A L B E R T – L U D W I G S – U N I V E R S I T Ä T F R E I B U R G IM B R E I S G A U

Examination Board

Master of Science in Economics Program

Analysing and forecasting financial risks under a trading perspective

Master Thesis

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1 Introduction

Analysing and forecasting financial risks of an asset mean i.a. forecasting the volatility of the asset in the future based on the information available up to the present timepoint. The forecasting of the Value at Risk and other quantile based measures belong here as well.

The trading perspective of my analysis can be interpreted as a way of collecting financial information about the relevant asset. This information refers to the asset price series up to the present timepoint. Sampling prices very often includes a lot of information and at the same time a lot of noise. It is however possible to find the optimal sampling frequency for collecting prices. Another aspect that has to be considered is the rule for collecting such information. Not only the prices themselves but also the number of trades and distribution of trades convey information.

In the past years more and more research was done related to high frequency data and capturing the heartbeat of the market through different sampling schemes. However, the new thing in my thesis is that the volatility signature plots of the different sampling scheme and frequency combinations have not yet been analysed. The goal of my empirical study is to compare the sampling schemes and frequencies, and choose the best one to forecast variances and VaR.

In my seminar paper *Multivariate volatilities and high frequency data: the latest developments* I dealt with the challenges of working with high frequency data in practice, especially solving the problems of high dimensionality when forecasting, which is computationally challenging. This topic directed my interest towards doing more research on high frequency data.

I proceed as follows. In section 2, I introduce the literature on high frequency financial data and intrinsic time sampling schemes. Section 3 includes my empirical results. I conclude in Section 4 and provide the more extensive tables and figures in the appendix. I attach the data and Python code in separate files. The libraries used in the code are referenced in the relevant parts of the code directly.

2 Presentation of the topic through the relevant literature

In this section I explain the importance of high frequency financial data and define the intrinsic time sampling schemes. I introduce the concept of Realized Volatility, starting with the theoretical foundations then continuing with the empirical findings on the price process and its influence on the RV estimator. I emphasize the importance of market microstructure noise. Finally, I present empirical results found in the literature on the intrinsic time sampling schemes and the optimal sampling frequency.

High frequency financial data refers to time-series data which is sampled at a fine time scale e.g. a few minutes or even seconds. The availability of this type of data improved a lot in the past years. Thanks to the development of technology and the appearance of electronic platforms and online databases it is relatively easy to obtain real time data, including stock prices, thus the research interest also increased in this field. Even though sampling at high frequency provides us with a high number of observations and lots of information, which are both advantageous, you have to be attentive when working with high frequency data. I elaborate on the possible risks and challenges later in this chapter.

There exist different schemes for sampling data at high frequencies. Besides the basic calendar time sampling (CTS) scheme, which samples price observations that are equidistant in calendar time, there exist other, so-called intrinsic time sampling schemes which deform the physical time and aim to capture the heartbeat of the market and the intensity of the markets activity. The intensity measures can include the number of trades, number of price changes and volatility levels etc. On the stock markets more intense trading activity and higher volatility is present at the beginning and at the end of the day compared to the rest of the day (Engle and Russell, 1998; Wu, 2012). Thus, sampling the prices in calendar time would not provide us with the appropriate information regarding the dynamics of the market. In the following, I present some of the existing intrinsic time sampling schemes with no completeness.

The Tick Time Sampling (TT) scheme selects transactions with regularly spaced number of ticks (Oomen, 2005, 2006). The non-zero Tick Time

Sampling scheme samples prices that are equidistant in the number of non-zero tick changes (Griffin and Oomen, 2008). The Time Transformation Sampling (TTS) scheme samples prices that are equidistant in the number of ticks averaged over all past trading days (Wu, 2012). This method takes into account the intraday pattern in the trading activity as a common property across all past and present trading days. The Duration Adjustment Sampling (DAS) scheme was introduced by Engle and Russell (1998) and it can be understood as a smoothed version of the TTS. The Transaction Time Sampling (TRTS) scheme samples prices with every transaction (Griffin and Oomen, 2008).

The Business Time Sampling (BTS) transactions are selected to ensure approximately equal volatility of the returns over each interval. The sample schemes presented so far are all based on explicit criteria while the BTS scheme depends on a latent, unobserved measure (Dong and Tse, 2014). The Weighted Standard Deviation Sampling (WSDS) scheme samples prices that are equidistant in the average volatility based on all past trading days (Boudt et al., 2011). The intraday volatility is computed as a weighted standard deviation of the intraday returns across all previous days. Further research could be done in the direction of alternative, not yet discovered sources of market activity for creating further types of intrinsic time sampling schemes.

The main purpose of sampling prices is to forecast the price movements in the future. However, it is not possible to appropriately forecast prices in the future, based on past information, but there are models that are able to decently forecast the future periods of either high or low volatilities. The main approaches for computing volatilities are based on past daily data like the GARCH models or the Risk Metrics model. The main disadvantage of these kind of models is that they do not incorporate the recent intraday trading behaviour since they are based solely on daily data. Further, the GARCH models are based on a deterministic parametric specification and have a fix form.

An alternative method that makes use of the intraday information and builds on high frequency data is called the Realized Volatility. The Realized Variance is calculated by summing over the intraday squared returns for each specific day. The availability of high frequency data has led to further improvements in realized models surveyed by Bauwens et al. (2006). Hansen and Huang (2016) show that using realized measures improves both in and out-of sample fit and model predictions, at the same time their model is able to include leverage effects. In order to obtain forecasts of future volatility from a RV time-series it is necessary to apply a long-memory model for example the HAR model of Corsi (2009).

The Realized Volatility approach theoretically provides an error-free estimation of the volatility according to Andersen et al. (1999) by assuming that the logarithmic price process is a standard continuous-time diffusion process. In theory, the realized variance is a consistent estimator of the 1-day integrated variance. These authors assume that the stochastic error of the measure can be reduced to zero by increasing the sampling frequency of the intraday returns. If there are no jumps in the log price process, then the Integrated Variance and the Quadratic Variation are equal and the RV is consistent (Barndorff-Nielsen and Shephard, 2002). Furthermore, Corsi et al. (2001) mathematically derive the RMSE formula at different frequencies, and by comparing them they clearly show that taking returns at small time intervals for measuring the daily volatility performs better compared to larger time intervals.

In contrary to the mathematical results, Corsi et al. (2001) found during their empirical analysis that there is actually a considerable systematic error in practice and the realized volatility computed from high frequency data is not unbiased and not a consistent estimator of the daily volatility computed with daily returns. In fact, the volatility estimator computed at a short time horizon is strongly biased. The explanation of Corsi et al. (2001) is when you shorten the time-scale of the observation, the logarithmic price process is not a diffusion process anymore. They study the autocorrelation of intraday returns and find that it is not negligible, which means that the empirical returns are not i.i.d. In other words, the intraday returns are serial correlated so the true price process cannot be a martingale and the RV is not consistent. This is directly related to microstructure effects that arise from the price formation process. Pohlmeier et al. (2012) mention several causes for market microstructure noise, such as price rounding effects, bid-ask bounces, gradual response of price to a block trade, strategic order flows and data recording mistakes.

Therefore, high number of observations decrease the measurement error but at the same time cause a bias in the estimated volatility due to more noise. Thus, there has to be a trade-off and evaluation of the two effects when choosing the optimal sampling frequency. For this purpose Andersen et al. (2000) suggest using the volatility signature plot, which plots the average of a very long sample of RV's against the sampling frequency and helps choosing the highest possible sampling frequency at which there is no substantial bias in the volatility. This is graphically the frequency where the average RV stabilizes.

Oomen (2005) studies the statistical properties of bias-corrected realized variances in the existence of market microstructure noise in the high-frequency data. His analysis is built on a pure jump process for asset prices and applies different sampling schemes such as the basic calendar time, business time and transaction time. The price process he uses consists of a martingale component and an i.i.d. distributed market microstructure noise. This pure jump specification allows to study the properties of the price process on different time scales and provides a closed form for the bias and MSE of the bias corrected realized variance. By using this form you can determine the optimal sampling frequency, measure the performance of bias corrected RV and compare the performance among the different sampling schemes. He mentions the diffusion limit, meaning that the jump intensity increases and the jump size decreases so the price process is built from a growing number of jumps with decreasing size. The pure jump process converges in distribution to diffusive processes studied by Hansen and Lunde (2006).

The pure jump specification that Oomen (2005) uses in his model, which is a discontinuous process, is not the standard way of modeling the efficient price process. In the previous literature the price process was described by a continuous semi-martingale process to ensure that the RV is a consistent estimator of the quadratic variation and integrated variance, see in Meddahi (2002), Andersen et al. (2003), Bandi and Russell (2006), Barndorff-Nielsen and Shephard (2004), Hansen and Lunde (2006), Zhang et al. (2005). However, in the discontinuous case the RV is only a consistent estimator of the quadratic variation but an inconsistent estimator of the integrated variance.

In the empirical analysis Oomen (2005) shows that sampling in business time and transaction time perform better compared to the commonly used calendar time, specifically they reduce the mean-squared error (MSE). He also explains that the reasons why transaction time sampling is better are: on one hand, the bias correction is more effective and on the other hand, regularly spaced returns provide a lower MSE on the transaction time scale than on a calendar time scale because the returns in transaction time are devolatized through an appropriate deformation of the time scale.

In their Monte Carlo study Dong and Tse (2014) find that the BTS returns perform better than the CTS and TTS returns when estimating the daily integrated volatility. They also show that the BTS scheme is closer to yielding i.i.d. Gaussian returns than the CTS or TTS meaning that it includes less jumps and the RV is closer to the integrated variance.

When Oomen (2005) introduces bias correction, the performance for high frequency data increases. If sampling in transaction time instead of calendar time the MSE can be further reduced and a better performance can be obtained. His empirical results support the mathematical and statistical reasoning. The optimal sampling frequency of the stock prices is 2.5 minutes which is reduced to 12 seconds after applying the bias correction and further decreased when using TTS instead of CTS. He emphasizes the importance of bias correction and choosing the right sampling scheme.

Oomen (2005) observed that the relative size of the noise compared to the efficient price innovation increases as the sampling frequency increases. Market microstructure noise makes the RV a biased estimator of the integrated variance of the efficient price process. There are many ways to get rid of this bias like subsampling or sparse sampling.

The returns in transaction time follow an MA(1) process coming from the noise component, returns in calendar time follow an ARMA(1,1) process. Oomen (2005) shows that taking the diffusion limit of the moments of returns in business time yields the moments of returns in transaction time and at the same time the diffusion limit can be considered a special case of moments of returns in calendar time, which can be calculated as a probability weighted average of moments of returns in transaction time. Thus, the results derived for CTS directly carry over to the transaction and business time.

Oomen (2005) compares the different sampling schemes and finds that the bias is the largest under TTS and smallest under CTS, at the same time the bias correction removes all bias under TTS but not under CTS or BTS because of the ARMA dependence of returns. He shows empirically that the bias-correction substantially reduces the MSE and increases the optimal sampling frequency. The benefits are higher from sampling in transaction time when the trade intensity pattern is volatile, meaning an irregular trading pattern. This can mean 5-40% reduction in MSE according to Oomen (2006).

Oomen (2006) derives a closed form for bias and MSE as a function of model parameters and the sampling frequency. He uses Poisson process to model the asset price as the sum of a finite number of jumps, each seen as a transaction return with the Poisson process counting the number of transactions. For the market microstructure noise he uses a flexible MA(q) dependence structure on the price increments. He finds that the optimal sampling frequency can be determined mostly based on the number of trades and the level of the market microstructure noise.

