StatComp Project 2: Scottish weather

Peter Ly (s1325633, peter-lyy)

1 Weather data

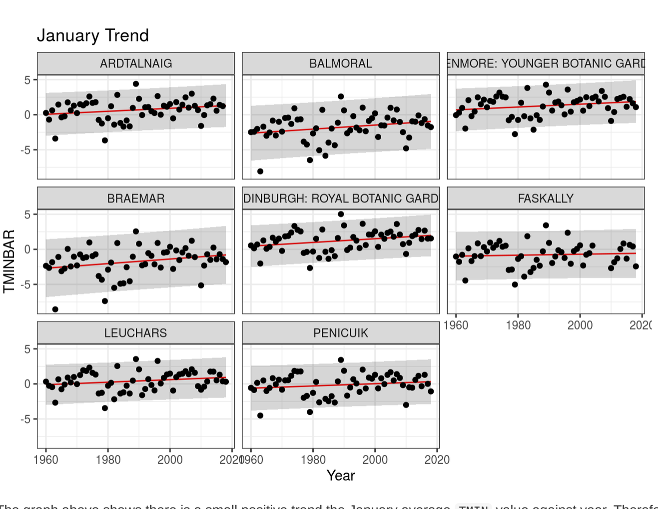
This report will be analysing data from the Global Historical Climatology Network at https://www.ncdc.noaa.gov/data-access/land-based-station-data/land-based-datasets/global-historical-climatology-network-ghcn. I will be specifically looking at data from eight weather stations in Scotland. The data includes the minimum temperature, `TMIN`, maximum temperature, `TMAX`, and precipitation, `PRCP`, recorded every day (when possible to) between 1960 - 2018. Below you will find the eight Scottish weather stations and their location in latitude, longitude and elevation(meters) above sea level, along with their respective IDs:

ID	Name	Latitude	Longitude	Elevation
UKE00105874	BRAEMAR	57.0058	-3.3967	339
UKE00105875	BALMORAL	57.0367	-3.2200	283
UKE00105884	ARDTALNAIG	56.5289	-4.1108	130
UKE00105885	FASKALLY	56.7181	-3.7689	94
UKE00105886	LEUCHARS	56.3767	-2.8617	10
UKE00105887	PENICUIK	55.8239	-3.2258	185
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	55.9667	-3.2100	26
UKE00105930	BENMORE: YOUNGER BOTANIC GARDE	56.0281	-4.9858	12

