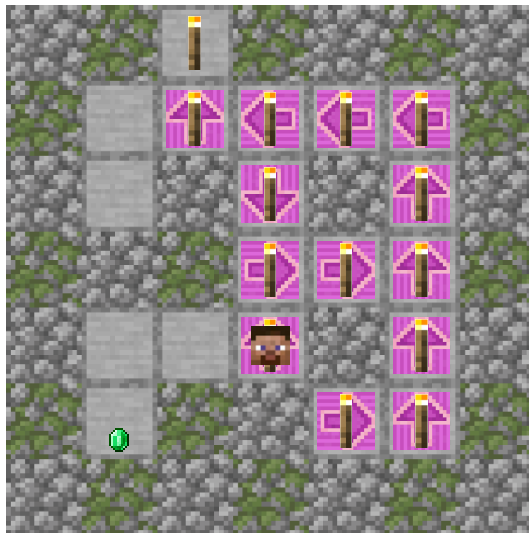
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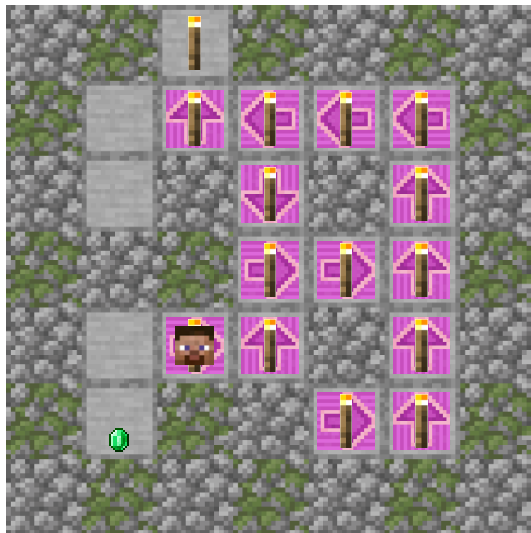
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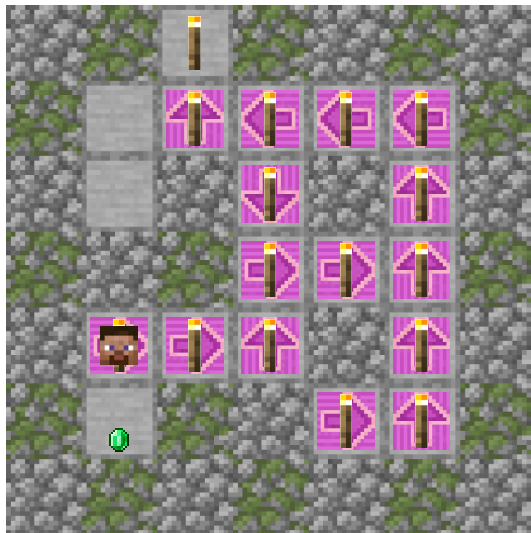
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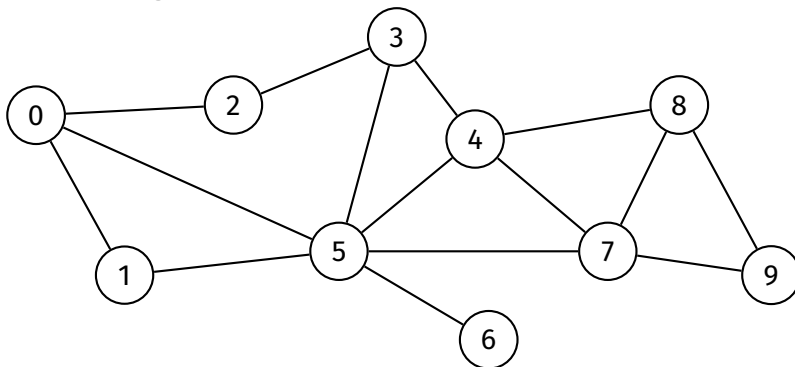
DFS Example

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Example



DFS starting at 0



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0

call stack

visit order

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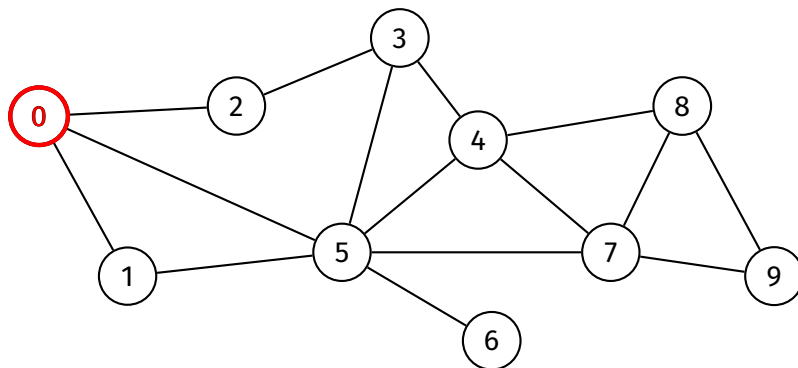
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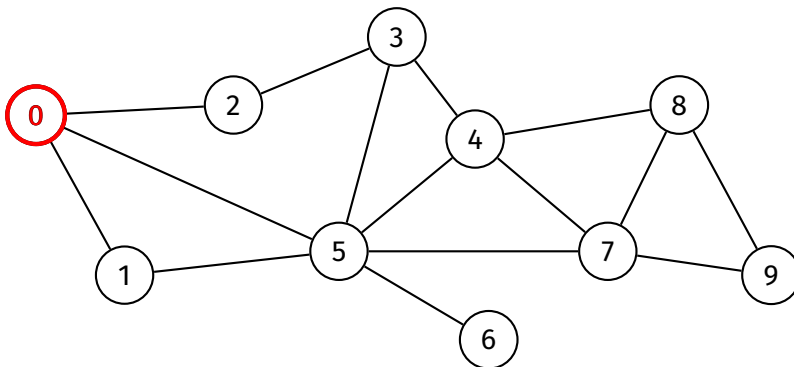
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0

visit order

dfs(0)

call stack

Mark 0 as visited



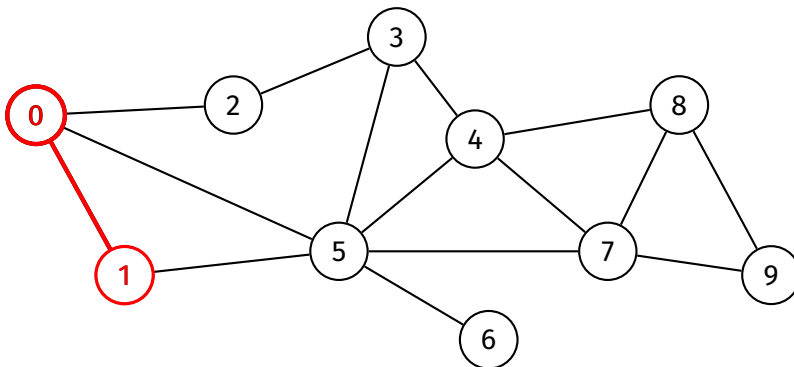
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

visit order 0

dfs(0)

call stack

1 has not been visited



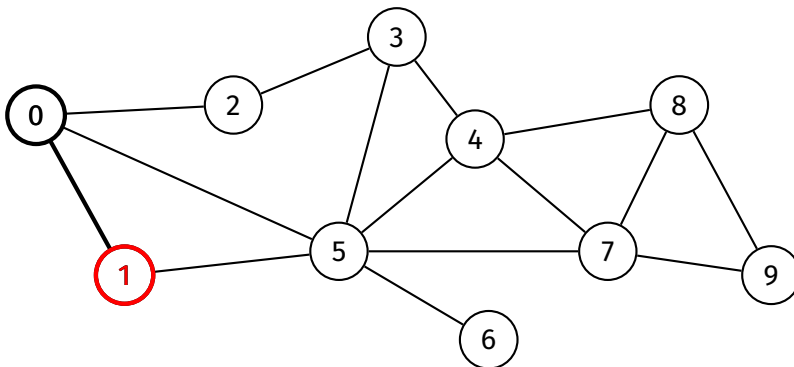
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

visit order 0

dfs(0)

call stack

Recurse into 1



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

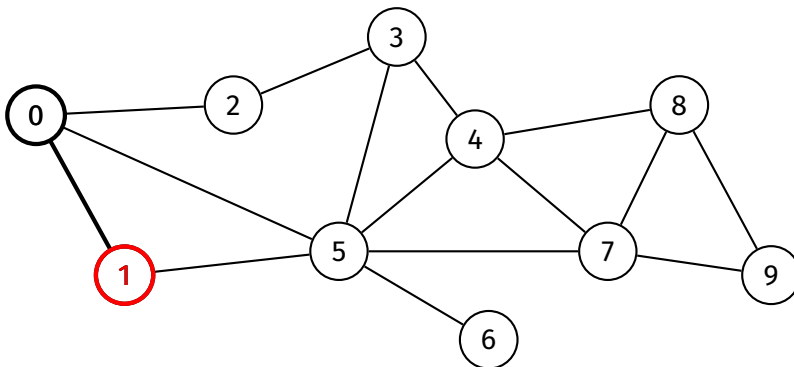
visit order 0

dfs(1)

dfs(0)

call stack

Mark 1 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

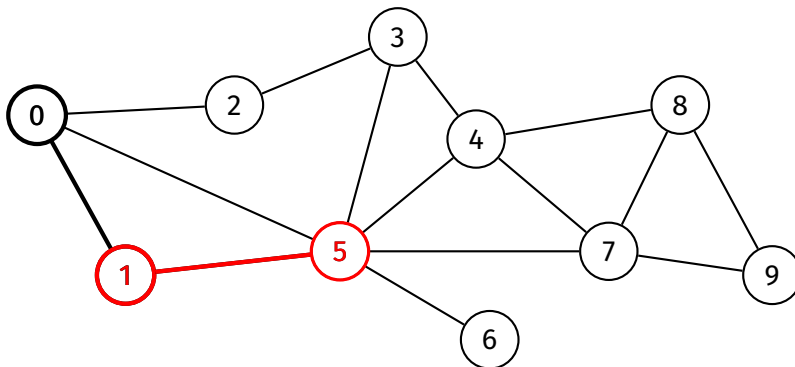
visit order 0 1

dfs(1)

