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A Scheme to Control the Speed of a DC Motor with Time Delay using LQR-PID controller

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***Abstract*— In this paper we have designed an optimal PID controller for the speed control of a DC motor. The optimal Linear Quadratic Regulator (LQR) based PID controller is derived analytically for the second order transfer function of the DC motor with time delay. The state weighting matrices of the LQR are used for finding the set of optimal PID gains. We have compared the results with the previously developed results of LQR-PID and found that the present approach of PID tuning gives the speed control of the DC motor more close to the desired damping ratio and natural frequency.**

Keywords-PID, pole-placement, dominant pole, LQR

I. INTRODUCTION

Existence of time-delay in a control loop reduces the phase margin of control systems and reduces relative stability. The existence of time-delay in a control loop is a source of instability and performance deterioration [1]. Most of the real time system contains time delay in their transfer function and we cannot neglect them. As the high performance is always desired from the controller, many researchers have worked on the tuning of controller for the system having time-delay and few recent techniques can be found in Ref. [2]. Proportional-Integral-Derivative (PID) controller is the most widely used controller in the industry today. The popularity of PID controller is due to its simplicity which uses only three parameters to tune i.e Proportional (K_p) term which controls the plant or system proportional to the input error, Integral (K_i) term which gives the change in the control input proportional to the integral of the error signal and the last one is the Derivative term (K_d) that controls the system by providing control signal proportional to the derivative of the error signal. Derivative action is used in some cases to speed up the response and to stabilize the system behavior [3]. In this paper we present the methodology of PID tuning based on the optimal approach of LQR and dominant pole placement technique applicable to second order with time delay (SOPTD) systems with user specified closed loop damping ratio 'ζ_{cl}' and natural frequency 'ω_{cl}'.

Although the PID controller is known to be the simplest and efficient controller but it requires effective and optimized tuning of the control parameter (K_p, K_i, K_d). Many PID controller tuning methods have been proposed in the literature and some of the popular methods are Ziegler–Nichols tuning, Cohen–Coon tuning, Internal model control, Direct Synthesis Method, neural networks based methodologies relay based auto tuning method etc[1,2,4].

Most of the real plants can be more closely approximated using second order plus time delay model compared to first order plus time delay (FOPTD) model [4]. He et al. [5] have proposed an analytical method to tune the PI/PID parameters in an optimal way using LQR techniques with user specified closed loop damping ratio and natural frequency for the FOPTD model. Authors have also extended their approach for SOPTD systems by equating the larger process pole with the derivative term of the PID controller and then applied the PI tuning approach using LQR to obtain other two parameters.

In the present paper we will discuss that taking the derivative gain of the PID controller equal to the one of the system poles does not provide optimal PID settings and hence does not give the closed loop time response satisfactory with user specified closed loop damping ratio and natural frequency. This technique also fails to produce satisfactory PID parameters for complex conjugate poles of the systems since a single complex pole of the process cannot be eliminated with a single complex zero of the controller, as they are always in conjugate pairs [6].

We have extended the PI/PID tuning method proposed by He et al. [5] for the first order plus time delay (FOPTD) model to develop the tuning procedure for SOPTD model using dominant pole placement approach [6,7]. Dominant pole placement design was first introduced by person [7] and further explained by [4]. In the dominant pole design a pair conjugate poles are chosen based on the requirements on the closed-loop response such as rise time and percentage overshoot.

In this paper we present the simulation results of the speed control of a DC motor presented in Ref [8] subjected to the time delay. The PID parameters are evaluated analytically and compare with the previous developed LQR-PID tuning method. The simulated results done in MATLAB shows remarkable closed loop time response with user defined closed loop performance measures up to the certain time delay. Section 2, first briefly presents the

LQR solution for time delay systems and then details of the PID tuning method for the SOPTD systems. In section 3 case study of a speed control of a DC motor is discussed followed by the conclusion.

