A Truncated SVD Approach for Fixed Complexity Spectrally Efficient FDM Receivers

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***Abstract*—Spectrally Efficient Frequency Division Multiplexing (SEFDM) systems aim to reduce the utilized spectrum by multiplexing non-orthogonal overlapped carriers. Since the per carrier transmission rate is maintained, SEFDM yields higher spectral efficiency relative to an equivalent Orthogonal Frequency Division Multiplexing (OFDM) system. Yet, due to the loss of the orthogonality, detection of the SEFDM system requires overly complex detectors. In this work, new SEFDM receivers that offer substantial complexity reduction with a competitive Bit Error Rate (BER) performance are presented. The Truncated Singular Value Decomposition (TSVD) is proposed as an efficient tool to overcome the ill conditioning of the system caused by the orthogonality collapse. The performance of the system with respect to the system size and spectrum saving is examined by extensive numerical simulations. It is shown that the TSVD detector outperforms linear detectors such as Zero Forcing (ZF) and Minimum Mean Squared Error (MMSE) detectors in terms of BER. Furthermore, a combination of TSVD with the Fixed Sphere Decoder (FSD) algorithm is proposed and tested for the first time. This novel FSD-TSVD receiver achieves near -optimum performance in terms of BER with a fixed and reduced complexity for systems with bandwidth savings of up to 40%.**

***Index Terms*—OFDM, spectral efficiency, SEFDM, truncated singular value decomposition, sphere decoder.**

I. Introduction

In the last decade, research in the communication systems area focused on the enhancement of spectrum utilization so that more bandwidth is accommodated to the users. In particular, recent efforts led to several multi carrier communication systems aiming to promote higher Spectral Efficiency (SE) than the well known Orthogonal Frequency Division Multiplexing (OFDM) systems. Early examples being the Fast OFDM (FOFDM) [1] and M-ary Amplitude Shift Keying (MASK) OFDM [2] offering half spectrum utilization, yet both are constrained to one dimensional modulation schemes such as BPSK and M-ary ASK. Moreover, Spectrally Efficient FDM (SEFDM) [3], High Compaction Multicarrier-Communication (HC-MCM) [4], Overlapped FDM (Ov-OFDM) [5], Precoded SEFDM [6] and Multi-stream Faster than Nyquist Signaling (FTN) [7], [8] systems promise variable spectral utilization savings for two dimensional modulations. All these systems share the concept of relaxing the orthogonality principle by reducing the spacing between the sub-carriers and/or the transmission symbol period. In essence, such SE enhancement approach is backed up by the Mazo work in [9], where Mazo established that it is possible to increase the signaling rate by 20% without experiencing any performance degradation. The focus of the work presented in this paper is on SEFDM [3]. However, the findings may be generalized to the other systems with the appropriate change of notation.

SEFDM aims to save the spectrum by reducing the spacing between the sub-carriers in frequency whilst maintaining the same per carrier rate. Notwithstanding, the deliberate collapse of orthogonality generates significant interference between the subcarriers that turns the detection of the signal to an overly complex problem. Maximum Likelihood (ML) SEFDM detectors have demonstrated an attractive BER performance. However, ML detection complexity dramatically increases with the increase in the system size. In addition, linear detection techniques such as Zero Forcing (ZF) and Minimum Mean Squared Error (MMSE) perform well only for small sized systems in high Signal to Noise Ratio (SNR) conditions [10]. Finally, sphere decoders provide optimum performance at a much reduced but random complexity [11] whose volatility depends on the noise and the system coefficient matrix properties.

