Selection Sort Bubble Sort Insertion Sort Summary Sorting Lists Appendix

COMP2521 25T2

Sorting Algorithms (II) Elementary Sorting Algorithms

Sim Mautner

cs2521@cse.unsw.edu.au

selection sort bubble sort insertion sort

Implementation
Analysis
Properties

Bubble Sort
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Method:

- Find the smallest element, swap it with the first element
- Find the second-smallest element, swap it with the second element
- ..
- Find the second-largest element, swap it with the second-last element

Each iteration improves the "sortedness" of the array by one element.

Example

Analysis Properties

Bubble Sort

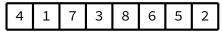
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Example

Selection Sort

Example

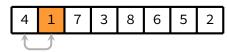
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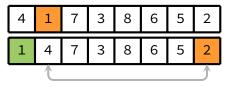
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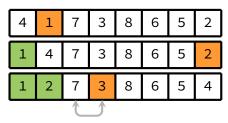
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Example

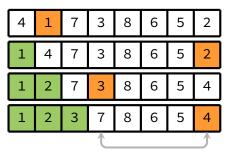
Analysis

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4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	2	7	3	8	6	5	4
1	2	3	7	8	6	5	4
1	2	3	4	8	6	5	7
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4 1 7 3 8 6 5 2 1 4 7 3 8 6 5 2 1 2 7 3 8 6 5 4 1 2 3 7 8 6 5 4 1 2 3 4 8 6 5 7 1 2 3 4 5 6 8 7		· ·						
1 4 7 3 8 6 5 2 1 2 7 3 8 6 5 4 1 2 3 7 8 6 5 4	1	2	3	4	5	6	8	7
1 4 7 3 8 6 5 2 1 2 7 3 8 6 5 4	1	2	3	4	8	6	5	7
1 4 7 3 8 6 5 2	1	2	3	7	8	6	5	4
	1	2	7	3	8	6	5	4
4 1 7 3 8 6 5 2	1	4	7	3	8	6	5	2
	4	1	7	3	8	6	5	2

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4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	2	7	3	8	6	5	4
1	2	3	7	8	6	5	4
1	2	3	4	8	6	5	7
1	2	3	4	5	6	8	7
1	2	3	4	5	6	8	7
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4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	2	7	3	8	6	5	4
1	2	3	7	8	6	5	4
1	2	3	4	8	6	5	7
1	2	3	4	5	6	8	7
1	2	3	4	5	6	8	7
1	2	3	4	5	6	7	8

C Implementation

```
Selection Sort
Implementation
```

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```
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```
void selectionSort(Item items[], int lo, int hi) {
    for (int i = lo; i < hi; i++) {</pre>
        int min = i;
        for (int j = i + 1; j <= hi; j++) {
            if (lt(items[j], items[min])) {
                min = j;
        swap(items, i, min);
```

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Cost analysis:

- ullet In the first iteration, n-1 comparisons, 1 swap
- In the second iteration, n-2 comparisons, 1 swap
- ...
- In the final iteration, 1 comparison, 1 swap
- $C = (n-1) + (n-2) + \ldots + 1 = \frac{1}{2}n(n-1) \Rightarrow O(n^2)$
- S = n 1

Cost is the same, regardless of the sortedness of the original array.

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Selection sort is unstable

- Due to long-range swaps
- For example, sort these cards by value:







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Unstable

Due to long-range swaps

Non-adaptive

Performs same steps, regardless of sortedness of original array

In-place

Sorting is done within original array; does not use temporary arrays

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Method:

- Make multiple passes from left (lo) to right
- On each pass, swap any out-of-order adjacent pairs
- Elements "bubble up" until they meet a larger element
- Stop if there are no swaps during a pass
 - This means the array is sorted

Bubble Sort

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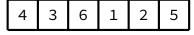
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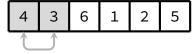
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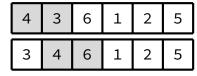
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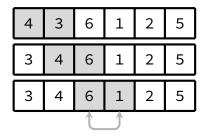
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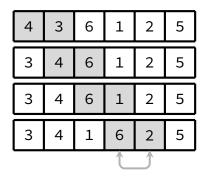
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Bubble Sort Example

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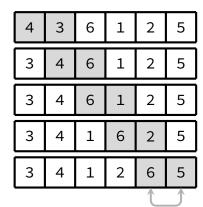
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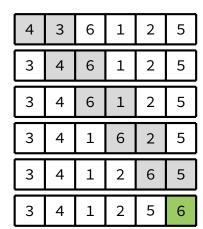
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Second pass

3 4 1 2 5 6

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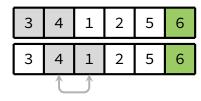
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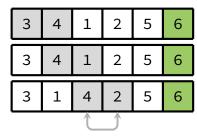
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Bubble Sort Example

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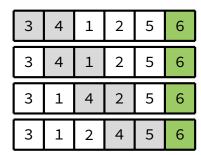
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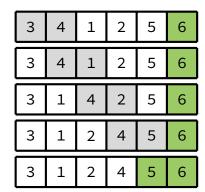
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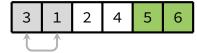
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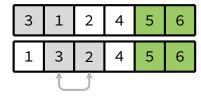
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Bubble Sort

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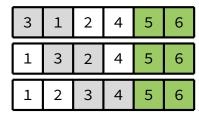
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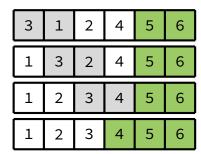
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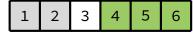
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Fourth pass



Bubble Sort

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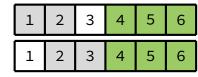
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Fourth pass



Example

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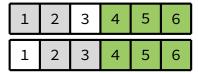
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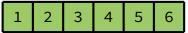
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Fourth pass



No swaps made; stop



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```
C Implementation
```

```
void bubbleSort(Item items[], int lo, int hi) {
    for (int i = hi; i > lo; i--) {
        bool swapped = false;
        for (int i = lo; j < i; j++) {
            if (gt(items[i], items[i + 1])) {
                swap(items, j, j + 1);
                swapped = true;
        if (!swapped) break;
```

