PhD Candidacy Exam Proposal:

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1 A Single Photon Source Enabled by Fano Interference between a Broadband Emitter and a High Q, Kerr Cavity

1.1 Introduction

Single photon sources play a prominent role in the field of quantum information science. [1, 2] In the QKD protocol, Alice encodes an information, for example, in the quibit of a photon polarization state. If Alice must securely transmit this quibit through a quantum channel to Bob, the reciever, then Alice's single photon source must be deterministic. In a case where Alice generates a probalistic two-photon state, then Eve, the eavesdropper, can glean information from one of the two photons (unbeknownst to Alice) while the second photon is transmitted to Bob. [3, 4] Moreover, since photons travel at the speed of light and interact weakly with the environment over long distances, enconding infromation in the quanutm state of a single photon (using degrees of freedom of polarization, momentum, or energy) is hihgly compatible and thus desirable in quantum communication application.

A deterministic single-photon source that emits a single photon *on demand*, with 100 % probability, is ideal. In practice, one evaluates the single-photon nature of a source by the ratio of the probability of single-photon to multi-photon emission, and thus single-photon sources lie on a spectrum of two main classes: deterministic and probabilistic sources. [5, 6] The former involves effective two level systems (quantum dots, single atoms, single ions) [7, 8, 9, 10, 11] that emit a single photon when excited by a resonant incident field; the latter involves, for example, parametric down conversion in waveguides (or four-wave mixing in optical fiber) systems [12, 13, 14] that emit a correlated pair of photons, where one photon heralds the other. The known difficulties with these systems involve trapping of a single ion or atom strongly coupled to cavities at cryogenic temperatures for deterministic single-photon sources, and care must be taken with the probabilistic sources to avoid generating multiple pairs of photons. Herein, we propose a theoretical basis for a novel system that circumvents ion trapping at ultra-cold temperatures, and this system is operative in the Purcell regime; then we calculate the second order correlation function as a measure of its single-photon nature.

Consider the following system: a high-quality cavity coupled to a broadband emitter. For example, ref [15] has a near sub-diffraction plasmon nanorod placed on the chip of a torodial silicon based microcavity, with a 10^7 Q-factor. The absorbtion crossection of the excited plasmon nanorod yields a Fano-lineshape slighly shifted about the resonace of the isolated mode of the microcavity. This Fano resonance is indicative of a hybrid mode of the hybridized plasmonic-photonic system. Moreover, not only is this system operative at room temperature, it is capable of measuring attometer shifts in the resonances of the microcavity modes. We propose the following modification: coating the cavity with polystyrene, which has a large third-order nonlinear susceptibility on the order of $10^{12}\mathrm{cm}^2/\mathrm{W}$. [16, 17]. (Polystyrene's third-order nonlinearity originates from the delocalization of the π-conjugated electrons along the polymer chains. [18, 19]) This achieves a kerr effect such that transmission of a photon from the pump laser (through the plasmon) to the cavity would result in the shift of the cavity's resonance (see figure). Thus the resonance of the nonlinear cavity now depends on the photon-number in the cavity.

A characteristic of this nonlinear plasmonic-photonic system is the cavity photon-number dependent Fano resonance, with a sharp resonant peak and anti-resonant dip corresponding to an enhanced and diminished plasmon absorption cross-section (see figure). At the anti-resonant frequency of diminished plasmon absorption, there is a transmision of photon into the cavity. Thus, a pump laser exciting the plasmon at the anti-resonance would yield, ideally, a single-photon occupation in the cavity; and for a non-linear cavity, this single-photon occupation would result to a shift of the Fano resonance (see yellow curve of figure). If this shifted Fano resonance (of the cavity mode with a single-photon occupation) now yields a peak of enhanced plasmon absorption that aligns with the pump laser, then any subsequent photon will be absorbed by the plasmon. Thus, the system exhibits a blockade effect for a two-photon transition at the resonant peak aligned with the pump laser, however, permiting a single-photon transition at the anti-resonant dip aligned with the pump laser. We set out to calculate the second order correlation function of such single- and two-photon states of the proposed nonlinear cavity-emitter system.