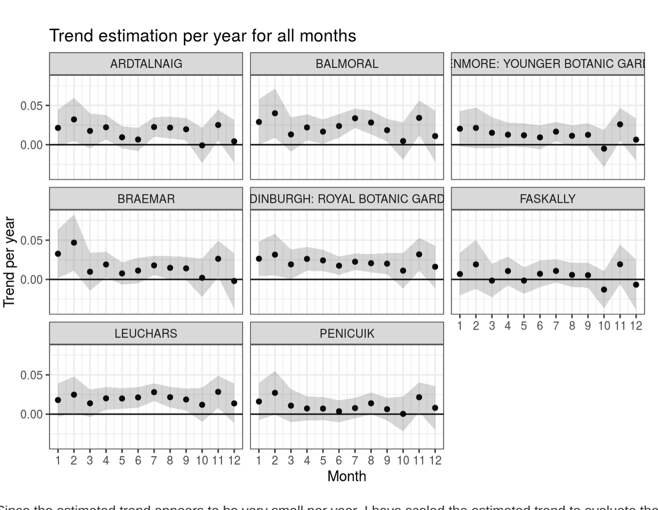
2 Climate trends

2.1 Monthly Trends

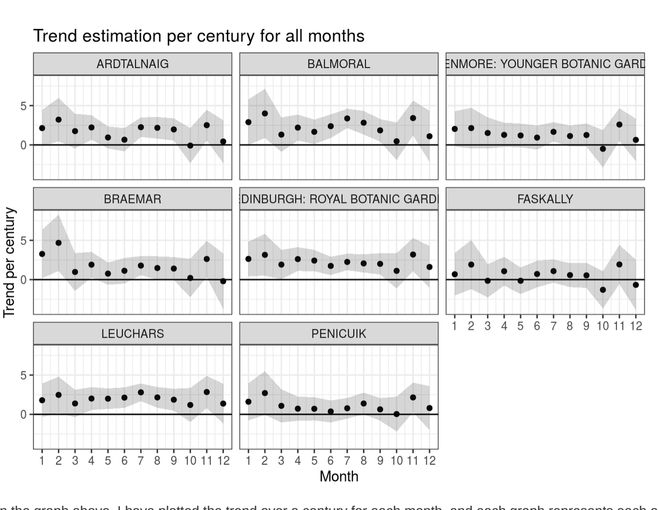
To assess whether there is a long-term temporal trend in the minimum temperature, `TMIN`, at each station for each month, I will first display whether a trend even exists in January for all stations. Below you will find a graph of the observed average TMIN for January against years, from 1960 to 2018. The red line represents an estimated linear model given by: $\overline{TMIN}_{m(t)} = \beta(Year) + c$ and the grey band is 95% predicition interval. β represents the long-term temporal trend and c represents the estimated average `TMIN` for that month, $\overline{TMIN}_{m(t)}$ at year 0. Therefore, for predicting $\overline{TMIN}_{m(t)}$ pre-1960 and post 2018 will be interpolating the data and should be done with caution.



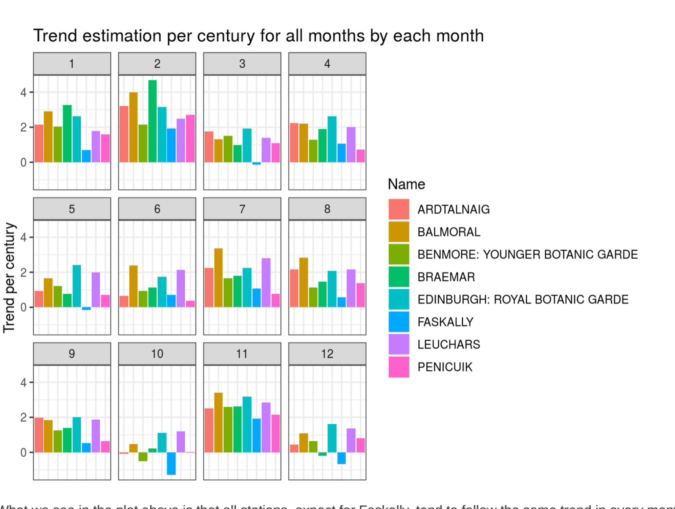
The graph above shows there is a small positive trend the January average `TMIN` value against year. Therefore, it would be worthwhile to apply a linear model to all months and evaluate the year coefficient to see the long-term temporal trend. The graphs below shows each estimated long-term temporal trend per year for each month at all 8 stations, with 95% confidence intervals:



Since the estimated trend appears to be very small per year, I have scaled the estimated trend to evaluate the change per century. The graph below shows the results:



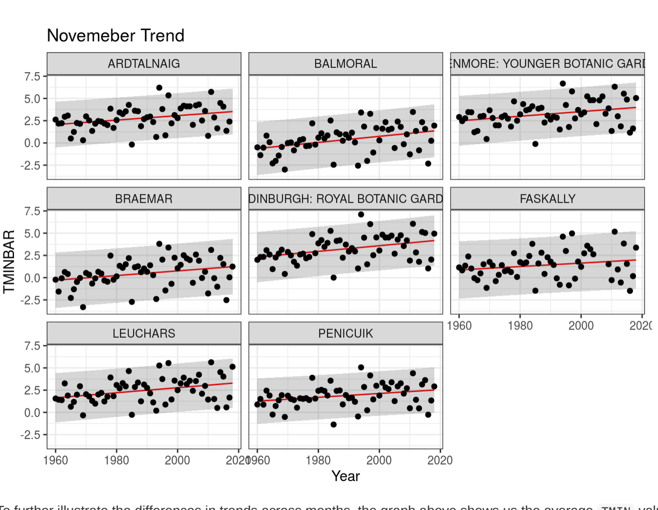
In the graph above, I have plotted the trend over a century for each month, and each graph represents each of the 8 stations. The grey band represents the 95% confidence interval for the trend.
The way in which we can interpret this graph is that the "Trend per century" represents an change in average minimum temperature, `TMIN`, for that month over a century. For example, the weather station at Braemar is recording an increase in average minimum temperature in February at approximately 5 degrees celsius per century.
What we observe is that there is generally a small positive trend over years for the average monthly `TMIN` value. This means as time goes on we will see a small increase in average monthly `TMIN` value. However the trend is not consistent for all months across all stations. The bar chart below will provide a visual representation of the stability of the trend within the same month across stations.