In this paper, a novel sub-optimal detector for SEFDM system, that is based on the Truncated Singular Value Decomposition (TSVD) [12] and [13], is proposed. The TSVD detector is shown to provide substantial BER improvement when compared to ZF and MMSE with a negligible increase in complexity. Furthermore, a novel hybrid receiver that combines TSVD with the Fixed complexity Sphere Decoder (FSD) is proposed. The FSD is a technique used in Multiple Input Multiple Output (MIMO) system to provide fixed complexity sphere wise detection [14], [15]. However, FSD does not guarantee an optimal solution like standard SD since it enumerates a fraction of the points within the sphere search space. In this work, we import the FSD technique from the MIMO context into SEFDM signal detection. Further, an improvement in the FSD error performance is attempted by applying TSVD for the initialization of the FSD algorithm. The performance of the system is tested in AWGN in order to demonstrate the feasibility of the proposal. Overall, the proposed FSD-TSVD detector enhances the error performance of the standard FSD without introducing any premium at its computational cost. An extension to fading channels could be straight forward using a joint channel equalization and detection approach as demonstrated in [16] for the original SEFDM system.

The rest of this paper is organized as follows: section II introduces the SEFDM signal model. Sections III and

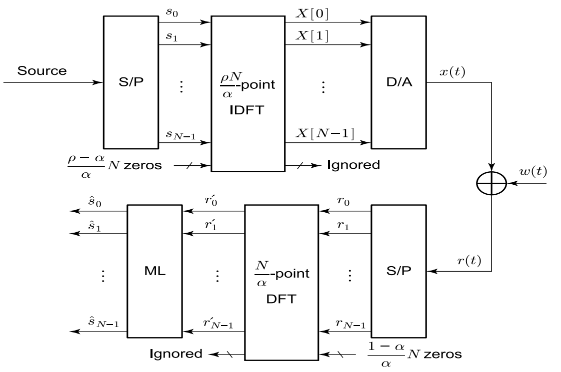

Fig. 1. SEFDM block diagram.

IV present the sub-optimal TSVD based detector and the combined FSD-TSVD detector respectively. Section V evaluates through simulations the performance of the proposed solutions. Finally, the paper is concluded in section VI. Throughout this paper vectors are denoted by uppercase characters and matrices by uppercase boldface characters.

II. Spectrally Efficient FDM Signal

The SEFDM signal consists of a stream of SEFDM symbols each carrying a block of N complex input symbols, denoted by $s = s_{\Re e} + js_{\Im m}$, transmitted within T seconds [3]. Each of the N complex input symbols modulates one of the non-orthogonal and overlapping sub-carriers, hence, giving the SEFDM signal $x(t)$ as

$$x\left(t\right) = \frac{1}{\sqrt{T}} \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} s_{l,n} exp\left(j2\pi n\alpha\left(t-lT\right)/T\right) \quad (1)$$

where α denotes the bandwidth compression factor defined as $\alpha = \Delta f T$, Δf is the frequency distance between the sub-carriers, T is the SEFDM symbol duration N is number of sub-carriers and $s_{l,n}$ denotes the symbol modulated on the n^{th} sub-carrier in the l^{th} SEFDM symbol. The fraction α determines the level of the bandwidth compression, with $\alpha = 1$ corresponding to an OFDM signal.

The discrete SEFDM signal model is derived by sampling the SEFDM symbol at $l = 0$ from (1) at (T/Q) intervals where $Q = \rho N$ and $\rho \geq 1$, the SEFDM symbol is then expressed as

$$X\left[k\right] = 1/\sqrt{Q} \sum_{n=0}^{Q-1} s_n \exp\left(j2\pi\alpha nk/Q\right). \quad (2)$$

where $X\left[k\right]$ is the k^{th} time sample of the first symbol of $x\left(t\right)$ in (1) and the factor $1/\sqrt{Q}$ is a normalization constant. Furthermore, the system may be expressed in matrix format as

$$X = \mathbf{\Phi} S, \quad (3)$$

where $X = \left[x_0, \cdots, x_{Q-1}\right]^{'}$ is a vector of time samples of $x\left(t\right)$ in (1) and $S = \left[s_0, \cdots, s_{N-1}\right]^{'}$ is a vector of input symbols, $\left[\cdot\right]^{'}$ denoting a vector or matrix transpose operation. The sampled SEFDM carriers are given by the matrix $\mathbf{\Phi}$ where $\mathbf{\Phi}$ is a $Q \times N$ matrix whose elements are $\phi_{k,n} = 1/\sqrt{Q} \exp\left(j2\pi\alpha nk/Q\right)$, for $0 \leq n < N$, $0 \leq k < Q$.