Oomen (2006) further raises awareness to the fact that it is generally important that market microstructure noise has a huge affect on the statistical properties of the realized variance. Andersen et al. (2000) were one of the first who have documented the evidence on the relationship between sampling frequency, market microstructure noise and the bias in RV. Oomen (2006) mathematically derives that an increase in the trading activity and a decrease in the level of market microstructure noise lead to a higher optimal sampling frequency.

From his empirical results Oomen (2006) concludes that the optimal sampling frequency varies daily due to the variation in the noise ratio and the market activity. For each day he estimates the intensity process plus the CPP-MA model parameters. Afterwards, he estimates the optimal sampling frequency and the associated MSE. He observes that the noise ratio has a steady decrease in time indicating better market efficiency and as a result of this the optimal sampling frequency increases. As a result the MSE of realized variance drops dramatically and it is smaller under TTS than under CTS. It is clear that sampling at a fixed frequency is not optimal and a data-driven approach such as the one outlined in his paper would be desirable. Bandi and Russell (2006) measure the economic value and the benefits of calculating and using the optimal sampling frequency each day instead of setting it to a fixed value as in common practice.

The connection between the optimal sampling frequency and the dynamics of the noise ratio is straightforward, a high noise ratio corresponds to a high frequency and vice versa. A regression of the number of prices on a constant plus the estimated noise ratio gives a regression of close to 80% according to the results by Oomen (2006).

Dimitriadis and Halbleib (2021) are the first to apply the intrinsic time approach on intensity measures to estimate and forecast daily VaR and ES measures directly stemming from high frequency data. RV generally assumes continuous-time but they build on a log price process which is a subordinated self-similar process indexed by the intrinsic time. The subordinated processes are very general, they can follow either continuous or pure-jump processes. It is empirically proven that they can capture important properties of financial returns like fat-tails, conditional heteroscedasticity and long memory. Generally the properties of the subordinator (a latent process) drive the properties of the subordinated process. Based on the subordination process assumption it is possible to include the intrinsic time as an index for the subordinated process. In a stochastic subordination process the subordinator is a stochastic transformation of the clock time 't' and assumed to be independent of X(t), the main stochastic process. The subordinator also defines the different patterns of paths being continuous or pure-jump processes. Due to the self-similarity assumption it is possible to switch between the different timescales by a scaling law e.g. 5 minute or 10 minute frequencies. It is also possible to combine these two assumptions and connect the intrinsic time idea and also scale the intraday measures to get daily measures – all of this holds for the quantiles and ES as well.

Dimitriadis and Halbleib (2021) have empirically shown that the BTS is the best, then TTS and tickTS. The regularly used CTS performs way worse in their analysis. They derived these results by comparing the models applied on the different sampling schemes on stocks w.r.t. their scores. They also found that their model is the best performing, with returns that are sampled at a high frequency (1 minute). Additionally, the intraday returns sampled at high frequencies in intrinsic time provide way better results compared to the calendar time sampling scheme when forecasting extreme risks. Furthermore, they have observed that for the stocks the volatility increases in the morning, while the trading activity peaks before the market closure.

In the next chapter I present the results of my empirical analysis and the interpretation of them.

3 Empirical Analysis

3.1 Analysing the Time Series

In this section I first describe the data that I used, then I study the price and log return series, calculate and inspect the realized volatility series and create a volatility signature plot to compare the intrinsic time sampling schemes to the CTS and derive some important results such as the amount of noise included in the data. I study the ACF, PACF functions, and finally I standardize the return series by the realized variance and look at the distribution properties of the standardized series.

3.1.1 Data Description

I use the IBM stock prices from January 2, 2001 to March 29, 2019 (4543 days) from the NYSE Trade and Quote (TAQ) database with the classic trading hours from 9:30:00 a.m. to 4:00:00 p.m. between Monday and Friday. The NYSE is an electronic hybrid market, where many transactions take place and it is highly liquid.

I sample the stock prices at five different frequencies such as 1 second, 10 second, 1 minute, 3 minute and 5 minute. These are equivalent to c=23.400, 2.340, 390, 130 and 78 returns per day.

I apply the CTS, BTS, TTS, TT, TRTS, WSD and DA sampling schemes, which I already introduced in the theoretical part of my thesis. I consider all combinations of frequencies and sampling schemes, in total 5x7=35 combinations.

3.1.2 Price and log return series

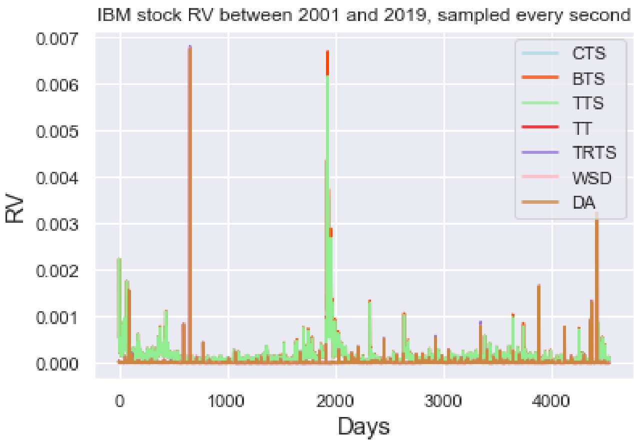
At 1 second sampling frequency a total of 4.543x23.401=106.310.743 prices can be observed in the whole sampling period. These prices are depicted in Figure 1. on the left, together with the daily log returns on the right. The mean of the prices changes over time which means that a shock causes a rather persistent change in the price. While on the second graph the daily returns move around a constant mean. It is preferred to work with returns instead of prices.

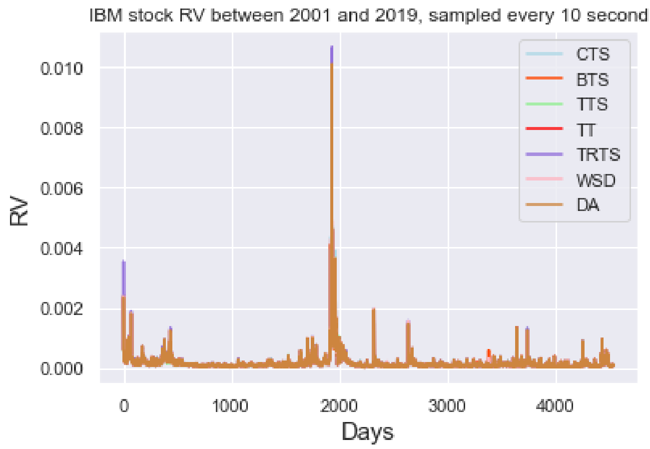
Figure 1: IBM stock prices and daily log returns in the period January 2, 2001 to March 29, 2019. Prices are sampled at 1 second frequency with different sampling schemes.

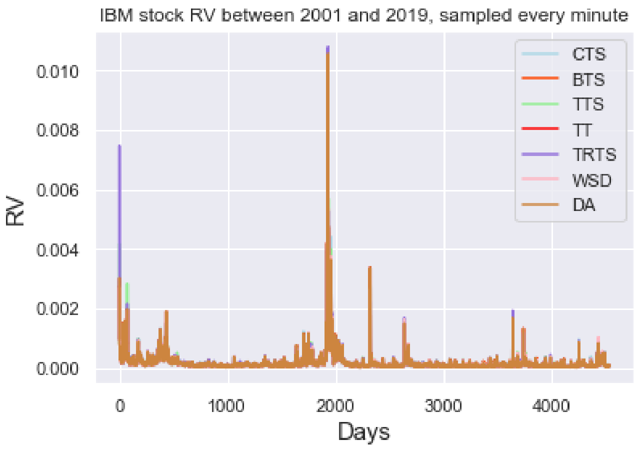
3.1.3 Realized Volatility series

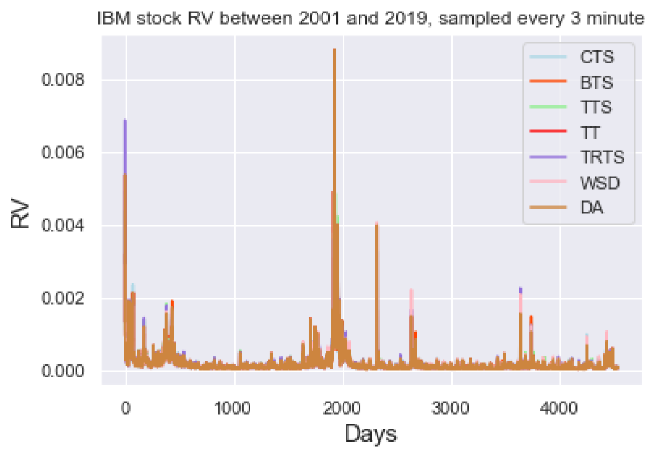
The Realized Volatility is the sum of the squared intraday returns for a particular day. The calculation is repeated for all days in the sampling period and the results are shown on the different graphs in Figure 2 for the different sampling frequencies.

It can be seen that the realized volatilities are time-varying, and they show some clustering in time as well. The graphs look very much alike for the different frequencies and it is not easy to get to a conclusion, thus I created a volatility signature plot as a next step.


IBM stock RV between 2001 and 2019, sampled every second
RV
Days
CTS
BTS
TTS
TT
TRTS
WSD
DA


IBM stock RV between 2001 and 2019, sampled every 10 second
RV
Days
CTS
BTS
TTS
TT
TRTS
WSD
DA


IBM stock RV between 2001 and 2019, sampled every minute
RV
Days
CTS
BTS
TTS
TT
TRTS
WSD
DA


IBM stock RV between 2001 and 2019, sampled every 3 minute
RV
Days
CTS
BTS
TTS
TT
TRTS
WSD
DA

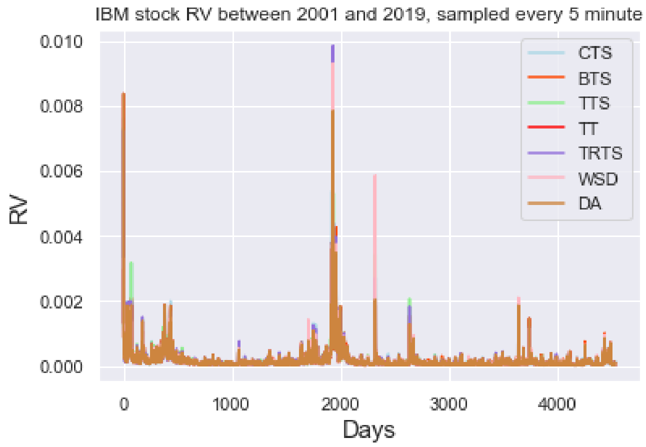


Figure 2: RV in the period January 2, 2001 - March 29, 2019.

3.1.4 Volatility signature plot

I created the volatility signature plot, which presents the average realized volatilities against the sampling frequency and solves the bias-variance trade-off. It provides intuition when deciding on the appropriate sampling frequency and sampling scheme. Figure 3 shows the volatility signature plot, where the integer *k* represents multiples of the 1 second interval.

I aggregated the average realized volatility values from 1 second up to 1800 seconds (30 minutes). I created a function where the average realized variance series is expressed as a function of the appropriate frequency, *k*. I standardized the variables with their mean and standard deviation. Then I applied a nonparametric estimation on the standardized average realized variance series and the frequency series. I used nonparametric estimation because there was no available information on the functional form. I used the Epanechnikov kernel which has the smallest Asymptotic Integrated Mean Squared Error, so it is the most optimal kernel. I applied local linear estimation which is good at solving the boundary problem that comes up in many nonparametric estimation methods. The result of the estimation is a curve for each sampling scheme, shown on the relevant volatility signature plot in Figure 3. In order to interpret the results of the estimation, I de-standardized the values.

First, look at the overall pattern in Figure 3. It can be seen that independent of the sampling scheme, all volatility signature plots stabilize around 5 minutes and start increasing again from 25 or 30 minutes. Thus the overall pattern is similar for any sampling scheme.