What we see in the plot above is that all stations, expect for Faskally, tend to follow the same trend in every month. This is seen by the similarity in heights per monthly plot. With the exception of Faskally where we tend to see dampened or negative trend compared to others when considering the average `TMIN` value per month. Furthermore, from the plot preceding the bar chart, the trend per century over the months at each station all tend to follow the same pattern. Including Faskally except it is shifted downward by about 2-3 degrees celsius. These features can be attributed to the location of the station compared to the others. Faskally is located between hills and elevated ground which is a unique geographic location.

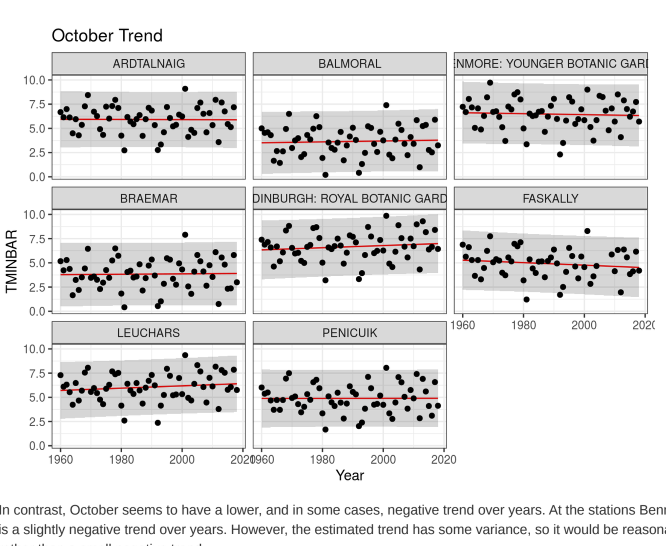
The most steady and largest trend across all stations exists in the months February and November. Conversely the least steady and lowest trends across all stations exist in the months October and December. We will be exploring these months further in the next section.

2.1.1 November Trends



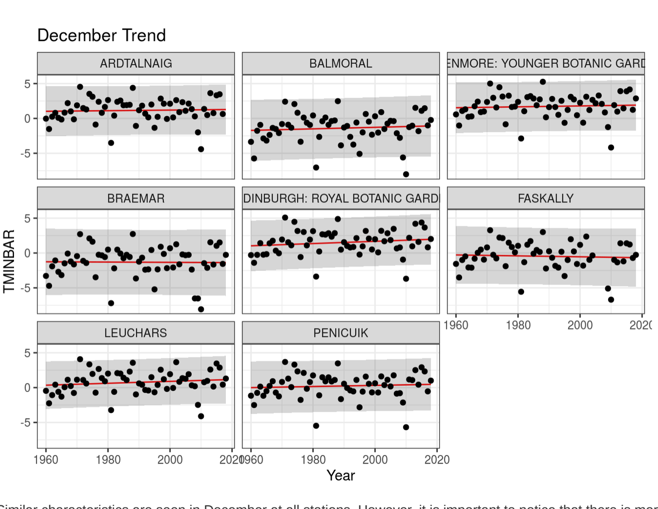
To further illustrate the differences in trends across months, the graph above shows us the average `TMIN` value against year for the month of November. We can see that there is a positive trend over time for all stations. This would indicate to us that in November, the average `TMIN` is increasing over time. Therefore, we see higher average minimum temperatures in November which means on average it is not as cold as it has been in previous years.

2.1.2 October Trends



In contrast, October seems to have a lower, and in some cases, negative trend over years. At the stations Benmore, Faskally and Ardtalnaig there is a slightly negative trend over years. However, the estimated trend has some variance, so it would be reasonable to assume that there is no trend rather than a small negative trend.

2.1.3 December Trends



Similar characteristics are seen in December at all stations. However, it is important to notice that there is more variability in the December estimated trend. The more extreme observed average `TMIN` values fall outside of the 95% prediction interval for the estimated model shown in the graphs above. In particular observed values below the 95% prediction interval

We can conclude there there is generally a small upward trend over years for the monthly average `TMIN` values at all stations in Scotland for most months. Where there is no positive long-term temporal trend, we see that there is little to no trend at all. This would be in line with what we generally see across the world. The trend seems the least prevalent in the November and December. Conversely, the trend seems more stable in the spring and summer seasons, with a steady increase over time. This means the average monthly `TMIN` value is slowly and steadily increasing over time, which is a sign of a slight increase in temperature. However, we are only considering the average monthly `TMIN` value, perhaps the daily `TMIN` or the average monthly `TMAX` value will present a different conclusion.