dfs(0)

call stack

5 has not been visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

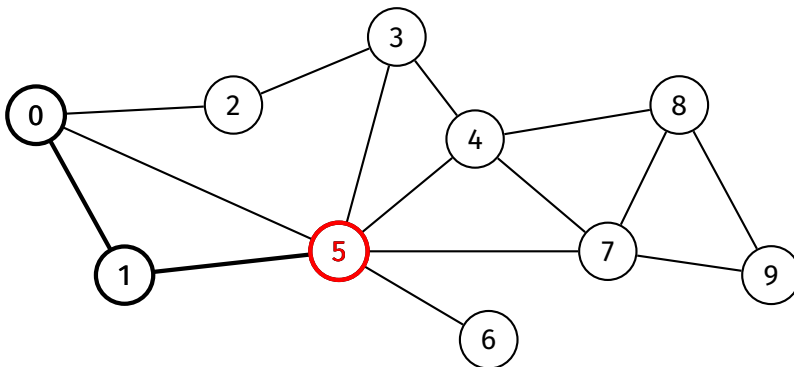
visit order 0 1

dfs(1)

dfs(0)

call stack

Recurse into 5



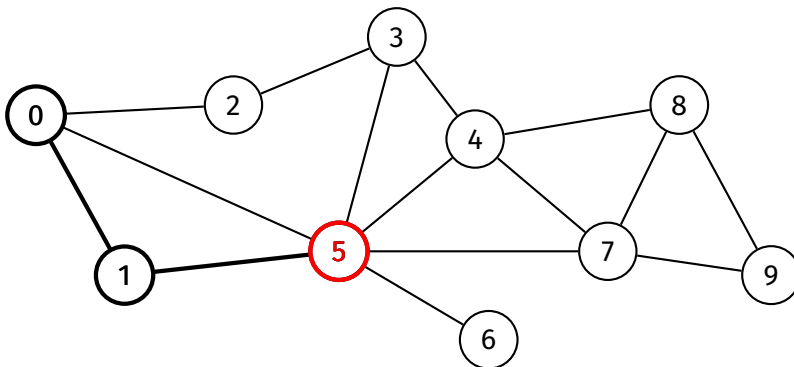
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

visit order 0 1

dfs(5)
dfs(1)
dfs(0)

call stack

Mark 5 as visited



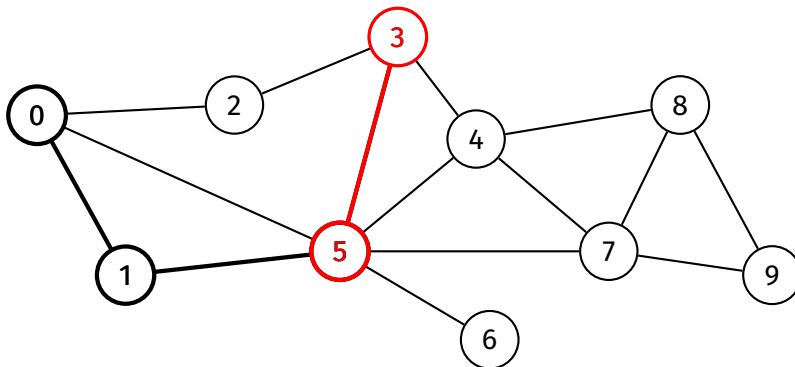
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

visit order 0 1 5

dfs(5)
dfs(1)
dfs(0)

call stack

3 has not been visited



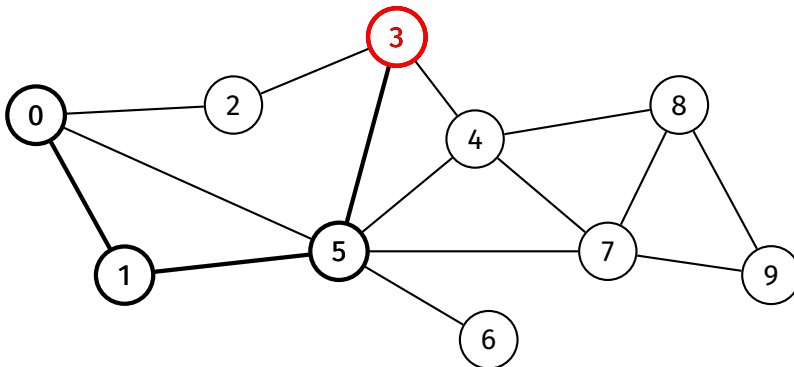
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

visit order 0 1 5

dfs(5)
dfs(1)
dfs(0)

call stack

Recurse into 3



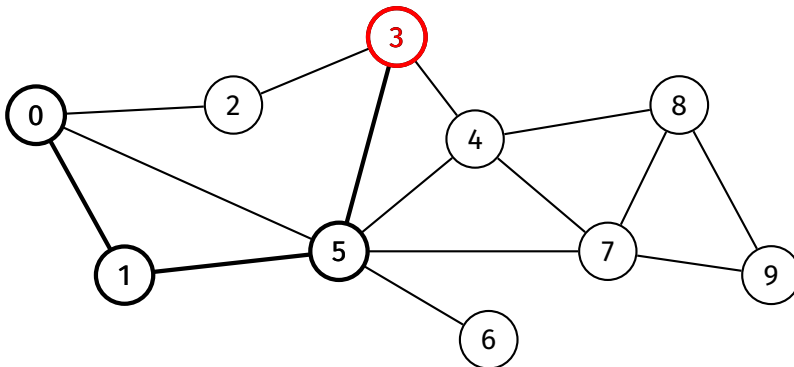
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

visit order 0 1 5

dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

Mark 3 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

visit order 0 1 5 3

dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

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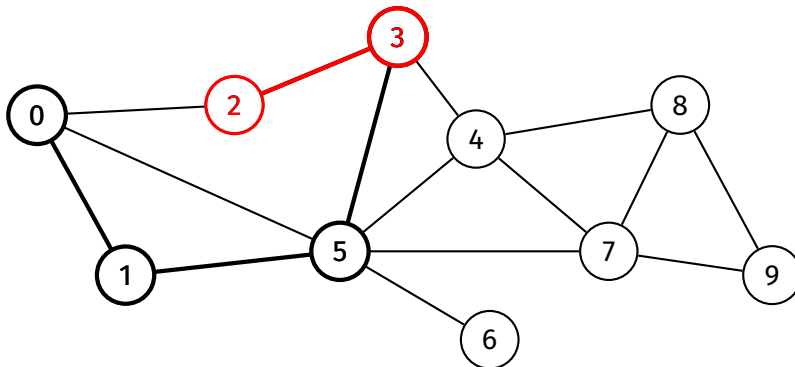
DFS

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Path-Checking

Example

2 has not been visited



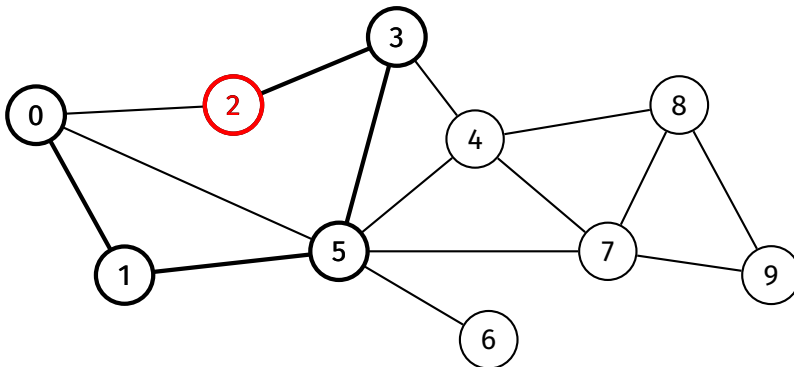
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

visit order 0 1 5 3

dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

Recurse into 2



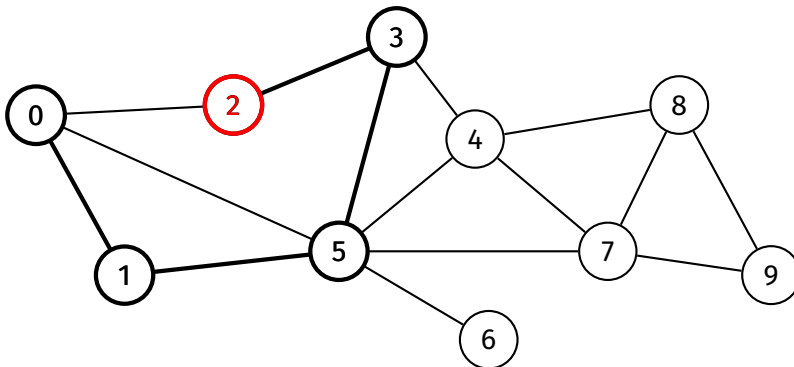
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

visit order 0 1 5 3

dfs(2)
dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

Mark 2 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

visit order 0 1 5 3 2

dfs(2)

dfs(3)