II. MATHEMATICAL PREMINARIES FOR OBTAINING OPTIMAL LQR-PID CONTROLLER

Fig. 1 shows the step response of a typical open loop system with time delay. The delay term can be evaluated from the curve by drawing a tangent line on the inflection point in the transient region and then calculating the distance of its intersection from origin on the time axis [4].

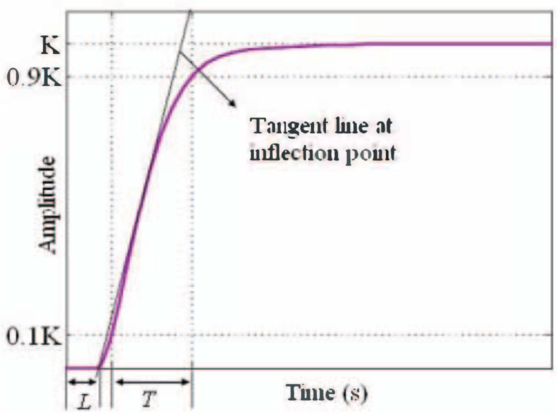

Fig. 1. Open Loop time response of the system with time delay (L)

A linear plant with time delay can be characterized by the state-space representation as

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t) + \mathbf{Bu}(t - L), \qquad t \geq 0 \tag{1}$$

where **A, B, X** and L are the corresponding state transition matrix, control matrix, state matrix and the time delay term respectively. For t < L, no control signal will be effective and only for t ≥ L the control signal comes into the picture. Equation (1) can be written as

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t), \qquad 0 \leq t < L \tag{2}$$

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t) + \mathbf{Bu}^m(t), \qquad t \geq L \tag{3}$$

where, $\mathbf{u}^m(t) = \mathbf{u}(t - L)$. Fig. 2 shows the Block diagram of the closed loop SOPTD system with PID controller. Representation of $\mathbf{X}(t)$ is given as

$$\mathbf{X}(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^T,$$

where

$$x_1(t) = \int e(t)\,dt, \; x_2(t) = e(t), \; x_3(t) = \frac{de(t)}{dt},$$

Since (3) is now delay free, one can easily apply the standard LQR approach [9] for delay free processes to find the optimum control vector **u**(t) subjected to the minimization of the cost function defined by

$$J = \int_0^\infty \left(\mathbf{X}^T(t)\,\mathbf{Q}\,\mathbf{X}(t) + \mathbf{u}^T(t)\,\mathbf{R}\,\mathbf{u}(t)\right) dt \tag{4}$$

where **Q** is the semi positive definite state weighting matrix and **R** is the positive definite control weighting matrix.

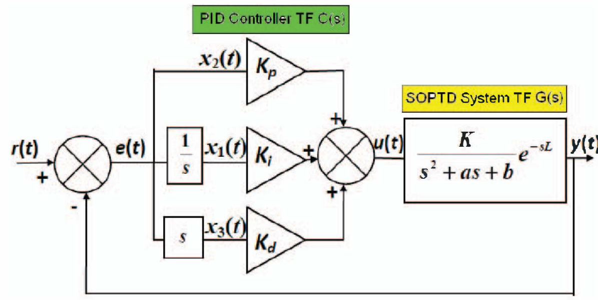

Fig. 2. Block diagram of the closed loop SOPTD system with PID controller

LQR solution to the above process gives the optimal control vector $\mathbf{u}^m$ (t) as

$$\mathbf{u}^m(t) = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{PX}(t), \qquad t \geq L \tag{5}$$

where **P** is the symmetric positive definite Riccati coefficient matrix which can be obtained by solving Continuous Algebraic Riccati Equation (CARE) given as

$$\mathbf{A}^T\mathbf{P} + \mathbf{PA} + \mathbf{Q} - \mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} = \mathbf{0} \tag{6}$$

In the case of a second order process the state weighting matrix **Q**, and the Riccati coefficient matrix **P** are generally taken as

$$\mathbf{Q} = \begin{bmatrix} Q_1 & 0 & 0 \\ 0 & Q_2 & 0 \\ 0 & 0 & Q_3 \end{bmatrix}, \; \mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{12} & p_{22} & p_{23} \\ p_{13} & p_{23} & p_{33} \end{bmatrix}. \tag{7}$$