2.2 Seasonally varying variability

To assess whether the temperature variability is the same for each month of the year I will be looking at the Balmoral station daily `TMIN` residuals from the month averages. I define the residuals as $R_t = T_t - \overline{T}_{m(t)}$, where T_t is `TMIN` at time t, and $\overline{T}_{m(t)}$ is the average `TMIN` value in the month of time t. I will be testing whether the daily `TMIN` residuals within a month are larger than the overall residual variance.
Our hypotheses are:

$$H_0 : \text{The residual variance is the same for all months of the year}$$
$$H_1 : \text{The residual variance for January is higher than the overall variance}$$

and similarly for all other month.
First, I will define the observed test statistic as

$$\text{Test statistic} = var(R_{m(t)}) - var(R_t)$$

which is to say, the difference in variance in a given month of time t and the variance of the whole set of residuals. Since this is a one-sided test to see if the variance of the residual from one month is larger than the variance of the set of residuals. I have formulated the test statistic to see the positive difference.
The observed test statistics are given in the table below:

Month	T_Month	H0	T_stat
1	16.235135	11.99151	4.2436238
2	15.618038	11.99151	3.6265266
3	11.645986	11.99151	-0.3455250
4	10.334631	11.99151	-1.6568802
5	9.904222	11.99151	-2.0872892
6	8.562389	11.99151	-3.4291220
7	7.708484	11.99151	-4.2830266
8	9.565547	11.99151	-2.4259643
9	11.405000	11.99151	-0.5865109
10	12.160173	11.99151	0.1686622
11	14.392107	11.99151	2.4005962
12	16.602199	11.99151	4.6106877

Since there is a large number of possible permutations, beyond the capabilities of my workstation, I will be performing a randomisation test to see if there is sufficient evidence to reject the null hypothesis.
The upper bound of the standard deviation of the p-value is given by $1/\sqrt{4M}$. As a result, I have performed 2500 permutations of the data to calculate the p-value to a standard deviation of 0.01.
Below you will find a table of the calculated p-values for each month at the Balmoral station:

Month	pvalue
1	0.0000
2	0.0000
3	0.7880
4	1.0000
5	1.0000
6	1.0000
7	1.0000
8	1.0000
9	0.9164
10	0.3464
11	0.0000
12	0.0000

We can conclude from the table above that there is sufficient evidence to reject the null hypothesis for January, February, November and December at the 5% significance level whilst accounting for the standard deviation of the p-value.
That is to say that the residual variance is not the same for those months of the year and it is higher than the overall variance.
What is interesting to note is that these months, November to January, are the winter months in Scotland. Therefore, we can interpret this as more variation in `TMIN` temperature in the winter season when compared to the rest of the year. This means that it can be colder and warmer compared it's monthly average in the winter months.

Another interesting observation is that during all other seasons the residual variance is not greater than the overall residual variance, however that is not to say it is not less than the overall residual variance. If I were to test for a different alternative hypothesis:

$$H_1 : \text{The residual variance for January is } \textbf{lower} \text{ than the overall variance}$$

and similarly for all other months. Then I would expect to see sufficient evidence to reject null hypothesis in the months between April-October at the 5% significance level.

To conclude, there is sufficient evidence to reject the null hypothesis and to say that the residual variance is not the same for all months of the year. Since if we reject the null hypothesis for any of the 12 tests, then we would reject the null hypothesis all together.

