# Statistical properties of material line elements in incompressible MHD turbulence

**name** , more names

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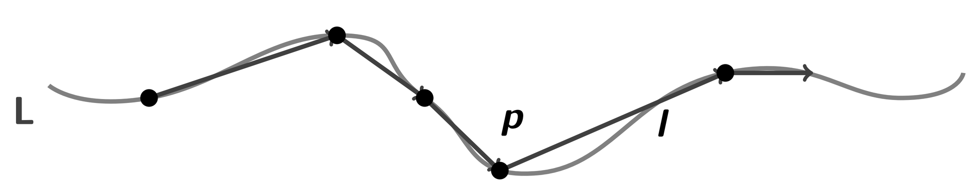
## Motivation

The deformation of material lines in turbulence is of fundamental interest and practical importance. Vortex lines and magnetic field lines in an inviscid fluid of high conductivity are examples of vector fields that are proportional to material line elements. It is known analytically and shown in hydrodynamic simulations (Girimaji & Pope 1990) that the length of material line elements increases exponentially in time. The stretching rate of line and surface elements are found to be significantly lower in MHD turbulence than in the hydrodynamic case. Moreover the results show that the material lines are primarily aligned along the direction of the magnetic field. Further the role of the magnetic field in material element deformation is investigated by injecting cross and magnetic helicity into the system.

## Line Element Simulation

A Material line is defined as a line that always consists of the same set of particles or fluid elements. In order to study the material line dynamics statistically the lines are simplified to infinitessimal elements (Batchelor [1]) which allows for a this one-point description of the material line elements.




A material line $L$ is approximated by line elements $l_i$ which are computed for for each lagrangian particle $p_i$.

The dynamic evolution of a line element $\boldsymbol{l}$ is given by

$$\frac{d\boldsymbol{l}}{dt} = \nabla \boldsymbol{u}\, \boldsymbol{l} = \boldsymbol{S}\boldsymbol{l} + \boldsymbol{\Omega}\boldsymbol{l},$$

where velocity gradient can be split into is the symmetric part $\boldsymbol{S}$ (strain-rate tensor) and an antisymmetric part $\boldsymbol{\Omega}$(rotation-rate tensor). The line stretching rate $\zeta$ is defined as

$$\zeta \equiv \frac{d\ln(l)}{dt} = S_{ij}\widehat{l_i l_j}.$$

In the simulation lagrangian velocity gradient data $\boldsymbol{V}$ is first gathered for each particle and then used to evolve the corresponding line elements through the matrix $\boldsymbol{B}$

$$\frac{d}{dt}\boldsymbol{B} = \boldsymbol{V}\boldsymbol{B}(t), \qquad \boldsymbol{B}(0) = \mathbb{1},$$

$$\boldsymbol{l}(t) = \boldsymbol{B}(t)\boldsymbol{l}(0).$$

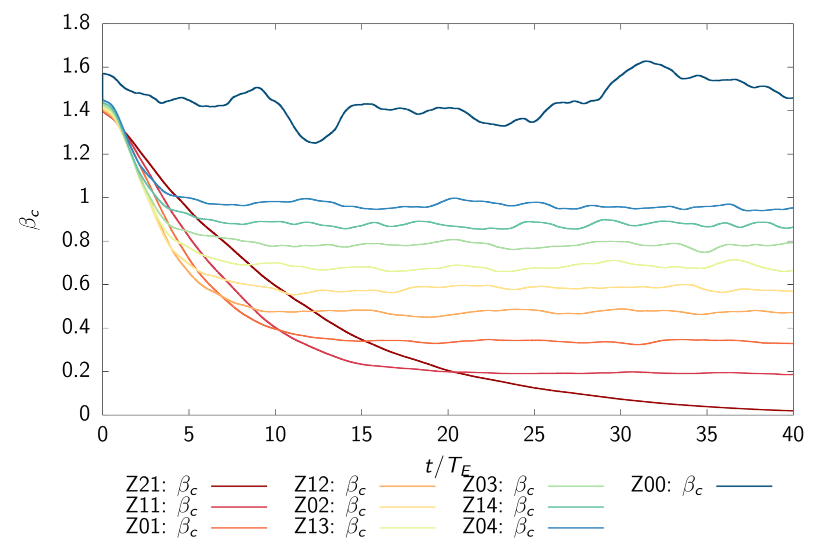
## References

[1] Batchelor, G. K. *The effect of homogeneous turbulence on material lines and surfaces.* Proc. R. Soc. Lond. A, 213(1114), 349-366, 1952.

