Statistical properties of material line elements in incompressible MHD turbulence

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Motivation

The deformation of material lines in turbulence is of fundamental interest and practical importance. Due to its diffusive character fluid particles material lines consisting of the same set of fluid particles tend to stretch while following the fluid motion. Vortex lines and magnetic field lines in an inviscid fluid of high conductivity are examples of vector fields that are proportional to material line elements. It is known analytically [1] and shown in hydrodynamic simulations [2] that the length of material line elements increases exponentially in time. In the present work the deformation of material lines is studied statistically by simulating infinitesimal material line elements in stationary incompressible magnetohydrodynamic (MHD) turbulence using velocity gradient time series. The velocity gradient data is obtained by tracking Lagrangian particles in a stochastically forced direct numerical simulation (DNS). In order to further understand the influence of the magnetic field on the material line deformation a method for injecting cross helicity has been devised to control the alignment of the magnetic and velocity field.

Line element simulation

A Material line is defined as a line that always consists of the same set of particles or fluid elements. In order to study the material line dynamics statistically the lines are simplified to infinitessimal elements (Batchelor [1]) which allows for a this one-point description of the material line elements.

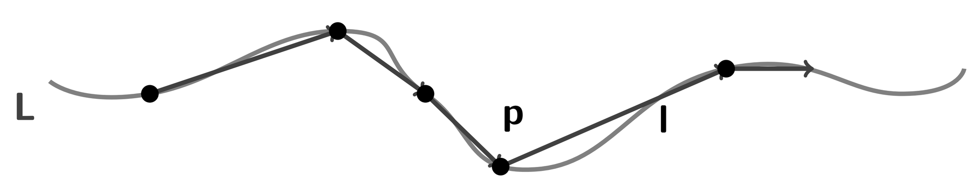

Figure 1: A material line L is approximated by line elements l which are computed for for each lagrangian particle p.

The dynamic evolution of a line element $\mathbf{l}$ is given by

$$\frac{d\mathbf{l}}{dt} = \nabla\mathbf{u}\,\mathbf{l} = \mathbf{S}\mathbf{l} + \mathbf{\Omega}\mathbf{l}, \tag{1}$$

where velocity gradient can be split into is the symmetric part $\mathbf{S}$ (strain-rate tensor) and an antisymmetric part $\mathbf{\Omega}$(rotation-rate tensor). The line stretching rate ζ is defined as

$$\zeta \equiv \frac{d\ln(l)}{dt} = S_{ij}\hat{l}_i\hat{l}_j. \tag{2}$$

In the simulation lagrangian velocity gradient data $\mathbf{V}$ is first gathered for each particle and then used to evolve the corresponding line elements through the matrix $\mathbf{B}$

$$\frac{d}{dt}\mathbf{B} = \mathbf{V}\mathbf{B}(t), \qquad \mathbf{B}(0) = \mathbb{1}, \tag{3}$$

$$\mathbf{l}(t) = \mathbf{B}(t)\mathbf{l}(0). \tag{4}$$

References

[1] Batchelor, G. K. *The effect of homogeneous turbulence on material lines and surfaces*. Proc. R. Soc. Lond. A, 213(1114), 349-366, 1952.

[2] Girimaji, S. S., Pope, S. B. *Material-element deformation in isotropic turbulence*. Journal of fluid mechanics, 220, 427-458, 1990.

Helicity injection

The velocity gradient is obtained by tracking lagrangian particles through tricubic interpolation in a direct numerical simulation of the incompressible MHD equations which are solved using the pseudo spectral method. Since the MHD equations are dissipative a stochastic forcing based on the Ornstein-Uhlenbeck Process was applied on large scales to keep the system in a stationary state.

Cross helicity

In order to further understand the role of the magnetic field in material line deformation the alignment of the velocity field $\mathbf{v}$ and the magnetic field $\mathbf{b}$ was controlled by injecting cross helicity H^C into the system,

$$H^C = \int \mathbf{v}\cdot\mathbf{b}\,dV. \tag{5}$$

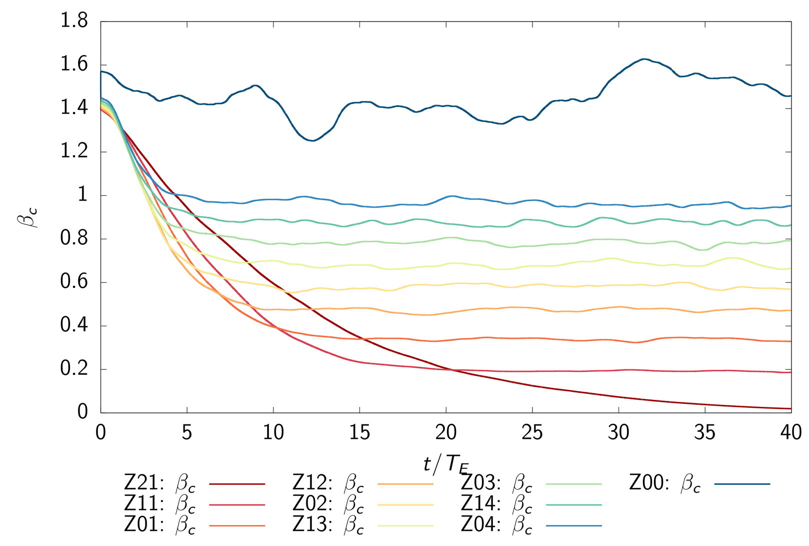

Figure 2: The alignment is shown for different values of σ_c: $H00: \sigma_c^f = 1, H11: \sigma_c^f = 0.6, H12: \sigma_c^f = 0.4, H13: \sigma_c^f = 0.2, H14: \sigma_c^f = 0.$

