

Algorithms

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Algorithm 4 Kyber.Encaps($pk = (\mathbf{t}, \rho)$)

```
1:  $m \leftarrow \{0, 1\}^{256}$ 
2:  $(\hat{K}, r) := G(H(pk), m)$ 
3:  $(\mathbf{u}, v) := \text{Kyber.CPA.Enc}((\mathbf{t}, \rho), m; r)$ 
4:  $c := (\mathbf{u}, v)$ 
5:  $K := H(\hat{K}, H(c))$ 
6: return  $(c, K)$ 
```

Algorithm 5 Kyber.Decaps($sk = (\mathbf{s}, z, \mathbf{t}, \rho), c = (\mathbf{u}, v)$)

```
1:  $m' := \text{Kyber.CPA.Dec}(\mathbf{s}, (\mathbf{u}, v))$ 
2:  $(\hat{K}', r') := G(H(pk), m')$ 
3:  $(\mathbf{u}', v') := \text{Kyber.CPA.Enc}((\mathbf{t}, \rho), m'; r')$ 
4: if  $(\mathbf{u}', v') = (\mathbf{u}, v)$  then
5:   return  $K := H(\hat{K}', H(c))$ 
6: else
7:   return  $K := H(z, H(c))$ 
8: end if
```

Algorithm 3 Kyber.CPA.Dec($sk = \mathbf{s}, c = (\mathbf{u}, v)$): decryption

```
1:  $\mathbf{u} := \text{Decompress}_q(\mathbf{u}, d_u)$ 
2:  $v := \text{Decompress}_q(v, d_v)$ 
3: return  $\text{Compress}_q(v - \mathbf{s}^T \mathbf{u}, 1)$ 
```

Algorithm 2 Kyber.CPA.Enc($pk = (\mathbf{t}, \rho), m \in \mathcal{M}$): encryption

```
1:  $r \leftarrow \{0, 1\}^{256}$ 
2:  $\mathbf{t} := \text{Decompress}_q(\mathbf{t}, d_t)$ 
3:  $\mathbf{A} \sim R_q^{k \times k} := \text{Sam}(\rho)$ 
4:  $(\mathbf{r}, \mathbf{e}_1, e_2) \sim \beta_\eta^k \times \beta_\eta^k \times \beta_\eta := \text{Sam}(r)$ 
5:  $\mathbf{u} := \text{Compress}_q(\mathbf{A}^T \mathbf{r} + \mathbf{e}_1, d_u)$ 
6:  $v := \text{Compress}_q(\mathbf{t}^T \mathbf{r} + e_2 + \lceil \frac{q}{2} \rceil \cdot m, d_v)$ 
7: return  $c := (\mathbf{u}, v)$ 
```

n	k	q	η_1	η_2	(d_u, d_v)	δ
256	2	3329	3	2	(10, 4)	2^{-139}
256	3	3329	2	2	(10, 4)	2^{-164}
256	4	3329	2	2	(11, 5)	2^{-174}

SC-assisted CCA

SC-info \sim POC \Rightarrow CCA on underlying CPA-PKE

POC or DFO: whether or not PKE.Dec result is equiv to a *reference* plaintext.

SC-info: SC-leakage of PRF or PRG in re-encryption (SHAKE/AES)

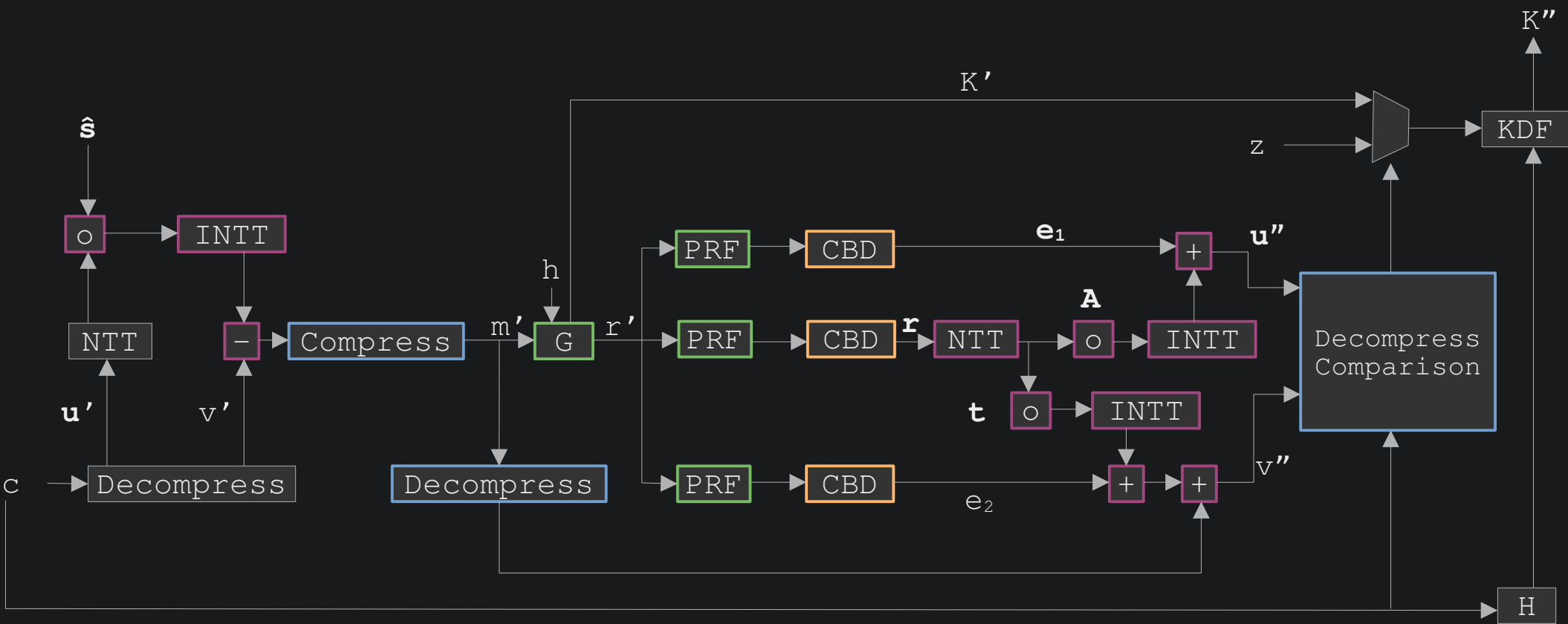
$$m = 0$$

$$\mathbf{c}' = (\mathbf{u}, v)$$

$$m' = v - \mathbf{s}^T \mathbf{u}, \quad \mathbf{u} = (\mathbf{u}_0 = \rho/2, \mathbf{u}_1 = 0), \quad v = (0, \dots, t, 0, \dots, 0)$$

$$\text{POC}(m, \mathbf{c}') = 1 \Leftrightarrow |\mathbf{s}_{0i} \times \rho/2 + t \times \rho/2| \leq \rho$$

POC: fixed vs random binary classifier.



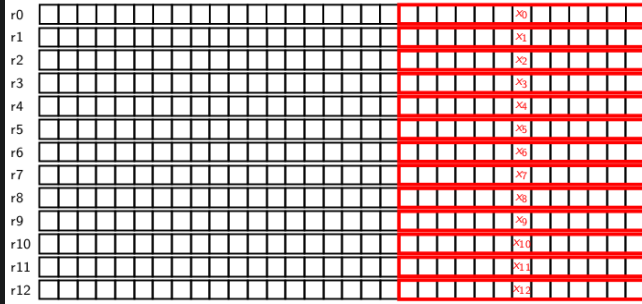
Masked Operations

[Doc](#)

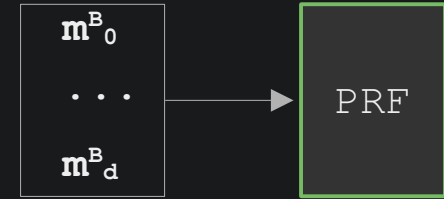
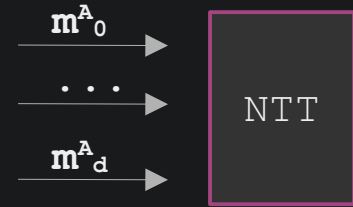
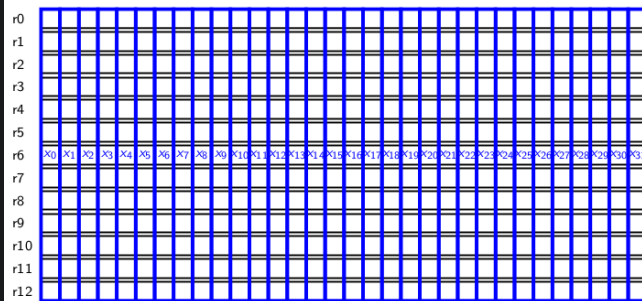
$$s = \sum^{d-1} m_d^A \bmod q$$