At the receiver, the SEFDM signal is received contaminated with Additive White Gaussian Noise (AWGN), $w\left(t\right)$, as

$$r\left(t\right) = x\left(t\right) + w\left(t\right). \quad (4)$$

The SEFDM receiver generates statistics of the incoming signal by correlating $r\left(t\right)$ with the conjugate carriers. These statistics are fed to a detector to generate estimates of the transmitted signal [3]. Fig. 1 depicts a block diagram for an SEFDM system. The signal generation and correlation is realized by the Inverse Discrete Fourier Transform (IDFT) [17] and Discrete Fourier Transform (FFT) [18] respectively. The SEFDM reception process can be expressed as

$$\begin{aligned} R &= \mathbf{\Phi}^{*} X + W_{\mathbf{\Phi}^{*}}, \\ &= \mathbf{C} S + W_{\mathbf{\Phi}^{*}}, \end{aligned} \quad (5)$$

where R is a $Q \times 1$ statistics vector, $\left[\cdot\right]^{*}$ denotes the conjugate transpose of the argument, $W_{\mathbf{\Phi}^{*}}$ is a $Q \times 1$ vector of AWGN noise samples correlated with the conjugate carriers and $\mathbf{C}$ denotes the cross correlation coefficient matrix defined as $\mathbf{C} = \left[c\left[m,n\right]\right]$, where $c\left[m,n\right]$ is derived by considering any two discretized SEFDM carriers, $\exp\left(j2\pi\alpha km/Q\right)$ and $\exp\left(j2\pi\alpha kn/Q\right)$ for $k = 0, ..., Q-1$ as:

$$\begin{aligned} c\left[m,n\right] &= \frac{1}{Q} \sum_{k=0}^{Q-1} e^{\frac{j2\pi\alpha km}{Q}} e^{-\frac{j2\pi\alpha kn}{Q}} \\ &= \frac{1}{Q} \left[\begin{array}{ll} Q & , m = n \\ \frac{1-e^{j2\pi\alpha(m-n)}}{1-e^{\frac{j2\pi\alpha(m-n)}{Q}}} & , m \neq n \end{array} \right]. \end{aligned} \quad (6)$$

The ML estimates $\hat{S}_{ML}$ of the originally sent symbols are given by the least square problem as:

$$\hat{S}_{ML} = \arg\min_{s \in M} \left\| R - \mathbf{C} S \right\|^2 \quad (7)$$

where $\left\|\cdot\right\|$ denotes the Euclidean norm and M is the constellation cardinality. However, the ML complexity grows dramatically with the increase in the number of carriers and/or the cardinality of the modulation alphabet. An improvement to the exhaustive enumeration techniques is given by sphere decoders that achieve ML solution with a smaller computational effort. However, SD suffers from a random complexity that becomes overly complex in the low SNR regimes. In the following section, we propose two detectors that aim to reduce and fix the complexity whilst achieving attractive BER performance.

III. The Truncated Singular Value Decomposition SEFDM Detector

The detection of the SEFDM signal requires the efficient handling of the intercarrier interference (ICI). ML detector produces optimal BER performance but overly complex. On the other hand linear detectors such as Zero Forcing (ZF) defined as

$$\hat{S}_{ZF} = \left\lfloor S + \mathbf{C}^{-1} N_{\Phi} \right\rceil, \quad (8)$$

and Minimum Mean Squared Error (MMSE) defined as

$$\hat{S}_{MMSE} = \left| \mathbf{C}^* \left(\mathbf{C}\mathbf{C}^* + \frac{1}{\text{SNR}}\mathbf{I} \right)^{-1} R \right|, \qquad (9)$$