In the optimal control it is a standard practice to design regulator by varying **Q** and keeping **R** fixed equal to (**R**=[r]=[1]) [10]. In order to find the optimal control for time delay system put $t = t + L$ in (5)

$$\mathbf{u}^m(t + L) = \mathbf{u}(t + L - L) = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{PX}(t + L)$$

For, $t + L \geq L$ which gives

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{PX}(t + L) \quad \text{for} \quad t \geq 0 \tag{8}$$

The time advance feature of state variable matrix leads to the decomposition of control vector **u**(t) in two parts one for t less than L and another for t greater than L i.e.

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}e^{(\mathbf{A}_c)t}e^{\mathbf{A}(L-t)}\mathbf{X}(t), \qquad 0 \leq t < L \tag{9}$$

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}e^{(\mathbf{A}_c)L}\mathbf{X}(t), \qquad t \geq L \tag{10}$$

where, $\mathbf{A}_c = \mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}$.

From (9) and (10) knowing the values of matrices **A**, **B** ,**P**, $e^{(\mathbf{A}_c)t}$ and $e^{\mathbf{A}(L-t)}$ one can calculate the optimal control **u**(t). It should be noted that (9) leads to the time varying control signal (coefficient of $\mathbf{X}(t)$) while (10) has constant control signal. We have not consider the control signal given by (9) in the present paper as it leads to the time varying and larger control signal initially. Large initial control signal leads to the problem of actuator saturation and time varying signal are generally difficult to implement in analog controller. So we have modified the control law and consider the same control signal initially which is valid for t greater than L. So in this paper the only control signal which is valid throughout is given as

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}e^{(\mathbf{A}_c)L}\mathbf{X}(t) \ , \qquad t \geq 0 \qquad (11)$$

So we do not have to find the values of $e^{\mathbf{A}(L-t)}$. Now we will discuss how to obtain these values. From Fig. 2 the control signal in terms of the state variable is given as

$$u(t) = K_p x_2(t) + K_i x_1(t) + K_d x_3(t) \qquad (12)$$

The transfer function (TF) of the PID controller can be express in s domain as

$$C(s) = \frac{u(s)}{e(s)} = K_p + \frac{K_i}{s} + K_d s \qquad (13)$$

In the case of unity output feedback system such as shown in Fig. 2, if we put the reference signal $r(t) = 0$, we have $e(t) = - y(t)$. With this condition, the second order TF with time delay can be written as

$$G(s) = \frac{y(s)}{u(s)} = \frac{K e^{-sL}}{s^2 + as + b} = \frac{-e(s)}{u(s)} \ , \qquad (14)$$

in which $a = 2\zeta_{ol}\omega_{ol}$ and $b = \omega_{ol}^2$, where ζ_{ol} and ω_{ol} are the damping ratio and natural frequency of the open loop plant respectively. Equation (14) can be expressed in terms of time variable as

$$\dot{x}_3(t) = -ax_3(t) - bx_2(t) - Ku(t - L) .$$

In terms of state-space formulation the derivative of the state variables can be written as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -b & -a \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -K \end{bmatrix} u(t - L) \qquad (15)$$

Comparing (15) with (1), it is straightforward to obtain matrices **A** and **B** as

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -b & -a \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ -K \end{bmatrix} \qquad (16)$$

The matrix $\mathbf{A_c}$ can be determined by setting the characteristic equation of the closed loop systems $\Delta(s) = |s\mathbf{I} - \mathbf{A_c}|$ equal to the desired closed loop equation $(s + m\zeta_{cl}\omega_{cl})(s^2 + 2s\zeta_{cl}\omega_{cl} + \omega_{cl}^2)$. Here we have utilized the help of dominant pole placement technique [4, 6] where the location of third pole is placed m times away from the real part of the dominant closed loop poles i.e. $m\zeta_{cl}\omega_{cl}$. We call this m as relative dominance and as per the published literature its value should be chosen ~ 3 or more. Matrix $\mathbf{A_c}$ can be evaluated in terms of parameters ζ_{cl},and ω_{cl} as

$$|s\mathbf{I} - \mathbf{A}_c| = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ \alpha p_{13} & b + \alpha p_{23} & s + a + \alpha p_{33} \end{vmatrix}$$
$$= (s + m\zeta_{cl}\omega_{cl})(s^2 + 2s\zeta_{cl}\omega_{cl} + \omega_{cl}^2) \qquad (17)$$