$$s = \oplus^{d-1} m_d^B$$

Canonical



Bitslice



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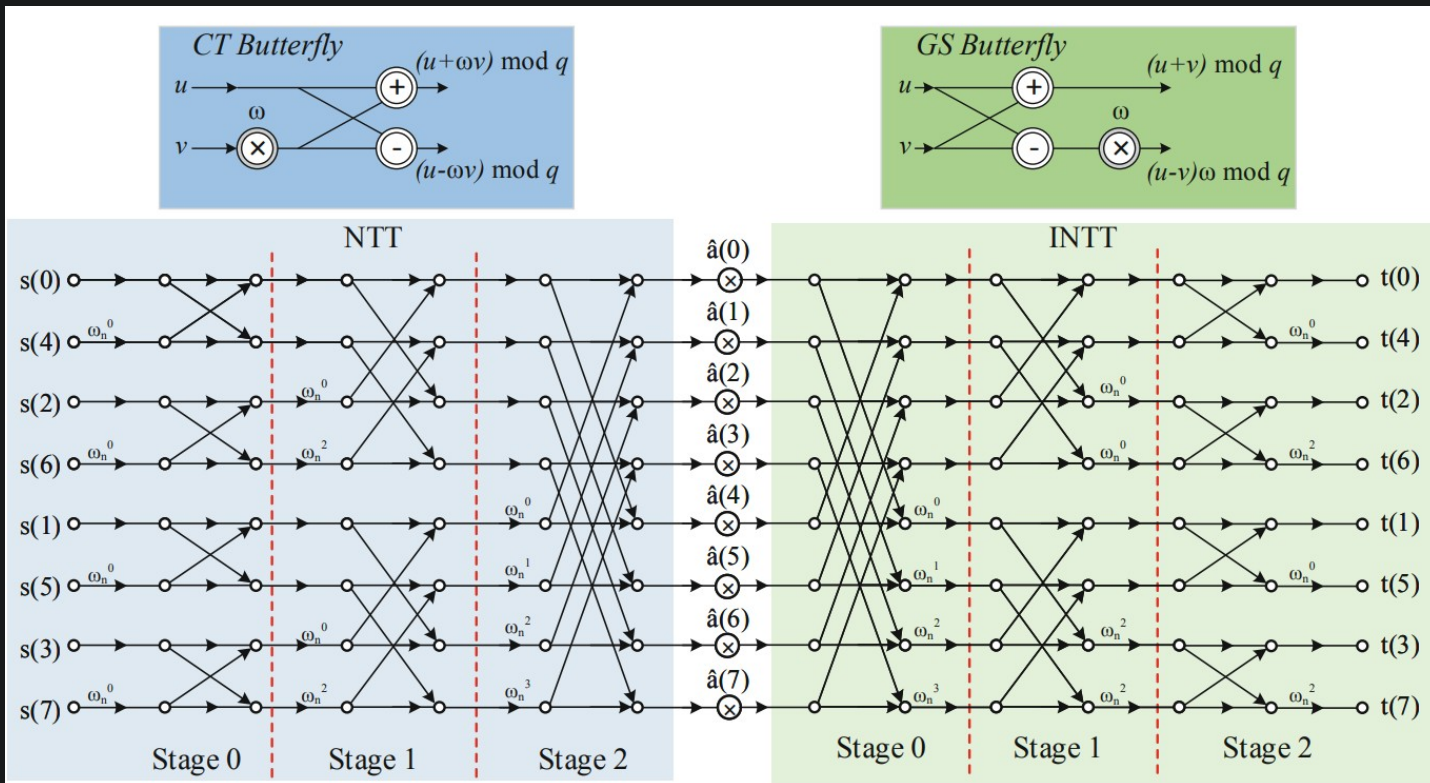
NTT & INTT

[Doc](#)

$$R_q = \mathbb{Z}_q[X] / (X^{256} + 1)$$

ω is the n -th root of unity, $\omega^2 = \omega$

Wrapped Convolution to avoid zero padding

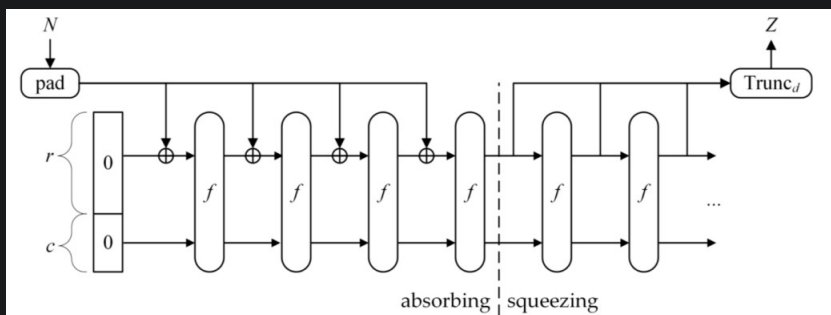


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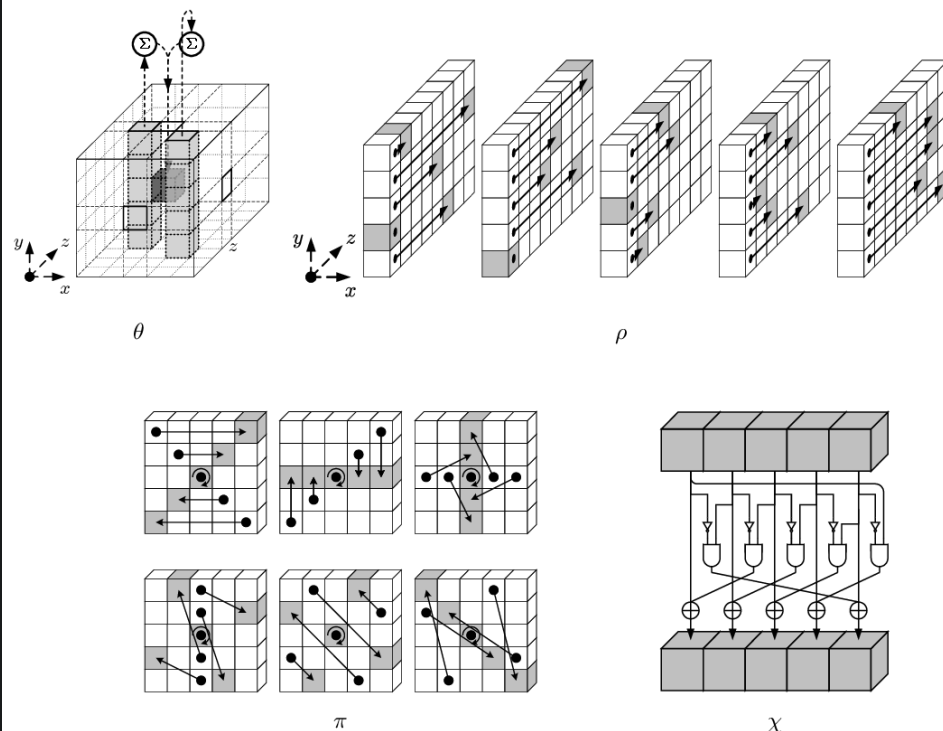
KECCAK (XOF, H, G, PRF, KDF)

[Doc](#)

θ : XOR bits w/ parities of 2 col
 ρ : Rotate bit by an offset
 π : Rearrange positions of the lanes
 χ : XOR bits w/ non-linear function of 2 other bits in its row
 ι : Modify some bits of lane(0,0)
(depend on round index)



B Keccak Permutation



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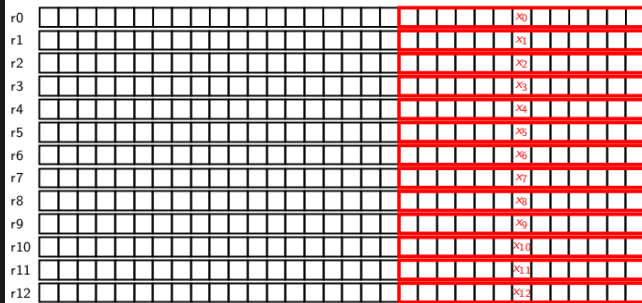
Masked Operations

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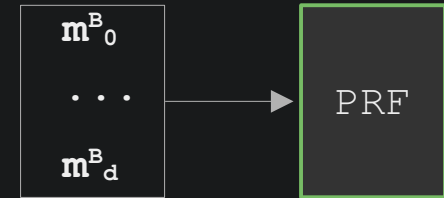
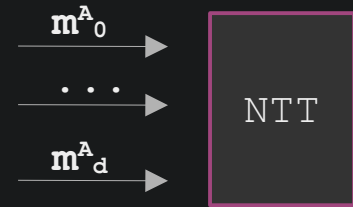
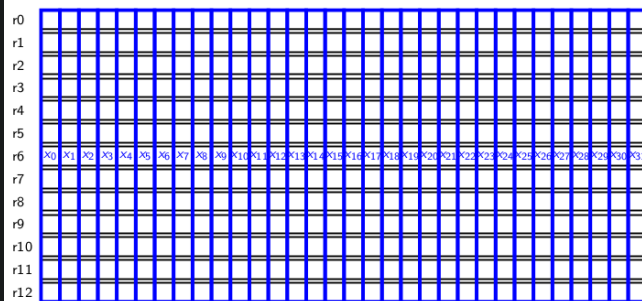
$$s = \sum^{d-1} m_d^A \bmod q$$

$$s = \oplus^{d-1} m_d^B$$

Canonical



Bitslice



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First experiment

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Distinguish decrypted message m' from *reference* message m
Even if m differs m' 1 bit,
 $\mathbf{r} = G(m)$ can differ $\mathbf{r}' = G(m')$ in several coefficients

First order masking:

$$\mathbf{r} = [r_0, \dots, r_{255}], r_i \in [-2, 2]$$

$$\mathbf{x} = [x_0, \dots, x_{255}], x_i \in [0, q], \mathbf{l}_0 = HW(\mathbf{x}) + \beta$$

$$\mathbf{y} = (\mathbf{r} - \mathbf{x}) \% q, \mathbf{l}_1 = HW(\mathbf{y}) + \beta$$

Goal: Distinguish $\mathbf{r}^0, \mathbf{r}^1$ given \mathbf{L}

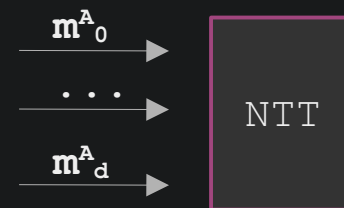
Leakage:

- Jointly (SASCA)
- Prod (normalized prod)
- Sum
- Abs diff

Noise from low to high & increasing #shares

Independent coefficients =>

- Equiv with whole key distinguishing
- Large #coeff <=> higher distinguishability
- Higher complexity (nonD&C)
- IT value can be deduced from IT of each coeff



Unprotected:

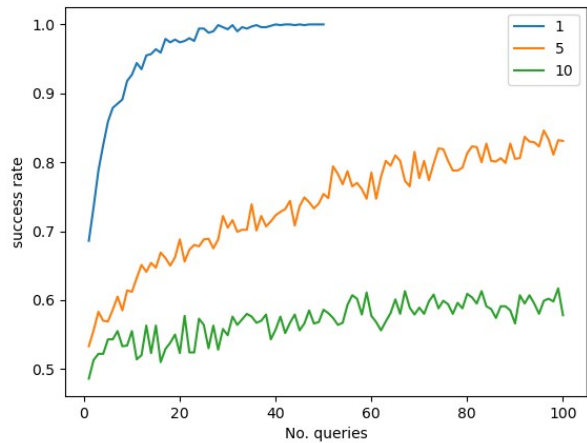
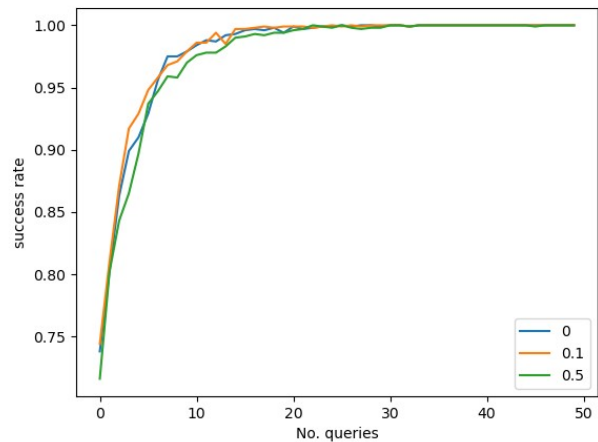
$$\mathbf{L} = HW(\mathbf{R}) + \beta$$

$$SR = 1 \text{ as soon as } HW(\mathbf{r}^0_i) \neq HW(\mathbf{r}^1_i)$$

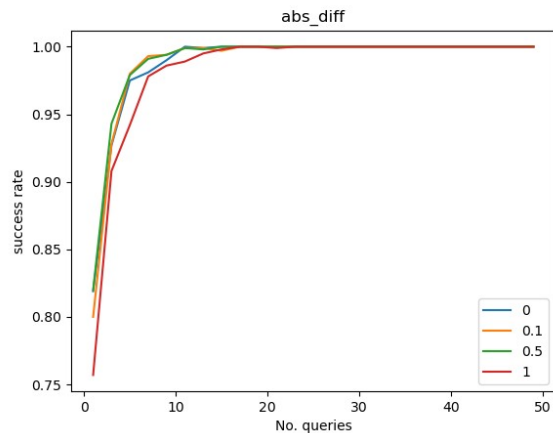
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First results-SR

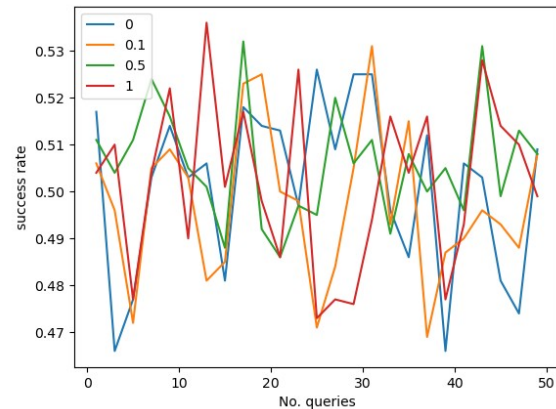
2 shares



abs_diff

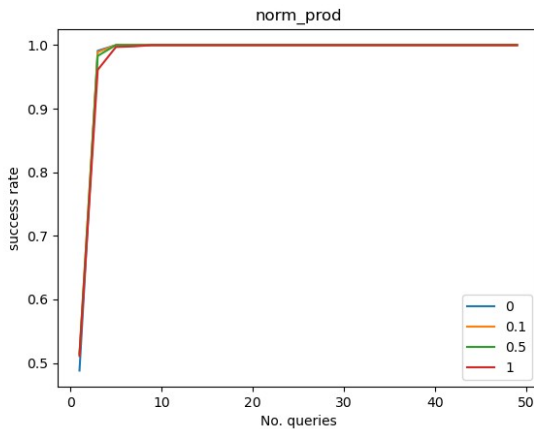


sum



prod

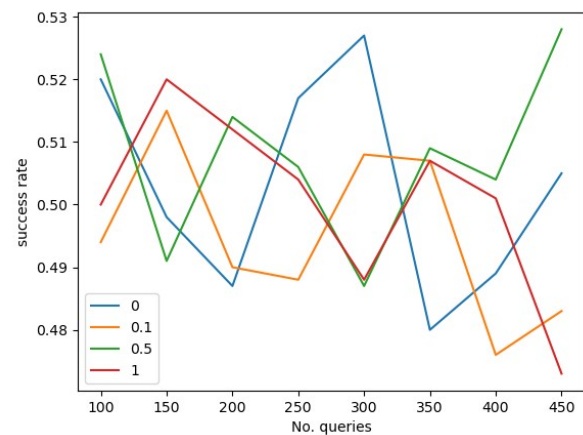
Normalized prod



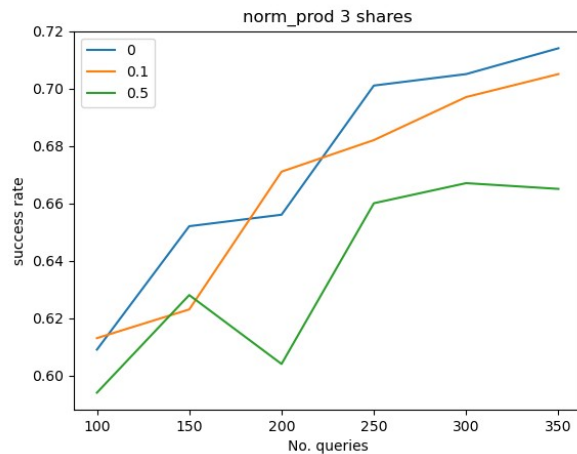
First results-SR

3 shares

prod



Normalized prod

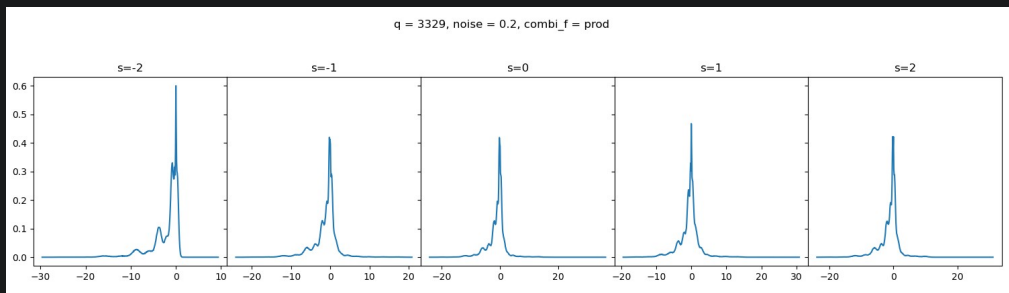
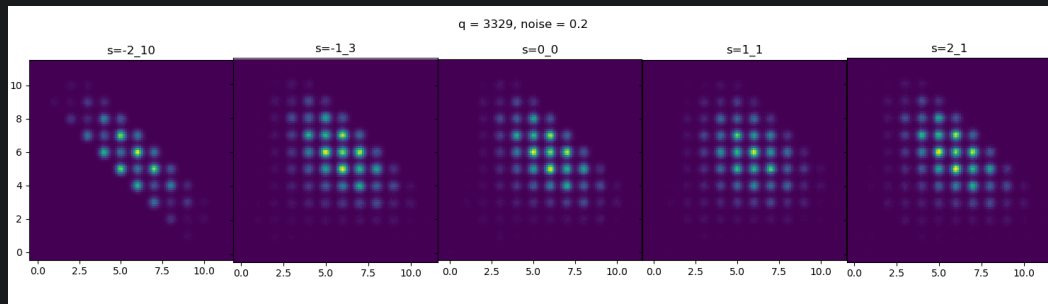
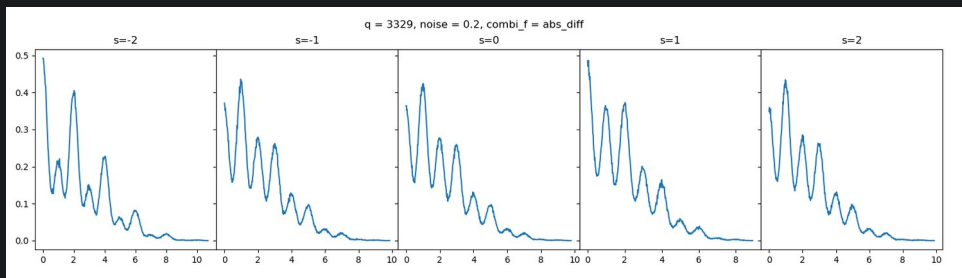


