Non-stable approximation to surviving daughters

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The Goodman-Keyfitz equation for the number of surviving daughters to a women aged a is

$$\int_0^a \ell_{a-x} m_x dx$$

where m_x are the female stable population fertility rates and l_{a-x} is the stable survival function.

The goal is to come up with an approximation to the non-stable case (i.e. a period approximation to the cohort case).

Ignoring ℓ_x for now, a non-stable version of the fertility rates could be indexed by t

$$\int_0^a \ell_{a-x} m_x(t-a+x) dx$$

or just writing cohort $g = t - a$

$$\int_0^a \ell_{a-x} m_x(g+x) dx$$

So we would use the age-specific fertility rates in year $g + x$. We want to find approximation to these cohort rates such that we can just index to one year i.e.

$$\int_0^a \ell_{a-x} m_x(g+A_m) dx$$

What is A_m?

Seems likely that A_m is the mean age at childbearing μ.

QUESTION I: Can I do this? Expand $m_x(t)$ around $t - x + \mu$, which gives

$$m_x(t) \approx m_x(t-x+\mu) + (x-\mu) m'(t-x+\mu)$$

Then

$$\begin{aligned}\int_0^a \ell_{a-x} m_x(g+x) dx &\approx \int_0^a \ell_{a-x} \left[m_x(g+\mu) + (x-\mu) m_x'(g+\mu) \right] dx \\ &= \int_0^a \ell_{a-x} m_x(g+\mu) + \int_0^a \ell_{a-x} (x-\mu) m_x'(g+\mu) dx\end{aligned}$$

Can I cancel out the second term because of the $x - \mu$ bit? Which would give me what I want

$$\int_0^a \ell_{a-x} m_x (g + x) dx \approx \int_0^a \ell_{a-x} m_x (g + \mu) dx$$

This would only solve the fertilty piece, would also need to work out survival piece...