Here $\alpha = r^{-1}K^2$ By comparing the coefficients of power of s from both sides of (17), the elements p_{13} , p_{23} and p_{33} can be expressed as

$$p_{13} = \frac{m\zeta_{cl}\omega_{cl}^3}{\alpha},$$

$$p_{23} = \frac{\omega_{cl}^2 + 2m\zeta_{cl}^2\omega_{cl}^2 - b}{\alpha},$$

$$p_{33} = \frac{(2+m)\zeta_{cl}\omega_{cl} - a}{\alpha} , \qquad (18)$$

The remaining three elements of the matrix **P** and three elements of the matrix **Q** can be obtain by solving CARE equation given by (6). Results are

$$p_{11} = \frac{m\zeta_{cl}\omega_{cl}^5(1 + 2m\zeta_{cl}^2)}{\alpha},$$

$$p_{12} = \frac{(2+m)m\zeta_{cl}^2\omega_{cl}^4}{\alpha},$$

$$p_{22} = \frac{(2\omega_{cl}^3\zeta_{cl} + 4m\zeta_{cl}^3\omega_{cl}^3 + 2m^2\zeta_{cl}^3\omega_{cl}^3 - ab)}{\alpha},$$

$$Q_1 = \frac{m^2\zeta_{cl}^2\omega_{cl}^6}{\alpha},$$

$$Q_2 = \frac{\omega_{cl}^4 + 4m^2\zeta_{cl}^4\omega_{cl}^4 - b^2 - 2m^2\zeta_{cl}^2\omega_{cl}^4}{\alpha},$$

$$Q_3 = \frac{4\zeta_{cl}^2\omega_{cl}^2 + m^2\zeta_{cl}^2\omega_{cl}^2 + 2b - a^2 - 2\omega_{cl}^2}{\alpha} \tag{19}$$

Finally the value of $e^{\mathbf{A}_c L}$ can be evaluated as

$$e^{(\mathbf{A}_c)t} = \ell^{-1}\left[(s\mathbf{I} - \mathbf{A}_c)^{-1}\right]$$

$$= \begin{bmatrix} f_{11}(t) & f_{12}(t) & f_{13}(t) \\ f_{21}(t) & f_{22}(t) & f_{23}(t) \\ f_{31}(t) & f_{32}(t) & f_{33}(t) \end{bmatrix}_{t=L} \tag{20}$$

Values of $f_{ij}(L)$ are the final values of the matrix elements of $e^{(\mathbf{A}_c)t}$ at $t = L$, where $i = j = 1, 2, 3$. The optimal control $\mathbf{u}(t)$ can be obtained by using (11) as

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{P}e^{(\mathbf{A}_c)L}\mathbf{X}(t)$$

$$\mathbf{u}(t) = r^{-1}K\begin{bmatrix} p_{13} \\ p_{23} \\ p_{33} \end{bmatrix}^T \begin{bmatrix} f_{11}(L) & f_{12}(L) & f_{13}(L) \\ f_{21}(L) & f_{22}(L) & f_{23}(L) \\ f_{31}(L) & f_{32}(L) & f_{33}(L) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \tag{21}$$

Comparing the coefficients of $x_1(t)$, $x_2(t)$ and $x_3(t)$ in (21) and (12) it is straightforward to obtain the PID parameters for $t \geq 0$ as

$$K_i = r^{-1}K\left(p_{13}f_{11}(L) + p_{23}f_{21}(L) + p_{33}f_{31}(L)\right),$$
$$K_p = r^{-1}K\left(p_{13}f_{12}(L) + p_{23}f_{22}(L) + p_{33}f_{32}(L)\right),$$
$$K_d = r^{-1}K\left(p_{13}f_{13}(L) + p_{23}f_{23}(L) + p_{33}f_{33}(L)\right). \tag{22}$$

