

**Assignment 06**

1. Consider a symmetric game where each player must decide whether to hunt or gather for food. If both players decide to hunt, they successfully capture an animal for meat, and they each get a payoff of 10. If both players gather, they will help each other find food faster and each will get payoff 7. If one player hunts and one gathers, the hunter will be injured by the prey and get payoff 0, while the gatherer will get some food and get payoff 5.

a. Draw the matrix representation of this game.

		Player 2	
		Hunt	Gather
Player 1	Hunt	+10, +10	0, +5
	Gather	+5, 0	+7, +7

b. Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.

No, none of the players have a strict dominant strategy, because both players are dependent on each other for them to have a good outcome. That is if both hunt together (where both get a +10 payoff) or gather (where both get a +7 payoff)

c. Identify any pure strategy Nash equilibria. Explain your answer.

There is a pure strategy Nash equilibrium when both decide to hunt together or gather, they get the maximum payoff. If they hunt and gather alone both, only one of them benefits and the other one does not have anything.

2. Two drivers are playing chicken, the game where they drive straight toward each other and see if one or both will swerve to avoid a crash. If one driver swerves and one stays straight, the driver who swerves is declared a "chicken" and gets payoff -10, while the driver who stays straight wins and gets payoff +10. If both drivers swerve, the game is a tie although they are both somewhat "chicken" and each gets payoff -5. If both drivers continue straight, they will crash and both get payoff -50.

a. Draw the matrix representation of this game.

		Driver 1	
		Swerve	Straight
Driver 2	Swerve	-5, -5	-10, +10
	Straight	+10, -10	-50, -50

b. Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.

No, none of the drivers have a strict dominant strategy, because both drivers are dependent on other to swerve to get the maximum payoff. If none of them swerve they will crash into each other or if both swerve, they will get negative payoffs

c. Identify any pure strategy Nash equilibria. Explain your answer.

There is a pure strategy Nash equilibrium when both drivers select opposite to what the other drivers chooses to get the payoff. It is the best option for the drivers to drive straight until the other driver chickens out, if no one chickens out both of them will crash and will be worse.

3. Company A and Company B make competing products. If they both keep production low, the monthly profit for each company will be \$16,000. If one company increases production and the other doesn't, the profit for the company that increases production will be \$20,000 while the profit for the one that didn't increase production will be \$12,000. If both companies increase production, the profit for each will be \$14,000.

a. Draw the matrix representation of this game.

		Company B	
		Low	High
Company A	Low	\$16,000, \$16,000	\$12,000, \$20,000
	High	\$20,000, \$12,000	\$14,000, \$14,000

b. Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.

The strict dominant strategy is that both of the companies have to aim for high production to get a better payoff, either of the companies have to compete with the other company for a high production. If company A has a low production compared to company B then company B gets the higher payoff, and the other way round.

c. Identify any pure strategy Nash equilibria. Explain your answer.

Company A and B both are choosing high production to get the maximum profit is a pure strategy Nash equilibria.

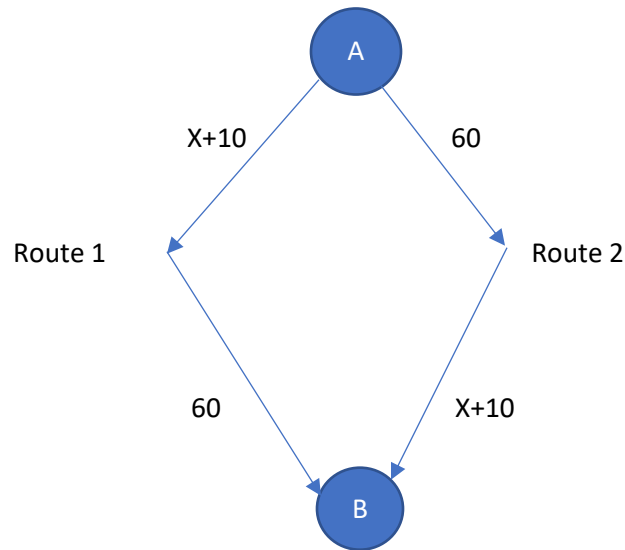
4. There are 80 cars which begin in city A and must travel to city B. There are two routes between city A and city B:

Route I: begins with a local street leaving city A, which requires a travel time in minutes equal to 10 plus the number of cars which use this street, and ends with a highway into city B which requires one hour of travel time regardless of the number of cars which use this highway.