Since the value H^C in Eq. (5) depends on the magnitude and the orientation of $\mathbf{v}$ and $\mathbf{b}$, the dynamical alignment $\sigma_C = H^C/|H^C|$ is shown in Fig. (2).

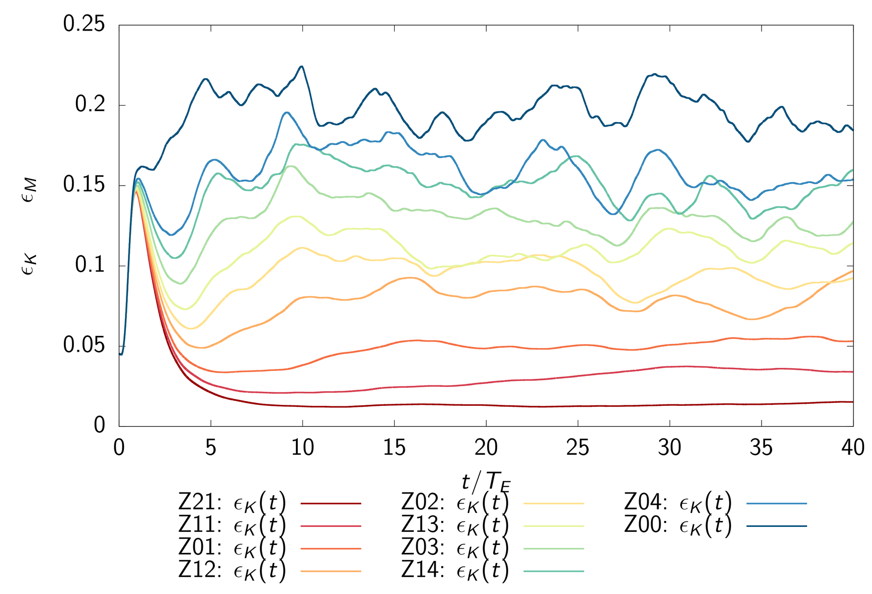

Figure 3: MHD stochastic forcing, dependency of energy dissipation evolution on cross helicity on .