III. SPEED CONTROL OF A DC MOTOR WITH TIME DELAY VIA OPTIMAL LQR-PID CONTROLLER

In order to test the effectiveness of the proposed methodology we have taken the standard transfer function used for the speed control of a DC motor from [8]. Here authors proposed the evaluation of PID controller with user specified damping ratio ($\zeta_{cl} = 0.8$) and natural frequency ($\omega_{cl} = 3$rad/s) for a second order transfer function without time delay. With the introduction of time delay L the TF becomes

$$G(s) = \frac{\dot{\theta}}{V} = \frac{2}{s^2 + 12s + 20}e^{-sL} \tag{23}$$

Here L represents the delay due to the motor as well as from the components in the forward path. In Fig. 3 we have shown that by the introduction of time delay L with same PID parameters (K_p = 27; K_i = 60; K_d =3.3318) one could not get the desired closed loop time response as without time delay. It can be easily seen from Fig. 3 that increasing the value of time delay L the closed loop system becomes more oscillatory and finally leads to instability.

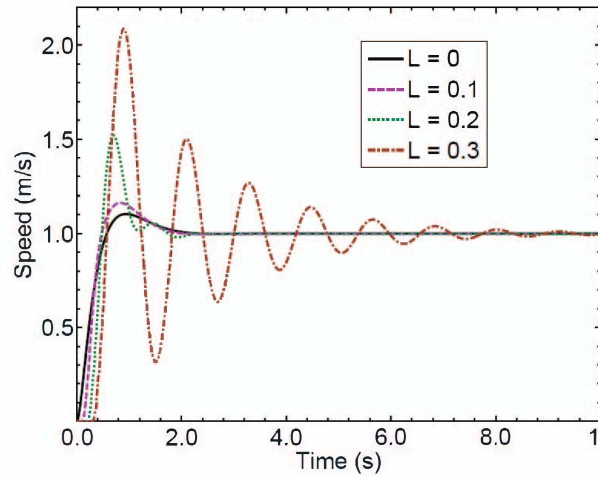

Fig. 3. Closed loop time response of the speed control of a DC motor with different L

From the inspection of (23) the roots of the system are real (-2 and -10). This implies the PID tuning for such SOPTD system can be obtained by the previous LQR-PID techniques as proposed in [5], by equating the derivative gain as one of the larger real system poles and then obtaining the K_p and K_i according to the FOPTD tuning procedure. The values of PID gains according to the previous method with same ζ_{cl} and ω_{cl} are given in Table I.

TABLE I: PID Parameters using formulation of [5]

L	K_p	K_i	K_d	ζ_{cl}	ω_{cl}
0.1s	1.1727	3.3418	10	0.8	3rad/s
0.2s	0.9488	2.388	10	0.8	3rad/s

Fig. 4 shows the closed loop time response of the speed control of a DC motor by using the un-optimized LQR-PID settings keeping K_d = 10 for different time delay L. It can be easily seen that at L = 0.1s, although the system is stable but it does not fulfill the required closed loop performance measures. At L = 0.2s the closed loop response becomes unstable. The gain margin (GM) and phase margin (PM) for L =0.1s is GM = 3.15dB and PM = 28.34deg and for L =0.2s these are negative (unstable response).

Observing Fig. 4, it could be concluded that it is not always possible to obtain the PID gains for SOPTD system by equating the larger process pole with the derivative term of the PID controller and then applying the PI tuning approach using LQR to obtain other two parameters. Fig. 5 shows the zoom view of Fig. 4 so that the existence of time delay L can be seen clearly. In Fig. 6 we have plotted the close loop time response obtained using the optimized LQR-

PID settings following the present methodology for the SOPTD system of DC motor speed control at different values of time delay *L*.