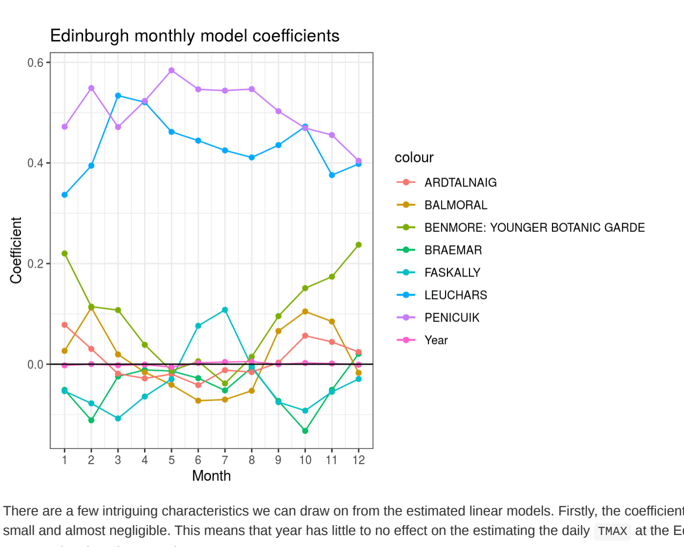
3 Spatial weather prediction

3.1 Estimation

The weather station I have chosen to estimate a linear model for is Edinburgh: Royal Botanic Garde. The linear model will predict the daily maximum temperature, `TMAX`, at the Edinburgh weather station using the 7 other stations and Year as covariates. I will estimate a linear model for each month of the year.
Below you will find a table of the coefficients of all covariates and the intercept of the estimated linear model:

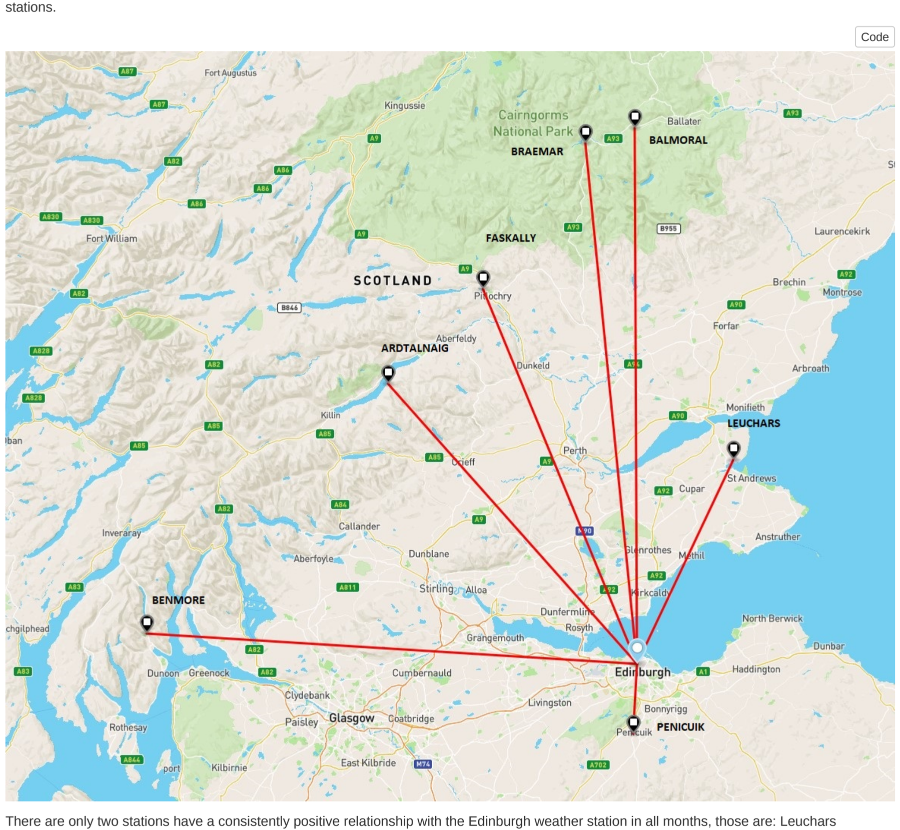
ID	Name	Month	(Intercept)	UKE00105874	UKE00105875	UKE00105884	UKE00105885	UKE00105886	UKE00105887	UKE00105930	Year
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	1	5.167786	-0.0510624	0.0264702	0.0782827	-0.0535950	0.3367438	0.4720412	0.2201165	-0.0022888
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	2	0.415828	-0.1114989	0.1124204	0.0303980	-0.0778315	0.3945294	0.5488249	0.1143426	0.0001623
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	3	4.748618	-0.0243217	0.0194527	-0.0188910	-0.1078024	0.5338197	0.4713329	0.1075609	-0.0020471
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	4	2.530060	-0.0114008	-0.0160266	-0.0282355	-0.0642628	0.5206657	0.5232977	0.0385885	-0.0008788
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	5	12.559714	-0.0136021	-0.0409086	-0.0196575	-0.0297159	0.4617616	0.5841268	-0.0132593	-0.0056310
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	6	-3.910384	-0.0277568	-0.0724955	-0.0415149	0.0763364	0.4443402	0.5463749	0.0058304	0.0026302
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	7	-7.070721	-0.0521135	-0.0702933	-0.0118494	0.1080776	0.4249245	0.5438046	-0.0382479	0.0044533
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	8	-7.845750	-0.0067258	-0.0527463	-0.0156752	-0.0042984	0.4109186	0.5468530	0.0146939	0.0050568
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	9	1.428979	-0.0730696	0.0659322	0.0031785	-0.0754671	0.4354722	0.5028338	0.0955135	-0.0001107
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	10	-4.650025	-0.1325062	0.1047731	0.0566988	-0.0922954	0.4722369	0.4693005	0.1511667	0.0024834
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	11	-2.160441	-0.0508737	0.0848657	0.0442595	-0.0549109	0.3758637	0.4555294	0.1739467	0.0013709
UKE00105888	EDINBURGH: ROYAL BOTANIC GARDE	12	2.513994	0.0207425	-0.0171255	0.0242396	-0.0293784	0.3980769	0.4042687	0.2374048	-0.0010620