Results

Strechting and alignment of line elements

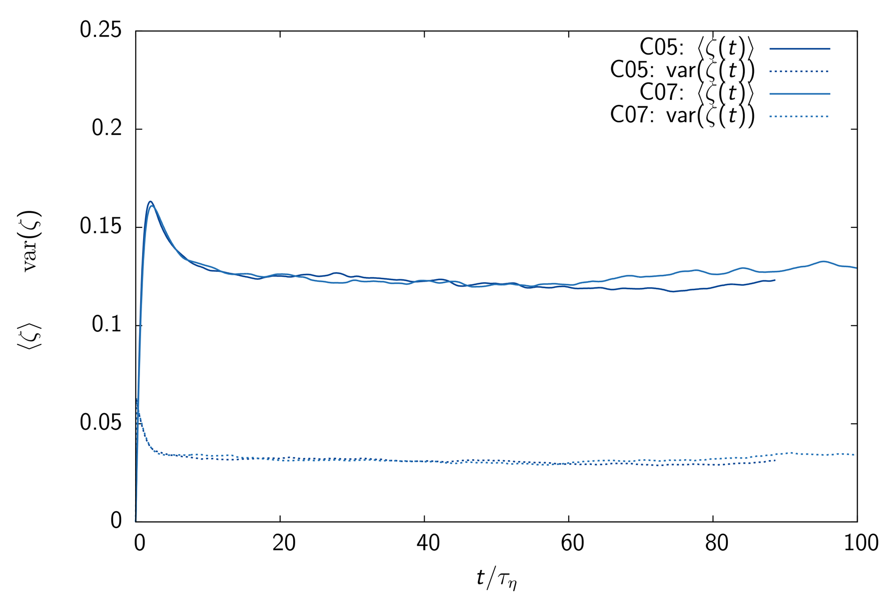

Figure 4: caption

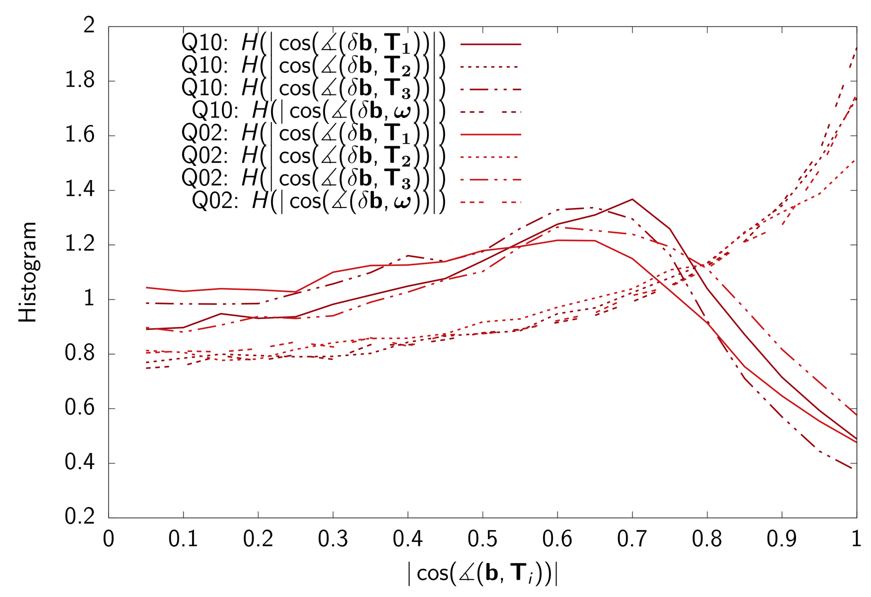

Figure 5: MHD p.d.fs for the angles between the local magnetic field and the line element orientation at steady state ($t/\tau_\eta = 20$).

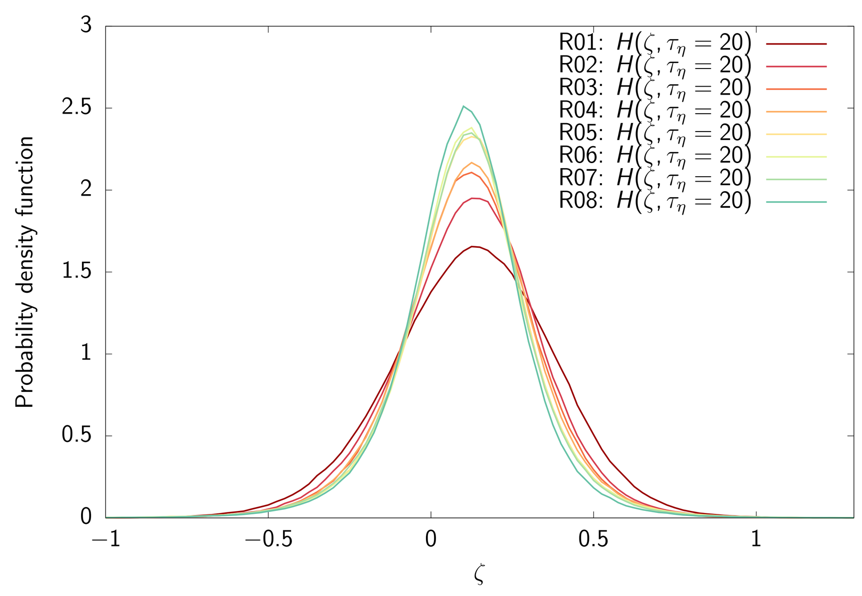

Figure 6: caption

Influence of helicities on line strechting

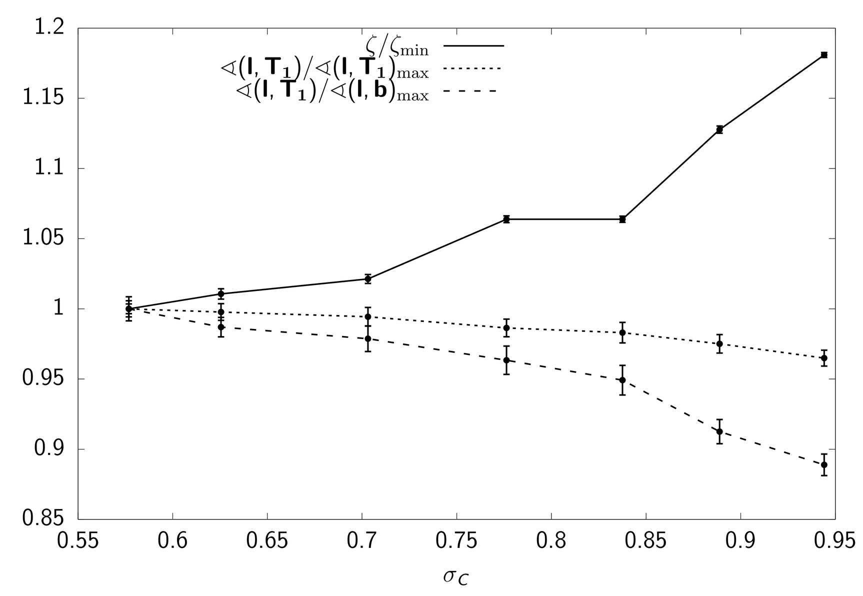

Figure 7: caption

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