First results

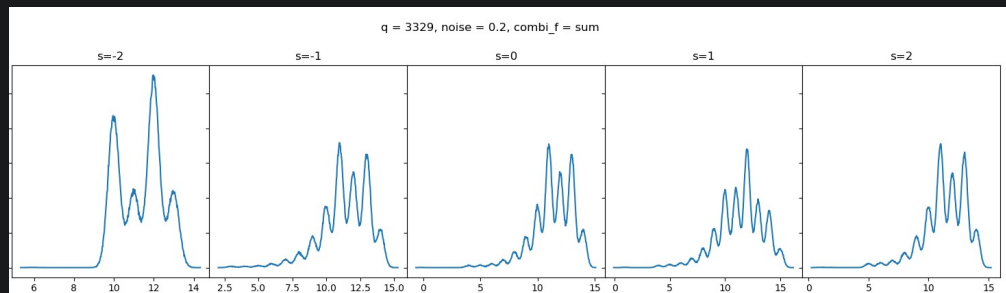
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Distribution:

abs_diff



norm_prod



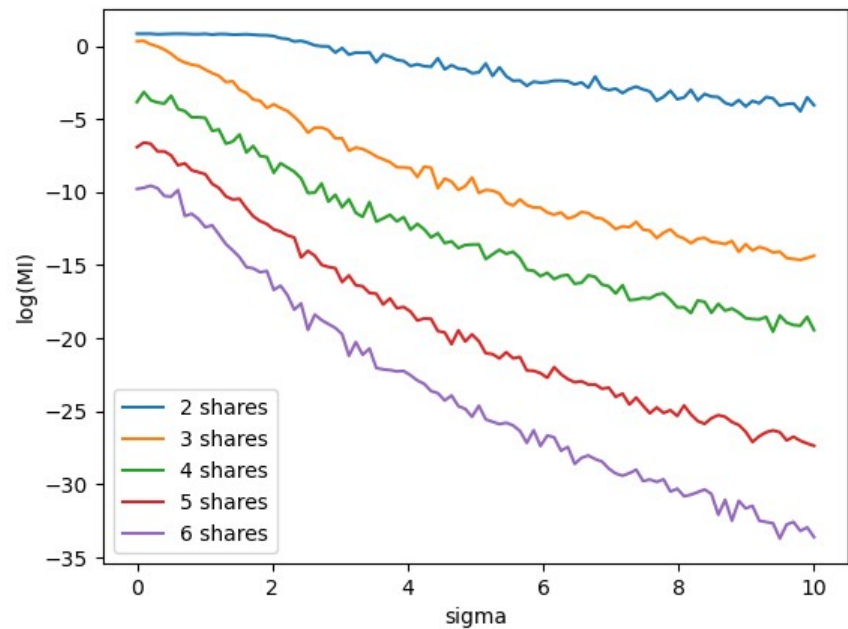
sum

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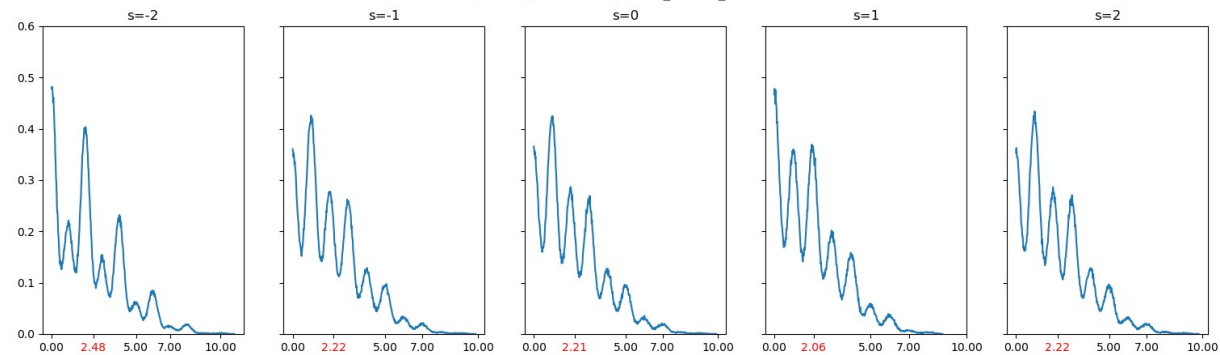
First results-MI

SASCA

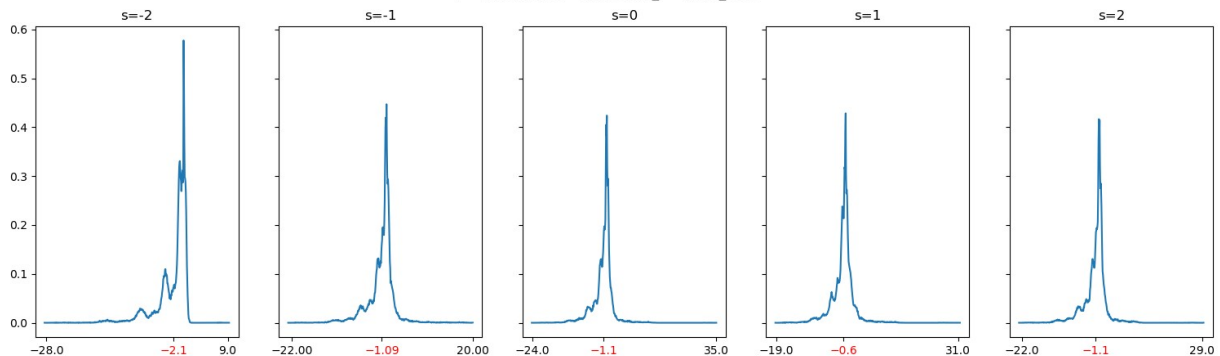
MI: prod sum



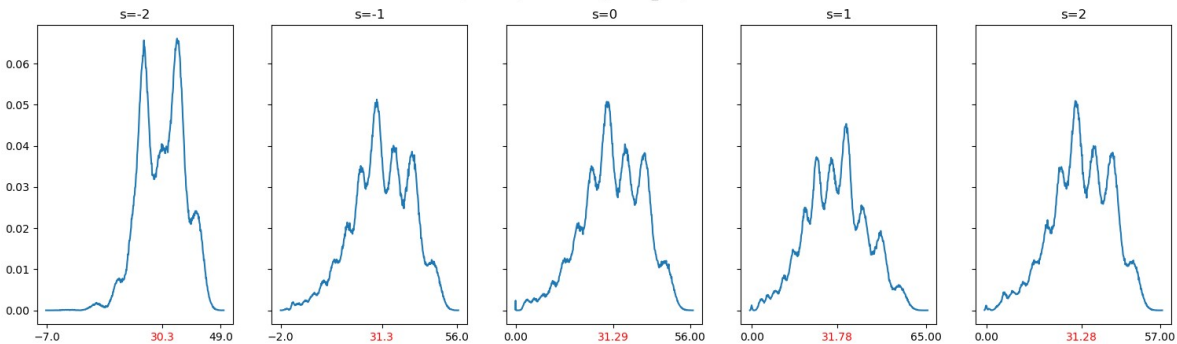
q = 3329, noise = 0.2, combi_f = abs_diff