where $|\cdot|$ is a slicer and QAM demapper, showed BER that is highly dependent on the system parameters α and N. This is mainly due to the ill conditioning of the matrix $\mathbf{C}$ that results in noise enhancement. The conditioning of a matrix is measured by its condition number defined as the ratio of the maximum singular value to the minimum one. As the condition number of a matrix grows, any solution that is based on an inverse of that matrix becomes sensitive to even very small perturbations (i.e. noise). Therefore, in this section the use of the Truncated Singular Value decomposition (TSVD) [12], [13], which is a popular method for inverting ill conditioned matrices, is proposed for SEFDM signals detection.

In essence, TSVD is a method to generate an approximated inverse of an ill conditioned matrix and hence solve a modified least squares problem that is less sensitive to perturbations. The approximated inverse is derived by finding the singular value decomposition of the argument matrix and then truncating its small singular values that overly multiply the noise. This is accomplished by finding first the SVD of matrix $\mathbf{C}$ given as $\mathbf{C} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^*$, where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices whose columns are the eigenvectors of $\mathbf{C}\mathbf{C}^*$ and $\mathbf{C}^*\mathbf{C}$ respectively and $\mathbf{\Sigma}$ =diag$(\sigma_1, \sigma_2, \cdots, \sigma_N)$, for σ_i is the i^{th} singular value of $\mathbf{C}$. The TSVD based pseudoinverse of $\mathbf{C}$, denoted by $\mathbf{C}_\xi$, is defined as

$$\mathbf{C}_\xi = \mathbf{V}\mathbf{\Sigma}_\xi^{-1}\mathbf{U}^*, \qquad (10)$$

where $\mathbf{\Sigma}_\xi^{-1} = \text{diag}(1/\sigma_1, 1/\sigma_2, \cdots, 1/\sigma_\xi, 0, \cdots, 0)$, ξ is the truncation index. The truncation index, ξ, defines the number of singular values that are accepted, and therefore determines the quality of the generated matrix and consequently the obtained solution. In other words, to calculate the pseudoinverse, TSVD filters out the elements in $\mathbf{\Sigma}^{-1}$ that correspond to small singular values in $\mathbf{\Sigma}$ statrting at index $\xi + 1$. The TSVD based detector is then defined as

$$\hat{S}_{TSVD} = |\mathbf{C}_\xi R|. \qquad (11)$$

The choice of the truncation index greatly affects the quality of the obtained solution and has been the focus of many researches [19], [20]. In [19] the optimum truncation index is stated to satisfy the picard condition. That is for a system expressed using the generalized singular value decomposition, the Fourier coefficients on the right hand side decay faster than the generalized singular values. In SEFDM particularly, it has been observed by simulation that αN of the eigenvalues of $\mathbf{C}$ are larger than or equal to 1 while $(1-\alpha)N$ of the eigenvalues quickly decay to values close to 0 [6]. Therefore, it is thought sensible to test for truncation indices around this benchmark.

IV. Combined Sphere Decoder and TSVD

Despite the favourable BER performance of the sphere decoding algorithm (SD) [11], the variable complexity and the sequential nature of the algorithm can greatly obstacle the implementation of the system. On the other hand the Fixed SD (FSD) fixes the complexity of the SD yet does not guarantee the optimal solution. In this section we first present the FSD algorithm applied to the SEFDM problem. Then, we propose a novel improvement of the FSD algorithm that decreases the gap between the optimal ML and the FSD estimates while maintaining fixed low complexity.