We develop the system's dynamics in Heisenberg picture. Heisenberg picture is suitable to calculate the quantum statistical correlation of the field of light transmitted throught the kerr cavity. This field correlation can be related to the experimentally observed spectral distribution, using the spectral response function. [21] Moreover, in the Heisenberg picture the dynamics of the quantum transverse field is analogous to the classical transverse field

[22], which allows for direct comparison to the classical system. [15] Then damping of the single-mode in the cavity field (and of the broad-band field) is described using Heisenberg-Langevin formalism. In so doing, this model provides a simple yet rigorous quantum approach (complementing the Louivillian formalism in the Linblad form, recently developed in Ref. [23, 24]) to account for damping of the discrete state (cavity mode) of Fano resonance, and the spectral distribution derived from this model is expressed in terms of Fano-parameters. [25]

1.2 Model

The Hamiltonian of the discrete kerr-cavity coupled to a broad-band emitter is the following $\mathcal{H} = \mathcal{H}_a + \mathcal{H}_b + \mathcal{H}_f + \mathcal{H}_I$. The derived Hamiltonian $\mathcal{H}_a$ is the energy of a single-mode dielectric kerr-cavity in free space [26], such that,

$$\begin{aligned} \mathcal{H}_a &= \frac{1}{2}\int_V \mathrm{d}^3\mathbf{x}\,\mathbf{P}\cdot\mathbf{E} \\ &= \frac{1}{2}\int_V \mathrm{d}^3\mathbf{x}(\mathbf{P}^{(1)} + \mathbf{P}^{(3)})\cdot\mathbf{E} \\ &= \frac{1}{2}\int_V \mathrm{d}^3\mathbf{x}\,4\pi(\chi^{(1)} + \frac{3}{4}\chi^{(3)} : \mathbf{E}\mathbf{E}^*)\times\mathbf{E}\cdot\mathbf{E} \end{aligned} \tag{1}$$

where $3/4\chi^{(3)}$ is the higher order non-linear suceptibility of a kerr-cavity [27], and the quantized field $\mathbf{E}$ is normalized with respect to the cavity's mode volume V. In the rotating wave approximation (and for a linear polarization), $\mathcal{H}_a$ simplifies as follows

$$\begin{aligned} \mathcal{H}_a &= \hbar\omega_k 4\pi^2\Big(\chi_{ef}^{(1)} + \frac{3}{4}\chi_{ef}^{(3)}\mathcal{E}^2\mathbf{a}^\dagger\mathbf{a}\Big)\times\Big(\mathbf{a}^\dagger\mathbf{a} + \frac{1}{2}\Big) \\ &= \hbar(\omega_c + U\mathbf{a}^\dagger\mathbf{a})\times\Big(\mathbf{a}^\dagger\mathbf{a} + \frac{1}{2}\Big) \\ &\approx \hbar(\omega_c + U < \mathbf{a}^\dagger\mathbf{a} >)\times\Big(\mathbf{a}^\dagger\mathbf{a} + \frac{1}{2}\Big). \end{aligned} \tag{2}$$

where $\mathcal{E} = \sqrt{2\pi\hbar\omega_k/V}$ is the amplitude of the quantized field.

The first-order approximation in Eq. (5) linearizes the non-linear term. This is based on the motivation that the resonant frequency of a kerr-cavity changes as a function of intesity $|\mathbf{E}|^2$. In this case, the resonant frequency depends on the average photon-number in the kerr-cavity

$$\omega_c^{NL}(N_c =< \mathbf{a}^\dagger\mathbf{a} >) = \omega_c + UN_c. \tag{3}$$

Thus the Hamilonian for the single-mode kerr-cavity $\mathcal{H}_a$, the bosonic broad-band field $\mathcal{H}_b$, and the free field $\mathcal{H}_f$ is as follows

$$\mathcal{H}_a = \hbar\omega_c^{NL}\Big(\mathbf{a}^\dagger\mathbf{a} + \frac{1}{2}\Big), \tag{4}$$

$$\mathcal{H}_b = \hbar\omega_p\Big(\mathbf{b}^\dagger\mathbf{b} + \frac{1}{2}\Big), \tag{5}$$

$$\mathcal{H}_f = \hbar\sum_j \omega_j\Big(\mathbf{f}_j^\dagger\mathbf{f}_j + \frac{1}{2}\Big), \tag{6}$$

The discrete cavity mode ω_c^{NL} and the broad-band mode ω_p both disipate energy to the free fields ω_k. Thus the interaction Hamiltonian $\mathcal{H}_I$ is as follows

$$\mathcal{H}_I = \hbar g(\mathbf{a}^\dagger\mathbf{b} + \mathbf{b}^\dagger\mathbf{a}) + \hbar\sum_j (V_j^a\mathbf{f}_j^\dagger\mathbf{a} + V_j^b\mathbf{f}_j^\dagger\mathbf{b} + \text{h.c.}). \tag{7}$$

We work in the purcell regime where $|V_a| \ll g \ll |V_b|$. Note that the original Fano problem is the limit where $|V_a| \to 0$.