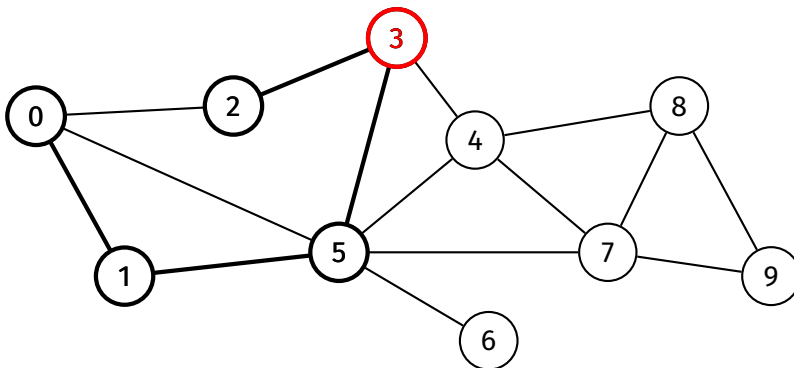
dfs(5)

dfs(1)

dfs(0)

call stack

Return



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

visit order 0 1 5 3 2

dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

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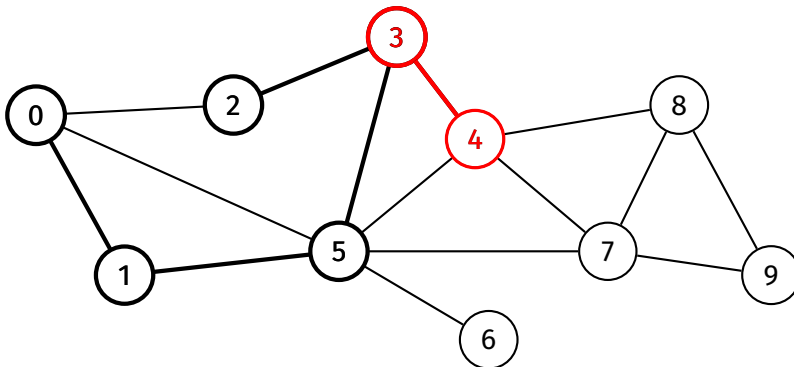
DFS

DFS Example

Path-Checking

Example

4 has not been visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

visit order 0 1 5 3 2

dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

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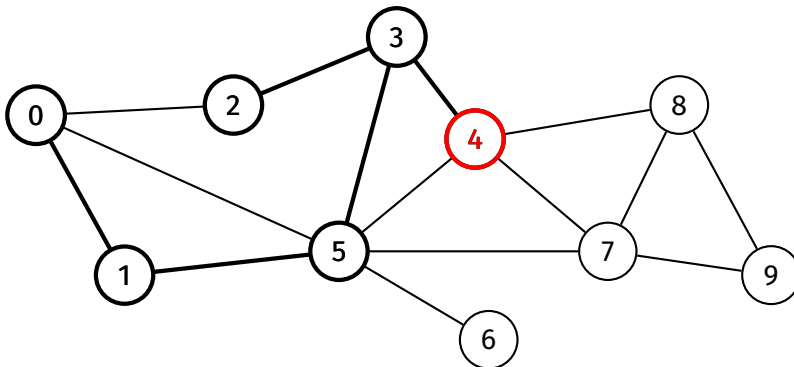
DFS

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Recurse into 4



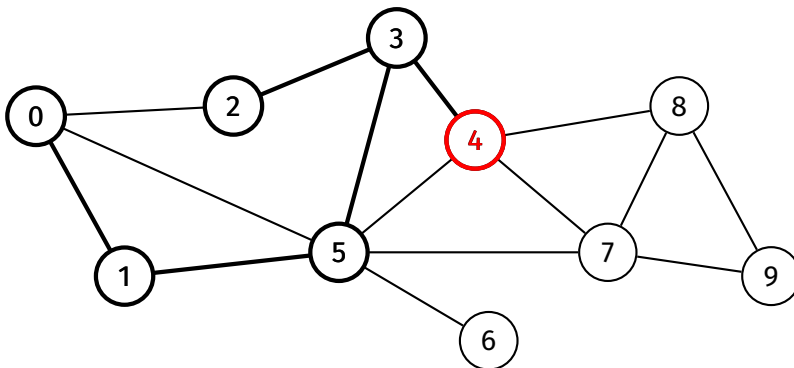
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

visit order 0 1 5 3 2

dfs(4)
dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

Mark 4 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0

visit order 0 1 5 3 2 4

dfs(4)
dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

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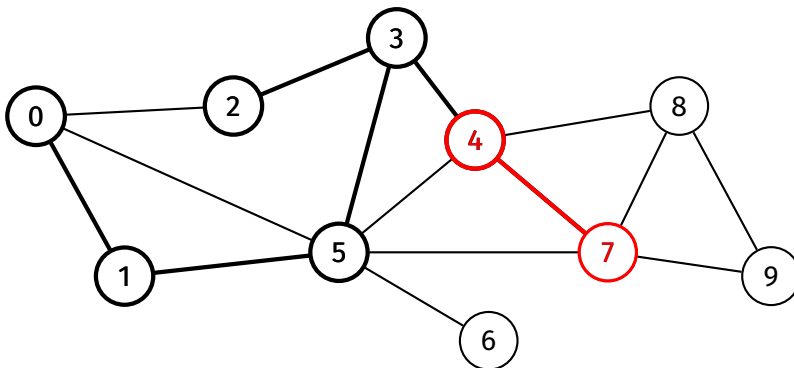
DFS

DFS Example

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Example

7 has not been visited



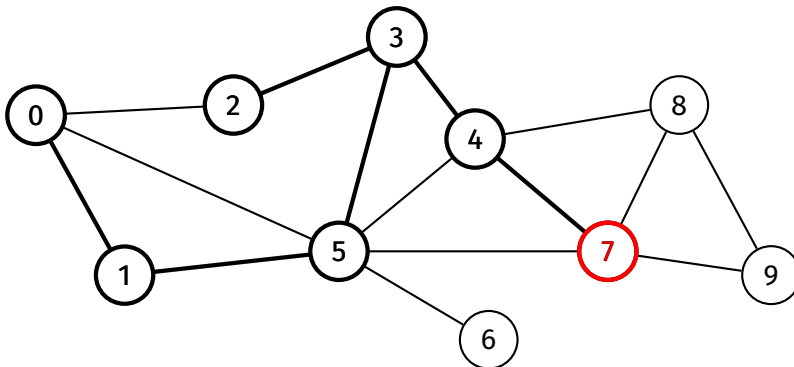
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0

visit order 0 1 5 3 2 4

dfs(4)
dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

Recurse into 7



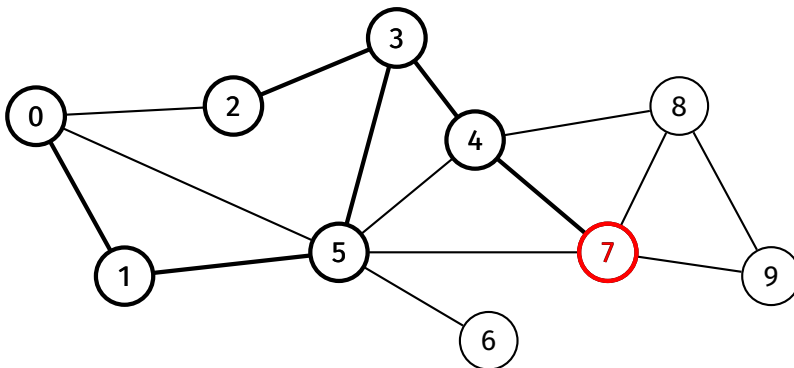
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0

visit order 0 1 5 3 2 4

dfs(7)
dfs(4)
dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

Mark 7 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

visit order 0 1 5 3 2 4 7

dfs(7)
dfs(4)
dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

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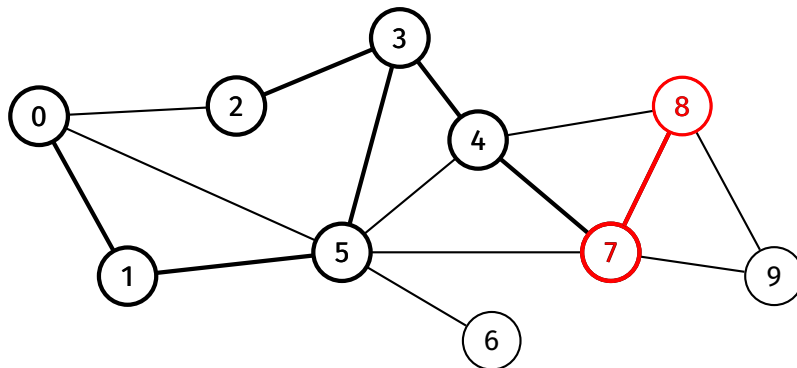
DFS

DFS Example

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Example

8 has not been visited



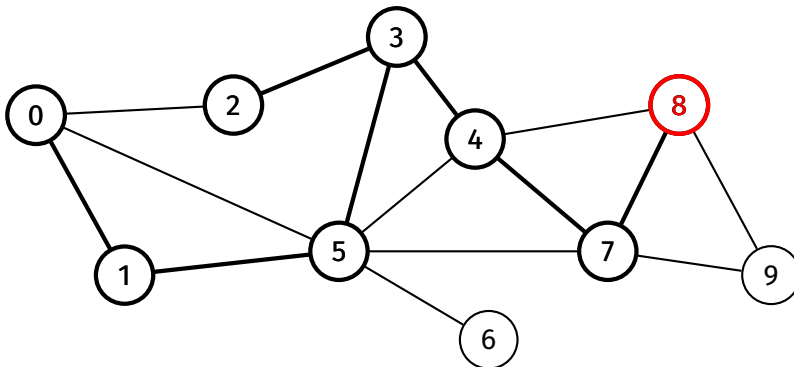
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

visit order 0 1 5 3 2 4 7

dfs(7)
dfs(4)
dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

Recurse into 8



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

visit order 0 1 5 3 2 4 7

dfs(8)

dfs(7)

dfs(4)

dfs(3)