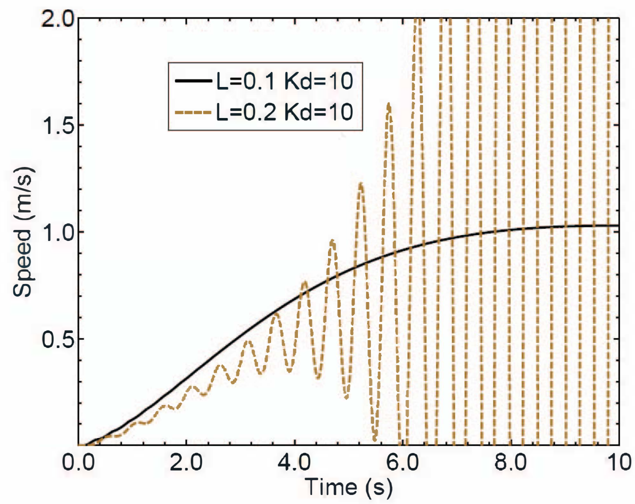

Fig. 4. Closed loop time response of the speed control of a DC motor with time delay following calculation of PID using [5].

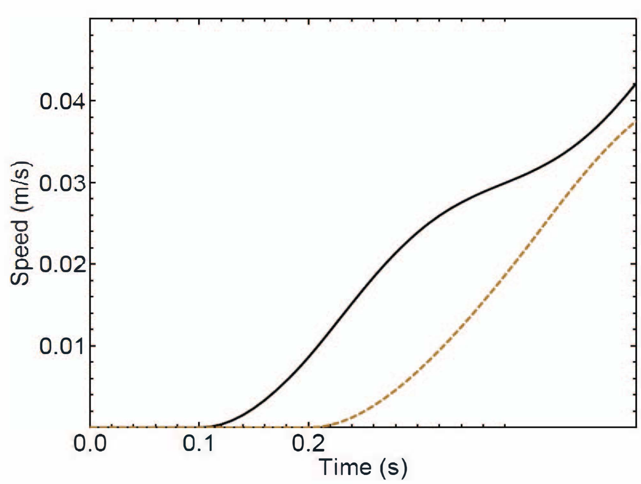

Fig. 5. Zoom view of Fig. 4 for clear visibility of time delay *L*.

From the simulation results presented in Fig. 6, it is easy to conclude that the present methodology gives much better closed loop time response as compared to the previously developed LQR-PID present system. The reason for this improvement lies in the fact that for each value of time delay a new set of optimized PID parameters is needed in contrast to the previous method where K_d is fixed for any value of *L*. The values of optimized PID settings with same ζ_{cl} and ω_{cl} are given in Table II for comparison where for different values of time deay *L* one gets different K_d along with the optimized sets of K_p and K_i.

TABLE II: PID parameters with present method

L	K_p	K_i	K_d	ζ_{cl}	ω_{cl}
0.1s	14.8576	33.0225	1.1287	0.8	3rad/s
0.2s	11.9241	23.9702	0.9588	0.8	3rad/s
0.3s	9.1788	16.5105	0.7681	0.8	3rad/s

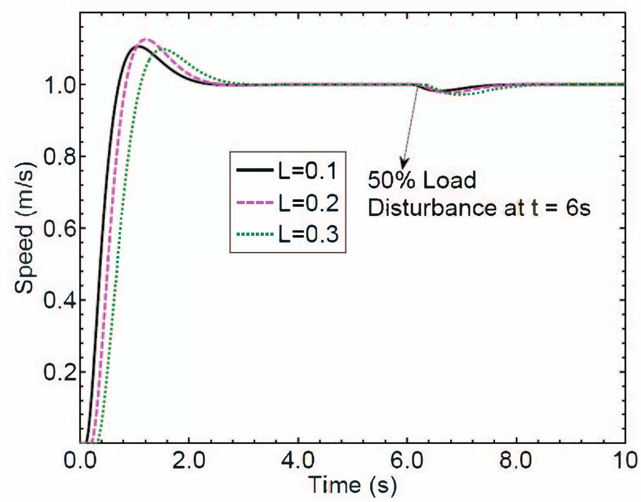

Fig. 6. Closed loop time response of the speed control of a DC motor with time delay using present methodology.