To illustrate the interesting characteristics of the models, I have constructed a graph below of the results plotted against the individual months:



There are a few intriguing characteristics we can draw on from the estimated linear models. Firstly, the coefficients for the year covariates are very small and almost negligible. This means that year has little to no effect on the estimating the daily `TMAX` at the Edinburgh weather station compared to the other covariates.

To assess the coefficient at each station on the daily `TMAX` prediction for the Edinburgh weather station I have included an image of the location of all 8 weather stations, using GeoJSON. The light blue marker is the Edinburgh weather station and the red lines are the distances to the other stations.



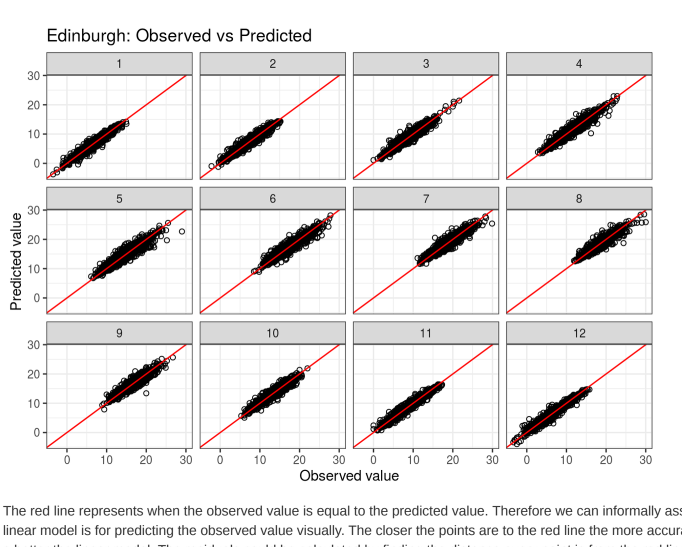
There are only two stations have a consistently positive relationship with the Edinburgh weather station in all months, those are: Leuchars (UKE00105886) and Penicuik (UKE00105887). The reason why we see the positive coefficients for Leuchars in our estimated model is because of the short distance between the two stations and the similarity in elevation from sea level. The distance between the two stations is approximately 50km, which makes it the second closest station. Furthermore, difference in elevation is approximately 16m making it also the second closest in elevation. As for Penicuik, it is the closest station to Edinburgh at approximately 16 km. This would explain the positive relationship between the two weather stations and their recorded `TMAX` temperature, despite having an elevation difference of approximately 160m.

On the other hand, the station Braemar (UKE00105874) has a small negative or negible relationship with the Edinburgh weather station. We can attribute this to the large difference in elevation, at 313m. The distance between the two is approximately 116 km, and which is similar to the distance to Benmore, and we will see later an interesting relationship between Edinburgh and Benmore stations. Therefore we can attribute the lack of a positive relationship due to the difference in elevation.

When assessing the coefficient for the Benmore: Younger Botanic Garde(UKE00105930) weather station we see it has a positive relationship with the Edinburgh station during the autumn and winter months (October-March) but little to no relationship during the rest of the seasons. This is due to the geographic similarities between the two stations. There is approximately 14m in elevation difference but the distance between the two stations is approximately 110km. Furthermore, both weather stations are situated at opposite coasts in Scotland. This helps explain the unusual relationship and the coefficient. What we see is that both stations experience similar daily `TMAX` during the autumn and winter months.