A. The Fixed Sphere Decoder (FSD)

As the conventional SD, FSD converts the SEFDM problem to an equivalent depth-first problem. The sphere decoder solves the problem expressed in (7) by examining only the points that exists within an N dimensional hypersphere of radius g such that:

$$\hat{S}_{SD} = \arg\min_{s \in M} \|R - \mathbf{C}S\|^2 \le g \qquad (12)$$

Expanding the norm argument and substiuting by $P = \mathbf{C^{-1}}R$ where P is the unconstrained ML estimate of S in (12) leads to

$$\hat{S}_{SD} = \arg\min_{s \in M} \left\{ (P - S)^* \mathbf{C}^*\mathbf{C}(P - S) \right\} \le g \qquad (13)$$

The transformation of the problem in (13) into a spherewise representaion is achieved by applying the Cholesky decomposition as chol$\{\mathbf{C}^*\mathbf{C}\} = \mathbf{L}^*\mathbf{L}$, [21] where $\mathbf{L}$ is an $N \times N$ upper triangular matrix. Consequently, (12) can be re-expressed as

$$\hat{S}_{SD} = \arg\min_{s \in M} \|\mathbf{L}(P - S)\|^2 \le g, \qquad (14)$$

The structure of $\mathbf{L}$ allows for the examining of the nodes at a given level by calculating the contribution of these nodes on the radius g, and hence discarding all nodes, and consequently their respective children nodes, that are outside the hypersphere of radius g. The SD search continues until the ML estimate is decided, therefore the complexity of the algorithm is variable and depends on the noise and the properties of the system. Therefore, the Fixed complexity Sphere Decoder (FSD) is introduced [14], [15].

FSD fixes the complexity of SD by restricting the search within a limited sub-space of the problem [14], [15]. At every level, a fixed number of nodes, termed here as the tree width, are examined. In this work, a number of nodes, corresponding to the tree width, at each level are chosen based on their distance from the centre point, decided by P, in a similar manner to the Schnorr-Euchner enumeration [22]. The FSD-ZF estimate is given by

$$\hat{S}_{FSD-ZF} = \arg\min_{\tilde{s} \in M} \left\| \mathbf{L}\left(P - \tilde{S}\right) \right\|^2 \le \breve{g}, \qquad (15)$$

where $\tilde{S} \subset \mathcal{H}$ for $\mathcal{H}$ the sub-space of the FSD algorithm and $\breve{g} = \left\| R - \mathbf{C}\hat{S}_{ZF} \right\|^2$. Fig. 2 illustrates the FSD algorithm where a 4 carrier system with BPSK symbols is presented. At each level, a fixed number of nodes (=2) is retained, whereas the discarded nodes at that level results in discarding all their children nodes. For this particular example, the FSD tree width is fixed to 2.

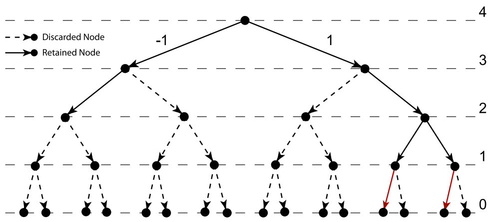

Fig. 2. The FSD tree search algorithm with tree width equal to 2.

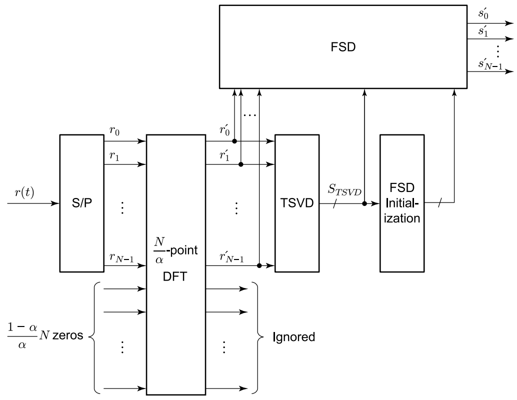

Fig. 3. The TSVD-SD detector block diagram.

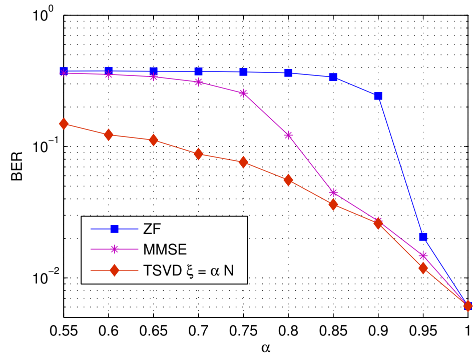

Fig. 4. BER performance of TSVD detector Vs α for 32 carrier system carrying 4QAM symbols.