Route II: begins with a highway leaving city A. This highway takes one hour of travel time regardless of how many cars use it, and ends with a local street leading into city B. This local street near city B requires a travel time in minutes equal to 10 plus the number of cars which use the street.

Show your work for all calculations.

a. Draw the network described above and label the edges with the travel time in minutes needed to move along the edge, in terms of  $x$ . Let  $x$  be the number of travelers who use Route 1. The network should be a directed graph as all roads are one-way.



b. What would be the travel time per car if all cars chose to use Route 1? Explain your answer.

For Route 1 the travel time will be  $= X+10+60$  where  $X$  is 80  
 $= 80+10+60$   
 $= 150$  minutes

c. Assume that cars simultaneously chose which route to use. Find the Nash equilibrium value of  $x$ , where  $x$  is the number of cars on each route. Also give the drive time per car and the total drive time for all cars.

If all the cars simultaneously chose which route to use then the value of  $X = 40$ , the Nash equilibrium value of  $x = 40$

Drive time of each car will be  $= 10 + 40 + 60 = 110$  minutes

Total drive time of all the cars  $= 110 \text{ minutes} \times 80 \text{ cars} = 8800$  minutes

d. Explain your answer to part c.

The Nash equilibrium will be exactly half of the total numbers of cars, in which 40 cars will take route 1 and 40 cars will take route 2. There is not strict domain strategy for any drivers. If all the drivers take only route 1 then it will take a total of 150 minutes, but if all the drivers split into equal parts then it will take 110 minutes which saves 40 minutes for each of the drivers.

Now the government builds a new (two-way) road connecting the nodes where local streets and highways meet. The new road is very short and takes no travel time. This adds two new routes.

Route III: consists of the local street leaving city A (on Route I), the new road, and the local street into city B (on Route II).

Route IV: consists of the highway leaving city A (on Route II), the new road, and the highway leading into city B (on Route I).

e. What would the travel time be per car if all cars chose Route III?

$$\begin{aligned}\text{For route 3} &= X + 10 + X + 10 \\ &= 80 + 10 + 80 + 10 \\ &= 180 \text{ minutes}\end{aligned}$$

f. What would the travel time be per car if all cars chose Route IV?

$$\begin{aligned}\text{For route 4} &= 60 + 60 \\ &= 120 \text{ minutes}\end{aligned}$$

g. What happens to total travel time as a result of the availability of the new road? Explain your answer.

If all the cars travel from route 4 (120 minutes) it will save them 30 minutes compared to route 1 and route 2 (150 minutes).

h. If you can assign travelers to routes, it's possible to reduce total travel time relative to what it was before the new road was built. That is, the total travel time of the 80 cars can be reduced (below that in the original Nash equilibrium from part c ) by assigning cars to routes. There are many assignments of routes that will accomplish this. Find one (where total travel time for all cars is lower than the Nash equilibrium total travel time). Explain why your reassignment reduces total travel time by giving the number of cars assigned to each of the Routes I, II, III, IV, the travel time per car on each route, and the total travel time of the 80 cars.

Route 1: 10 cars = 80 mins/car

Route 2: 10 cars = 80 mins/car

Route 3: 10 cars = 40 mins/car

Route 4: 50 cars = 120 mins/car

$$\begin{aligned}\text{Total time of all the cars} &= (80 * 10) + (80 * 10) + (40 * 10) + (120 * 50) \\ &= 8000 \text{ mins}\end{aligned}$$

This reassignment of cars reduces the total time of 800 mins.