[2] Girimaji, S. S., Pope, S. B. *Material-element deformation in isotropic turbulence.* Journal of fluid mechanics, 220, 427-458, 1990.

## Forced Helicity Injection

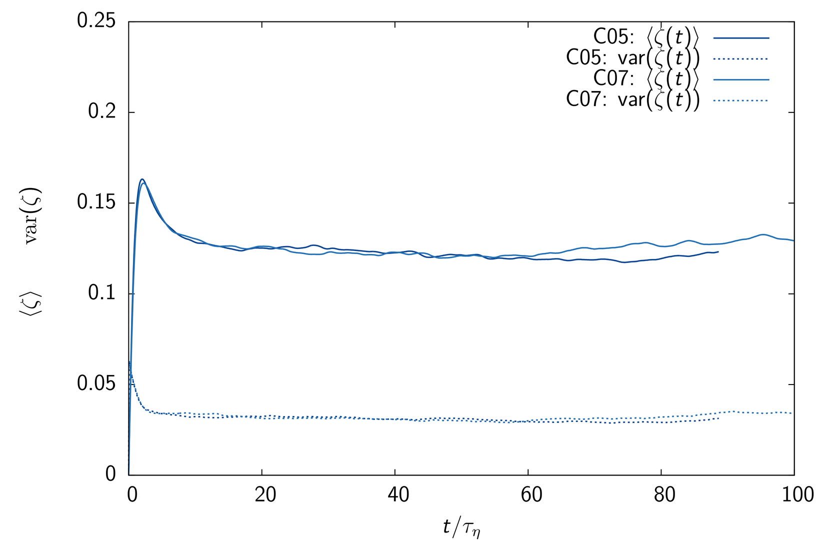
- pseudo spectral code
- ou forcing on large scales
- helicity forcing on small scales
- ideal invariant, how it reduces dynamics of the system



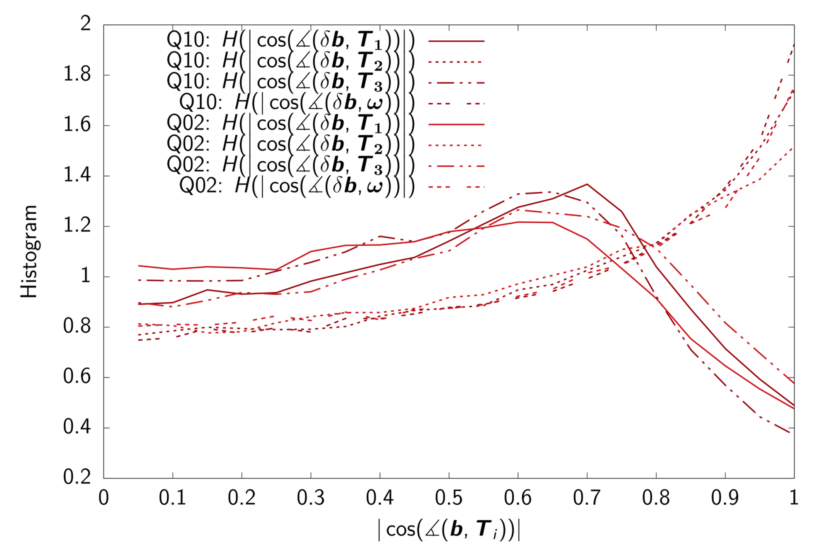
The alignment is shown for different values of $\sigma_c$:
$H00 : \sigma_c^f = 1, H11 : \sigma_c^f = 0.6, H12 : \sigma_c^f = 0.4, H13 : \sigma_c^f = 0.2, H14 : \sigma_c^f = 0.$

## Results

### Strechting and alignment of line elements



caption



MHD p.d.fs for the angles between the local magnetic field and the line element orientation at steady state ($t/\tau_\eta = 20$).

### Influence of helicities on line strechting

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- ...