Statistical properties of material line elements in incompressible MHD turbulence

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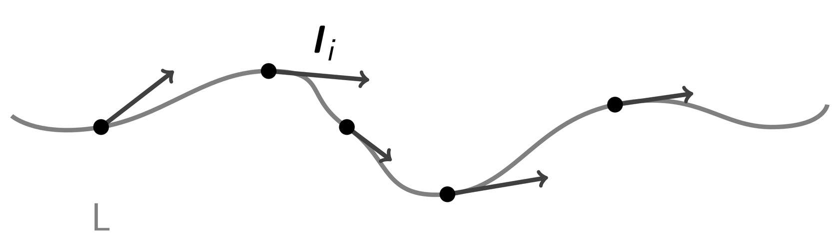
Faculty

Motivation

The deformation of material lines in turbulence is of fundamental interest and practical importance. Due to its diffusive character fluid particles material lines consisting of the same set of fluid partices tend to stretch while following the fluid motion. Vortex lines and magnetic field lines in an inviscid fluid of high conductivity are examples of vector fields that are proportional to material line elements. It is known analytically [1] and shown in hydrodynamic simulations [2] that the length of material line elements increases exponentially in time. In the present work the material line deformation is studied statistically throug infinitesimal material line elements are studied numerically in stationary incompressible magnetohydrodynamic (MHD) turbulence using velocity gradient time series. The velocity gradient data is obtained by tracking Lagrangian particles in a stochastically forced direct numerical simulation (DNS). In order to further understand the influence of the magnetic field on the line deformation a method for injecting cross helicity has been devised to controll the alignment of the magnetic and velocity field.

Line Element Simulation



The dynamics of a line element is given by

$$\frac{d\boldsymbol{l}}{dt} = \nabla \boldsymbol{u}\, \boldsymbol{l} = \boldsymbol{S}\boldsymbol{l} + \boldsymbol{\Omega}\boldsymbol{l}.$$

The line and surface stretching rates ζ and ξ are defined respectively as

$$\zeta \equiv \frac{d\ln(l)}{dt} = S_{ij}\widehat{l_i l_j}, \qquad \xi \equiv \frac{d\ln(A)}{dt} = -S_{ij}\widehat{n}_i\widehat{n}_j, \ \boldsymbol{A} = \boldsymbol{l_1} \times \boldsymbol{l_2}$$

From the lagrangian velocity gradient data $\boldsymbol{V}$ the evolution of the line elemens can be computed by the following equations

$$\boldsymbol{l}(t) = \boldsymbol{B}(t)\boldsymbol{l}(0),$$

$$\frac{d}{dt}\boldsymbol{B} = \boldsymbol{V}\boldsymbol{B}(t), \qquad \boldsymbol{B}(t=0) = \nVdash.$$

References

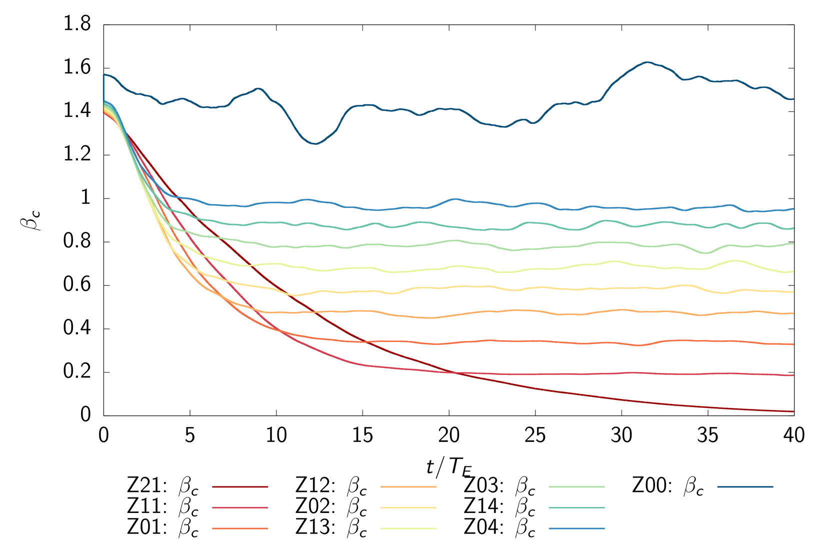
[1] Batchelor, G. K. *The effect of homogeneous turbulence on material lines and surfaces.* Proc. R. Soc. Lond. A, 213(1114), 349-366, 1952.

