Statistical properties of material line elements in incompressible MHD turbulence

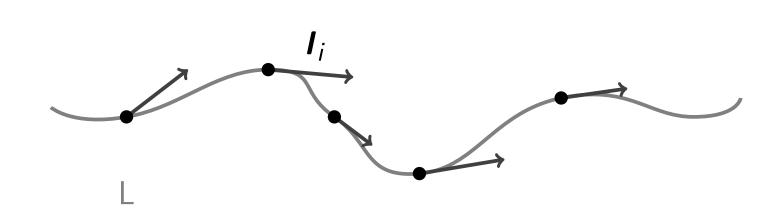
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Motivation

The deformation of material lines in turbulence is of fundamental interest and practical importance. Vortex lines and magnetic field lines in an inviscid fluid of high conductivity are examples of vector fields that are proportional to material line elements. It is known analytically and shown in hydrodynamic simulations (Girimaji & Pope 1990) that the length of material line elements increases exponentially in time. The stretching rate of line and surface elements are found to be significantly lower in MHD turbulence than in the hydrodynamic case. Moreover the results show that the material lines are primarily aligned along the direction of the magnetic field. Further the role of the magnetic field in material element deformation is investigated by injecting cross and magnetic helicity into the system.

Line Element Simulation



The dynamics of a line element is given by

$$rac{doldsymbol{I}}{dt}=
ablaoldsymbol{u}oldsymbol{I}=oldsymbol{S}oldsymbol{I}+oldsymbol{\Omega}oldsymbol{I}.$$

The line and surface stretching rates ζ and ξ are defined respectively as

$$\zeta \equiv rac{d \ln(I)}{dt} = S_{ij} \widehat{I_i} \widehat{I_j}, \qquad \xi \equiv rac{d \ln(A)}{dt} = -S_{ij} \widehat{n}_i \widehat{n}_j, \; \boldsymbol{A} = \boldsymbol{I_1} \times \boldsymbol{I_2}$$

From the lagrangian velocity gradient data $oldsymbol{V}$ the evolution of the line elemens can be computed by the following equations

$$I(t) = B(t)I(0),$$

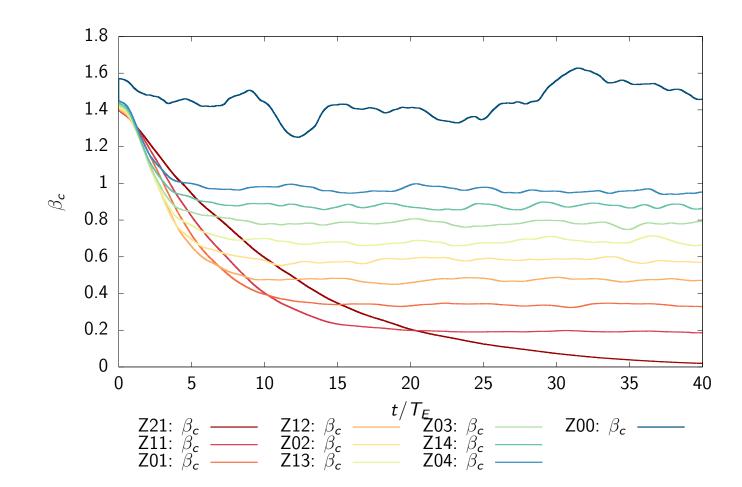
$$\frac{d}{dt}\mathbf{B}=\mathbf{V}\mathbf{B}(t), \qquad \mathbf{B}(t=0)=\mathbb{H}.$$

References

- [1] Batchelor, G. K. The effect of homogeneous turbulence on material lines and surfaces. Proc. R. Soc. Lond. A, 213(1114), 349-366, 1952.
- [2] Girimaji, S. S., Pope, S. B. Material-element deformation in isotropic turbulence. Journal of fluid mechanics, 220, 427-458, 1990.
- [3] Helicity

Forced Helicity Injection

Cross helicity



The temporal evolution of the alignment is shown for different values of σ_c : H00 : $\sigma_c^f = 1$, H11 : $\sigma_c^f = 0.6$, H12 : $\sigma_c^f = 0.4$, H13 : $\sigma_c^f = 0.2$, H14 : $\sigma_c^f = 0$

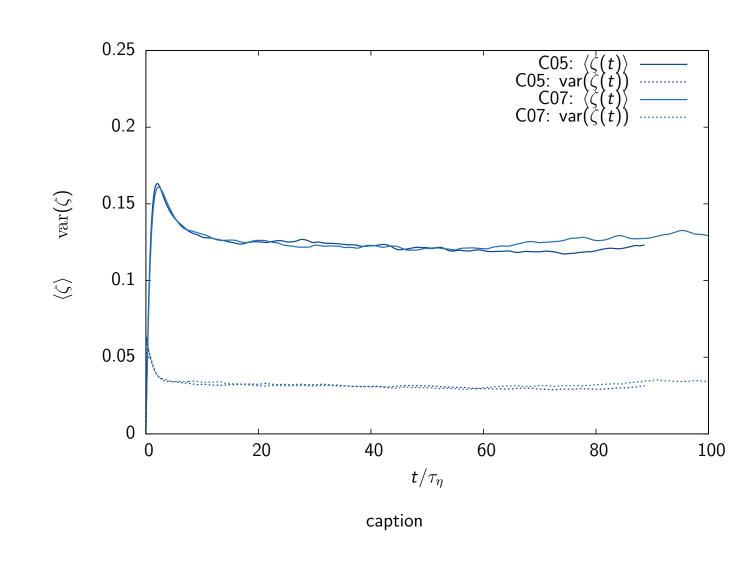
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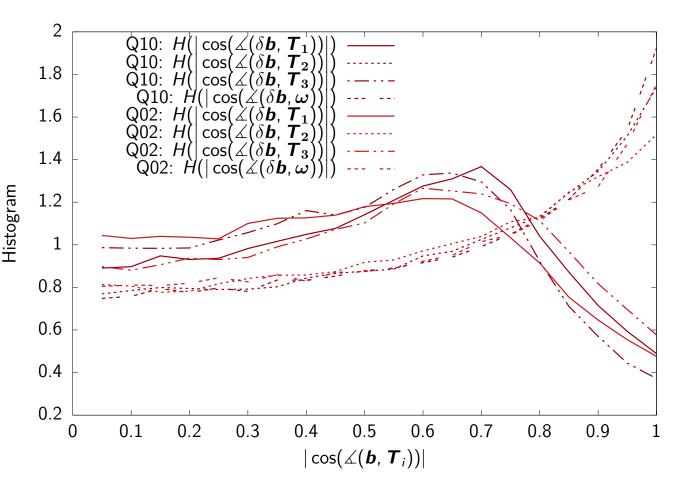
Magnetic helicity

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Results

Strechting and alignment of line elements





MHD p.d.fs for the angles between the local magnetic field and the line element orientation at steady state ($t/ au_{\eta}=20$).