Bubble Sort Analysis

Selection Sort

Bubble Sort Analysis

Insertion Sort

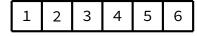
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Best case: Array is sorted

- Only a single pass required
- n-1 comparisons, no swaps
- Best-case time complexity: O(n)



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Worst case: Array is reverse-sorted

- n-1 passes required
 - First pass: n-1 comparisons
 - Second pass: n-2 comparisons
 - ..
 - Final pass: 1 comparison
- Total comparisons: $(n-1) + (n-2) + ... + 1 = \frac{1}{2}n(n-1)$
- Every comparison leads to a swap $\Rightarrow \frac{1}{2}n(n-1)$ swaps
- Worst-case time complexity: $O(n^2)$

6 5	4	3	2	1
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Bubble Sort

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Average-case time complexity: $O(n^2)$

- It can be proven that for a randomly ordered array, bubble sort needs to perform $\frac{1}{4}n(n-1)$ swaps on average $\Rightarrow O(n^2)$
 - See appendix for details
- Can show empirically by generating random sequences and sorting them

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Stable

Comparisons are between adjacent elements only Elements are only swapped if out of order

Adaptive

Bubble sort is $O(n^2)$ on average, O(n) if input array is sorted

In-place

Sorting is done within original array; does not use temporary arrays

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Method:

- Take first element and treat as sorted array (of length 1)
- Take next element and insert into sorted part of array so that order is preserved
 - This increases the length of the sorted part by one
- Repeat for remaining elements

Insertion Sort

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Selection Sort

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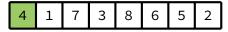
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Insertion Sort

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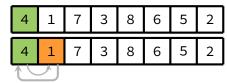
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Insertion Sort

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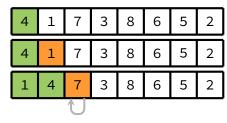
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4	1	7	3	8	6	5	2
4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	4	7	3	8	6	5	2

Selection Sort

Bubble Sort

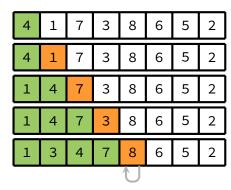
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4	1	7	3	8	6	5	2
4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	3	4	7	8	6	5	2
1	3	4	7	8	6	5	2

Selection Sort

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4	1	7	3	8	6	5	2
•							_
4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	3	4	7	8	6	5	2
1	3	4	7	8	6	5	2
1	3	4	6	7	8	5	2
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Selection Sort

Bubble Sort

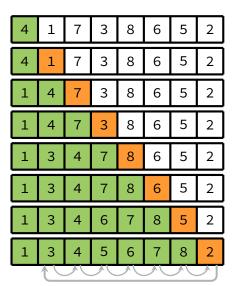
Insertion Sort

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Insertion Sort

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4	1	7	3	8	6	5	2
4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	3	4	7	8	6	5	2
1	3	4	7	8	6	5	2
1	3	4	6	7	8	5	2
1	3	4	5	6	7	8	2
1	2	3	4	5	6	7	8