The limitation of the SD search in a sub-set of the points within the initial hypersphere relates the performance of the detector to the quality of the initial estimate since the latter determines the size of the sphere and consequently how close this subset of points is to the optimal solution. Therefore, a combined receiver that first estimates the transmitted symbols using TSVD and thence uses this estimate to instantiate the FSD part of the detection is proposed.

B. The Combined FSD-TSVD Receiver

The FSD constrains the subspace of the search covered by the original SD algorithm, thus fixes the complexity, but no longer guarantees optimal solution. Noting that the FSD relies on the unconstrained ML estimate P obtained by matrix division, it is proposed here to combine the TSVD detector with the Fixed Sphere Decoder (FSD) to enhance the BER performance.

Combining the FSD with the TSVD enhances the accuracy of the detector as the searched points are tested against the TSVD estimate which is of better quality than the ZF or MMSE estimate. In terms of the complexity the TSVD can be computed using a QR factorization of the matrix $\mathbf{C}$ [23], which can be exchanged with the Cholesky decomposition in the FSD algorithm. Fig. 3 depicts the combined FSD-TSVD detector. The detector starts by finding the TSVD estimate of the sent symbols as in (11). This estimate is then used to calculate the radius that positions the TSVD estimate at the surface of the sphere as

$$\hat{g} = \left\| R - \mathbf{C}\hat{S}_{TSVD} \right\|^2 . \qquad (16)$$

The FSD-TSVD algorithm, then enumerates a fixed number of points at each level. The FSD-TSVD estimate is given by

$$\hat{S}_{FSD-TSVD} = \arg\min_{\tilde{s}\in M} \left\| \mathbf{L}\left(\hat{P} - \tilde{S}\right) \right\|^2 \leq \hat{g} \qquad (17)$$

where $\hat{P}$ is the unconstrained TSVD estimate defined as $\hat{P} = \mathbf{C}_{\xi}R$ and $\tilde{S} \subset \mathcal{H}$ for $\mathcal{H}$ the sub-space of the FSD algorithm. If no node is discovered within the sphere then

$$\hat{S}_{FSD-TSVD} = \hat{S}_{TSVD}. \qquad (18)$$

The proposed FSD-TSVD detector has two main differences from the original SD detector. The first is that the complexity of the FSD-TSVD is fixed to a number of nodes equal to the FSD tree width. The second, is that the FSD-TSVD measures the distance of the nodes with the FSD sub-space from the unconstrained TSVD estimate $\hat{P}$. This last difference distinguishes the proposed FSD-TSVD from the conventional FSD algorithm presented in section (IV-A) and termed therein as FSD-ZF. Fig. 3 depicts the block diagram of the FSD-TSVD detector. The FSD initialization block ensures that the FSD uses the reference point $\hat{P}$ and calculates $\hat{g}$.

V. Performance Investigations

The performance of the TSVD detector and the FSD-TSVD was evaluated by extensive numerical simulations. The simulations were carried in AWGN channel to ensure that the proposed techniques are fundamentally valid. For fading channels it is believed that using joint channel equalization and detection is possible. The performance of the proposed detectors was examined for different system size and level of bandwidth compression and is compared with the SEFDM detectors reported in [10], [11].

A. TSVD sub-Optimal detector

The TSVD detector as in (11) is examined for different system settings. Fig. 4 shows the BER performance of a TSVD detector for different values of α. It is evident that the TSVD greatly reduces the BER compared to other linear detectors specifically ZF and MMSE. However, the performance is still not close to OFDM which corresponds to $\alpha = 1$.