The CTS has higher realized variance than BTS, TTS and WSD when sampling with the highest possible frequency. Thus CTS includes more noise relative to some of the intrinsic time sampling schemes. TRTS and DA have higher variances than CTS near the 1 second sampling frequency so not all intrinsic time sampling schemes are better compared to CTS.

The estimated curve for the CTS is steeper than the intrinsic time sampling schemes, meaning that there is a large decay in the realized variance (and amount of noise included) as you sample the data less frequently. There is also a decay in the variance for the intrinsic time sampling but relatively slow decay, the curves are somewhat flatter. The BTS has a way slower decay and its curve it much more flatter than the others. Thus the data sampled by BTS includes less noise relative to the other sampling schemes.

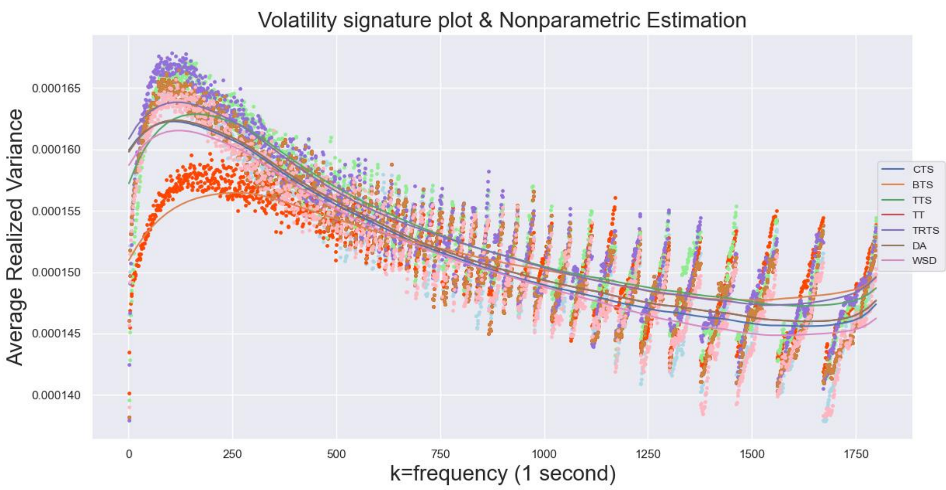


Figure 3. Volatility Signature plot, calculated in the period between January 2, 2001 and March 29, 2019, calculated from the intraday log returns sampled every 1 second with all sampling schemes in different colors.

For better visibility I printed the confidence intervals individually for each sampling scheme. Figure 4. shows the results of the nonparametric estimation with its confidence interval for the Business Time Sampling. The other sampling schemes are presented in the Appendix.

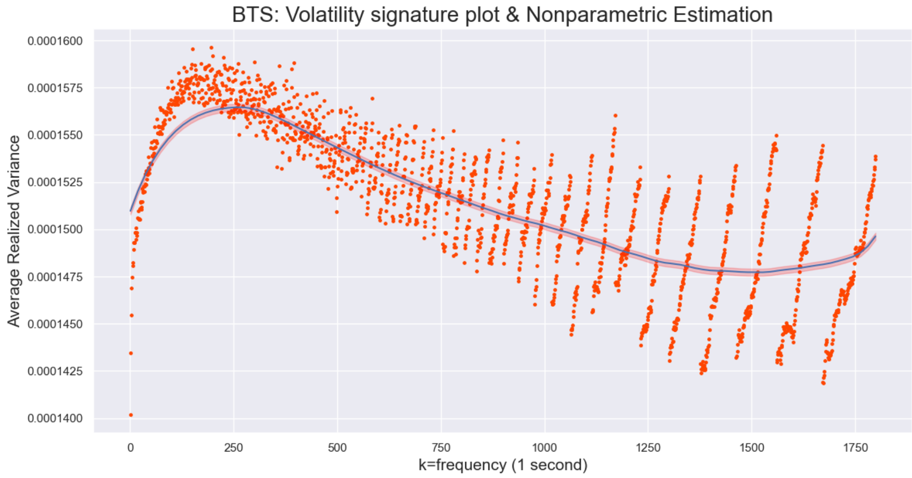


Figure 4. Volatility Signature plot with the confidence interval in pink around the curve, in the period between January 2, 2001 and March 29, 2019, calculated from the intraday log returns sampled every 1 second with Business Time Sampling scheme.

I show the volatility signature plot for the logarithm of the RV in Figure 5. The same conclusions can be drawn as for Figure 3. The order of steepness and magnitude remain unchanged.

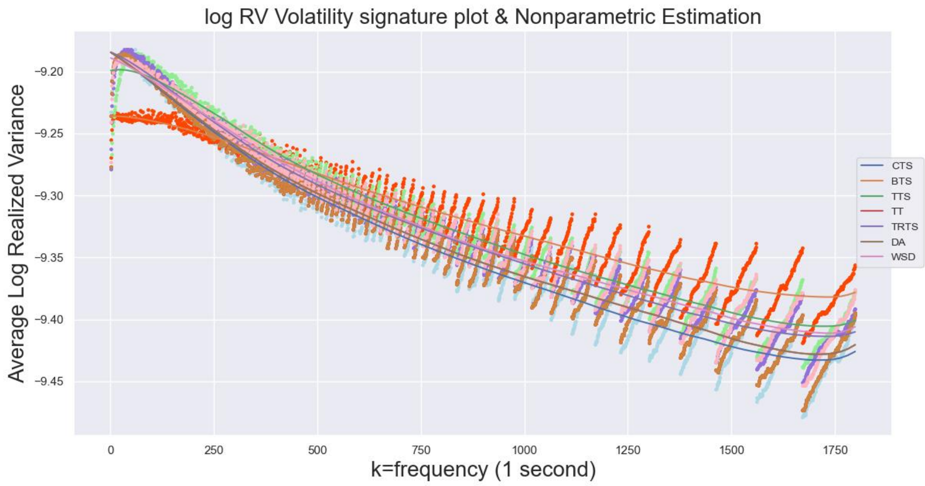


Figure 5. Volatility Signature plot of the log RV, calculated in the period between January 2, 2001 and March 29, 2019, calculated from the intraday log returns sampled every 1 second for all sampling schemes in different colors.

3.1.5 ACF,PACF functions

It can be observed in Figure 6. that there is significant autocorrelation at 5% significance level in the realized variances calculated from 1 second CTS data up to 100 lags, where 1 lag represents 1 day. It is clear from the ACF and PACF plots that the return process has a long memory, meaning that a shock has a persistent effect. This property is common for financial returns and provides intuition to use RV based models for forecasting. The significant autocorrelation in the intraday returns means that the empirical returns are not i.i.d. This is directly related to microstructure effects that arise from the price formation process.

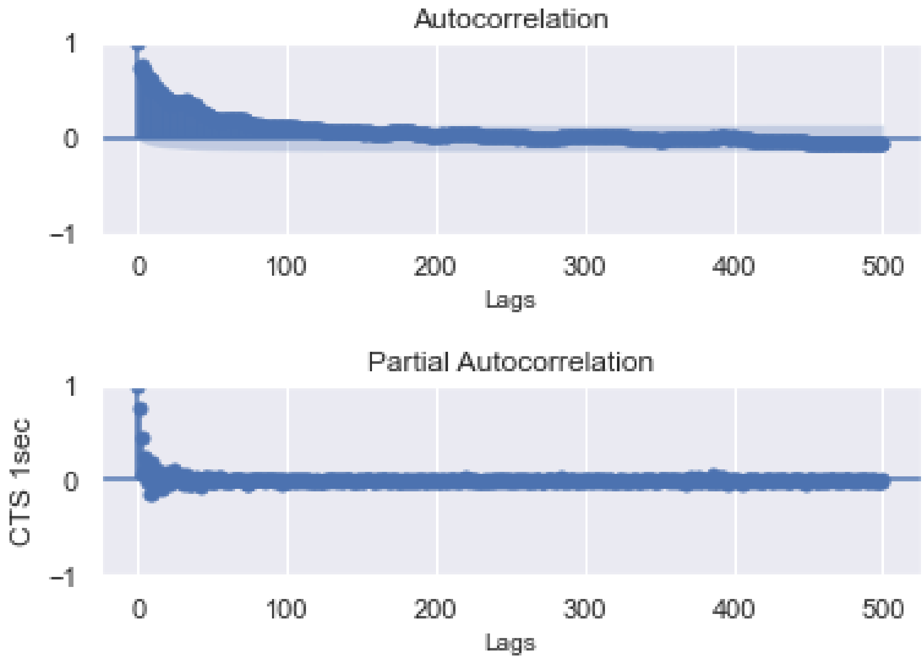


Figure 6: ACF and PACF for the data sampled every second up to 500 lags at 5 %. calculated in the period between January 2, 2001 and March 29, 2019. The plots produced by sampling at other frequencies and sampling schemes look very similar and thus not presented here.

3.1.6 Distribution properties of the standardized return series

The log return series can be standardized by the corresponding realized variances simply by dividing each element in the series by the square root of the RV calculated from prices sampled at different frequencies and with different sampling schemes. I always divide the returns with the RV calculated from the same sampling scheme and frequency e.g. the 1 second CTS return series with the RV calculated from the 1 second CTS squared returns.

The histograms in Figure 7 indicate that the distribution of the standardized return series (second graph) is more similar to the normal distribution, than the original return series. I present here the 1 second CTS results, the plots produced by sampling at other frequencies look very similar.

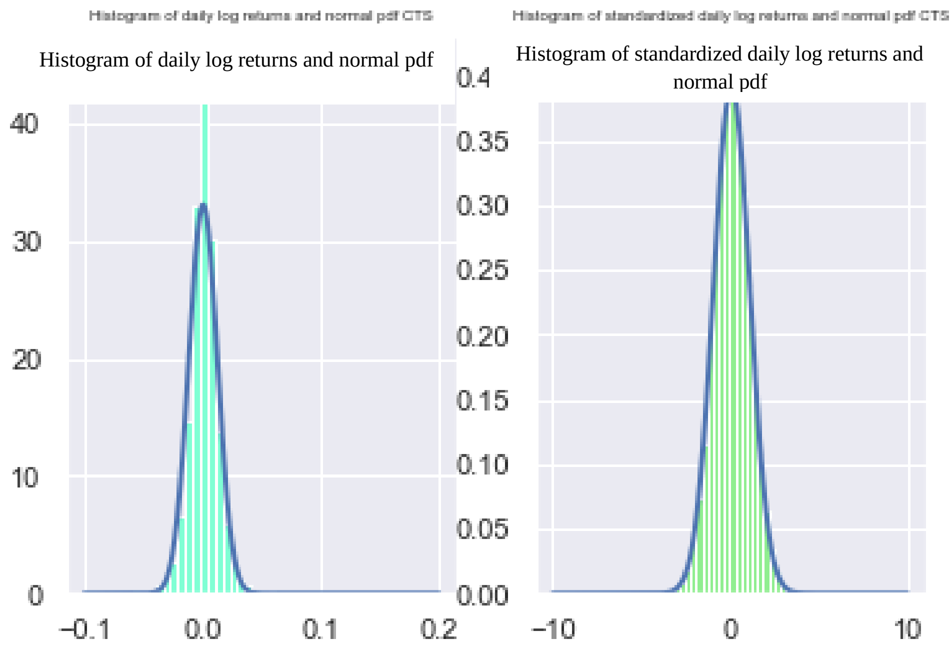


Figure 7: Histograms of the (standardized) daily log returns, plotted together with the normal distribution probability density function for comparison purposes. The data is sampled every second by CTS between January 2, 2001 and March 29, 2019.