Insertion Sort

C Implementation

Selection Sort **Bubble Sort**

Insertion Sort

Implementation Analysis

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Sorting Lists

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int i = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        items[j] = item;
```

Insertion Sort Analysis

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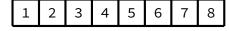
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Sorting Lists

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Best case: Array is sorted

- Inserting each element requires one comparison
- n-1 comparisons
- Best-case time complexity: O(n)



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Worst case: Array is reverse-sorted

- Inserting *i*-th element requires *i* comparisons
 - Inserting index 1 element requires 1 comparison
 - Inserting index 2 element requires 2 comparisons
 - ..
- Total comparisons: $1 + 2 + ... + (n-1) = \frac{1}{2}n(n-1)$
- Worst-case time complexity: $O(n^2)$

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Average-case time complexity: $O(n^2)$

- Same reason as for bubble sort
- Can show empirically by generating random sequences and sorting them

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Stable

Elements are always inserted to the right of any equal elements

Adaptive

Insertion sort is $\mathcal{O}(n^2)$ on average, $\mathcal{O}(n)$ if input array is sorted

In-place

Sorting is done within original array; does not use temporary arrays

Summary of Elementary Sorts

Selection Sort Bubble Sort

Insertion Sort

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	Tin	ne complex	Prop	perties	
	Best	Average	Worst	Stable	Adaptive
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	No	No
Bubble sort	O(n)	$O(n^2)$	$O(n^2)$	Yes	Yes
Insertion sort	O(n)	$O(n^2)$	$O(n^2)$	Yes	Yes

Aside: Sorting Linked Lists

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Selection sort:

- Let L = original list, S = sorted list (initially empty)
- Repeat the following until L is empty:
 - Find the node V containing the largest value in L, and unlink it
 - Insert V at the front of S

Bubble sort:

- Traverse the list, comparing adjacent values
 - If value in current node is greater than value in next node, swap values
- Repeat the above until no swaps required in one traversal

Insertion sort:

- Let L = original list, S = sorted list (initially empty)
- For each node in *L*:
 - Insert the node into S in order

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Bubble sort average case Insertion sort walkthrough

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Bubble Sort

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Bubble sort avera

Insertion sort

Note: Not required knowledge in COMP2521!

New concept: inversion

An inversion is a pair of elements from a sequence where the left element is greater than the right element.

For example, consider the following array:

The array contains 5 inversions: (4, 2), (4, 1), (4, 3), (2, 1), (5, 3)

Bubble Sort - Proof of $\mathcal{O}(n^2)$ Average Case

Continued

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Bubble sort average

Insertion sort walkthrough

Observation:

• In bubble sort, every swap reduces the number of inversions by 1

The goal of the proof: Show that the average number of inversions in a randomly sorted array is $O(n^2)$.

- This implies the number of swaps required by bubble sort is $O(n^2)$...
- Which implies that the average-case time complexity of bubble sort is $O(n^2)$ or slower
 - (but we know that it can't be slower than $O(n^2)$ since the worst-case time complexity of bubble sort is $O(n^2)$)

Continued

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Bubble sort average

case Insertion sort

In a randomly sorted array:

- The minimum possible number of inversions is 0 (sorted array)
- The maximum possible number of inversions is $\frac{1}{2}n(n-1)$ (reverse-sorted array)

Continued

Selection Sort

Insertion Sort

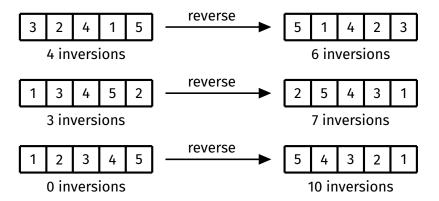
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Bubble sort average case

Let k be the number of inversions in a random permutation. By reversing this permutation, one can obtain a permutation with $\frac{1}{2}n(n-1)-k$ inversions.

For example, suppose n=5:



Continued

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Bubble sort average case

Thus, if we take all the possible permutations of an array and pair each permutation with its reverse, the total number of inversions in each pair is $\frac{1}{2}n(n-1)$.

This implies that the average number of inversions across all permutations is $\frac{1}{4}n(n-1)$, which is $O(n^2)$.

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Bubble sort aver

Insertion sort walkthrough Sort the following array:

4 2 1 5 3

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Bubble sort average case

Insertion sort walkthrough

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

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Bubble sort averagese

Insertion sort

walkthrough

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

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```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

item

Selection Sort
Bubble Sort
Insertion Sort

Summary

Sorting Lists

Appendix

Bubble sort avera
case
Insertion sort
walkthrough

```
[0] [1] [2] [3] [4]
lo hi
4 2 1 5 3 2 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort **Bubble Sort**