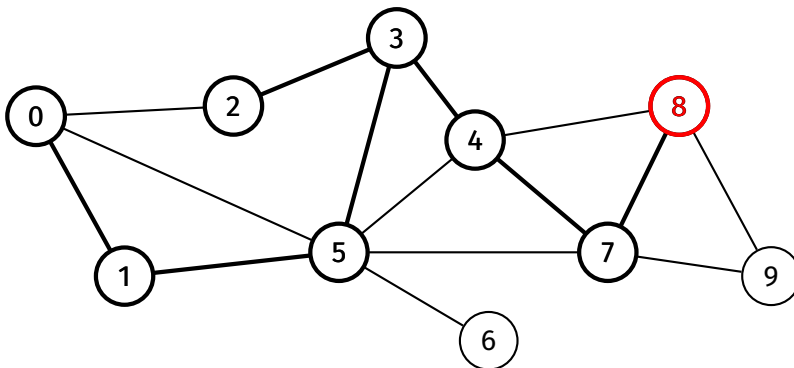
dfs(5)

dfs(1)

dfs(0)

call stack

Mark 8 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	0

visit order 0 1 5 3 2 4 7 8

dfs(8)

dfs(7)

dfs(4)

dfs(3)

dfs(5)

dfs(1)

dfs(0)

call stack

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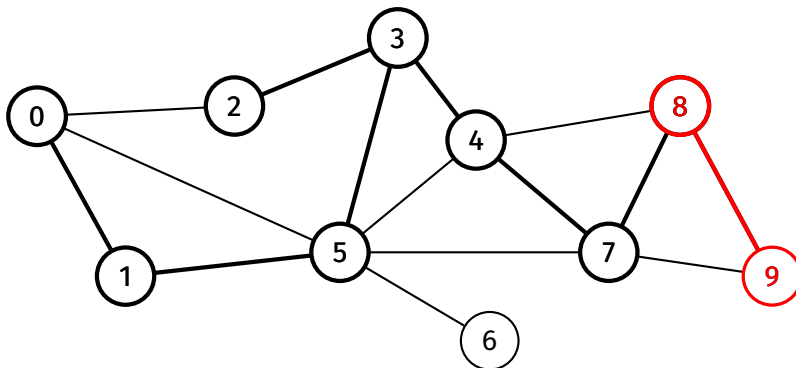
DFS

DFS Example

Path-Checking

Example

9 has not been visited



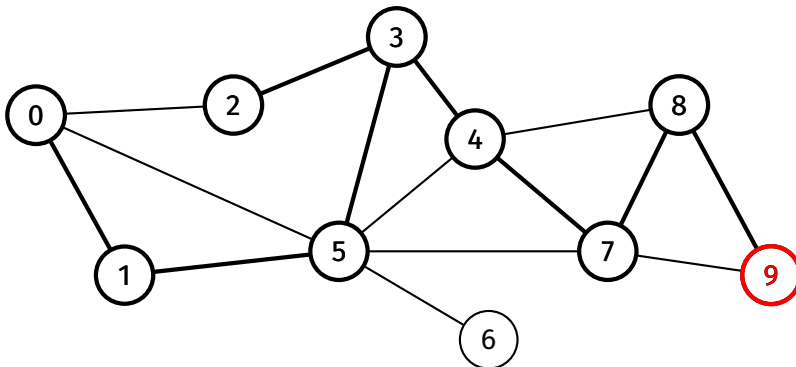
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	0

visit order 0 1 5 3 2 4 7 8

dfs(8)
dfs(7)
dfs(4)
dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

Recurse into 9



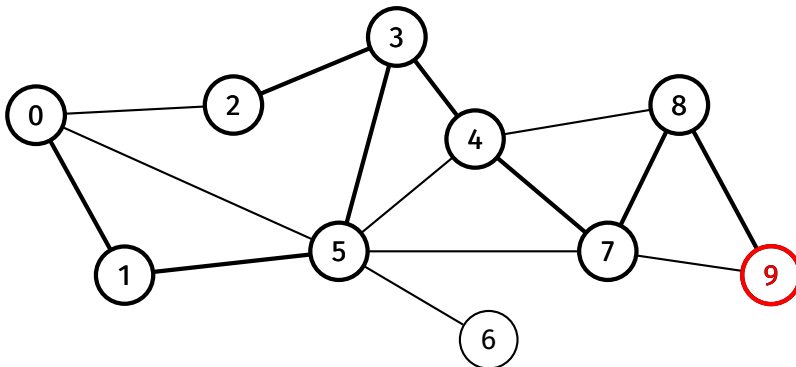
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	0

visit order 0 1 5 3 2 4 7 8

dfs(9)
dfs(8)
dfs(7)
dfs(4)
dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

Mark 9 as visited



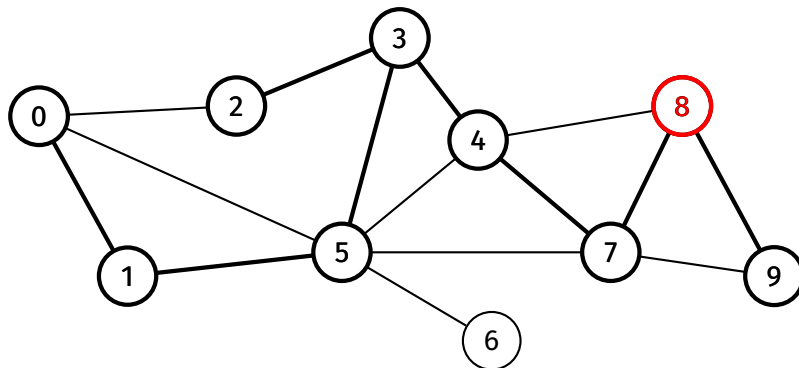
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

dfs(9)
dfs(8)
dfs(7)
dfs(4)
dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

Return



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

dfs(8)

dfs(7)

dfs(4)

dfs(3)

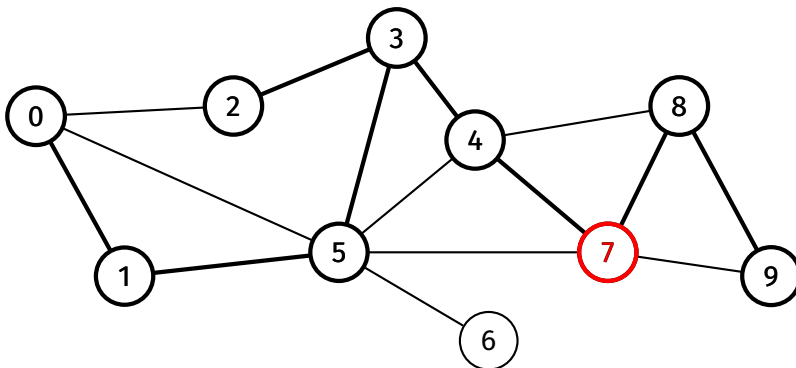
dfs(5)

dfs(1)

dfs(0)

call stack

Return



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

dfs(7)

dfs(4)

dfs(3)

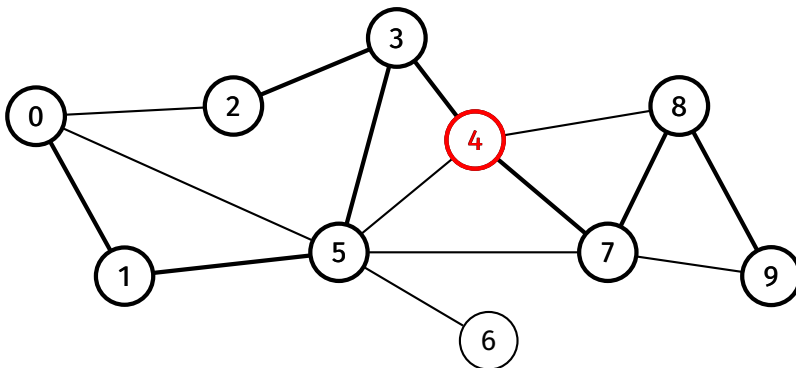
dfs(5)

dfs(1)

dfs(0)

call stack

Return



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

dfs(4)

dfs(3)

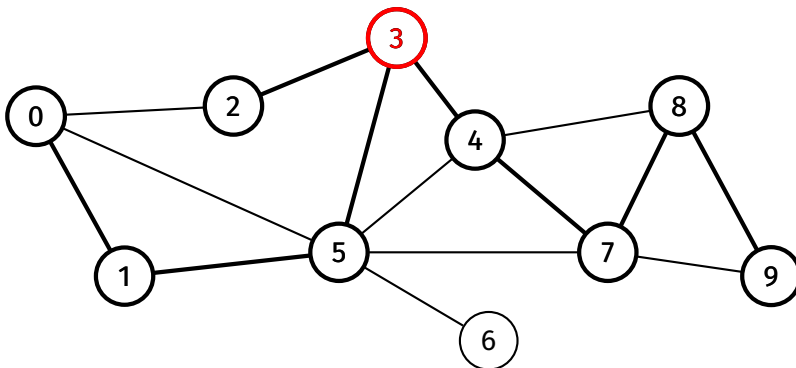
dfs(5)

dfs(1)

dfs(0)

call stack

Return



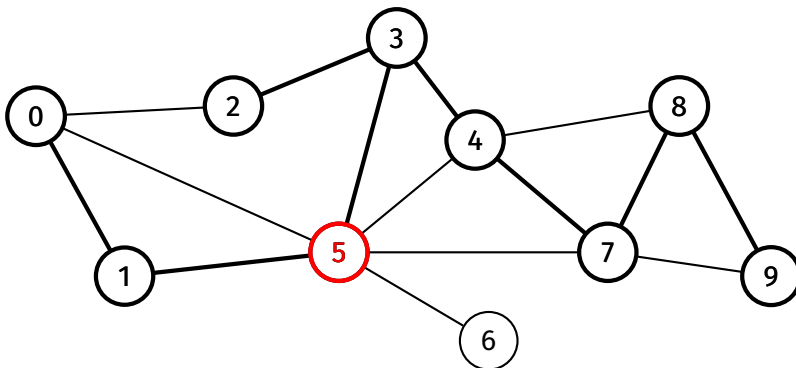
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

dfs(3)
dfs(5)
dfs(1)
dfs(0)