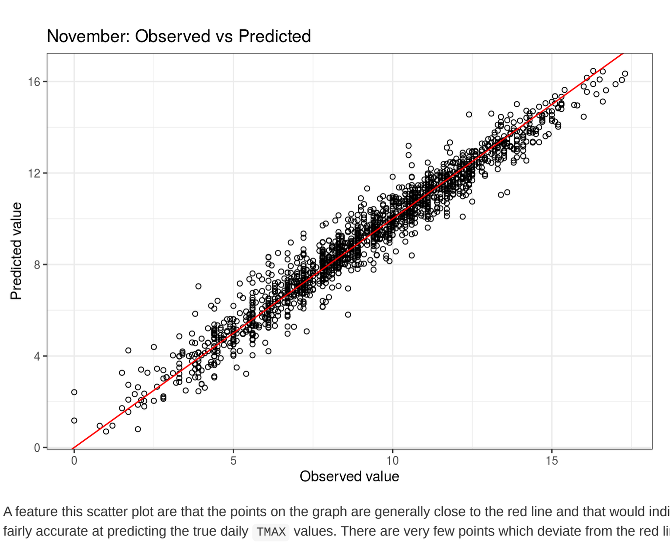
Finally, the Faskally (UKE00105885) weather station has a interesting geographic location when comparing to the Edinburgh weather station. With the elevation difference being 68m and the distance being 90km, we might expect a small positive coefficient. However, this is not the case, since all coefficients for that station is below 0.1 in all months expect July. The cause of this may be the geographic location of the Faskally. This weather station is located on a slight elevation and surround by larger hills and mountains. The valley like positioning of this station has had an influence on the `TMAX` recorded.

To demonstrate how effective the linear models for each month is at predicting the daily `TMAX`, I have plotted the observed value at the Edinburgh weather station against the predicted value from the linear model. Below we see the scatter plot for all months:



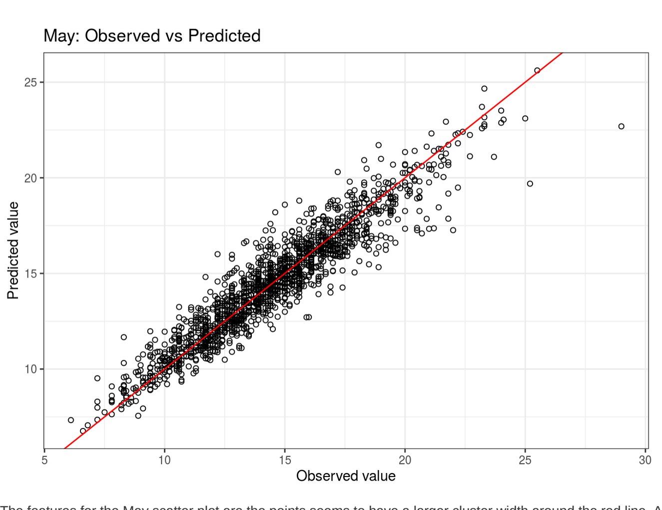
The red line represents when the observed value is equal to the predicted value. Therefore we can informally assess how good the estimated linear model is for predicting the observed value visually. The closer the points are to the red line the more accurate the prediction is and therefore a better the linear model. The residuals could be calculated by finding the distance every point is from the red line. Predicted values which are less than the observed value fall below the red line, and predicted values which are greater than the observed values are above the red line.

I would like to bring to your attention two models, firstly the November model:



A feature this scatter plot are that the points on the graph are generally close to the red line and that would indicate the estimated linear model is fairly accurate at predicting the true daily `TMAX` values. There are very few points which deviate from the red line.

The second model is the May Model:



The features for the May scatter plot are the points seems to have a larger cluster width around the red line. Also there are some points that the under predicted by up to 5 degrees.

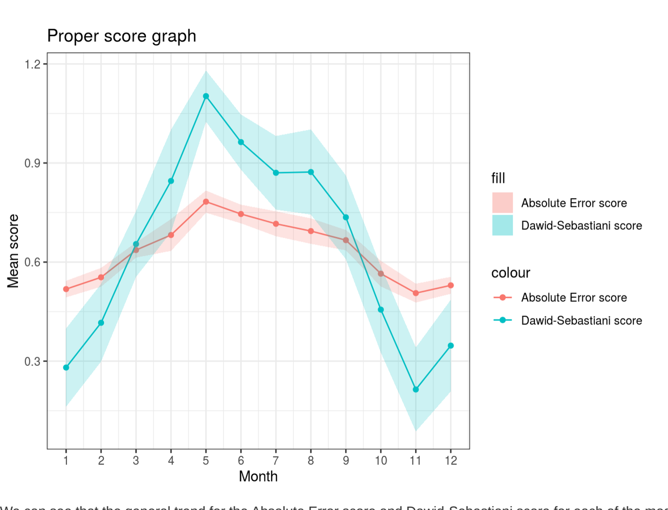
Informally, we can see that the November model is better at predicting the true value than the May model for the Edinburgh weather station. In the next section, I will be formally assessing this in the next section using the Absolute Error score and Dawid-Sebastiani score.