Deriving the Heisenberg-Langevin equation of motion for the slowly varying operator $\mathbf{A} = \mathbf{a}e^{i\omega_c^{NL}t}$, and then transforming back to the non-slowly varying operator $\mathbf{a} = \mathbf{A}e^{-i\omega_c^{NL}t}$ yields

$$\dot{\mathbf{a}} = -i(\omega_c^{NL} - i\gamma_c/2)\mathbf{a} - ig\mathbf{b}, \tag{8}$$

$$\dot{\mathbf{b}} = -i(\omega_p - i\gamma_p/2)\mathbf{b} - ig\mathbf{a}, \tag{9}$$

$\gamma_{c,p}$ accounts for both radiative and non-radiative damping as described in Ref [28]. (Note that the above equation has assumed an evacuated initail reservoir state.)

In the experiment, the broad-band mode is driven by the external field of a monochromatic laser operating at ω. The spectral distribution is derived from the spectral response function, such that,

$$S(\omega) = \frac{1}{\pi}\int_0^\infty \mathrm{d}\tau\, e^{i\omega\tau} < \mathbf{b}^\dagger(t_0 = 0)\,\mathbf{b}(t_0 + \tau) > . \tag{10}$$

We find that the spectral reponse function can be interpreted as the steady state solution of the average photon-number in the broad-band mode rotating in the frame of a drive frequency, i.e.

$$S(\omega) =< \tilde{\mathbf{b}}_{ss}^\dagger\tilde{\mathbf{b}}_{ss} > \tag{11}$$

$$\tilde{\mathbf{b}}_{ss}^\dagger = \mathbf{b}_{ss}^\dagger\, e^{i\omega_L t}; \quad \tilde{\mathbf{a}}_{ss}^\dagger = \mathbf{a}_{ss}^\dagger\, e^{i\omega_L t}; \tag{12}$$

$$\dot{\mathbf{b}}_{ss} = 0 = -i(\omega_p - i\gamma_p/2)\mathbf{b}_{ss} - ig\mathbf{a}_{ss} + iE_{drive}(e^{i\omega_L t} - e^{-i\omega_L t}). \tag{13}$$

and E_{drive} is the amplitude of the monochromatic laser operating at a drive frequency $\omega = \omega_L$. The reduced spectral response yields

$$\begin{aligned} \frac{S(\omega=\omega_L)}{S(\omega=\omega_L)_{g=0}} &= \frac{< \tilde{\mathbf{b}}^\dagger_{ss}\tilde{\mathbf{b}}_{ss} >}{< \tilde{\mathbf{b}}^\dagger_{ss}\tilde{\mathbf{b}}_{ss} >_{g=0}} \\ &= \left(1 + \frac{\gamma_c}{\gamma_p}\frac{g^2}{(\omega_L - \omega_c^{NL})^2 + (\gamma_c/2)^2}\right)\left|\frac{q+\epsilon}{\epsilon + i}\right|. \end{aligned} \tag{14}$$

The first term is the Fano profile, where $\epsilon = (\omega_L - \omega_{eff})/(\gamma_{eff}/2)$ and $q = (\omega_c^{NL} - i\gamma_c/2 - \omega_{eff})/(\gamma_{eff}/2)$; ω_{eff}, γ_{eff}, and q are the Fano parameters. The second term is the Lorentz distribution due to the line-width broadening of the discrete state. The spectral distribution reduces to the Fano profile in the limit $\gamma_c \propto |V_a| \to 0$.

Since we are interested in single-photon blockade due to the kerr nonlinearity, $U \propto \chi^{(3)}_{eff}$, we calculate the second order correlation function $g^{(2)}$ of light transmitted through the kerr-cavity coupled to the driven broad-band emitter, at the steady state, as a function of U, i.e.,