call stack

Return



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

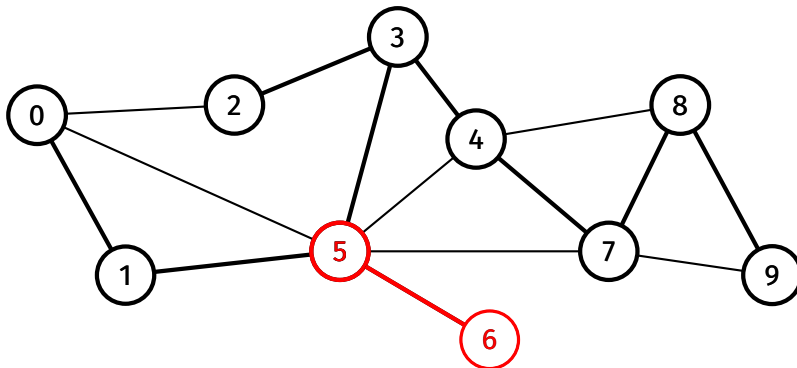
dfs(5)

dfs(1)

dfs(0)

call stack

6 has not been visited



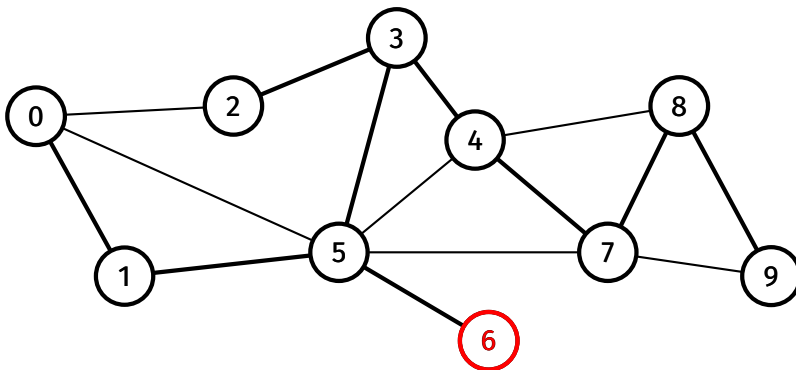
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

dfs(5)
dfs(1)
dfs(0)

call stack

Recurse into 6



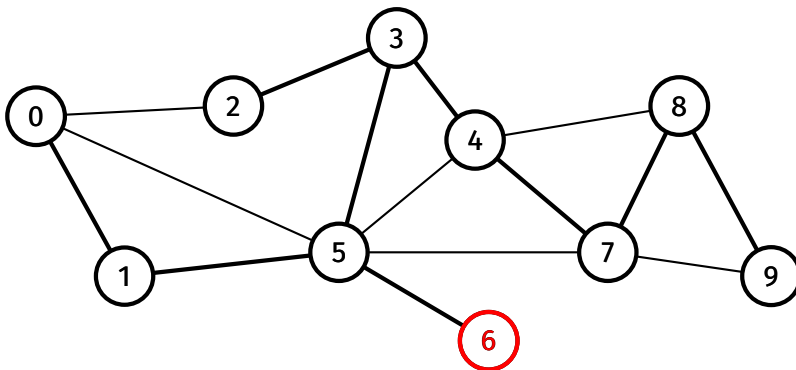
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

dfs(6)
dfs(5)
dfs(1)
dfs(0)

call stack

Mark 6 as visited



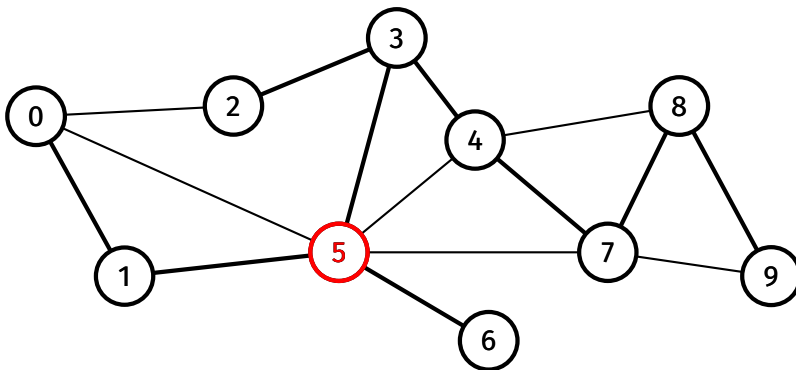
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

visit order 0 1 5 3 2 4 7 8 9 6

dfs(6)
dfs(5)
dfs(1)
dfs(0)

call stack

Return



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

visit order 0 1 5 3 2 4 7 8 9 6

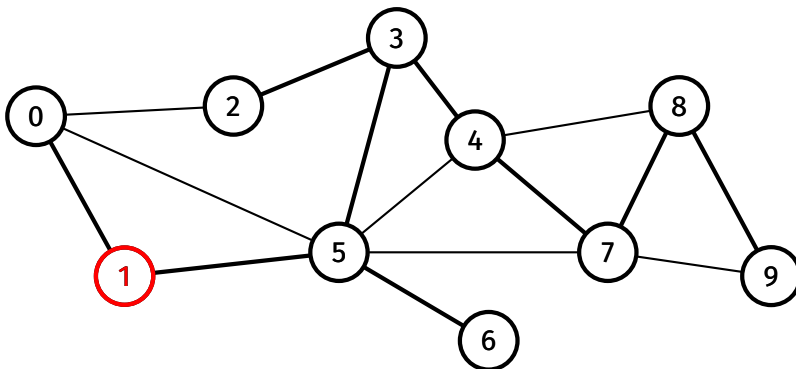
dfs(5)

dfs(1)

dfs(0)

call stack

Return



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

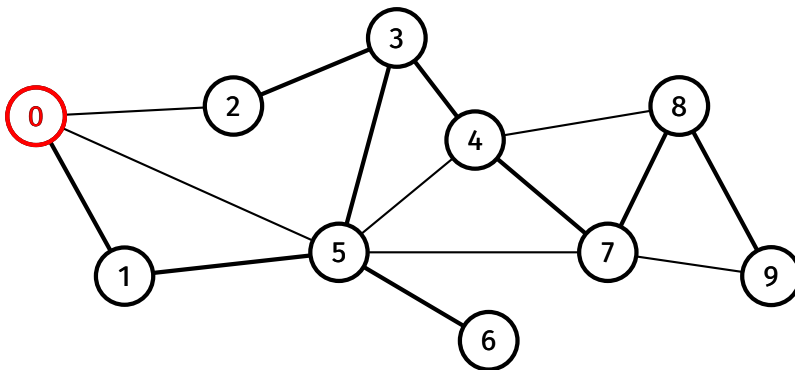
visit order 0 1 5 3 2 4 7 8 9 6

dfs(1)

dfs(0)

call stack

Return



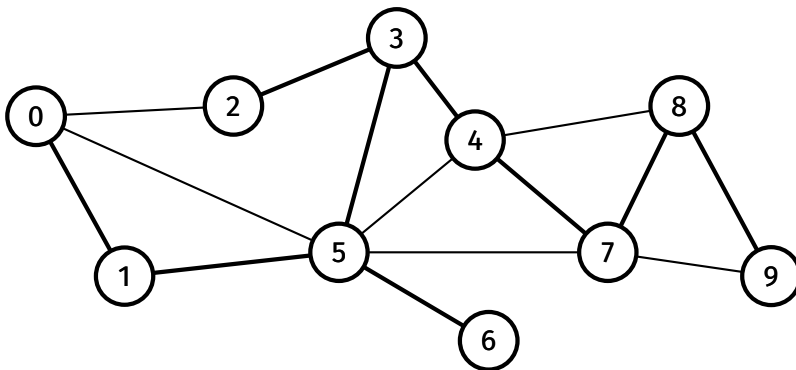
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

visit order 0 1 5 3 2 4 7 8 9 6

dfs(0)

call stack

Return

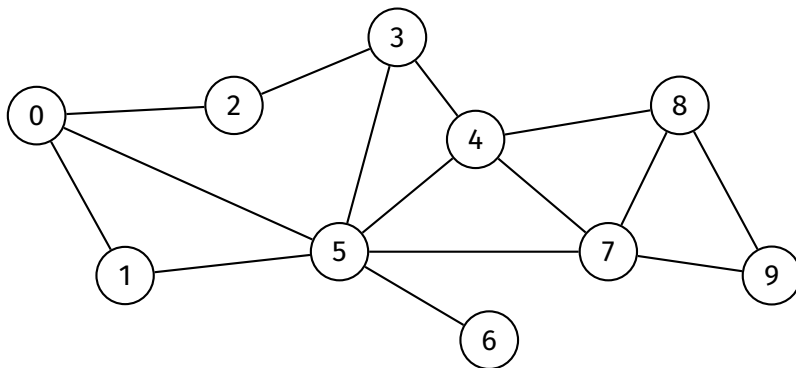


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

visit order 0 1 5 3 2 4 7 8 9 6

call stack

Is there a path between 0 and 7?