To understand the standardized return series better, I present the summary statistics of all sampling schemes and frequencies in Table 1. It is obvious that each standardized series are closer to the standard normal distribution (zero mean, standard deviation 1, zero Skewness and Kurtosis 3). However, it is not easy to differentiate between the different sampling combinations, regarding the question: which one is ’more normal’ distibuted. For example 1s CTS has a standard deviation closer to the value 1 but Kurtosis further from the value 3 compared to the 5 minute BTS distribution.

		CTS		BTS		TTS		TT		TRTS		WSD		DA	
		stdlr	dailyret	stdlr	dailyret	stdlr	dailyret	stdlr	dailyret	stdlr	dailyret	stdlr	dailyret	stdlr	dailyret
1 second	**Mean**	0,08	0,00	0,08	0,00	0,08	0,00	0,077	0,00	-0,03	0,00	0,077	0,00	0,077	0,00
	Std	1,02	0,01	1,01	0,01	1,02	0,01	0,965	0,00	1,22	0,00	0,967	0,00	0,965	0,00
	Skew.	0,08	0,18	0,08	0,18	0,08	0,18	0,065	3,57	-0,11	3,16	0,063	3,00	0,065	3,57
	Kurt.	3,42	8,59	3,41	8,59	3,42	8,59	3,339	62,91	2,90	53,47	3,346	55,39	3,339	62,91
10 second	**Mean**	0,077	0,001	0,080	0,001	0,078	0,001	0,077	0,001	0,078	0,001	0,077	0,001	0,077	0,001
	Std	0,965	0,012	0,985	0,012	0,986	0,012	0,965	0,012	0,967	0,012	0,967	0,012	0,965	0,012
	Skew.	0,063	0,180	0,069	0,180	0,060	0,180	0,065	0,180	0,072	0,180	0,063	0,180	0,065	0,180
	Kurt.	3,334	8,586	3,324	8,586	3,395	8,586	3,339	8,586	3,313	8,586	3,346	8,586	3,339	8,586
1 minute	**Mean**	0,079	0,001	0,082	0,001	0,080	0,001	0,080	0,001	0,080	0,001	0,081	0,001	0,080	0,001
	Std	0,942	0,012	0,970	0,012	0,946	0,012	0,947	0,012	0,943	0,012	0,949	0,012	0,947	0,012
	Skew.	0,068	0,180	0,084	0,180	0,081	0,180	0,079	0,180	0,079	0,180	0,090	0,180	0,079	0,180
	Kurt.	3,070	8,586	3,124	8,586	3,073	8,586	3,056	8,586	3,044	8,586	3,082	8,586	3,056	8,586
3 minute	**Mean**	0,079	0,001	0,083	0,001	0,081	0,001	0,081	0,001	0,079	0,001	0,082	0,001	0,081	0,001
	Std	0,946	0,012	0,962	0,012	0,944	0,012	0,950	0,012	0,947	0,012	0,950	0,012	0,950	0,012
	Skew.	0,062	0,180	0,087	0,180	0,075	0,180	0,077	0,180	0,067	0,180	0,085	0,180	0,077	0,180
	Kurt.	2,894	8,586	2,974	8,586	2,893	8,586	2,916	8,586	2,916	8,586	2,955	8,586	2,916	8,586
5 minute	**Mean**	0,079	0,001	0,082	0,001	0,078	0,001	0,077	0,001	0,077	0,001	0,082	0,001	0,077	0,001
	Std	0,951	0,012	0,961	0,012	0,952	0,012	0,956	0,012	0,952	0,012	0,958	0,012	0,956	0,012
	Skew.	0,047	0,180	0,074	0,180	0,056	0,180	0,042	0,180	0,038	0,180	0,075	0,180	0,042	0,180
	Kurt.	2,817	8,586	2,925	8,586	2,852	8,586	2,862	8,586	2,808	8,586	2,896	8,586	2,862	8,586

Table 1: Summary statistics of the daily returns and the standardized daily returns. The data is sampled between January 2, 2001 and March 29, 2019.

Next I apply the Jarque-Bera test on the standardized log returns and present the results in Table 2. The null hypothesis states that the standardized log return series is normally distributed. It cannot be rejected at 5% significance for the green cells. The 1 second and 10 second sampling frequencies are never normally distributed, while the 1, 3 and 5 minute sampling frequencies provide normally distributed standardized log returns for most sampling schemes.

		CTS	BTS	TTS	TT	TRTS	WSD	DA
1 second	**t**	39,033	36,275	38,983	25,989	10,981	8,235	6,863
	p-value	0,000	0,000	0,000	0,000	0,004	0,017	0,033
10 second	**t**	24,214	23,529	32,362	24,992	22,499	25,597	24,992
	p-value	0,000	0,000	0,000	0,000	0,000	0,000	0,000
1 minute	**t**	4,420	8,242	5,966	5,307	5,127	7,362	5,307
	p-value	0,110	0,016	0,051	0,070	0,077	0,025	0,070
3 minute	**t**	5,047	5,800	6,478	5,770	4,789	5,864	5,770
	p-value	0,080	0,055	0,039	0,056	0,091	0,053	0,056
5 minute	**t**	8,064	5,215	6,494	4,928	8,080	6,258	4,928
	p-value	0,018	0,074	0,039	0,085	0,018	0,044	0,085

Table 2. Jarque-Bera test results for the standardized daily log returns at 5% significance.

3.1.7 Sample properties of the RV

I analyse the sample properties of the RV series and find that it is far from being normally distributed, see Table 3.

		CTS	BTS	TTS	TT	TRTS	WSD	DA
1 second	**Mean**	-9,279	-9,269	-9,279	-9,239	-12,195	-12,199	-12,202
	Std	0,760	0,764	0,760	0,758	1,001	0,988	0,976
	Skew.	0,931	0,955	0,931	0,936	0,750	0,806	0,745
	Kurt.	4,846	4,871	4,845	4,700	6,278	6,332	6,265
10 second	**Mean**	-9,191	-9,235	-9,231	-9,191	-9,199	-9,195	-9,191
	Std	0,779	0,782	0,780	0,777	0,786	0,777	0,777
	Skew.	1,058	1,030	1,040	1,054	1,038	1,055	1,054
	Kurt.	4,998	4,918	5,041	4,994	4,912	5,025	4,994
1 minute	**Mean**	-9,182	-9,233	-9,186	-9,191	-9,186	-9,194	-9,191
	Std	0,834	0,821	0,829	0,837	0,838	0,828	0,837
	Skew.	0,973	1,004	0,986	0,973	0,981	1,002	0,973
	Kurt.	4,487	4,571	4,565	4,467	4,518	4,552	4,467
3 minute	**Mean**	-9,219	-9,236	-9,212	-9,223	-9,219	-9,218	-9,223
	Std	0,870	0,850	0,864	0,872	0,876	0,864	0,872
	Skew.	0,902	0,956	0,920	0,903	0,886	0,941	0,903
	Kurt.	4,230	4,313	4,241	4,246	4,196	4,271	4,246
5 minute	**Mean**	-9,246	-9,246	-9,240	-9,253	-9,247	-9,243	-9,253
	Std	0,885	0,863	0,882	0,884	0,886	0,872	0,884
	Skew.	0,845	0,919	0,870	0,834	0,842	0,892	0,834
	Kurt.	4,120	4,223	4,211	4,116	4,135	4,189	4,116

Table 3. Descriptive statistics of the RV between January 2, 2001 and March 29, 2019.

3.2 Estimation and Forecasting

In this subchapter I apply 5 different models with 2 different distributions for each, on the sample of returns and realized variances based on different sampling schemes and frequencies to forecast volatilities in the out-of-sample period. After estimating the model parameters and forecasting, I evaluate the results by RMSFE,MAPE and MCS (Hansen et al., 2011). Further, I measure the VaR and compare the models and sampling combinations by means of backtesting of the VaR.

3.2.1 In and out-of-sample periods

I split the observation period into in and out-of sample periods for the proceeding calculations. I estimate the models in the in-sample period, then forecast the different measures in the out-of-sample period. Finally, I compare the forecasted and the actual values in the out-of sample period to evaluate forecasting performance. The in-sample period starts from 2nd January 2001 until 29th December 2017 and the out-of-sample period goes from January 2 2018 until 29th March 2019. The in-sample period consists of 4233 days, while the forecasting period has 309 days.

3.2.2 Model estimation and forecasting

First, in the in-sample period I applied the GARCH, EGARCH, T-GARCH, HAR and Risk Metrics models. I considered two different types of distributions for the forecasting, namely the normal and the Student's t distributions.

The GARCH (Generalized Autoregressive Heteroscedastic) models are conditional volatility models which estimate the daily variances as a function of past information, specifically lagged squared shocks and lagged conditional variance, thus they have a deterministic form. Further, the Exponential GARCH (EGARCH) and the Threshold GARCH (T-GARCH) models are able to capture leverage effects. Leverage effect is the asymmetric response of future variances to positive and negative shocks. The forecasting is done by substituting the estimated parameters and conditional measures in time 't' into the estimated functional form, which directly returns the conditional variance at time 't+1'. Based on the information set up to time 't', the conditional expectation of the variances can be computed recursively. I used the arch package from Sheppard (2021) in my code for the garch models. The Risk Metrics model was first introduced by J.P.Morgan in 1996 and it is very popular, it simplifies the calculations.

In order to use the benefits of high frequency data, I calculated the realized variances, and to obtain forecasts of future volatility from the RV time-series I applied a long-memory model, specifically the HAR (Heterogeneous Autoregressive) model of Corsi (2009). According to this model the risk is a cummulation of traders that trade at different frequencies. The volatility is estimated by daily, weekly and monthly volatilities.

3.2.3 Forecast evaluation

One of the most common methods for evaluating the forecasting performance of econometric models is comparing the forecasted values by means of Root Mean Squared Forecasting Error and Mean Average Prediction Error. I present the results in Table 4.

The proxy for the calculations is the relevant RV for the specific sampling scheme and frequency e.g. I compare the forecasted variance based on the 1 second CTS prices to the RV calculated from the 1 second CTS. I present the values for the 1 second sampling frequency in Table 4. and the other frequencies in the Appendix.

The HAR model has the lowest values compared to other models at each frequency. This is shown in Table S1. in the Appendix. Furthermore, the lowest RMSFE and MAPE values among all frequencies, sampling schemes and models is the 1 second frequency with TTS and HAR model, colored in blue in Table 4.

		garch	garch_t	egarch	egarch_t	tgarch	tgarch_t	RM	HAR
CTS	RMSFE	0,003474	0,003531	0,003468	0,003539	0,003464	0,003517	0,006799	0,00256
	MAPE	0,00263	0,002695	0,002696	0,002743	0,002668	0,002716	0,005904	0,00177
BTS	RMSFE	0,003459	0,003518	0,003433	0,003507	0,003429	0,003484	0,006732	0,002591
	MAPE	0,002609	0,002681	0,002652	0,002703	0,002626	0,00268	0,005832	0,001801
TTS	RMSFE	0,003474	0,00353	0,003468	0,003539	0,003464	0,003516	0,006799	0,002559
	MAPE	0,00263	0,002695	0,002695	0,002743	0,002667	0,002716	0,005903	0,00177
TT	RMSFE	0,003468	0,003525	0,003457	0,003529	0,003452	0,003505	0,006779	0,002561
	MAPE	0,002621	0,002688	0,002686	0,002733	0,002658	0,002707	0,005883	0,001773
TRTS	RMSFE	0,003474	0,003531	0,003468	0,003539	0,003464	0,003517	0,006799	0,00256
	MAPE	0,00263	0,002695	0,002696	0,002743	0,002668	0,002716	0,005904	0,00177
WSD	RMSFE	0,003458	0,003515	0,003444	0,003516	0,00344	0,003493	0,006758	0,002565
	MAPE	0,002612	0,002679	0,002671	0,002719	0,002642	0,002693	0,005862	0,001782
DA	RMSFE	0,003468	0,003525	0,003457	0,003529	0,003452	0,003505	0,006779	0,002561
	MAPE	0,002621	0,002688	0,002686	0,002733	0,002658	0,002707	0,005883	0,001773

Table 4. Root Mean Squared Forecasting Error and Mean Average Prediction Error for the 1 second sampling data for all mentioned sampling schemes and models in the forecasting period from January 2, 2018 until 29th March, 2019.