$$\begin{aligned} g^{(2)} &\equiv \frac{< n_a, n_b|\, \tilde{\mathbf{a}}^\dagger_0 \tilde{\mathbf{a}}^\dagger_{ss}\tilde{\mathbf{a}}_{ss}\tilde{\mathbf{a}}_0\, |n_a, n_b >}{(< \tilde{\mathbf{a}}^\dagger_0\tilde{\mathbf{a}}_0 >)^2} \\ &= \frac{n_a < n_a - 1, n_b|\, \tilde{\mathbf{a}}^\dagger_{ss}\tilde{\mathbf{a}}_{ss}\, |n_a - 1, n_b >}{n_a^2} \\ &= \frac{1}{n_a}\frac{g^2}{\left|\omega_L - \omega_c^{NL}(N_c = 1) + i\gamma_c/2\right|^2} < n_a - 1, n_b|\, \tilde{\mathbf{b}}^\dagger_{ss}\tilde{\mathbf{b}}_{ss}\, |n_a - 1, n_b > \\ &= \frac{g^2}{n_a}\frac{E^2_{drive}}{\left|\big(\omega_L - \omega_c^{NL}(N_c = 1) + i\gamma_c/2\big)\big(\omega_L - \omega_p + i\gamma_p/2\big) - g^2\right|^2}. \end{aligned} \tag{15}$$

For single-photon blockade, the photon-number in the cavity is set to one, such that, $\omega_c^{NL}(N_c = 1) = \omega_c + U$, and the amplitude of the drive field to populate the cavity mode with a single photon is such that

$$\begin{aligned} < n_a = 1|\, \tilde{\mathbf{a}}^\dagger_{ss}\tilde{\mathbf{a}}_{ss}\, |n_a = 1 > &= 1 \\ &= \frac{g^2\, E^2_{drive}}{\left|\big(\omega_L - \omega_c^{NL}(N_c = 0) + i\gamma_c/2\big)\big(\omega_L - \omega_p + i\gamma_p/2\big) - g^2\right|^2}. \end{aligned} \tag{16}$$

Thus to popoulate the cavity with a single-photon occupation at steady state, and with a resonant drive ($i.e., \omega_L = \omega_c$), then

$$E_{drive} = \left|(i\gamma_c/2)(\omega_c - \omega_p + i\gamma_p/2) - g^2\right|/g. \tag{17}$$

1.3 Analysis

The two-photon blockade effect depends on the Fano resonance shift induced by the kerr cavity. The system is designed such that a monochromatic pump laser aligns with the anti-resonant frequency (corresponding to a diminished plasmon absorption) of a single-photon transition but aligns with the resonat frequency (corresponding to an enhanced plasmon absorption) of a two-photon transition; the former state transmits into the cavity while latter state is absorbed by the plasmon. Thus the degree of nonlinearity depends on the line-width between the anti-resonant dip and the resonant peak of the Fano line-shape, such that,

$$\begin{aligned} \lambda_1 - \lambda_2 &= \lambda_1 - (\lambda_1 + 2\pi c/\Gamma) \\ \Delta\lambda &= 2\pi c/\Gamma \end{aligned} \tag{18}$$

where Γ, the FWHM, is approximately equal to the line-width between the anti-resonant dip and the resonant peak of the Fano-lineshape; λ_1 and λ_2 are the wavelengths of the modes associated with the single- and two-photon transitions, respectively. Assuming the whispering-gallery modes of the torodial kerr-cavity as standing waves, then $\lambda_{1,2}/n_{1,2} = 2L/m$, where n_2 is the refractive index of the kerr-cavity [20]:

$$\begin{aligned} n_2 &= n_1 + \Delta n I \\ &= n_1 + \frac{3\chi^{(3)}}{8n_1 c\epsilon_0} I. \end{aligned} \tag{19}$$

The kerr effect is induced by the local intensity I from the pump laser populating the kerr-cavity. From the above equations we find the relation:

$$\begin{aligned} \frac{\Delta\lambda}{\lambda_1/n_1} &= \Delta n I \\ &= \frac{3\chi^{(3)}}{8n_1 c\epsilon_0} I \end{aligned} \tag{20}$$

For a pump laser operating at an intensity $\approx 80\,\mathrm{GW/cm^2}$, with a third order susceptibility $\approx 1.15 \times 10^{-12}\ \mathrm{cm^2/W}$, [16] and for a cavity mode $\lambda_1/n_1 \approx 1550\,\mathrm{nm}$, the desired blockade effect can be achieved for a two-photon state whose resonance is shifted by $\Delta\lambda \approx$. This value serves as an upper bound since the fano-lineshape has a narrow line-width on the order of an attometer. This is advantageous because the desired blockade effect could be achieved for a kerr-cavity with a much smaller third order suceptibily and a faint laser beam (whose coherent state could have an average photon-number in the range $\mu = 1 - 4$).

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