

Diffeology is differential geometry reloaded. Its objects are not only anymore finite dimensional manifolds, but they are also singular spaces: irrational tori, or orbifolds, or set of leaves of foliations. They may be, as well, sets of smooth functions of infinite dimensions, or groups of diffeomorphisms, and finally they can of course be ordinary manifolds.

Essentially, a diffeology on a set X , declares which parametrizations, in X , are smooth. That is, what does mean, for a family of elements of X , to be smoothly parametrized by a set of numbers. For the sake of coherence, with what does mean ordinarily to be smooth, in the world of real numbers, these smooth parametrizations satisfy three simple axioms: covering, smooth compatibility and locality.

By moving the perspective, from ordinary manifolds to general diffeological spaces, we are rewarded with a strongly stable category, regarding the most common set theoretic operations: sums, products, subsets and quotients. Moreover, spaces of differentiable maps become immediately diffeological spaces, for the functional diffeology, as well as powersets of diffeological spaces. In other words, Diffeology is a remarkably simple Cartesian closed category. As many example show, this flexible property is not paid a high price, since objects like irrational tori, which are trivial in almost any other generalized approach, are highly non trivial as diffeological objects.

The imperative categorical properties of diffeology, make many theorems and constructions natural. We can use intensively the space of smooth paths, or loops, of diffeological spaces without leaving the category, which is appreciable and brings deep simplifications. Using for example differential calculus on space of loops, reduces numerous classical theorems to their simplest expression, and highlight their intrinsic nature. At the same time they give the right generalization for any diffeological space.

Homotopy, homology, cohomology, De Rham calculus, fiber bundles, orbifolds, covering, symplectic geometry, moment maps, all these classical constructions find their way and natural place in Diffeology, nothing is lost, and a lot is embraced. Many heuristic constructions find a rigorous expression and become a part of the general formal picture.

Along the book, many examples are given, from singular spaces to spaces of infinite dimensions, and through these examples and exercises, the reader can familiarize with the specific technics developed in diffeology, because diffeology is not just a formalism but also a working tool. Equipped with this experience, he will be able to extend this theory beyond the limits of this book.