[2] Girimaji, S. S., Pope, S. B. *Material-element deformation in isotropic turbulence.* Journal of fluid mechanics, 220, 427-458, 1990.

Forced Helicity Injection

Cross helicity

$$H_C = \int dV\, \boldsymbol{v} \cdot \boldsymbol{b}$$



The temporal evlution of the alignment is shown for different values of σ_c: $H00 : \sigma_c^f = 1$, $H11 : \sigma_c^f = 0.6$, $H12 : \sigma_c^f = 0.4$, $H13 : \sigma_c^f = 0.2$, $H14 : \sigma_c^f = 0$.

Magnetic helicity

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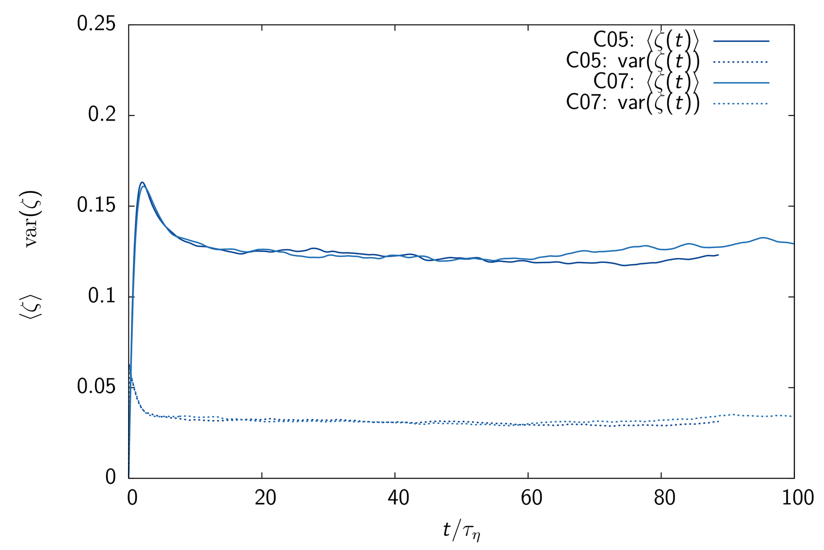
Results

Strechting and alignment of line elements



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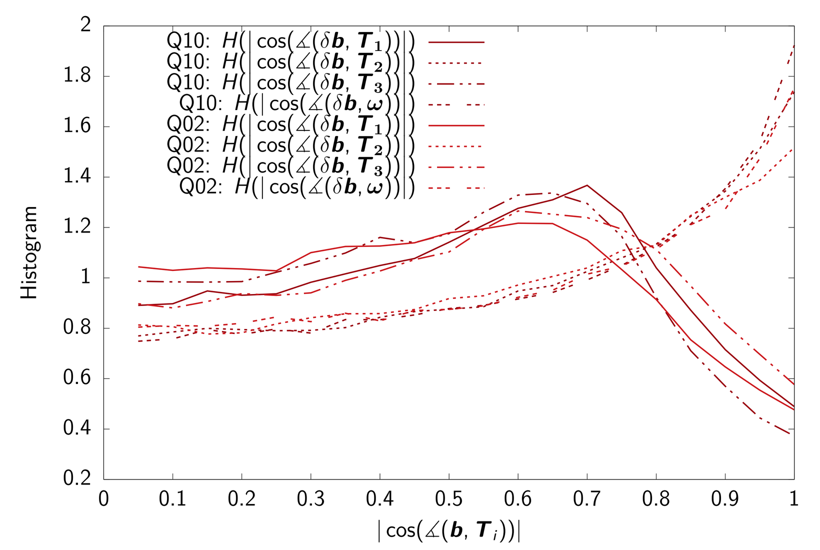

MHD p.d.fs for the angles between the local magnetic field and the line element orientation at steady state ($t/\tau_\eta = 20$).

Influence of helicities on line strechting

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