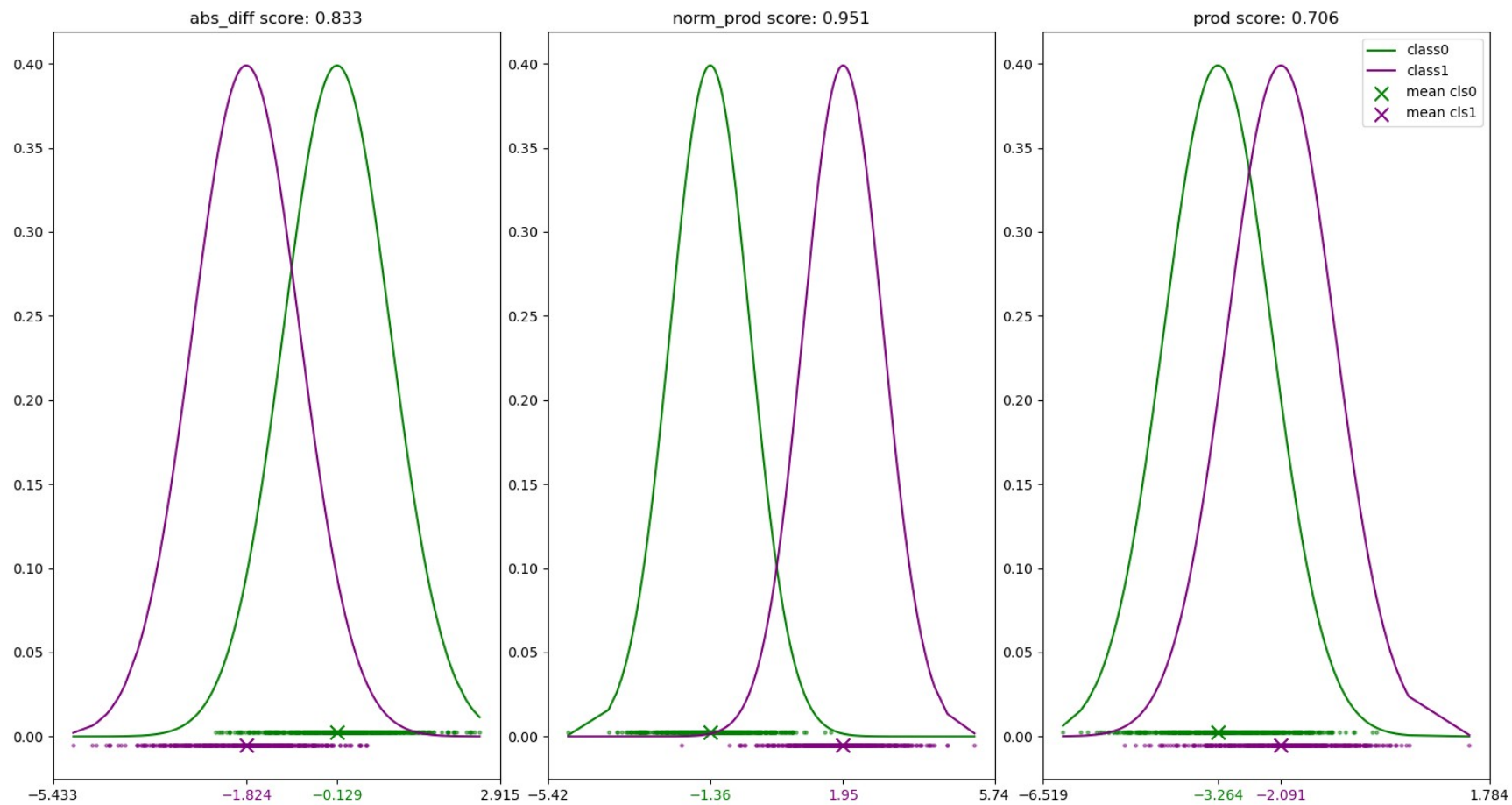


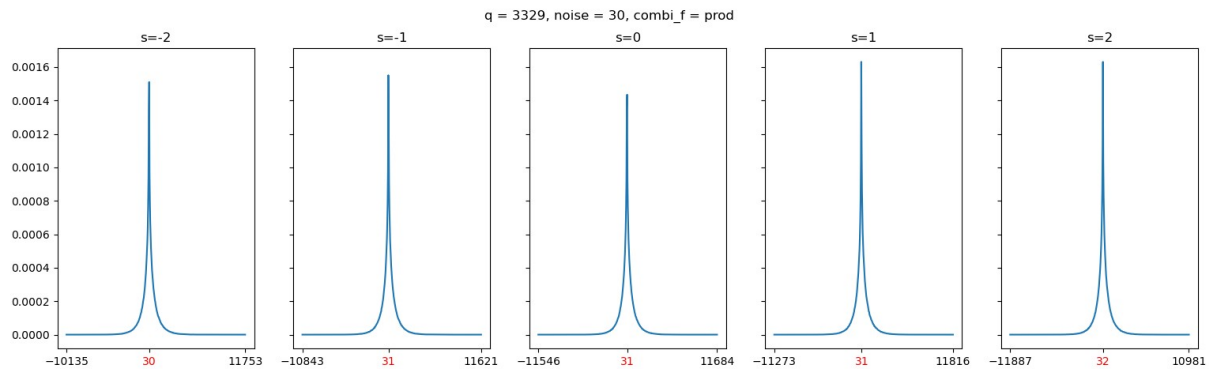
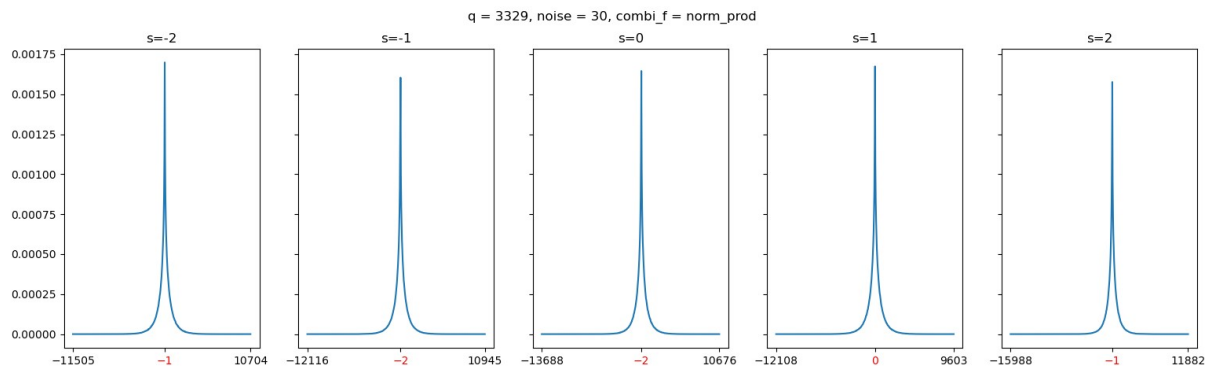
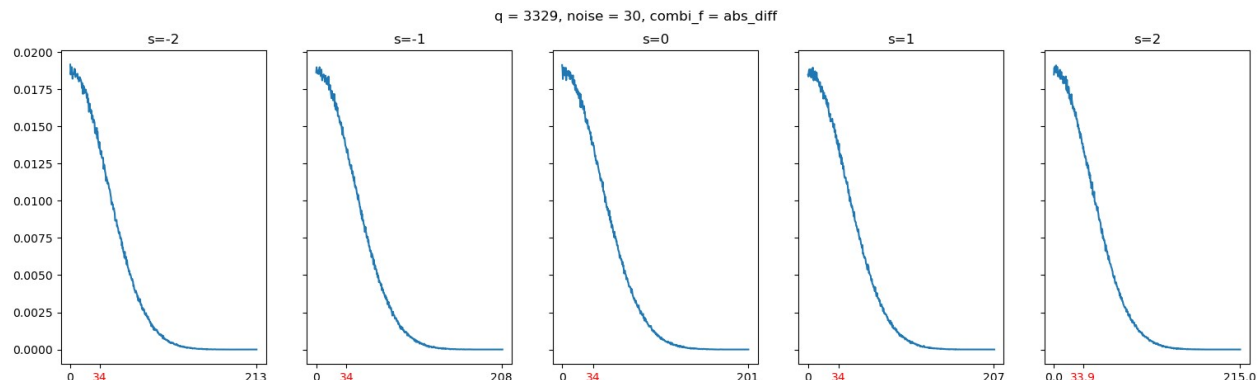
q = 3329, noise = 0.2, combi_f = norm_prod

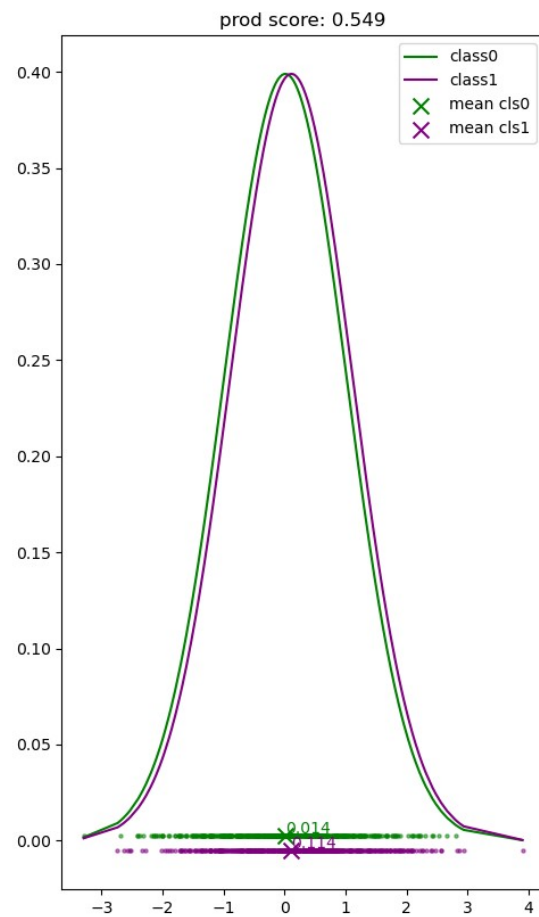
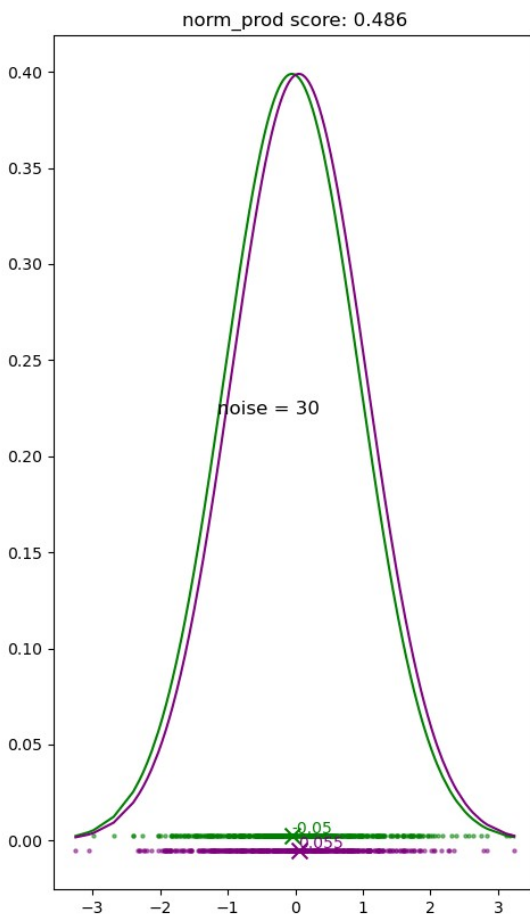
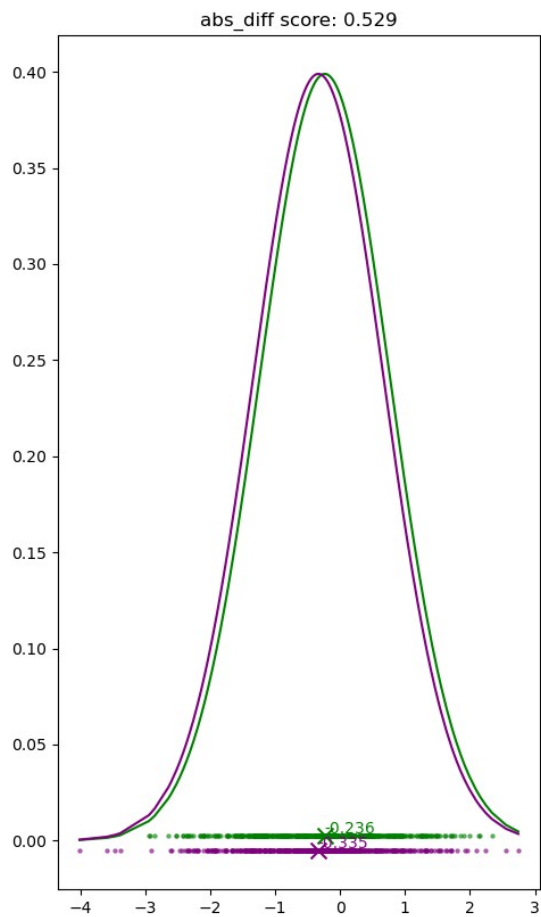