Another method for evaluating the forecasting performance is the MCS. The null hypothesis for the Model Confidence Set is that at 5% significance level the models have equal forecasting performance (in pairs). This is tested for each model at each frequency and sampling scheme. The results for the 1 second sampling frequency are presented in Table 5. The rest of the results is presented in the Appendix. The 3 minute BTS RV is taken as a proxy because it is an unbiased estimator of the true variance as shown in the volatility signature plot in Figure 3.

It is easy to see from Table 5. that all garch models other than garch_t perform equal to the HAR when sampling at 1s frequency with any sampling scheme, except the BTS, at 5% significance level. The null hypothesis is not rejected for the model-sampling scheme combinations where the cells are colored in green.

The MCS for other frequencies are presented in Table S2. in the Appendix and shows that the null hypothesis is never rejected for the HAR, regardless of the sampling scheme and frequency. This can be interpreted as the HAR model’s forecasting performance is not significantly different if you change the sampling scheme and/or frequency. Thus, it can be advantageous to use high frequency data with intrinsic time sampling schemes and gain the benefits mentioned earlier in the theoretical part (Oomen (2005, 2006), Dong and Tse (2014)).

		CTS	BTS	TTS	TT	TRTS	WSD	DA
1 second sampling	Model name	p-value	p-value	p-value	p-value	p-value	p-value	p-value
	RM	0	0	0	0	0	0	0
	garch_t	0,02	0,021	0,028	0,025	0,034	0,028	0,023
	garch	0,054	0,046	0,059	0,052	0,066	0,05	0,053
	tgarch_t	0,065	0,046	0,061	0,059	0,079	0,056	0,071
	egarch_t	0,065	0,046	0,062	0,059	0,079	0,056	0,071
	tgarch	0,065	0,046	0,073	0,059	0,079	0,056	0,072
	egarch	0,065	0,046	0,073	0,059	0,079	0,056	0,072
	HAR	1	1	1	1	1	1	1

Table 5. Model Confidence Set for the the 1 second sampling for all mentioned sampling schemes and models in the forecasting period (January 2, 2018 until 29th March, 2019). The 3 minute BTS Realized Variance is considered as a proxy for the true variation.

3.2.4 VaR and ES

The VaR is a loss bound that at most will be exceeded with probability 'p' (Pohlmeier et al., 2012). For the calculations I used the formula for log returns: $VaR(p,l) = V_t \times [e^{\left(\mu_t + \sigma_t \times \Phi^{-1}(p)\right)} - 1]$ (Pohlmeier et al., 2012), where V_t is the value of the asset, which I fix at V_t=1, p is the probability level, l=1 is the length of the forecasting period, μ_t =0 is the mean, σ_t is the standard deviation and $\Phi^{-1}(p)$ is the cumulative distribution function. Thus the above formula simplifies to: $VaR(p) = e^{\left(\sigma_t \times \Phi^{-1}(p)\right)} - 1$ (1.) where $\Phi^{-1}(p)$ is either the cdf of the Student's t distribution or cdf of the normal distribution and $\sigma_t$can be obtained directly by taking the square root of the forecasted variances from earlier. I present the results for 5% probability level which is most often used in Econometrics (Dimitriadis and Halbleib, 2021).

Table 6. presents the results of the VaR forecasts at 1 second sampling fr**e**quency with TTS sampling scheme. I chose this sampling because it gives the best results for forecast evaluation according to Table 4. The interpretation is as follows: with 95% probability on January 2, 2018 you would not make a loss worse

than 0.012 dollar for each dollar invested in the IBM stock, based on the forecasting by the garch model.

Moreover, I present the forecasted VaR values in the whole out-of-sample period in Figure 8. It can be observed that the t-distribution forecasts a greater loss than the normal distribution in general. The HAR model with normal distribution provides the smallest loss, while the Risk Metrics model with t-distribution predicts the greatest loss among the studied models.

	VaR(5%)
garch	0.012
garch_t	0.013
egarch	0.013
egarch_t	0.014
tgarch	0.013
tgarch_t	0.015
RM	0.015
RM_t	0.020
HAR	0.008
HAR_t	0.011

Table 6: The forecasted Value at Risk in the first out-of-sample period (January 2, 2018) based on the different models at 5% significance, calculated from data sampled at 1 second frequency with TTS.

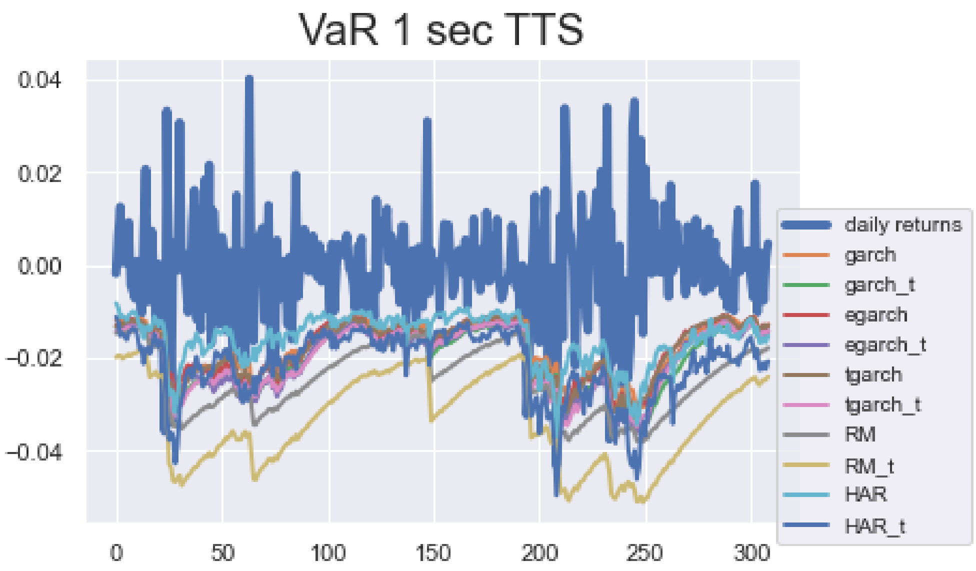


Figure 8. The forecasted Value at Risk in the out-of-sample period based on the different models at 5% significance, calculated from data sampled at 1 second frequency with TTS.

The VaR does not account for the distribution of the loss, thus the shape of the tail of the profit-loss distribution is unknown. To solve this problem, I consider another quantile based estimator, the Expected Shortfall. The Expected Shortfall is the expected loss, conditional on the fact that the VaR threshold has already been crossed (Pohlmeier et al., 2012). It is also called conditional or average value at risk or expected tail loss. The ES is recommended by the Basel Committee since 2016. Its advantage is that it can differentiate between positions with the same VaR but different distributions. By using a more fat-tailed distibution e.g. t-distribution relative to normal distribution, (Pohlmeier et al., 2012) you can forecast higher losses better.

The Student's t-distribution always gives a higher expected loss compared to the normal distribution. The highest ES is predicted by the Risk Metrics model with t-distribution, and the smallest ES by the-HAR model with normal distribution. In this context, the Risk Metrics model is the most conservative and the HAR is the least conservative model.

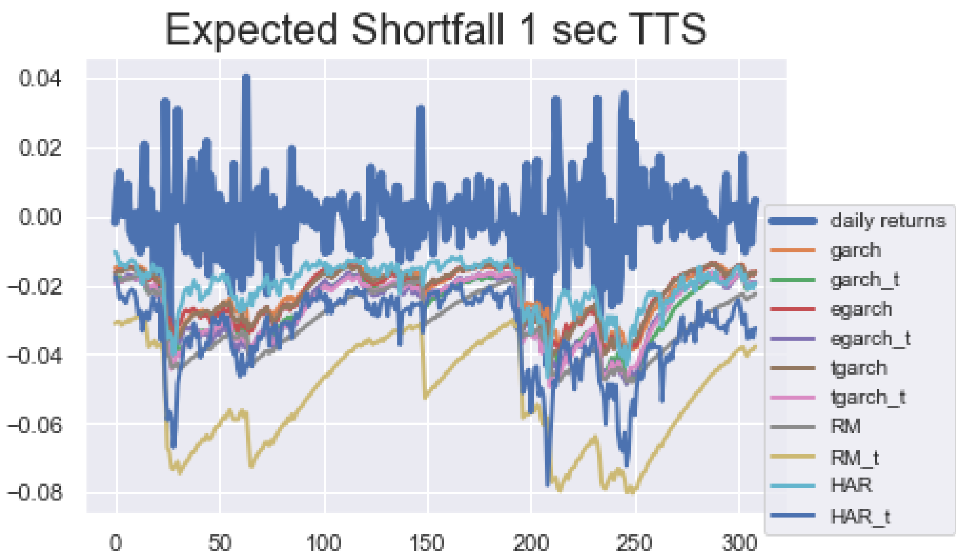


Figure 9. The forecasted Expected Shortfall in the out-of-sample period based on the different models at 5% significance, calculated from data sampled at 1 second frequency with TTS.

3.3 Backtesting VaR

3.3.1 Unconditional backtest

In order to validate the models I apply backtesting on the forecasted VaRs. First I implement the unconditional backtest (coverage test), which is based on counting the number of hits. A hit in period ’t’ means that the actual loss is greater than the forecasted VaR in absolute value, thus the model could not forecast the actual loss in period ’t’. The number of hits divided by the number of periods is called the hit ratio. Optimally, the hit ratio is equal to the nominal probability level ’p’, then the model is correctly specified. The unconditional backtest tests whether these two are equal. Formally, the null hypothesis is that E[H_t] = p, where $H_t = 1_{\{r_t < \widehat{VaR}_t(\mathrm{p})\}}$, t = T +1, . . .T+k is the sequence of hits (Pohlmeier et al., 2012). You can simply interpret the null hypothesis as expecting the number of hits to be equal to alpha× number of out of sample periods. Optimally you do not reject the null hypothesis.

I present the backtesting results at 5% significance level, including the total number of hits in the out-of sample period, the t-statistics and the p-values. The backtesting results for the garch models and the Risk Metrics model remain unchanged among the different sampling schemes and frequencies and are presented in Table 7. separately from the HAR model, for which the results vary in Table 8.

From analysing the results of Table 7. I conclude that the null hypothesis is never rejected for the garches and always rejected for the Risk Metrics model. The hit ratio is below the nominal probability level for the Risk Metrics, the model is too conservative.

Further, the Student’s t distribution provides more conservative forecasts for the VaR than the normal distribution due to the fat tail property of the Student’s t in general which translates into lower number of hits. The nominal level of the hits is 0.05×309=15,45. From Table 7. it is clear that the garch model with normal distribution is the closest to this value with 16 hits.

	HIT	t	p
garch	16	0,144	0,886
garch_t	10	-1,423	0,155
egarch	12	-0,901	0,368
egarch_t	10	-1,423	0,155
tgarch	14	-0,378	0,705
tgarch_t	10	-1,423	0,155
RM	7	-2,206	0,027
RM_t	3	-3,250	0,001

Table 7. Unconditional Backtest results for $VaR_{5\%}$in the out-of-sample period January 2nd 2018 to 29th March 2019 for the garches and Risk Metrics model. In the green cells the null hypothesis is not rejected.

The null hypothesis is never rejected for the HAR with normal distribution, except at the 1 second sampling frequency but it is still quite close to the 0,05 significance level with p-value 0,049 as shown in Table 8. The null hypothesis is always rejected for the HAR with Student's t distribution, it provides too few hits compared to the nominal level.