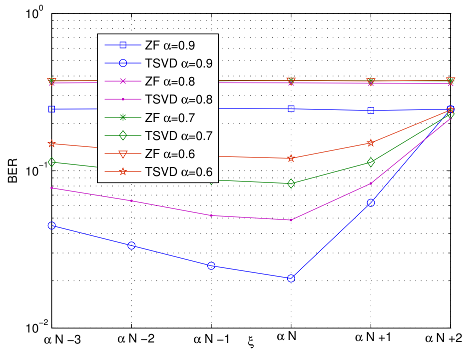

Fig. 5. BER performance for TSVD detector Vs the truncation index ξ for a 32 carriers system carrying 4QAM symbols.

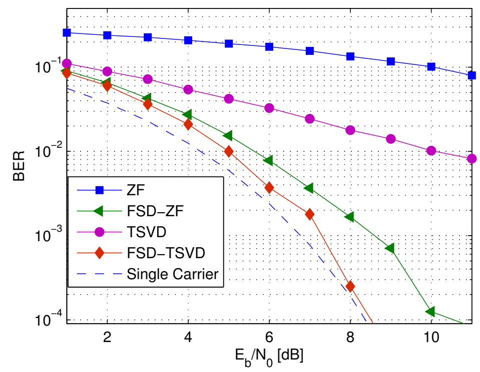

Fig. 6. BER performance of different detectors for a 12 carrier SEFDM system for $\alpha = 0.8$ carrying 4QAM symbols.

Fig. 5 illustrates how the BER performance is dependent on the truncation index ξ defined in section III. The figure shows that for different α values there is an optimum value for the truncation index ξ that corresponds to lowest achievable BER. It is observed that this optimum value is approximately αN which is consistent with the value anticipated from the conditioning of the $\mathbf{C}$ matrix. The complexity of the TSVD detector is of similar order as ZF as it only requires the inversion of the matrix which is usually carried with the aid of the SVD.

Although the TSVD detector provides substantial BER reduction, the BER performance is still not comparable to OFDM. These results are provided here to demonstrate the BER performance enhancement offered by the TSVD which is the base for more performance enhancement with the combined FSD-TSVD detector.

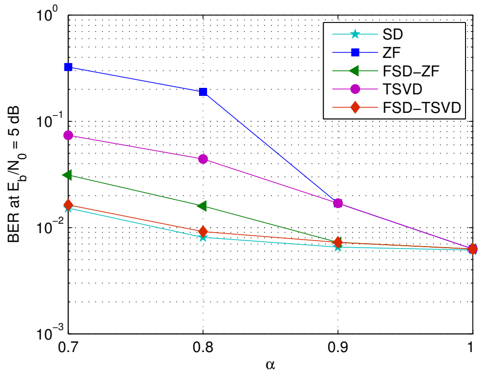

Fig. 7. BER performance of different detectors for a 12 carrier system carrying 4QAM symbols for $\alpha = 0.7, \ldots, 1$, $\alpha = 1$ corresponds to OFDM, maximum tree width 7.

B. FSD-TSVD Detector

Performance of the FSD-TSVD detector is examined for different number of carriers and levels of bandwidth compression. For comparison purposes results for ZF, a combined FSD and ZF (FSD-ZF), TSVD and original SD are plotted where applicable. Fig. 6 shows that the FSD-TSVD detector yields substantial BER reduction compared to FSD-ZF, TSVD and ZF detectors. The FSD-TSVD detector outperformed even the FSD-ZF detector as anticipated. This is mainly due to the better quality of the TSVD estimate fed to the FSD-TSVD detector.

Fig. 7 shows the BER performance with respect to α. Again the figure confirms that the FSD-TSVD detector greatly improves the BER performance and almost approaches the performance of the original SD. In addition, the FSD-TSVD detector outperforms the original SD in terms of complexity. On the other hand, the FSD-ZF detector also offers fixed complexity, however, achieves inferior BER performance to the FSD-TSVD detector.