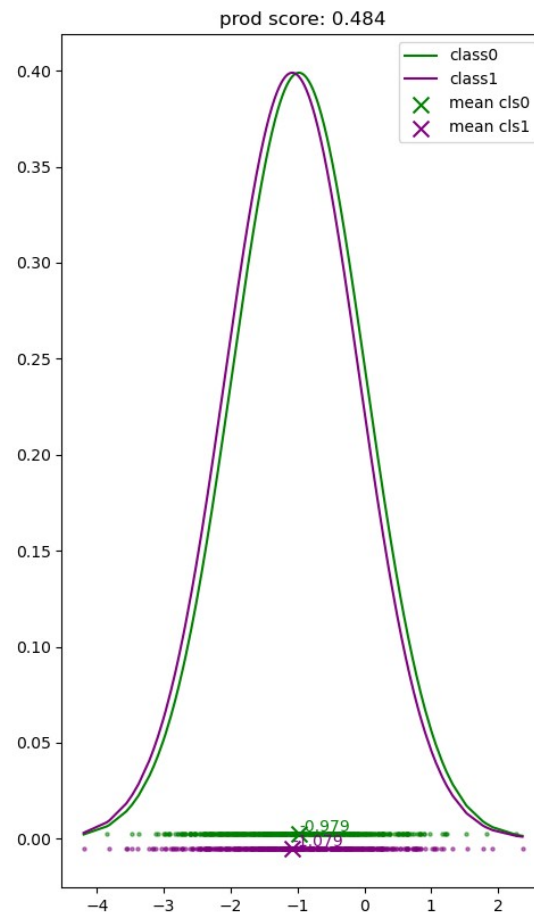
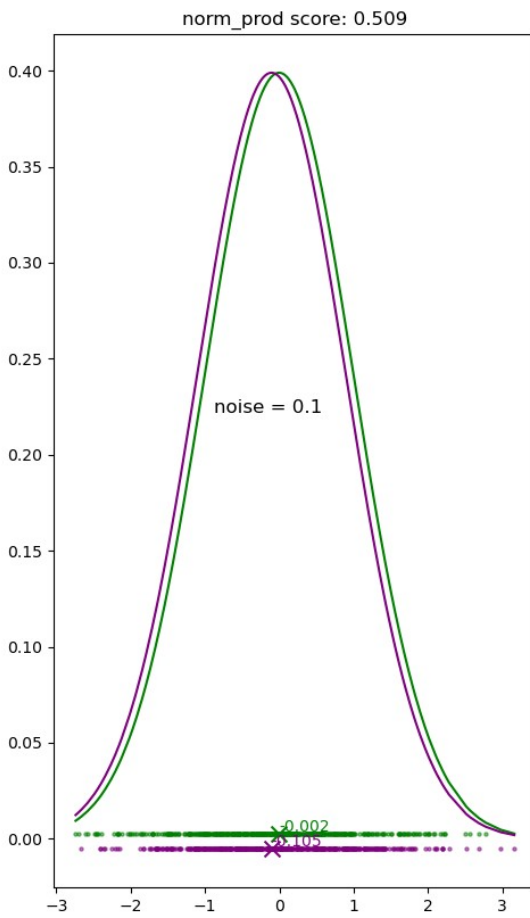
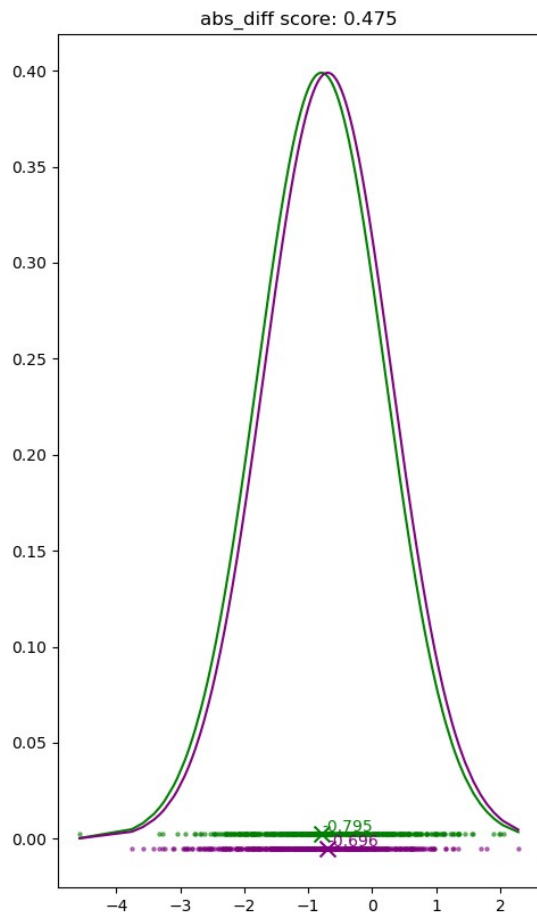
q = 3329, noise = 0.2, combi_f = prod