3.2 Assessment

To assess the predictive power of each of the 12 models, I will be estimating the absolute error score and the Dawid-Sebstiani scores for each of the models. To estimate these scores I will be using a 10-fold cross-validation. This means I will be randomingly splitting the data into 10 subsets; using 9 subsets to train the model and the remaining subset will be used to validate the model. The model will predict `TMAX` values for the validating data. From the predictions I will estimate the Absolute Error scores and Dawid-Sebastiani scores. Repeating this procedure 10 times will allow me to perform a 10-fold randomised cross-validation. Therefore I will be using all 10 subsets of the data as validating data once, for each of the 12 models. By taking the average of the scores, we will have estimated the Absolute Error score and Dawid-Sebastiani and its standard deviation.
It must be noted that in an ideal scenario, the size of the data for each month would be divisible by 10, so that the sizes of all 10 subsets could be the same. However, this is not the case and resulted in some subsets being one observation larger than others.
Below you will find a table of my results from the 10-fold cross-validation estimation of the Absolute Error and Dawid-Sebastiani scores:

Month	mean_ae	sd_ae	mean_ds	sd_ds
1	0.5181940	0.0110458	0.2806736	0.0523727
2	0.5538829	0.0125432	0.4161371	0.0519623
3	0.6364767	0.0101371	0.6545114	0.0439229
4	0.6819980	0.0212737	0.8457265	0.0685682
5	0.7829038	0.0147842	1.1026196	0.0342579
6	0.7455558	0.0124558	0.9633287	0.0369342
7	0.7159372	0.0166349	0.8703512	0.0492911
8	0.6938882	0.0170385	0.8726990	0.0569880
9	0.6663782	0.0136517	0.7355718	0.0560319
10	0.5648981	0.0171957	0.4559902	0.0575430
11	0.5060140	0.0125778	0.2142110	0.0561354
12	0.5296148	0.0113340	0.3470394	0.0613644

The results of the table are better analysed by the following graph:



We can see that the general trend for the Absolute Error score and Dawid-Sebastiani score for each of the model are the same. That is to say that the linear models estimating the `TMAX` values in the winter months (October-February) are generally better models than the summer months models. Both the Absolute Error score and Dawid-Sebastiani scores support this statement.
The May model produces the worst scores which is highlighted by the peak in the graph. The best performing model is the November model, which is highlighted by the lowest points on the graph.
An interesting feature of these results is that the Dawid-Sebastiani shows a more exaggerated trend when looking at the monthly model against scores compared to the Absolute Error score. An explanation for this is that the calculation of the Absolute Error score is:

$$S_{AE}(F, y) = |y - \hat{y}_F|$$

where, y is the true value and $\hat{y}_F$ is a point estimate under F, the predictive median. This means when calculating the absolute error score only considers the error, which is the difference between the predicted value and the true value.

In contrast, the calculation of Dawid-Sebastiani score is given by:

$$S_{DS}(F, y) = \frac{(y - \mu_F)^2}{\sigma_F^2} + log(\sigma_F^2)$$

where μ_F is the predictive mean and σ_F^2 is the predictive variance under F. This means that the score also depends on the predictive variance which the the Absolute Error score did not include. We can see from the formula above that a larger predictive variance, σ_F^2, will decrease the first term but increase second term of the Dawid-Sebastiani score. Considering we will generally be dealing with standard deviations less than 1, then we will also be dealing with variance less than 1. As a result, as variance increases the first term will decrease and the second term will increase, however the second term will always be negative due to taking the log of the variance.
Consequently, we see a lower and better Dawid-Sebastiani scores when the predictive variance is lower, like in the November model. On the other hand, we see larger and worse Dawid-Sebastiani scores when the predictive variance is higher. For example, the May model has a standard deviation and variance larger than 1, which makes the log term positive. This is why the trend of the scores are similar between the Absolute Error score and Dawid-Sebastiani score, however the latter has a penalty for large variance in predictions.

4 Code appendix

4.1 Function definitions

4.2 Analysis code