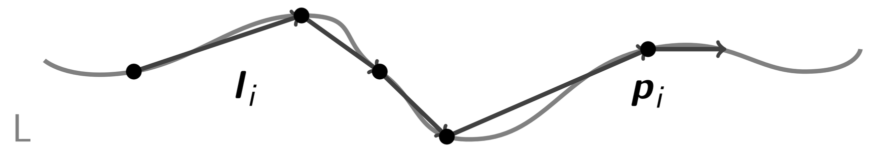
# Statistical properties of material line elements in incompressible MHD turbulence

**name** , more names

## Motivation

The deformation of material lines in turbulence is of fundamental interest and practical importance. Vortex lines and magnetic field lines in an inviscid fluid of high conductivity are examples of vector fields that are proportional to material line elements. It is known analytically and shown in hydrodynamic simulations (Girimaji & Pope 1990) that the length of material line elements increases exponentially in time. The stretching rate of line and surface elements are found to be significantly lower in MHD turbulence than in the hydrodynamic case. Moreover the results show that the material lines are primarily aligned along the direction of the magnetic field. Further the role of the magnetic field in material element deformation is investigated by injecting cross and magnetic helicity into the system.

## Line Element Simulation



The dynamics of a line element is given by

$$\frac{d\boldsymbol{l}}{dt} = \nabla \boldsymbol{u}\, \boldsymbol{l} = \boldsymbol{S}\boldsymbol{l} + \boldsymbol{\Omega}\boldsymbol{l}.$$

The line and surface stretching rates $\zeta$ and $\xi$ are defined respectively as

$$\zeta \equiv \frac{d\ln(l)}{dt} = S_{ij}\widehat{l_i}\widehat{l_j}, \qquad \xi \equiv \frac{d\ln(A)}{dt} = -S_{ij}\widehat{n}_i\widehat{n}_j, \; \boldsymbol{A} = \boldsymbol{l}_1 \times \boldsymbol{l}_2$$

From the lagrangian velocity gradient data $\boldsymbol{V}$ the evolution of the line elemens can be computed by the following equations

$$\boldsymbol{l}(t) = \boldsymbol{B}(t)\boldsymbol{l}(0),$$

$$\frac{d}{dt}\boldsymbol{B} = \boldsymbol{V}\boldsymbol{B}(t), \qquad \boldsymbol{B}(t=0) = \mathbb{1}.$$

Strain rates..

## References

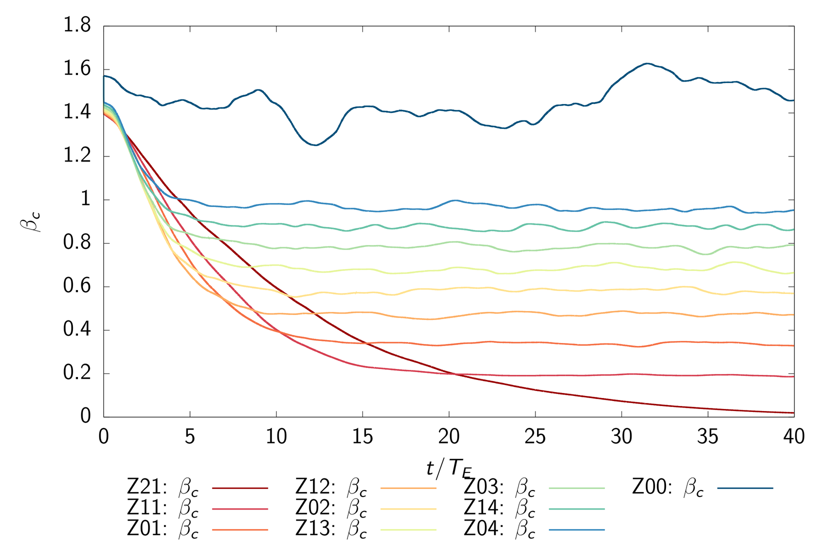

[1] Batchelor, G. K. *The effect of homogeneous turbulence on material lines and surfaces.* Proc. R. Soc. Lond. A, 213(1114), 349-366, 1952.