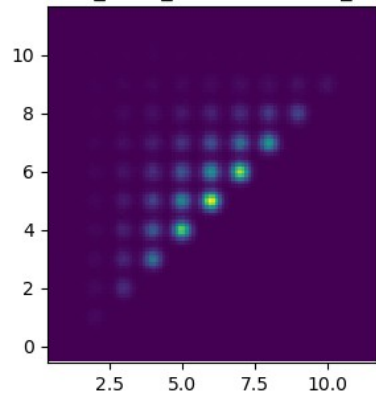




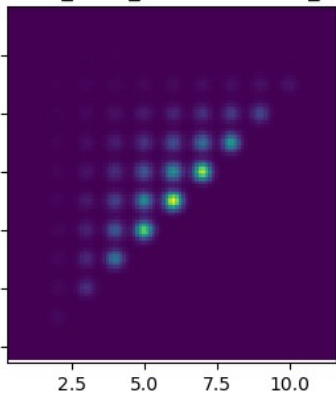




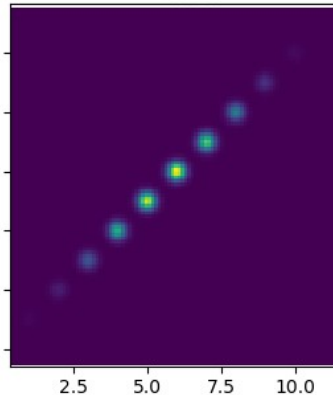
s=-2_3327_110011111111_10



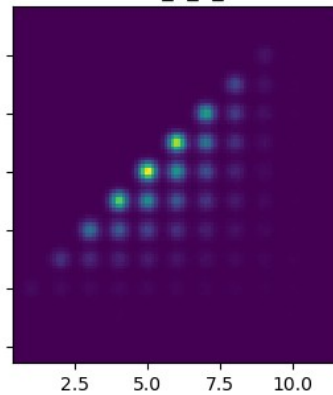
s=-1_3328_110100000000_3



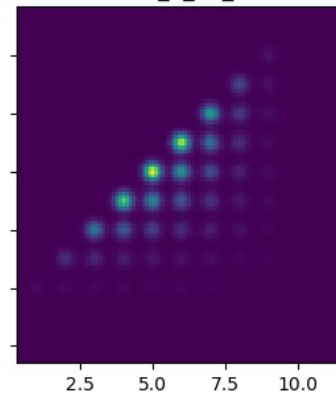
s=0_0_0_0



s=1_1_1_1

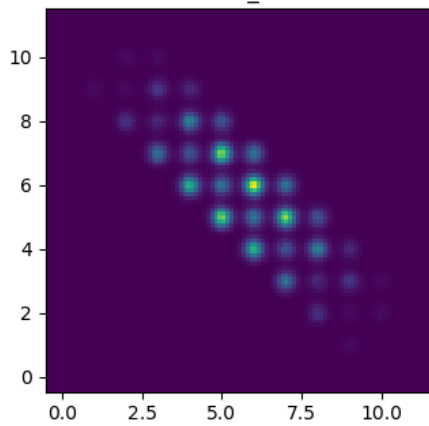


s=2_2_10_1

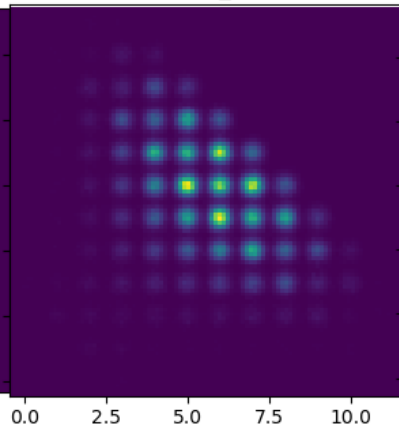


q = 3329, noise = 0.2

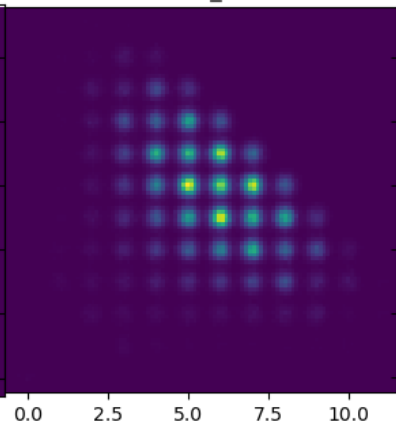
s=-2_10



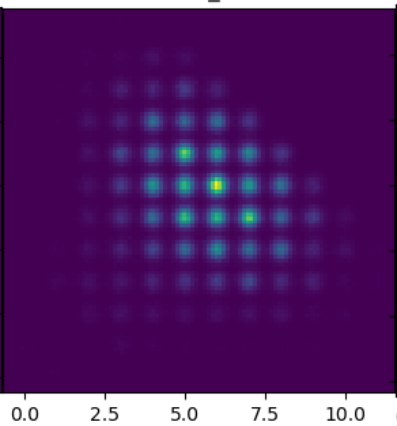
s=-1_3



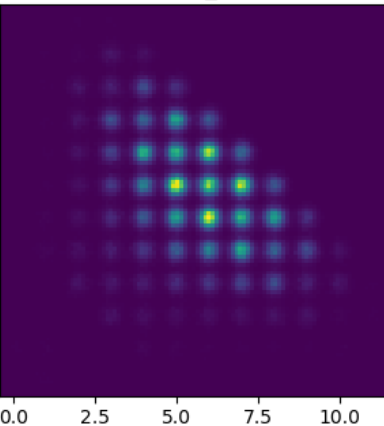
s=0_0



s=1_1



s=2_1



$$\begin{aligned}
f(\mathbf{l}|s) &= \sum_{r \in [0,q]} f(\mathbf{l}|s,r) \cdot p(r) \\
&= \sum_r f([l_r, l_{ms}|s, r) \cdot p(r) \\
&= \sum_r f([l_r, l_{ms}|r, ms) \cdot p(r) \\
&= \sum_r f(l_r|r) \cdot f(l_{ms}|ms) \cdot p(r) \\
&= \frac{1}{q} \sum_r f(l_r|r) \cdot f(l_{ms}|ms) \\
L_r|R &\sim \mathcal{N}(HW(R), \sigma^2), \quad L_{ms}|MS \sim \\
&\mathcal{N}(HW(MS), \sigma^2)
\end{aligned}$$

$$\begin{aligned}
p(s|\mathbf{l}) &= \frac{f(\mathbf{l}|s)}{\sum_{s^* \in \mathcal{S}} f(\mathbf{l}|s^*)} \\
\tilde{\mathbf{s}} &= [\tilde{s}_0, \tilde{s}_1, \dots, \tilde{s}_{255}] \\
\check{\mathbf{l}} &= [\check{\mathbf{l}}_0, \check{\mathbf{l}}_1, \dots, \check{\mathbf{l}}_{255}], \quad \check{\mathbf{l}}_i = [l_{r_i}, l_{ms_i}] \\
p(\tilde{\mathbf{s}}|\check{\mathbf{l}}) &= \prod_{i=0}^{255} p(\tilde{s}_i|\check{\mathbf{l}}_i) \\
\arg \max_{\tilde{\mathbf{s}}} \sum \log(p(\tilde{\mathbf{s}}|\check{\mathbf{l}}))
\end{aligned}$$

Target schoolbook multiplications (incomplete NTT) (KC)

Exploit leakage of storing resulting coefficients in memory

$n = 256 = 2^8$, $q = 3329 = 13 \cdot n + 1 \Rightarrow 7$ layers incomplete NTT followed by 2-coefficient schoolbook multiplication.
 $\mathbf{a} \mapsto \prod^{128} (a_{0i} + a_{1i}X)$

For each result coefficient that is stored in memory:

$c_{0i} = a_{0i}b_{0i} + \zeta a_{1i}b_{1i}$, $c_{1i} = a_{0i}b_{1i} + a_{1i}b_{0i}$, where a_{0i}, a_{1i} are known,

Two resulting coefficient are stored by the same instruction instead of consecutively

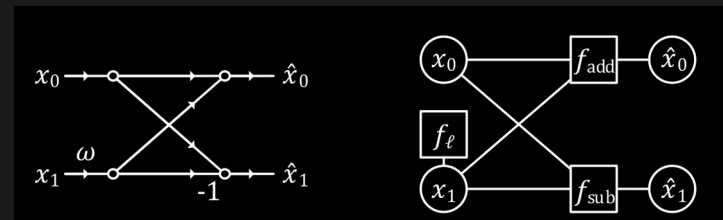
\Rightarrow Need to guess 2 coefficient at once within the range $(-q/2, q/2] \Rightarrow q^2$ combination of guessing values.

\Rightarrow Select the pair of values resulting in the largest correlation coefficient as key guess.

SASCA on INNT [Doc](#)

Target the computation of $\text{INTT}(\hat{\mathbf{s}}^T \circ \text{NTT}(\mathbf{u}'))$ (KC)

1. Build template for MulMod input
($q.n/2 = 983168$ templates for all possible combination of operand/twiddle)
2. Template matching on each butterfly.
3. Combine SC-info over entire INTT using BP.



Notes:

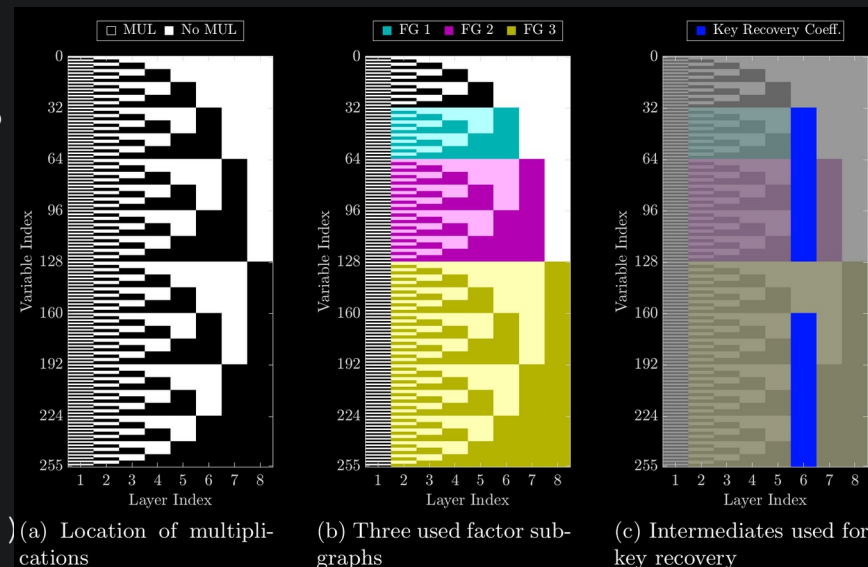
- Multiplication operation is the source of SC-info
- Apply BP on disjoint subgraphs => incomplete key recovery

=> Lattice reduction for key recovery

- In masking: BP twice to get the intermediates in all INNT invocations.

Cost:

- 983168 templates
 - 100 traces for each template
 - Lattice reduction: 5min (>160 coeff), hours (~150)
- Unsuccessful (<140)



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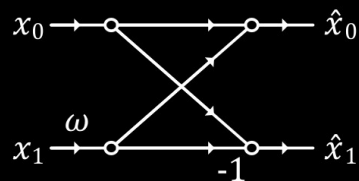
SASCA on NNT

[Doc](#)

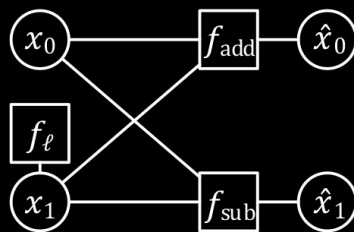
Target the computation of $\text{NTT}(\mathbf{r})$ then compute the decrypted message $m' = \text{Decode}(c_2 - \mathbf{t}^T \circ \mathbf{r})$.

Improvements (wrt SASCA on INNT):

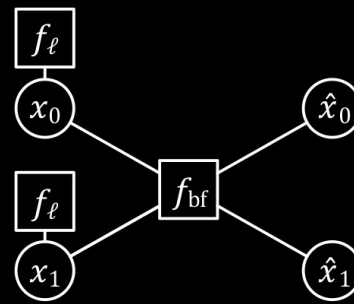
- \mathbf{r} has narrower support ($\eta = 4$)
- HW templates instead of ID templates (70000x less)
- Cluster factors of same butterfly into one (f_{add} , f_{sub} $\Rightarrow f_{bf}$) \Rightarrow avoid small loopy BP



(a) Single butterfly



(b) FG of [23]



(c) Our FG

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SC-assisted CCA.

Target the output of INNT: `fqmul()` function

- Secret key coefficients are in small range ($\eta = 2$, $[-2, 2]$)
- Select appropriate ciphertext s.t HW(INNT output) is partitioned
- Query chosen ciphertexts \Rightarrow classify PoI into different classes \Rightarrow recover key coefficients based on the partitions.

\Rightarrow full key recovery using 4 traces.

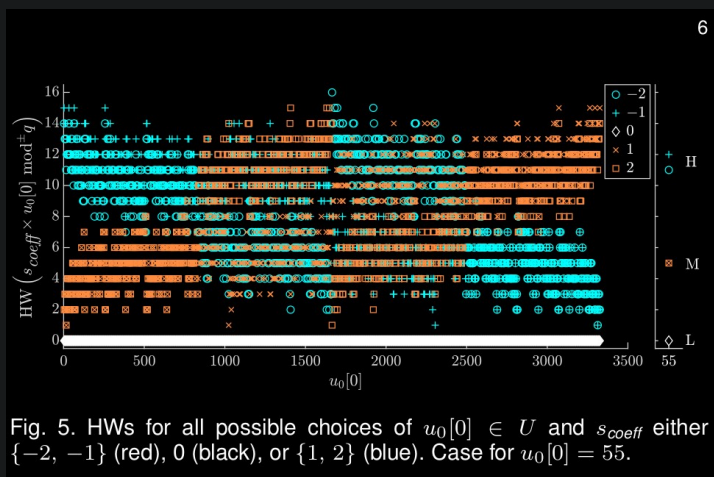


Fig. 5. HWs for all possible choices of $u_0[0] \in U$ and s_{coeff} either $\{-2, -1\}$ (red), 0 (black), or $\{1, 2\}$ (blue). Case for $u_0[0] = 55$.