call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0

Graph
Traversal

BFS

DFS

Ideas/Issues

Appendix

BFS

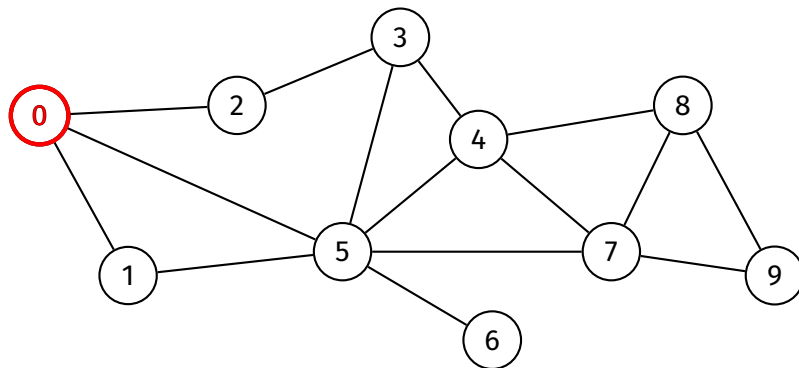
BFS Example

DFS

DFS Example

Path-Checking

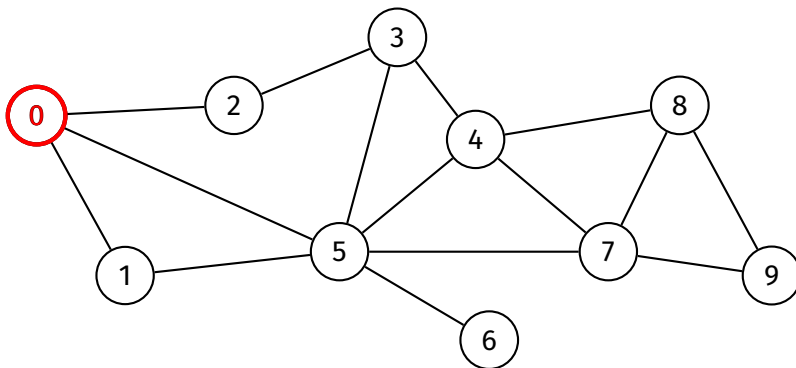
Example

**path(0, 7)?**

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0

Mark 0 as visited

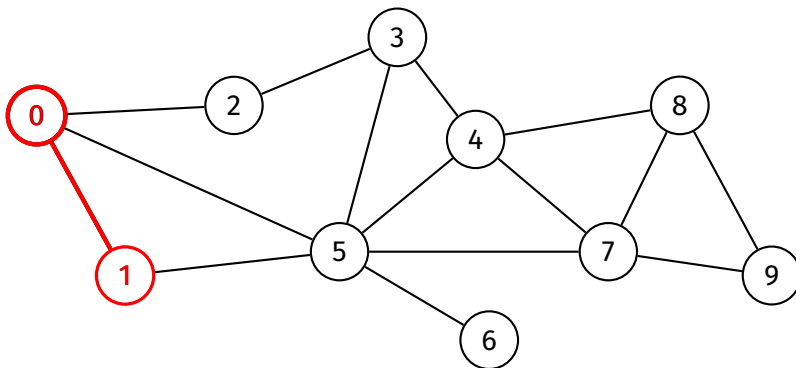


path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

1 has not been visited

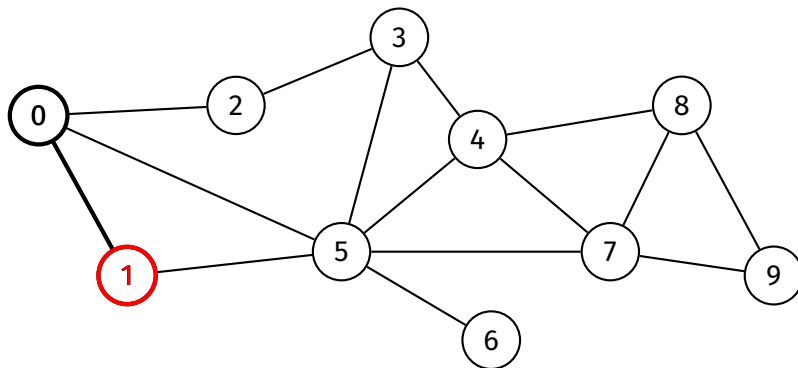


path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

Recurse into 1



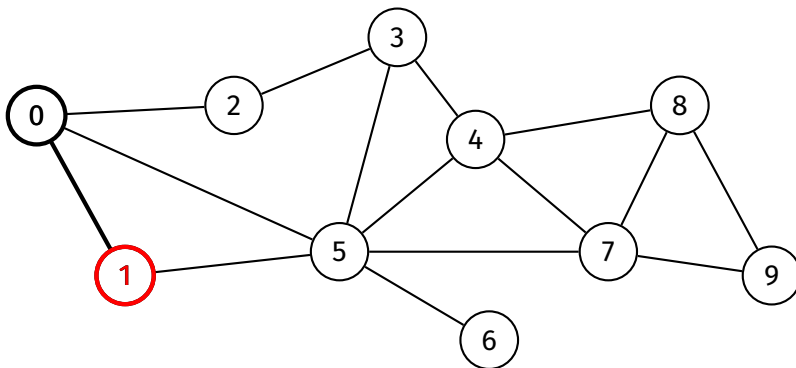
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

Mark 1 as visited



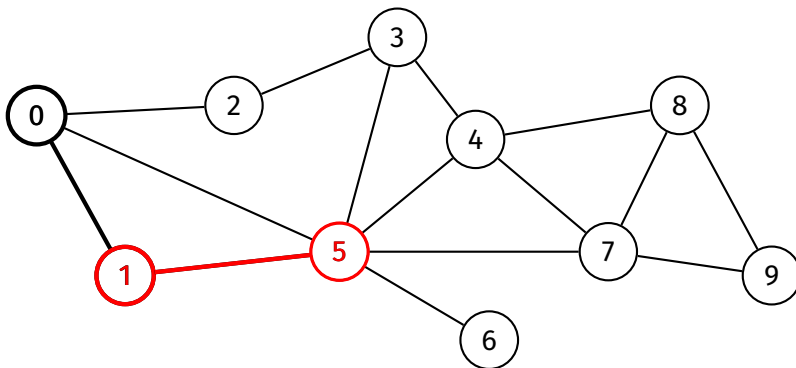
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

5 has not been visited



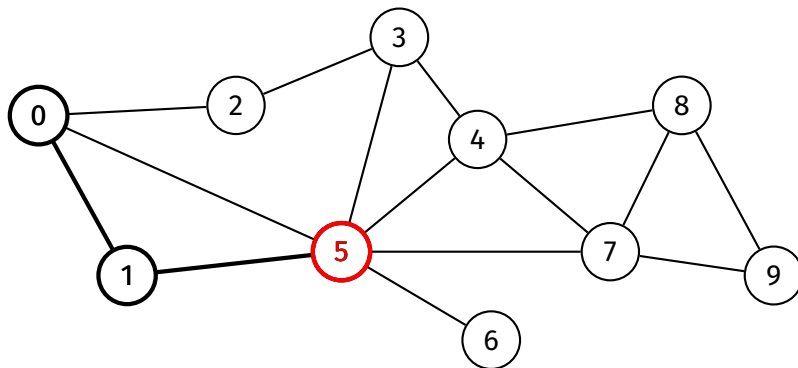
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

Recurse into 5



path(5, 7)?

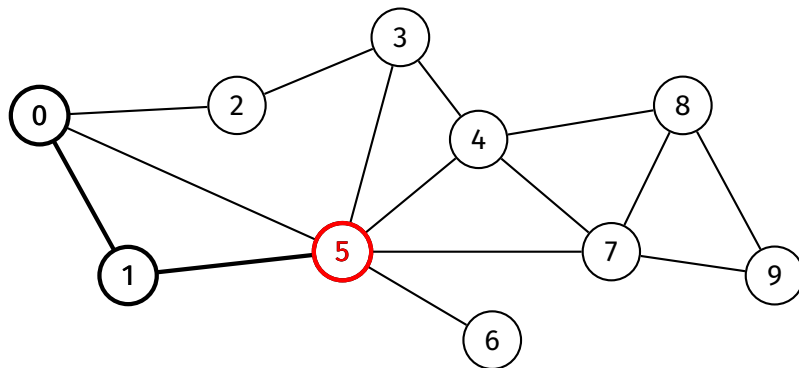
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

Mark 5 as visited



path(5, 7)?

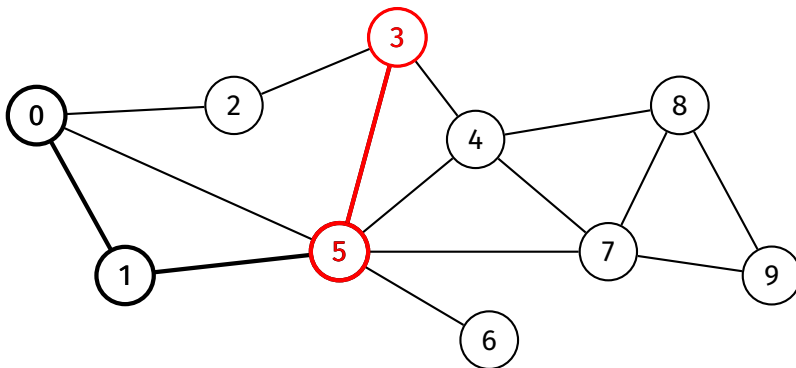
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

3 has not been visited



path(5, 7)?

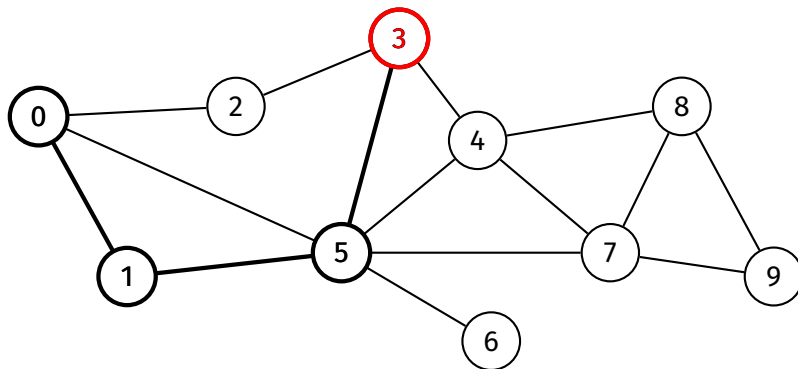
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

Recurse into 3



path(3, 7)?

path(5, 7)?

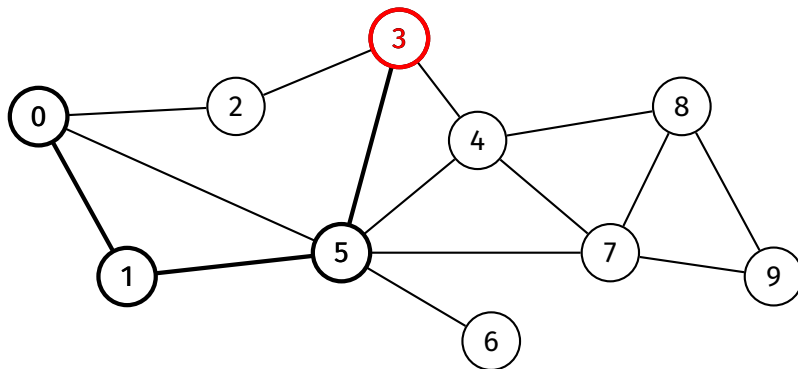
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

Mark 3 as visited



path(3, 7)?

path(5, 7)?