[2] Girimaji, S. S., Pope, S. B. *Material-element deformation in isotropic turbulence.* Journal of fluid mechanics, 220, 427-458, 1990.

[3] Helicity


## Forced Helicity Injection

### Cross helicity



The temporal evlution of the alignment is shown for different values of $\sigma_c$: $H00 : \sigma_c^f = 1$, $H11 : \sigma_c^f = 0.6$, $H12 : \sigma_c^f = 0.4$, $H13 : \sigma_c^f = 0.2$, $H14 : \sigma_c^f = 0$.

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### Magnetic helicity

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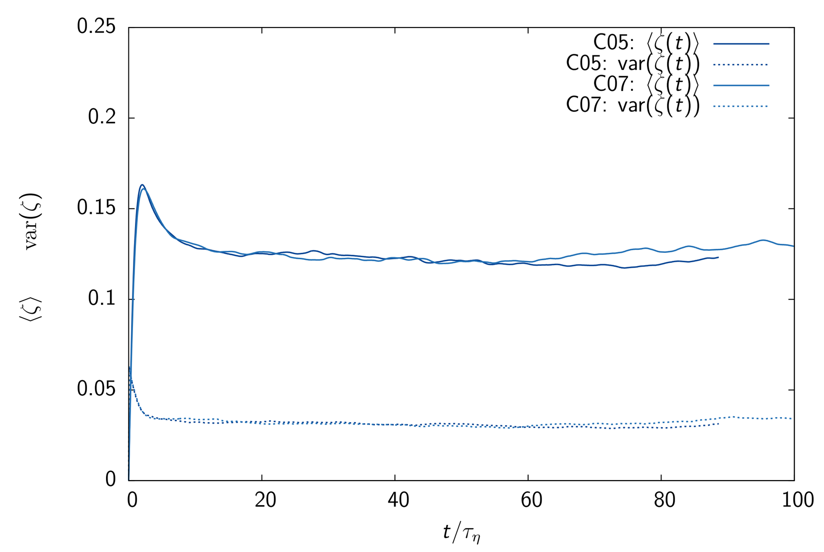
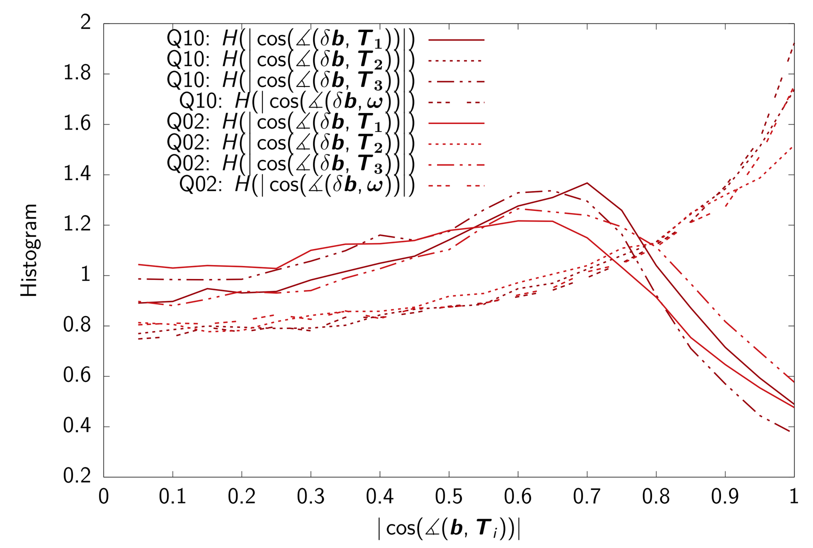
## Results

### Strechting and alignment of line elements



caption



MHD p.d.fs for the angles between the local magnetic field and the line element orientation at steady state ($t/\tau_\eta = 20$).

### Influence of helicities on line strechting

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