5. There are a number of instances world-wide where elimination of a freeway has been a big improvement for the area. Write a short (200 word) essay describing, in your own words, one such instance and the reasoning behind and results of the freeway elimination. Your essay must describe a specific highway. Give its name and location, and discuss the reasons behind the elimination, and the results of its elimination.

Out of the most instances where there has been elimination of freeways Seattle's Alaskan way Vaiduct has been rebuilding the freeways to a new underground tunnel. The purpose for this was to bring more public space and pedestrian walkways along the waterfront. The decision was made after the freeway had encounter an earthquake attack, which had damaged the freeway. Apart from that it being a freeway it was always traffic jammed this caused a lot of problem to the local people. The state and city government had considered several options to rebuild the freeway, but a deep bored tunnel project was selected in 2009 and the southern part of the freeway was demolished back in 2011. The tunnel was even made available for motorization, and it allowed the common public to walk and even cycle over there at the time of its opening. After the construction of the new highway there were several concerns regarding what if there is an earthquake again? Will it be able to withstand it? the new construction was created with that thought in mind. There is currently a new development in progress to improve the waterfront area to maximize the utilization which includes a park and is planned to be completed by 2022.

6. Consider an auction in which there is one seller who wants to sell one unit of a good and a group of bidders who are each interested in purchasing the good. The seller will run a sealed-bid, second-price auction. There will be around a dozen other bidders in addition to your firm. All bidders have independent, private values for the good. Your firm's value for the good is \$5,000.

a. What bid should your firm submit? Explain your answer.

As the auction is sealed-bid, second-price auction (where winner pays for the second bid amount) the firm should bid for the goods at \$5000, if the firm loses the payoff will be 0 but if the firm wins then it wins the goods for lower price than \$5000

b. How does your bid depend on the number of other bidders who show up?

For the firm to win they need to bid the highest value with the assumption that none of the other bidders will bid for more than \$5000, just in case of any of the other bidders do pay a higher amount the firm loses the bid and the payoff will be 0.

7. A seller wants to sell one unit of a good to some bidders, by running a sealed-bid second-price auction. Assume that there are two bidders who have independent, private values  $v$  which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both  $1/2$ . (If there is a tie at a bid of  $x$  for the highest bid the winner is selected at random from among the highest bidders and the price is  $x$ .)

NOTES: (1) When you have multiple identical top bids (a multi-way tie), then the highest and second highest will be that same bid. That means the winner will pay the top bid, because that is both the first- and second-place bid value.

(2) Do not use the formulas in section 9.7 of the text to calculate expected revenue. You should use the probabilities for the revenue for each of the bidding combinations: e.g. in 7a the probability that the two bidders each bid 3 (with a revenue of 3) is  $1/4$ .

a. Show that the seller's expected revenue is  $6/4$ .

Probability of two bidders bidding for the same bid  $P(\text{same bid}) = 1/4$   
the probabilities of 1 and 3 are both  $P(\text{bid of 1 or 3}) = 1/2$

the seller's expected revenue =  $\frac{1}{4} + \frac{1}{4} + \frac{1}{2} + \frac{1}{2} = \frac{6}{4} = 1.5$

b. Suppose that there are three bidders who have independent, private values  $v$  which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both  $\frac{1}{2}$ . What is the seller's expected revenue in this case? Explain your answer.

There will be 8 possible outcomes for 3 bidders with getting either 1 or 3

$(3,3,3), (1,1,3), (1,3,3), (1,1,1), (3,1,1), (3,3,1), (1,3,1), (3,1,3)$

With all the possible combinations the seller's expected revenue is greater than 2

c. Briefly explain why changing the number of bidders affects the seller's expected revenue. Would the expected revenue increase or decrease? Explain your answer.

As we can see from the numbers above a (1.5) and b (<2), we see that as the numbers of bidders increase the expected revenue of the seller also increases. As the numbers of bidders increase there will be more numbers of bids for the goods and each bidder puts a different price estimation, which increase the probability and overall increases the expected revenue.