		1 second sampling			10 seconds sampling			1 minute sampling			3 minute sampling			5 minute sampling		
		HIT	**t**	**p-value**	**HIT**	**t**	**p-value**	**HIT**	**t**	**p-value**	**HIT**	**t**	**p-value**	**HIT**	**t**	**p-value**
CTS	**HAR**	23	1,971	0,049	19	0,927	0,354	20	1,188	0,235	18	0,666	0,506	19	0,927	0,354
	HAR_t	7	-2,206	0,027	4	-2,989	0,003	5	-2,728	0,006	5	-2,728	0,006	5	-2,728	0,006
BTS	**HAR**	23	1,971	0,049	19	0,927	0,354	20	1,188	0,235	21	1,449	0,147	19	0,927	0,354
	HAR_t	6	-2,467	0,014	5	-2,728	0,006	6	-2,467	0,014	6	-2,467	0,014	5	-2,728	0,006
TTS	**HAR**	23	1,971	0,049	19	0,927	0,354	19	0,927	0,354	20	1,188	0,235	19	0,927	0,354
	HAR_t	7	-2,206	0,027	5	-2,728	0,006	5	-2,728	0,006	6	-2,467	0,014	5	-2,728	0,006
TT	**HAR**	23	1,971	0,049	19	0,927	0,354	19	0,927	0,354	18	0,666	0,506	21	1,449	0,147
	HAR_t	7	-2,206	0,027	5	-2,728	0,006	5	-2,728	0,006	5	-2,728	0,006	6	-2,467	0,014
TRTS	**HAR**	23	1,971	0,049	19	0,927	0,354	19	0,927	0,354	21	1,449	0,147	20	1,188	0,235
	HAR_t	7	-2,206	0,027	5	-2,728	0,006	5	-2,728	0,006	5	-2,728	0,006	5	-2,728	0,006
WSD	**HAR**	23	1,971	0,049	19	0,927	0,354	19	0,927	0,354	20	1,188	0,235	20	1,188	0,235
	HAR_t	7	-2,206	0,027	5	-2,728	0,006	4	-2,989	0,003	5	-2,728	0,006	5	-2,728	0,006
DA	**HAR**	23	1,971	0,049	19	0,927	0,354	19	0,927	0,354	18	0,666	0,506	21	1,449	0,147
	HAR_t	7	-2,206	0,027	5	-2,728	0,006	5	-2,728	0,006	5	-2,728	0,006	6	-2,467	0,014

Table 8. Unconditional Backtest results for $VaR_{5\%}$in the out-of-sample period January 2^{nd} 2018 to 29^{th} March 2019 for the HAR model with normal and Student's t distributions. In the green cells the null hypothesis is not rejected.

To sum up the unconditional backtest, the garch models and the HAR model with normal distribution perform well, while the Risk Metrics and the HAR model with Student's t distribution are too conservative and forecast bigger VaR than optimal, leading to fewer hits. Unconditional backtesting could not differentiate between sampling schemes or frequencies, if a model is good (bad), then the VaR forecast is good (bad) at any sampling scheme and frequency combination.

3.3.2 Independence test

In this section I analyse the VaR by means of the conditional backtest. I apply the Independence test by Christoffersen (1998), which tests the null hypothesis that there is no temporal dependence in the VaR exceedences. I also calculate the Probability of Failure(POF). Besides, I apply the Conditional Coverage Test with the null hypothesis that the hit rate is equal to the nominal VaR level after taking into account the potential temporal dependence.

I present the backtesting results at 5% significance level, including p-values for independence test, the POF and the p-values for the conditional coverage test (CC). The backtesting results for the garch models and the Risk Metrics model remain unchanged among the different sampling schemes and frequencies and they are presented in Table 9. separately from the HAR model, for which the results vary in Table 10.

The independence test cannot be rejected for the egarch and tgarch with normal distribution and the Risk Metrics with both normal and Student's t distribution at 5% significance. So there is no temporal dependence in the VaR exceedences at any frequency-sampling scheme combination when forecasting VaR with these models. However, there is significant temporal dependence in the VaR exceedences for the rest of the models presented in the table.

The conditional coverage test cannot be rejected for the garch, egarch and tgarch with normal distribution at 5% significance. So the hit rate is equal to the nominal VaR level after accounting for the temporal dependence (if there was any e.g. for garch we rejected the null hypothesis for the independence test). However, the hit rate is not considered equal to the nominal VaR level for the other models.

	garch	garch_t	egarch	egarch_t	tgarch	tgarch_t	RM	RM_t
Indep	0,045	0,033	0,074	0,033	0,143	0,033	0,137	0,808
POF	0,887	0,129	0,349	0,129	0,701	0,129	0,014	0,000
CC	0,132	0,033	0,131	0,033	0,317	0,033	0,016	0,000

Table 9. Conditional Backtest results for $VaR_{5\%}$in the out-of-sample period January 2nd 2018 to 29th March 2019 for the garches and Risk Metrics model. In the green cells the null hypothesis is not rejected.

The results in Table 10. indicate that the independence test cannot be rejected for the HAR with normal distribution at 5% significance at any sampling scheme and frequency combination. So there is no temporal dependence in the VaR exceedences at any frequency-sampling scheme combination when forecasting VaR with HAR and normal distribution. However, there is significant temporal dependence in the VaR exceedences for the HAR with Student's t distribution for all sampling schemes at 1 second frequency, for CTS at 10 seconds, for BTS and WSD at 1 minute, BTS at 3 minutes. There is no temporal dependence for the HAR_T at 5 minutes.

The conditional coverage test cannot be rejected for the HAR with normal distribution at 5% significance. So the hit rate is equal to the nominal VaR level after accounting for the temporal dependence. However, the hit rate is never considered equal to the nominal VaR level for the HAR with Student's t distribution.

	1 second sampling													
	CTS		**BTS**		**TTS**		**TT**		**TRTS**		**WSD**		**DA**	
	HAR	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**
Indep	0,331	0,006	0,331	0,003	0,331	0,006	0,331	0,006	0,331	0,006	0,331	0,006	0,331	0,006
POF	0,065	0,014	0,065	0,005	0,065	0,014	0,065	0,014	0,065	0,014	0,065	0,014	0,065	0,014
CC	0,114	0,001	0,114	0,000	0,114	0,001	0,114	0,001	0,114	0,001	0,114	0,001	0,114	0,001
	10 second sampling													
	CTS		**BTS**		**TTS**		**TT**		**TRTS**		**WSD**		**DA**	
	HAR	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**
Indep	0,123	0,034	0,123	0,059	0,123	0,059	0,123	0,059	0,123	0,059	0,123	0,059	0,123	0,059
POF	0,370	0,000	0,370	0,002	0,370	0,002	0,370	0,002	0,370	0,002	0,370	0,002	0,370	0,002
CC	0,203	0,000	0,203	0,001	0,203	0,001	0,203	0,001	0,203	0,001	0,203	0,001	0,203	0,001
	1 minute sampling													
	CTS		**BTS**		**TTS**		**TT**		**TRTS**		**WSD**		**DA**	
	HAR	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**
Indep	0,162	0,059	0,162	0,003	0,123	0,059	0,123	0,059	0,123	0,059	0,123	0,034	0,123	0,059
POF	0,255	0,002	0,255	0,005	0,370	0,002	0,370	0,002	0,370	0,002	0,370	0,000	0,370	0,002
CC	0,197	0,001	0,197	0,000	0,203	0,001	0,203	0,001	0,203	0,001	0,203	0,000	0,203	0,001
	3 minute sampling													
	CTS		**BTS**		**TTS**		**TT**		**TRTS**		**WSD**		**DA**	
	HAR	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**
Indep	0,090	0,059	0,210	0,003	0,162	0,094	0,090	0,059	0,210	0,059	0,162	0,059	0,090	0,059
POF	0,516	0,002	0,169	0,005	0,255	0,005	0,516	0,002	0,169	0,002	0,255	0,002	0,516	0,002
CC	0,193	0,001	0,177	0,000	0,197	0,005	0,193	0,001	0,177	0,001	0,197	0,001	0,193	0,001
	5 minute sampling													
	CTS		**BTS**		**TTS**		**TT**		**TRTS**		**WSD**		**DA**	
	HAR	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**	**HAR**	**HAR_t**
Indep	0,123	0,059	0,123	0,059	0,123	0,059	0,210	0,094	0,162	0,059	0,162	0,059	0,210	0,094
POF	0,370	0,002	0,370	0,002	0,370	0,002	0,169	0,005	0,255	0,002	0,255	0,002	0,169	0,005
CC	0,203	0,001	0,203	0,001	0,203	0,001	0,177	0,005	0,197	0,001	0,197	0,001	0,177	0,005

Table 10. Conditional Backtest results for $VaR_{5\%}$in the out-of-sample period January 2nd 2018 to 29th March 2019 for the HAR model with normal and Student's t distributions. In the green cells the null hypothesis is not rejected.

4 Conclusion

In theory, the realized variance is a consistent estimator of the 1-day integrated variance. However the empirical study of Corsi et al. (2001) shows that there is a considerable systematic error and the realized volatility is not unbiased and not consistent. This is directly related to microstructure effects that arise from the price formation process. Oomen (2006) mathematically derives that an increase in the trading activity and a decrease in the level of market microstructure noise lead to a higher optimal sampling frequency.

High number of observations decrease the measurement error but at the same time cause a bias in the estimated volatility due to more noise. Thus, there has to be a trade-off for which Andersen et al. (2000) suggest using the volatility signature plot. The connection between the optimal sampling frequency and the dynamics of the noise ratio is clear, a high noise ratio corresponds to a high frequency and vice versa. A regression of the number of prices on a constant plus the estimated noise ratio gives a regression of close to 80% according to Oomen (2006).

Furthermore, Oomen (2005) empirically shows that sampling in business time and transaction time perform better compared to the commonly used calendar time. Dong and Tse (2014) find that the BTS returns perform better than the CTS and TTS. They also show that the BTS scheme is closer to yielding i.i.d. Gaussian returns than the CTS or TTS meaning that it includes less jumps and the RV is closer to the integrated variance. Dimitriadis and Halbleib (2021) have empirically shown that the BTS is the best, then TTS and tickTS. The regularly used CTS performs way worse in their analysis. Besides, the intraday returns sampled at high frequencies in intrinsic time provide way better results compared to the calendar time sampling scheme when forecasting extreme risks. Further research could be done in the direction of alternative, not yet discovered sources of market activity for creating further types of intrinsic time sampling schemes.

In the empirical part of my thesis I compared the sampling schemes and frequencies, and chose the best one to forecast variances and VaR. Besides the Garch and Risk Metrics models that are based on past daily data, I applied the

HAR model with Realized Variance to make use of the intraday information and build on the information contained in high frequency data.

Based on the volatility signature plot in Figure 3., the BTS includes less noise relative to the other sampling schemes. Further, the CTS has higher realized variance than BTS, TTS and WSD when sampling with the highest frequency. So we can conclude that the calendar time sampling includes relatively more noise and it is not optimal to use CTS as opposed to the common practice. The least biased estimator can be obtained when sampling at 3 minutes with BTS.

From the forecast evaluation (RMSFE) it is clear that the 1 second sampling frequency with TTS sampling scheme is the best when forecasting volatilities. Among the models the HAR model performs best. According to the MCS at 1 second frequency the forecasted variances with different sampling schemes and frequencies can be considered identical at 5% significance level when taking the 3 minute BTS RV as proxy. The HAR model's forecasting performance is not significantly different when changing the sampling scheme and/or frequency.

According to the unconditional backtesting results it is important to choose a model which performs well and it does not make a significant difference which frequency and sampling scheme you apply with that model.