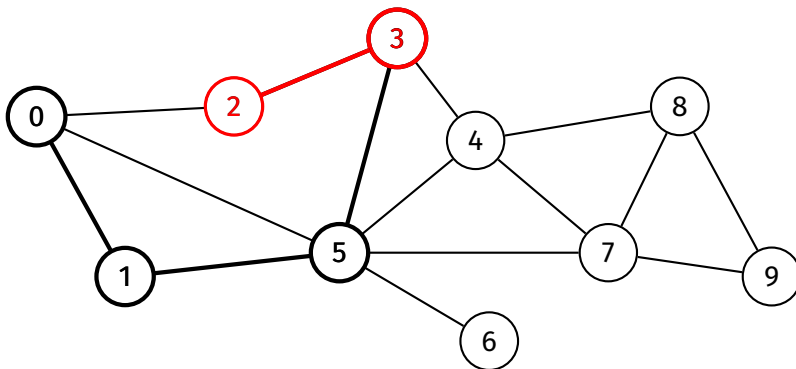
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

2 has not been visited



path(3, 7)?

path(5, 7)?

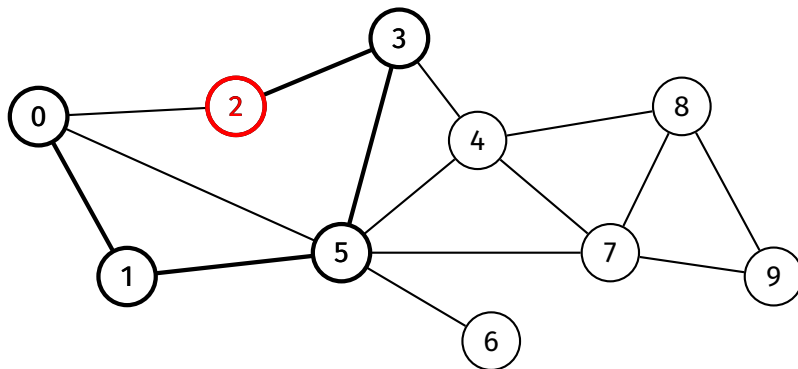
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

Recurse into 2



path(2, 7)?

path(3, 7)?

path(5, 7)?

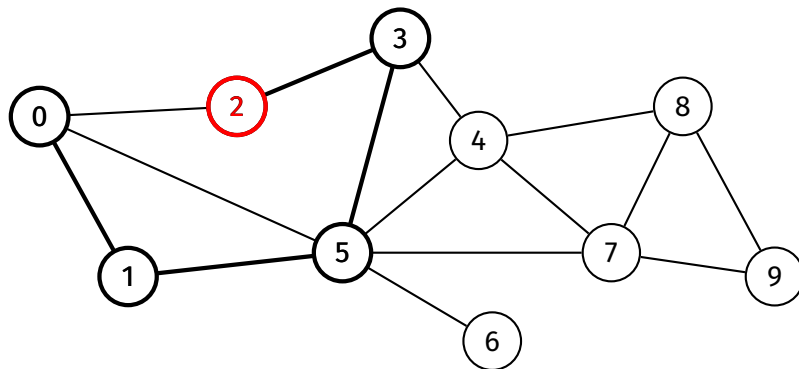
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

Mark 2 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

path(2, 7)?

path(3, 7)?

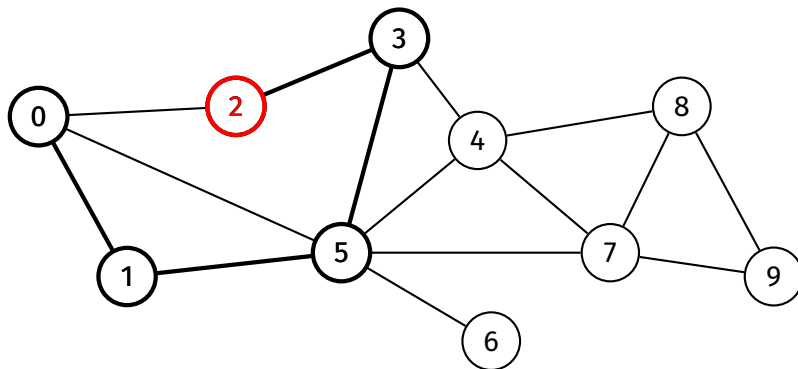
path(5, 7)?

path(1, 7)?

path(0, 7)?

call stack

Return false



path(2, 7)?

path(3, 7)?

path(5, 7)?

path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

Graph
Traversal

BFS

DFS

Ideas/Issues

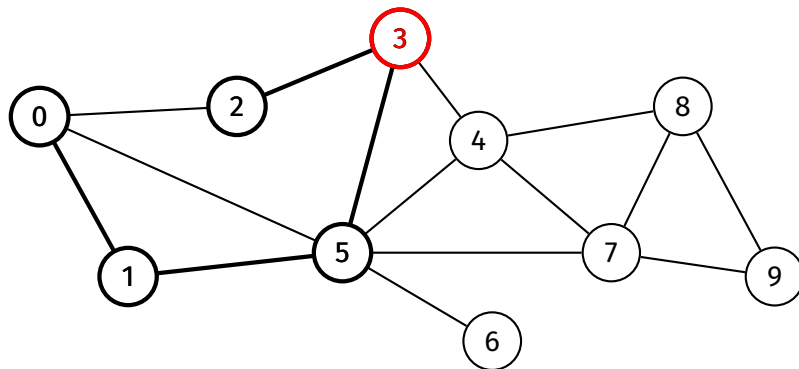
Appendix

BFS

BFS Example

DFS

DFS Example

Path-Checking
Example

path(3, 7)?

path(5, 7)?

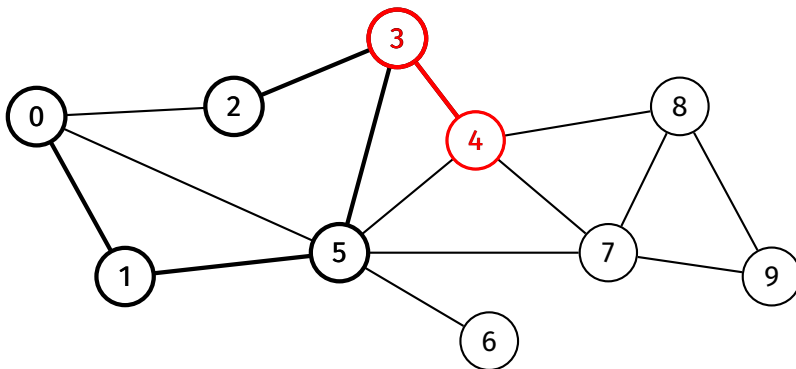
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

4 has not been visited



path(3, 7)?

path(5, 7)?

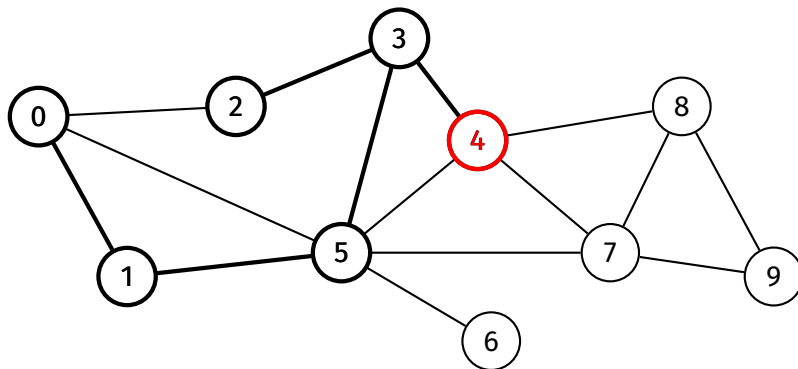
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

Recurse into 4



path(4, 7)?

path(3, 7)?

path(5, 7)?

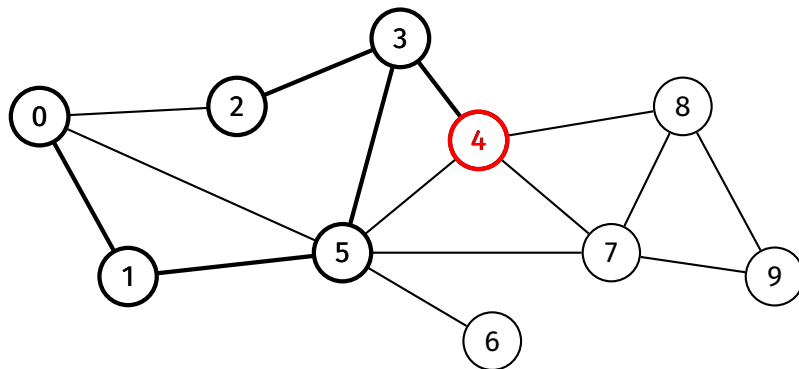
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

Mark 4 as visited



path(4, 7)?

path(3, 7)?

path(5, 7)?

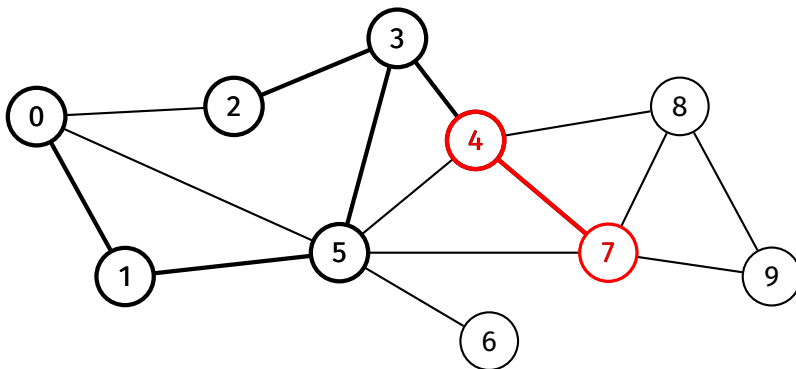
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0

7 has not been visited



path(4, 7)?

path(3, 7)?

path(5, 7)?