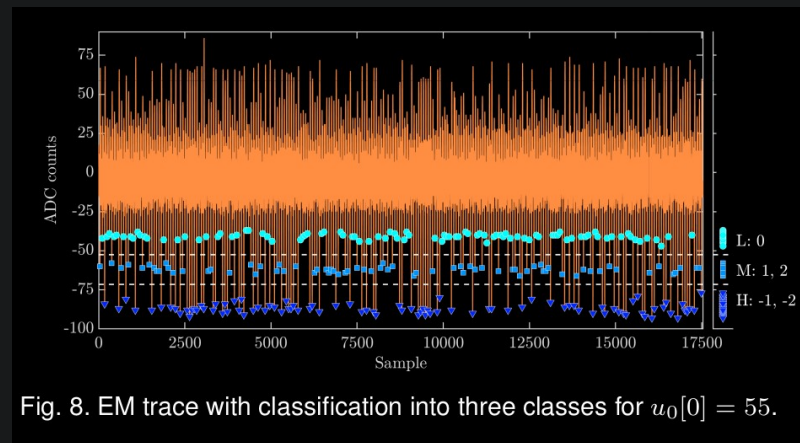


Fig. 8. EM trace with classification into three classes for $u_0[0] = 55$.

SPA Bytewise-storage

[Doc](#)

Message recovery => session key recover
(Chosen) Message recovery => long term key recover

Target any operation involves storage of decrypted message in memory:

- Message Encoding/Decoding
- KeccakAbsorb

1. Determiner-leakage: Distinguish mask value correspond to message bit => recover message bit.

2. Incremental storage: Distinguish intermediate value (HW) after each update (per bit)=> recover message bit.

3. Bytewise storage:

Exploit ciphertext malleability: Bit-Flip or Message-Rotation

Recover message bit by comparing the difference between original and modified decrypted message process => recover message word

SC-assisted CCA

- Choose ciphertext s.t the decrypted message has strong relation with secret key
- Recover message during decaps (message decoding) using SC-info (BPCO)
- Recover secret key from the relation with the decrypted message

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SC-assisted CCA.

SC-info \Leftrightarrow Binary Plaintext Checking Oracle

-Craft ciphertext s.t decrypted message only depends upon one coefficient of the secret key.

- Decrypted message can only be 0 or 1.

- Target hash operation over decrypted message (towards the end of hash computation).

- EM(G) : BPCO

=> key recover

Table 1: Unique distinguishability of every key candidate based on the validity of the codeword for the chosen values of (k_u, k_v) for our attack on the R5ND_1KEM_5d variant of IND-CCA secure Round5 KEM. **O** and **X** refer to valid ($c'' = 0$) and invalid codewords ($c'' = 1$) respectively.

Secret Coeff.	$c'' = 0$ (O) / $c'' = 1$ (X)	
	(k_u, k_v)	
	(21,3)	(12,1)
-1	X	X
0	X	O
1	O	O

$$c''_i = \begin{cases} \mathcal{D}(s[1]), & \text{if } i = 0 \\ 0, & \text{for } 1 \leq i \leq \mu - 1 \end{cases}$$

Extend to Parallel BPCO

SC-assisted CCA.

SC-info \Leftrightarrow Plaintext Checking Oracle.

Target any operations that show the different behaviors in processing reference/modified decrypted message:

- PRF
- Poly comparison

Ciphertext is correctly decrypted and decoded if noise is below a threshold.
The size of the noise provides info about the original noise (original ciphertext)-
linearly depends on secret key.

- Adding small noise into original ciphertext.
- SC-info \Leftrightarrow POC (whether modified ciphertext decrypted, decoded to original decrypted message).
- Query modified ciphertext + PCO + solve linear equations (key dependent) \Rightarrow KC

SPA Message Decode

[Doc](#)

(Chosen) Message recovery => long term key recover
Target `poly_frommsg()` function in decoding step (Decompress)

SC-assisted CCA

- Choose ciphertext s.t the decrypted message has strong relation with secret key
- Recover message during decaps (message decoding) using SC-info (BPCO)
- Recover secret key from the relation with the decrypted message

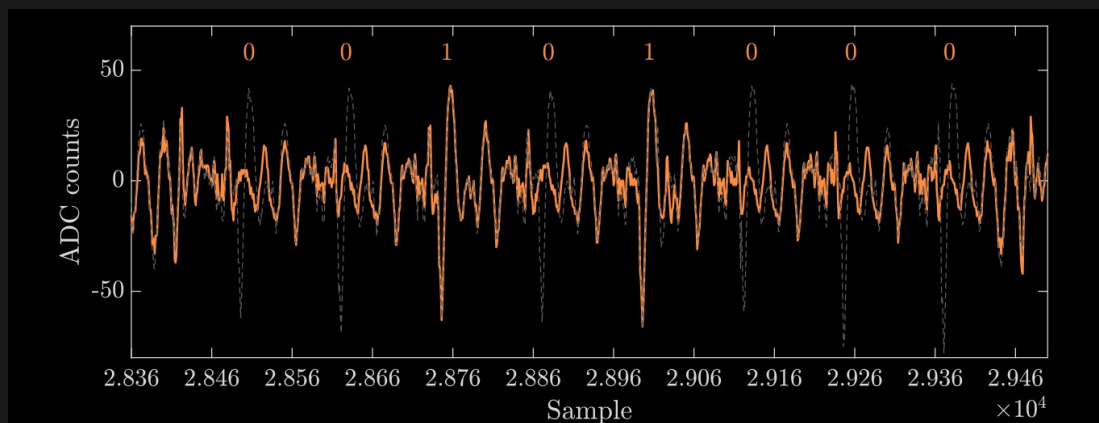


Fig. 11. Example trace at -00 (blue) showing the differences between message bit = 0 and 1. Reference trace r_1 (with all bits set 1) in gray.

$$\begin{cases} \mathbf{m}_i^{211} = 1, & \text{iff } s_0[i] = -2; \\ \mathbf{m}_i^{419} - \mathbf{m}_i^{211} = 1, & \text{iff } s_0[i] = -1; \\ \mathbf{m}_i^{2705} - \mathbf{m}_i^{2913} = 1, & \text{iff } s_0[i] = 1; \\ \mathbf{m}_i^{2913} = 1, & \text{iff } s_0[i] = 2; \\ \text{else,} & \text{iff } s_0[i] = 0. \end{cases}$$

$$\mathbf{s}_0 = (-2) \cdot \mathbf{m}^{211} + (-1) \cdot (\mathbf{m}^{419} - \mathbf{m}^{211}) \\ + 1 \cdot (\mathbf{m}^{2705} - \mathbf{m}^{2913}) + 2 \cdot \mathbf{m}^{2913}.$$

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DOC

SASCA on INTT: [Single-trace side-channel attacks on masked lattice-based encryption, Peter Pessl and Robert Primas](#)

SASCA on NTT : [More practical single-trace attacks on the number theoretic transform, Peter Pessl and Robert Primas](#)

SPA on INTT & KC on Message Decoding:

[Magnifying Side-Channel Leakage of Lattice-Based Cryptosystems with Chosen Ciphertexts: The Case Study of Kyber](#)

Byte-wise storage:

[On Exploiting Message Leakage in \(few\) NIST PQC Candidates for Practical Message Recovery and Key Recovery Attacks](#)

CCA-SPA_PCO:

[Curse of Re-encryption: A Generic Power/EM Analysis on Post-Quantum KEMs](#)

[A key-recovery timing attack on post-quantum primitives using the Fujisaki-Okamoto transformation and its application on FrodoKEM](#)

[Attacking and Defending Masked Polynomial Comparison for Lattice-Based Cryptography](#)

CCA-SPA_BPCO: [Generic Side-channel attacks on CCA-secure lattice-based PKE and KEMs](#)

CCA-SPA_Parallel_BPCO:

[Pushing the Limits of Generic Side-Channel Attacks on LWE-based KEMs - Parallel PC Oracle Attacks on Kyber KEM and Beyond](#)

[Multiple-Valued Plaintext-Checking Side-Channel Attacks on Post-Quantum KEMs](#)

[Efficient SCA on Masked Implementation.](#)

SCA on Masked Implementation

Problems with masked implementation:

- PDF is a mixture distribution of shares.
- Bitslice: each native word has 1 bit info of the intermediate variable (or i -th bit all shares)

Goals

- PDF modeling
- PI evaluation
- Validate with GE

Tools

- GMTA
- MLP
- SASCA/ESASCA/GESASCA/MLPSASCA
- EM
- KDE
- RF

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Distinguishers

Tools	Pros	Cons
GMTA	<ul style="list-style-type: none">- Simple estimations	<ul style="list-style-type: none">- Exp templates.- Gaussian assumption (bitslice)- Randomness knowledge
SASCA	<ul style="list-style-type: none">- Linear templates- Assumption adaptive (ESASCA, G-ESASCA, MLP-ESASCA)	<ul style="list-style-type: none">- Complex setup
MLP	<ul style="list-style-type: none">- Estimate full PDF- No randomness knowledge- Weak PoIs requirement	<ul style="list-style-type: none">- High profiling complexity (vague bounds)
KDE, RF		

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Variable representation

$$\mathbb{Z}_q[X] / (X^{256}+1)$$

$$\mathbf{x} = (x_0, x_1, \dots, x_{255})$$

$$\mathbf{s} = (s_0, s_1, \dots, s_k) \leftarrow B_{\eta}^k, \quad s_i \in R_q$$

$$\mathbf{c} = (\mathbf{u}, \mathbf{v})$$