Based on the independence test there is no temporal dependence in the VaR exceedences at any frequency-sampling scheme combination when forecasting VaR with egarch and tgarch with normal distribution and the Risk Metrics with both normal and Student's t distribution. However, there is significant temporal dependence in the VaR exceedences for the rest of the models.

In line with the conditional coverage test results, the hit rate is equal to the nominal VaR level after accounting for the temporal dependence for the garch, egarch and tgarch with normal distribution at 5% significance. However, the hit rate is not considered equal to the nominal VaR level for the other models. Besides, the conditional coverage test cannot be rejected for the HAR with normal distribution at 5% significance. However, the hit rate is never considered equal to the nominal VaR level for the HAR with Student's t distribution.

The novelty of my thesis is that I analysed the volatility signature plots of the different sampling scheme and frequency combinations. It would be interesting to see what other sampling schemes exist and how they perform compared to the schemes I worked with. Finally, I would like to thank Prof. Dr. Roxana Halbleib for the help and guidance on writing this paper.

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Appendix

Volatility signature plots with Confidence Intervals

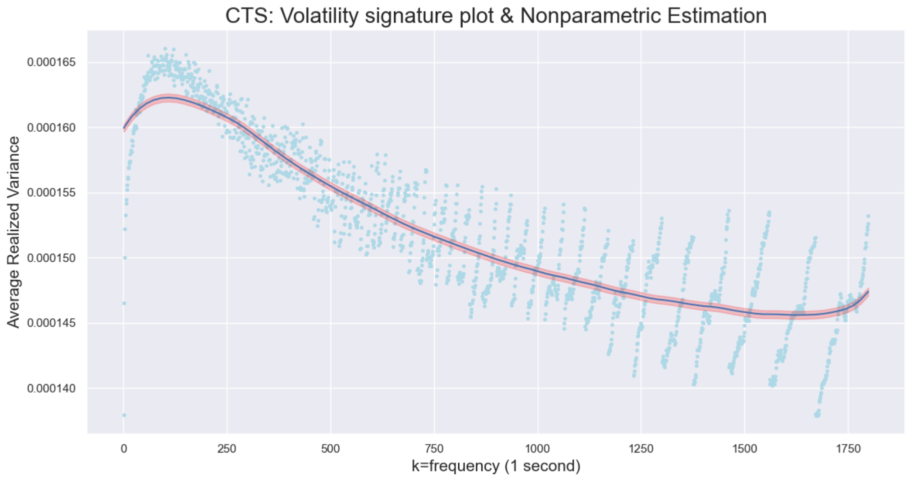

Figure S1. Volatility Signature plot with the confidence interval in pink around the curve, in the period between January 2, 2001 and March 29, 2019, calculated from the intraday log returns sampled every 1 second with Calendar Time Sampling scheme.

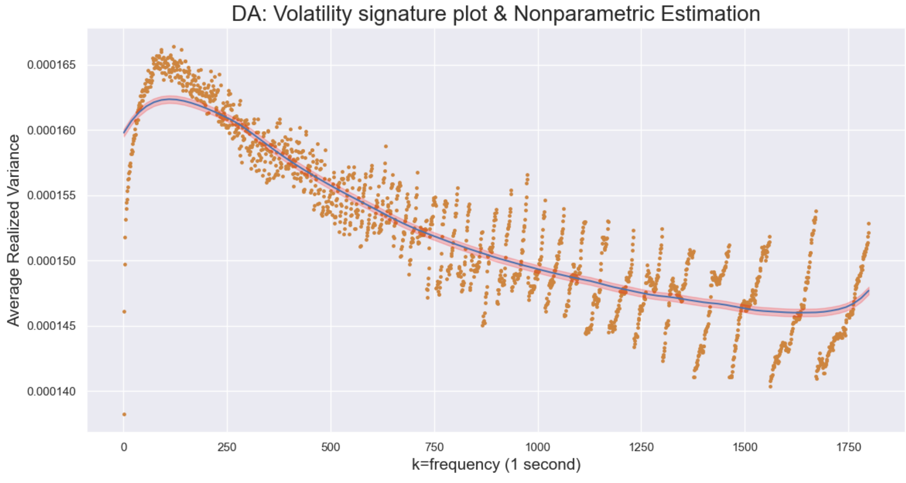

Figure S2. Volatility Signature plot with the confidence interval in pink around the curve, in the period between January 2, 2001 and March 29, 2019, calculated from the intraday log returns sampled every 1 second with Duration Adjustment Sampling scheme.

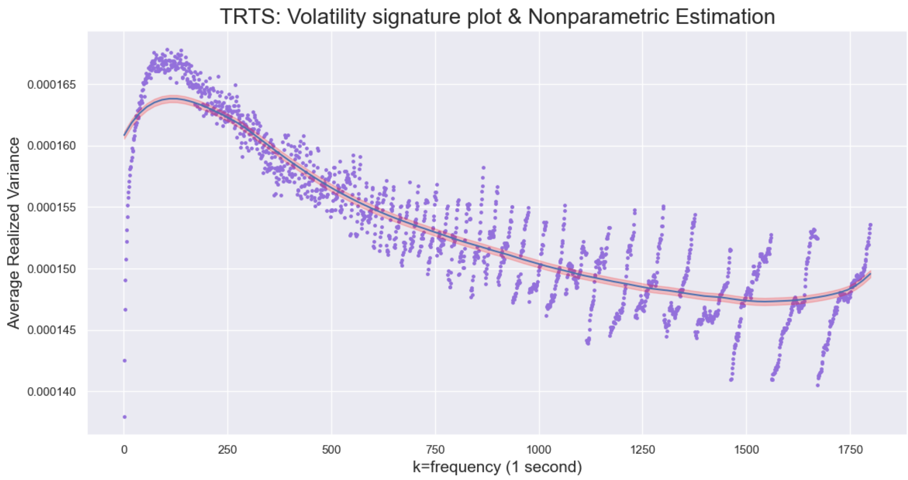

Figure S3. Volatility Signature plot with the confidence interval in pink around the curve, in the period between January 2, 2001 and March 29, 2019, calculated from the intraday log returns sampled every 1 second with Transaction Time Sampling scheme.

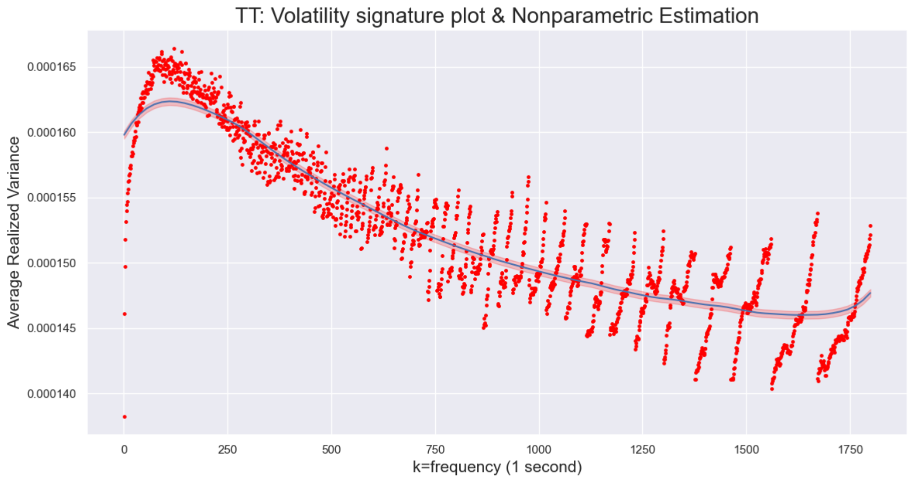

Figure S4. Volatility Signature plot with the confidence interval in pink around the curve, in the period between January 2, 2001 and March 29, 2019, calculated from the intraday log returns sampled every 1 second with Tick Time Sampling scheme.

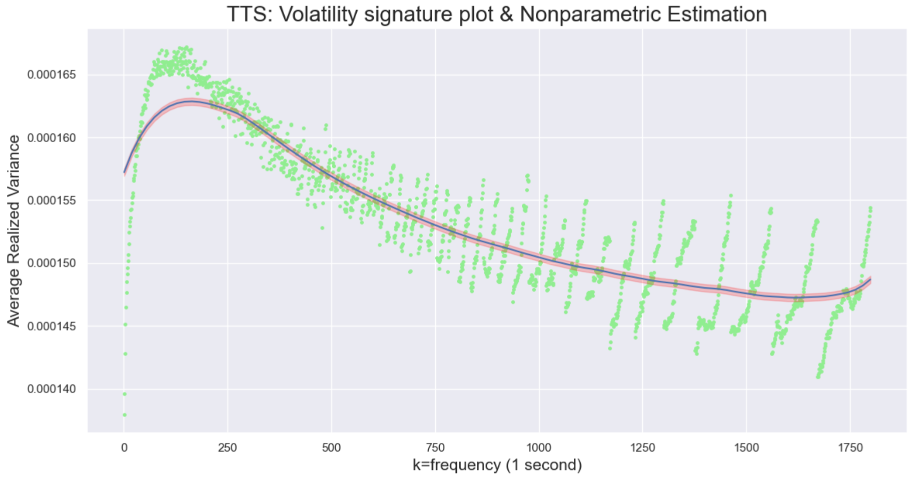

Figure S5. Volatility Signature plot with the confidence interval in pink around the curve, in the period between January 2, 2001 and March 29, 2019, calculated from the intraday log returns sampled every 1 second with Time Transformation Sampling scheme.

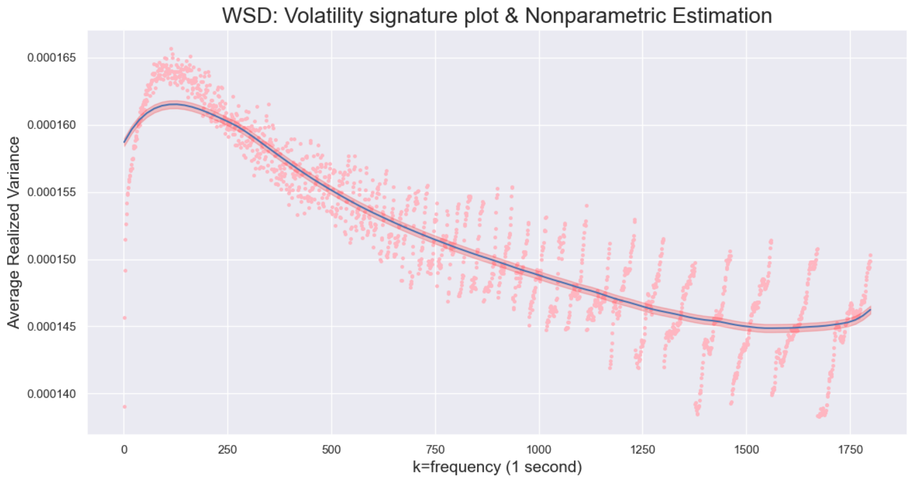

Figure S6. Volatility Signature plot with the confidence interval in pink around the curve, in the period between January 2, 2001 and March 29, 2019, calculated from the intraday log returns sampled every 1 second with Weighted Standard Deviation Sampling scheme.