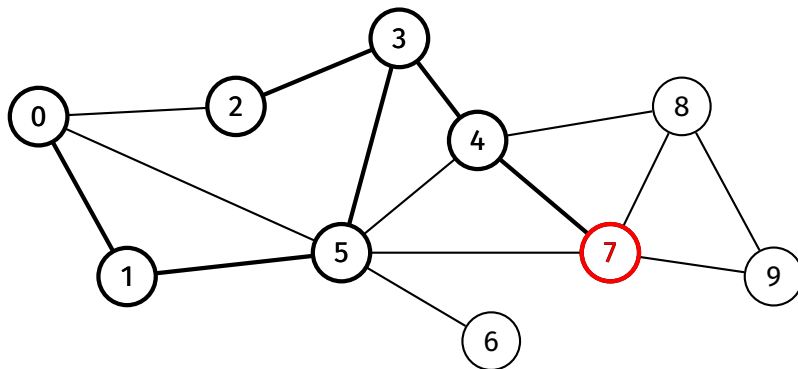
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0

Recurse into 7



path(7, 7)?

path(4, 7)?

path(3, 7)?

path(5, 7)?

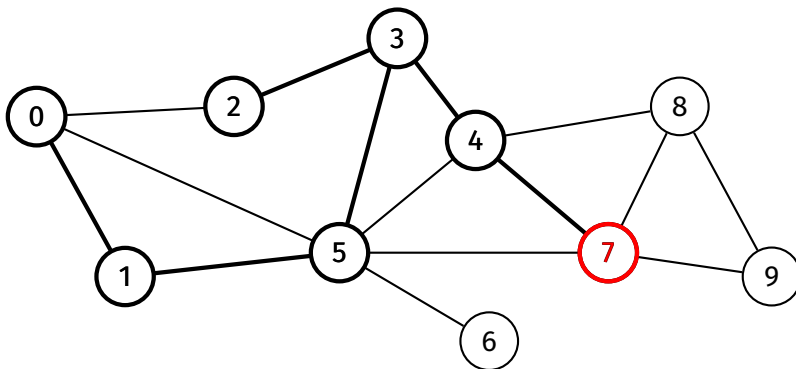
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0

Mark 7 as visited



path(7, 7)?

path(4, 7)?

path(3, 7)?

path(5, 7)?

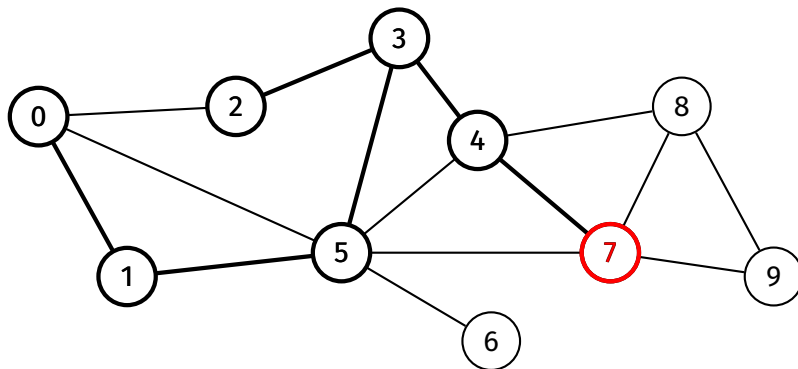
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

Return true



path(7, 7)?

path(4, 7)?

path(3, 7)?

path(5, 7)?

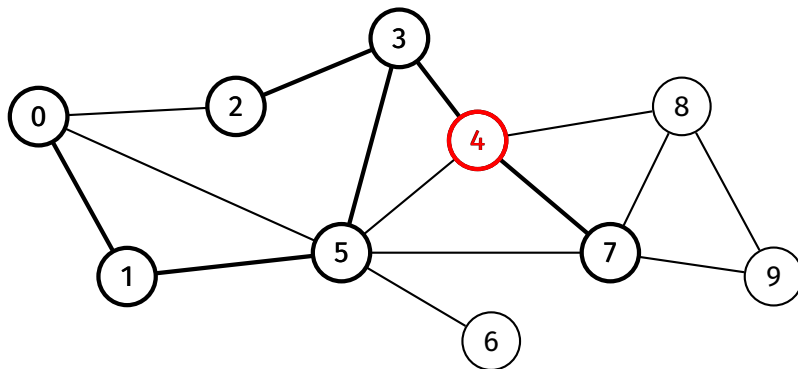
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

Return true



path(4, 7)?

path(3, 7)?

path(5, 7)?

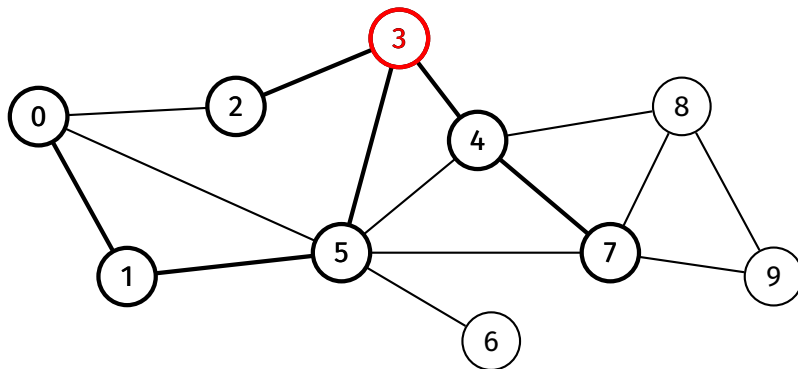
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

Return true



path(3, 7)?

path(5, 7)?

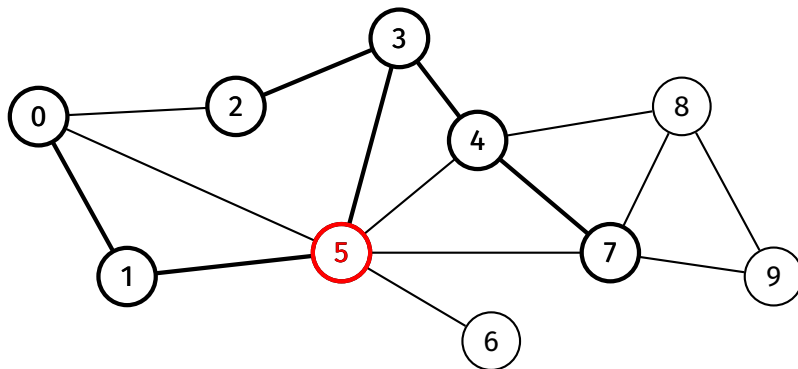
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

Return true



path(5, 7)?

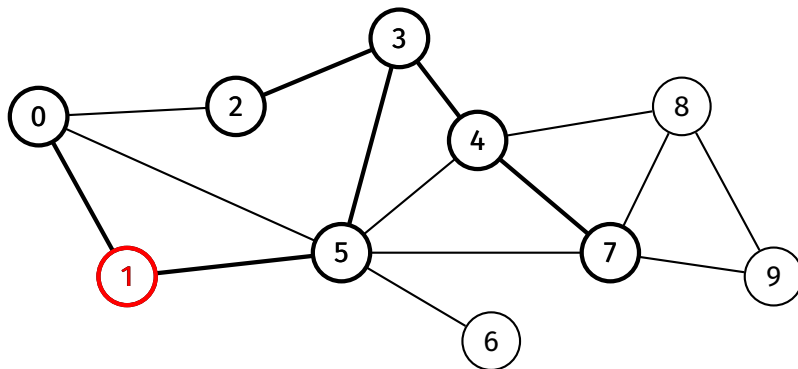
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

Return true



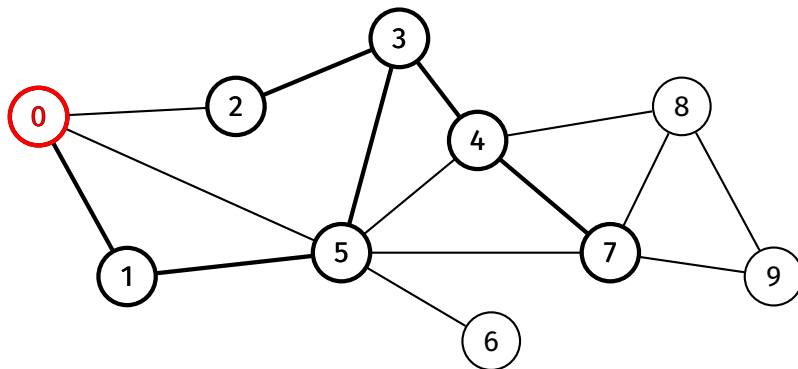
path(1, 7)?

path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

Return true

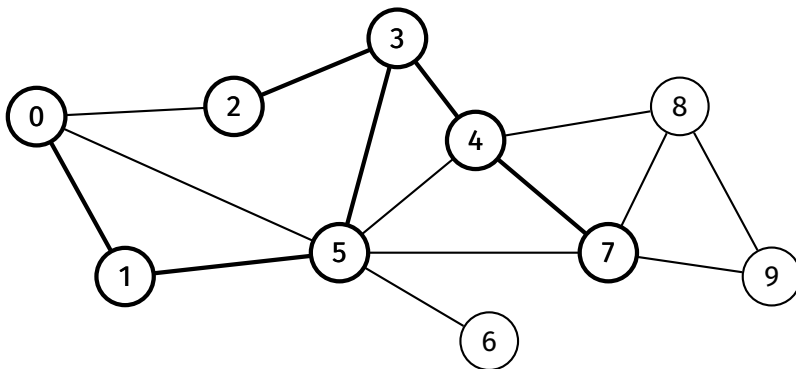


path(0, 7)?

call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

Answer: Yes



call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0