Fig. 8 depicts the BER performance of the FSD-TSVD detector for different values of α for a fixed complexity. The figure indicates that for fixed complexity there will be some performance degradation. This is due to the conditioning of the system that worsens with the decrease in α values. Furthermore, Fig. 9 illustrates how the BER performance can be enhanced by increasing the complexity for $\alpha = 0.7, 0.8$ for E_b/N_0 =7dB.

Fig. 10 depicts the BER performance of the FSD-TSVD detector with different complexities for $E_b/N_0 = 1, \ldots, 9$ dB. It is clear from Fig. 9 and Fig. 10 that the BER performance improves with the increase in complexity up-til a saturation point beyond which there is minor BER improvement

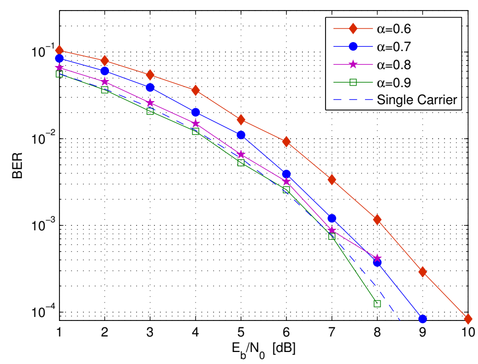

Fig. 8. BER performance of FSD-TSVD detector for a 12 carrier SEFDM system carrying 4QAM symbols for $\alpha = 0.6, \ldots, 0.9$ maximum tree width 6.

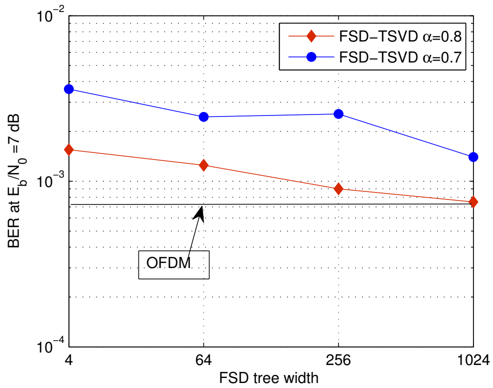

Fig. 9. BER performance of the FSD-TSVD for different tree width for 10 carriers SEFDM system carrying 4QAM symbols with $\alpha = 0.7, 0.8$.

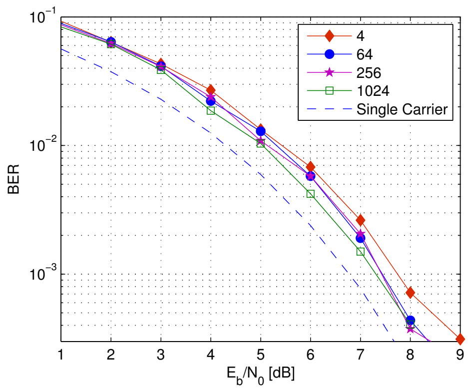

Fig. 10. BER performance of the FSD algorithm for different tree width for a 16 carriers SEFDM system carrying 4QAM symbols for $\alpha = 0.8$.

VI. Conclusions

In this paper, the use of the Truncated Singular Value Decomposition (TSVD) for the detection of Spectrally Efficient FDM (SEFDM) signals is proposed. Numerical simulations show that the standalone TSVD based detector achieves a sub-optimal solution of the SEFDM least squares problem that is more immune to small perturbations than ZF and MMSE, hence yielding a better BER performance. Furthermore, the complexity of the TSVD detector is of the same order with the ZF one with the added complexity for calculating the TSVD based inverse incorporated in the inversion process necessary for ZF as well. In addition, we designed a novel detector for SEFDM signal that combines the TSVD and the fixed Sphere Decoder (FSD). Simulation results show that the proposed FSD-TSVD detector achieves superior BER performance to other SEFDM detection techniques and performs close to the standard SD. Moreover, it constitutes an enhancement of the standard FSD since it improves FSD error performance while maintaining its merits of low and mainly fixed complexity.

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