RMSFE and MAPE

			garch	garch_t	egarch	egarch_t	tgarch	tgarch_t	RM	HAR
1 second	CTS	RMSFE	0,003474	0,003531	0,003468	0,003539	0,003464	0,003517	0,006799	0,002560
		MAPE	0,002630	0,002695	0,002696	0,002743	0,002668	0,002716	0,005904	0,001770
	BTS	RMSFE	0,003459	0,003518	0,003433	0,003507	0,003429	0,003484	0,006732	0,002591
		MAPE	0,002609	0,002681	0,002652	0,002703	0,002626	0,002680	0,005832	0,001801
	TTS	RMSFE	0,003474	0,003530	0,003468	0,003539	0,003464	0,003516	0,006799	0,002559
		MAPE	0,002630	0,002695	0,002695	0,002743	0,002667	0,002716	0,005903	0,001770
	TT	RMSFE	0,003468	0,003525	0,003457	0,003529	0,003452	0,003505	0,006779	0,002561
		MAPE	0,002621	0,002688	0,002686	0,002733	0,002658	0,002707	0,005883	0,001773
	TRTS	RMSFE	0,003474	0,003531	0,003468	0,003539	0,003464	0,003517	0,006799	0,002560
		MAPE	0,002630	0,002695	0,002696	0,002743	0,002668	0,002716	0,005904	0,001770
	WSD	RMSFE	0,003458	0,003515	0,003444	0,003516	0,003440	0,003493	0,006758	0,002565
		MAPE	0,002612	0,002679	0,002671	0,002719	0,002642	0,002693	0,005862	0,001782
	DA	RMSFE	0,003468	0,003525	0,003457	0,003529	0,003452	0,003505	0,006779	0,002561
		MAPE	0,002621	0,002688	0,002686	0,002733	0,002658	0,002707	0,005883	0,001773
10 second	CTS	RMSFE	0,003442	0,00351	0,003349	0,003427	0,003348	0,003406	0,006448	0,002791
		MAPE	0,002488	0,002559	0,002441	0,002511	0,002445	0,002518	0,005537	0,001934
	BTS	RMSFE	0,003397	0,003467	0,00333	0,003408	0,003327	0,003386	0,006572	0,002684
		MAPE	0,002494	0,002571	0,002479	0,002543	0,002471	0,002537	0,005671	0,001853
	TTS	RMSFE	0,003406	0,00347	0,003333	0,003408	0,003333	0,003389	0,006484	0,002712
		MAPE	0,002504	0,002578	0,002479	0,002543	0,002476	0,002542	0,005579	0,001884
	TT	RMSFE	0,003421	0,003485	0,003335	0,003411	0,003332	0,003387	0,00643	0,002758
		MAPE	0,00247	0,002545	0,002437	0,002508	0,002429	0,002498	0,005522	0,001892
	TRTS	RMSFE	0,003436	0,003501	0,003352	0,003428	0,003348	0,003404	0,006447	0,002773
		MAPE	0,002507	0,00258	0,002466	0,002531	0,002465	0,002531	0,005542	0,001935
	WSD	RMSFE	0,003406	0,003475	0,003318	0,003397	0,003309	0,003368	0,006418	0,002754
		MAPE	0,002465	0,002537	0,002436	0,002502	0,002426	0,002492	0,005516	0,001896
	DA	RMSFE	0,003421	0,003485	0,003335	0,003411	0,003332	0,003387	0,00643	0,002758
		MAPE	0,00247	0,002545	0,002437	0,002508	0,002429	0,002498	0,005522	0,001892
1 minute	CTS	RMSFE	0,003562	0,003634	0,003493	0,003571	0,003504	0,003562	0,006719	0,002918
		MAPE	0,002599	0,002666	0,002574	0,002639	0,002573	0,002638	0,005826	0,001944
	BTS	RMSFE	0,003384	0,003458	0,003343	0,003422	0,003354	0,003412	0,006729	0,002637
		MAPE	0,002525	0,002601	0,002541	0,002608	0,002529	0,0026	0,005861	0,001801
	TTS	RMSFE	0,003437	0,003505	0,003374	0,003449	0,003394	0,003448	0,006687	0,002784
		MAPE	0,002512	0,002578	0,002502	0,002564	0,002502	0,002567	0,005825	0,001865
	TT	RMSFE	0,003419	0,003486	0,003357	0,003432	0,00337	0,003424	0,006681	0,002755
		MAPE	0,002516	0,002586	0,002487	0,002556	0,002485	0,002551	0,005819	0,00186
	TRTS	RMSFE	0,003488	0,003557	0,003438	0,003514	0,003451	0,003506	0,006734	0,002815
		MAPE	0,002542	0,002609	0,002539	0,002601	0,002535	0,002596	0,005877	0,001871
	WSD	RMSFE	0,003431	0,003506	0,003369	0,003448	0,003376	0,003436	0,00667	0,002737
		MAPE	0,002491	0,002566	0,002502	0,002569	0,002485	0,002558	0,005793	0,001832
	DA	RMSFE	0,003419	0,003486	0,003357	0,003432	0,00337	0,003424	0,006681	0,002755

		MAPE	0,002516	0,002586	0,002487	0,002556	0,002485	0,002551	0,005819	0,00186
3 minutes	CTS	RMSFE	0,003559	0,003628	0,003504	0,003577	0,003511	0,003563	0,00673	0,002953
		MAPE	0,002605	0,002677	0,002598	0,002654	0,002597	0,002653	0,005866	0,00195
	BTS	RMSFE	0,003433	0,003502	0,003389	0,003464	0,003401	0,003456	0,006783	0,002697
		MAPE	0,002525	0,002604	0,002519	0,002586	0,002517	0,002581	0,005938	0,001776
	TTS	RMSFE	0,003416	0,003482	0,003381	0,003452	0,003404	0,003453	0,00675	0,00279
		MAPE	0,002571	0,002653	0,002583	0,002647	0,002582	0,002648	0,005923	0,001913
	TT	RMSFE	0,003384	0,003454	0,003336	0,003412	0,003346	0,0034	0,006707	0,002724
		MAPE	0,002587	0,002662	0,002576	0,002639	0,002566	0,002626	0,005897	0,001853
	TRTS	RMSFE	0,003485	0,003554	0,003443	0,003514	0,003461	0,003512	0,006779	0,002871
		MAPE	0,002594	0,002668	0,002588	0,002645	0,002588	0,002646	0,005973	0,001949
	WSD	RMSFE	0,00363	0,003705	0,00356	0,00364	0,003567	0,003629	0,006811	0,002974
		MAPE	0,002621	0,002692	0,002612	0,002674	0,002613	0,002673	0,005912	0,001943
	DA	RMSFE	0,003384	0,003454	0,003336	0,003412	0,003346	0,0034	0,006707	0,002724
		MAPE	0,002587	0,002662	0,002576	0,002639	0,002566	0,002626	0,005897	0,001853
5 minutes	CTS	RMSFE	0,003481	0,00355	0,00343	0,003503	0,003439	0,003492	0,006736	0,002933
		MAPE	0,002673	0,002753	0,002656	0,002716	0,002648	0,002708	0,005922	0,002033
	BTS	RMSFE	0,00351	0,003584	0,003455	0,003533	0,003468	0,003527	0,006844	0,002744
		MAPE	0,002604	0,002685	0,00259	0,002661	0,002586	0,002651	0,005989	0,001836
	TTS	RMSFE	0,00348	0,00355	0,003424	0,003498	0,003437	0,003491	0,006774	0,002869
		MAPE	0,002636	0,002711	0,00262	0,002674	0,002624	0,002681	0,005928	0,001972
	TT	RMSFE	0,003437	0,003502	0,00339	0,003457	0,003404	0,003453	0,006746	0,002944
		MAPE	0,002625	0,002705	0,00263	0,002689	0,00262	0,002686	0,00597	0,002028
	TRTS	RMSFE	0,003479	0,003546	0,003449	0,003521	0,003452	0,003504	0,006801	0,002895
		MAPE	0,002664	0,002737	0,002669	0,002723	0,002671	0,002724	0,005964	0,002
	WSD	RMSFE	0,003486	0,003565	0,00344	0,003519	0,003438	0,003499	0,006788	0,002866
		MAPE	0,002576	0,002665	0,002562	0,002631	0,002565	0,002629	0,005961	0,001903
	DA	RMSFE	0,003437	0,003502	0,00339	0,003457	0,003404	0,003453	0,006746	0,002944
		MAPE	0,002625	0,002705	0,00263	0,002689	0,00262	0,002686	0,00597	0,002028

Table S1. Root Mean Squared Forecasting Error and Mean Average Prediction Error in the period from January 2, 2018 until 29th March, 2019. The lowest values in each row are marked with blue.

Model Confidence Set

In Table S2. the p-values that are above 0.05 are marked with green. For those frequency-sampling scheme-model combinations the null hypothesis cannot be rejected at 5% significance.

		CTS	BTS	TTS	TT	TRTS	WSD	DA
	Model name	p-value	p-value	p-value	p-value	p-value	p-value	p-value
1 second	RM	0	0	0	0	0	0	0
	garch_t	0,02	0,021	0,028	0,025	0,034	0,028	0,023
	garch	0,054	0,046	0,059	0,052	0,066	0,05	0,053
	tgarch_t	0,065	0,046	0,061	0,059	0,079	0,056	0,071
	egarch_t	0,065	0,046	0,062	0,059	0,079	0,056	0,071
	tgarch	0,065	0,046	0,073	0,059	0,079	0,056	0,072
	egarch	0,065	0,046	0,073	0,059	0,079	0,056	0,072
	HAR	1	1	1	1	1	1	1
10 second	RM	0	0	0	0	0	0	0
	garch_t	0,011	0	0,007	0,003	0,01	0,009	0,003
	garch	0,024	0,004	0,015	0,016	0,024	0,019	0,016
	tgarch_t	0,036	0,012	0,035	0,033	0,035	0,03	0,03
	egarch_t	0,04	0,017	0,035	0,033	0,035	0,03	0,03
	tgarch	0,04	0,017	0,035	0,033	0,038	0,036	0,033
	egarch	0,04	0,017	0,035	0,033	0,038	0,036	0,033
	HAR	1	1	1	1	1	1	1
1 minute	RM	0	0	0	0	0	0	0
	garch_t	0	0	0	0,002	0	0	0,001
	garch	0,002	0,001	0,001	0,004	0,002	0,002	0,005
	tgarch_t	0,01	0,004	0,003	0,007	0,006	0,004	0,007
	egarch_t	0,01	0,004	0,004	0,007	0,006	0,004	0,007
	tgarch	0,012	0,004	0,004	0,007	0,006	0,004	0,007
	egarch	0,012	0,004	0,004	0,007	0,006	0,004	0,007
	HAR	1	1	1	1	1	1	1
3minute	RM	0	0	0	0	0	0	0
	garch_t	0	0	0,001	0	0	0	0
	garch	0,001	0,001	0,001	0	0,001	0,001	0
	tgarch_t	0,005	0,001	0,004	0	0,002	0,006	0,007
	egarch_t	0,005	0,001	0,004	0	0,002	0,007	0,007
	tgarch	0,005	0,001	0,004	0,002	0,002	0,007	0,007
	egarch	0,005	0,001	0,004	0,002	0,002	0,007	0,007
	HAR	1	1	1	1	1	1	1
5 minute	RM	0	0	0	0	0	0	0
	garch_t	0,001	0	0,001	0	0,001	0	0,001
	garch	0,002	0	0,001	0	0,003	0,001	0,002
	tgarch_t	0,004	0,006	0,002	0,004	0,003	0,005	0,006
	egarch_t	0,004	0,006	0,002	0,004	0,004	0,006	0,006
	tgarch	0,008	0,006	0,003	0,004	0,004	0,006	0,006
	egarch	0,008	0,006	0,003	0,004	0,004	0,006	0,006
	HAR	1	1	1	1	1	1	1

Table S2. MCS in the period from January 2, 2018 until 29th March, 2019.

Declaration of Independent Work According to the Official Examination Rules

I hereby declare that I have written this paper without any unauthorized help and without using any other means than those indicated. All passages that are taken verbatim or in spirit from publications have been marked as such. The submitted master thesis was neither completely nor in essential parts subject of another examination. The electronic version of the submitted master thesis is consistent in content and formatting with the printed paper copies.

Helga Pataki