Insertion Sort Summary

Sorting Lists

walkthrough

```
[0]
      \lceil 1 \rceil
              [2] [3]
                            [4]
                             hi
lo
                              3
                                                  item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[i] = items[i - 1];
        items[i] = item;
```

Selection Sort Bubble Sort

Bubble Sort Insertion Sort

Summary
Sorting Lists

Appendix

Bubble sort average case

Insertion sort walkthrough

```
[0] [1] [2] [3] [4]

lo hi

4 4 1 5 3 2 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
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            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

Appendix

Bubble sort average case

Insertion sort
walkthrough

```
[0] [1] [2] [3] [4]

lo hi

4 4 1 5 3 2 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort
Bubble Sort
Insertion Sort

Summary

Sorting Lists

```
[0] [1] [2] [3] [4]

lo hi

2 4 1 5 3 2 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort
Bubble Sort
Insertion Sort

Summary

Sorting Lists

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort

Summary
Sorting Lists

```
[0] [1] [2] [3] [4]

lo hi

2 4 1 5 3 1 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

```
[0] [1] [2] [3] [4]

lo hi

2 4 1 5 3 1 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
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        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

```
[0] [1] [2] [3] [4]

lo hi

2 4 1 5 3 1 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort
Bubble Sort
Insertion Sort

Summary

Sorting Lists

```
[0] [1] [2] [3] [4]
lo hi
2 4 4 5 3 1 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

```
[0] [1] [2] [3] [4]

lo hi

2 4 4 5 3 1 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

```
[0] [1] [2] [3] [4]

lo hi

2 2 4 5 3 1 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

Appendix

Bubble sort average case

walkthrough

```
[0] [1] [2] [3] [4]
lo hi
2 2 4 5 3
j i
```

```
1 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

```
[0] [1] [2] [3] [4]

lo hi

2 2 4 5 3 1 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

```
[0] [1] [2] [3] [4]
lo hi

1 2 4 5 3 1 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
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        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

```
[0] [1] [2] [3] [4]

lo hi

1 2 4 5 3 5 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort
Bubble Sort
Insertion Sort

Summary

Sorting Lists

```
[0] [1] [2] [3] [4]
lo hi

1 2 4 5 3 5 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

Appendix

Bubble sort average case

walkthrough

```
[0] [1] [2] [3] [4]
lo hi

1 2 4 5 3 5 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort
Bubble Sort
Insertion Sort

Summarv

Sorting Lists

```
[0] [1] [2] [3] [4]
lo hi

1 2 4 5 3 5 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort
Bubble Sort
Insertion Sort

Summary

Sorting Lists

walkthrough

Appendix

Bubble sort average case

```
[0] [1] [2] [3] [4]

lo hi

1 2 4 5 3 5 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
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}
```

Selection Sort
Bubble Sort
Insertion Sort

Summary
Sorting Lists

Appendix

Bubble sort average case

walkthrough

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
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        }
        items[j] = item;
    }
}
```

Selection Sort
Bubble Sort
Insertion Sort

Summary

Sorting Lists

```
3 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

Appendix

Bubble sort average case

Insertion sort walkthrough

```
[0] [1] [2] [3] [4]
lo hi

1 2 4 5 3
i
j
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

item

Selection Sort Bubble Sort

Insertion Sort
Summary

Sorting Lists

```
[0] [1] [2] [3] [4]
lo hi

1 2 4 5 3 3 item
j
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort Bubble Sort

Insertion Sort

Sorting Lists

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
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        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort
Bubble Sort
Insertion Sort

Summarv

Sorting Lists

```
3 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
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        for (; j > lo && lt(item, items[j - 1]); j--) {
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        }
        items[j] = item;
    }
}
```

Selection Sort
Bubble Sort
Insertion Sort

Summarv

Sorting Lists

```
3 litem
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort
Bubble Sort
Insertion Sort

Summary

Sorting Lists

```
[0] [1] [2] [3] [4] hi hi  
1 2 4 4 5  
i j i
```

```
3 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
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}
```

Selection Sort Bubble Sort

Insertion Sort

Summary

Sorting Lists

```
3 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
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            items[j] = items[j - 1];
        }
        items[j] = item;
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}
```

Selection Sort
Bubble Sort
Insertion Sort

Summary

Sorting Lists

```
3 item
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
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        int j = i;
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        }
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Selection Sort
Bubble Sort
Insertion Sort

Summary

Sorting Lists

```
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            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Selection Sort
Bubble Sort
Insertion Sort

Summary

Sorting Lists

walkthrough

Appendix

Bubble sort averag
case

```
[0] [1] [2] [3] [4]
lo hi

1 2 3 4 5
```

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```