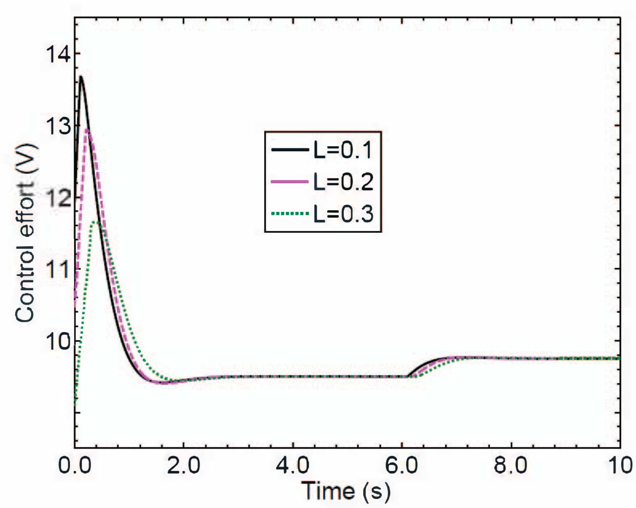

Fig. 7. LQR-PID Controller Response using present method.

We like to point out that above PID settings are evaluated at a fixed value of third pole location by considering $m = 4$. However, one can further improve the close loop time response by precisely tuning the value of *m*. From Fig. 6 one can easily observe that for three different values of time delay *L*, the percentage overshoot and settling time are almost identical and are very close to the required closed loop performance measures ($\zeta_{cl} = 0.8$) and ($\omega_{cl} =$ 3rad/s). Fig. 7 shows the LQR-PID controller response obtained using present method for the DC motor speed control at different values of time delay *L*. Here we see that as the time delay *L* increases we need less control effort. This advantage can be exploited in cases where there is limitation in the actuator saturation. One can design a PID controller intentionally by introducing some time delay in the forward path. However, this way one can slightly increase the rise time.

TABLE III: GM and PM obtained with present method.

L	GM(dB)	PM(deg)
0.1s	37.51	64.18
0.2s	27.15	58.88
0.3s	24.35	60.13

Table III gives the numerical values of the robustness measures of the closed loop control system. One can easily distinguish that with the present tuning method for time delay L =0.1s the GM are almost 10 times more while the PM are more than twice than the previous LQR-PID method (for L =0.1s: GM = 3.15dB and PM = 28.34deg).

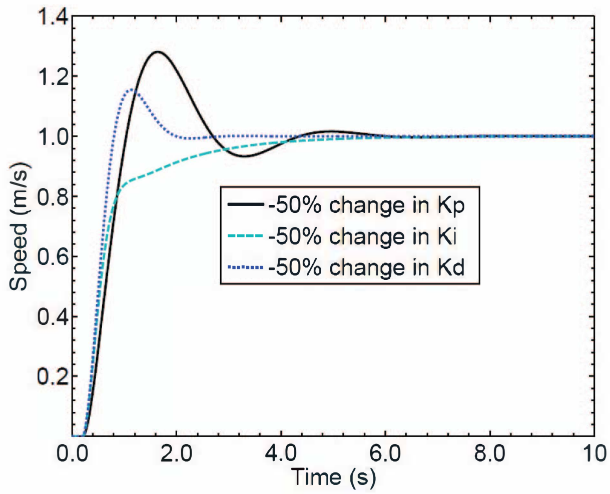

Fig. 8. Closed loop time response when the values of PID controller parameters are decreased by 50%.

In order to check the robustness of the PID controller we perturbed the PID controller parameters (K_p , K_i and K_d) by ± 50% from the actual values and performed simulation to check the behaviour of closed loop time response. Fig. 8 shows the closed loop time response when the values of PID controller parameters are decreased by 50% with same DC motor speed control TF at L=0.2s. It can be seen that a decrease in proportional term causes an extra larger overshoot where as a decrease in the value of K_i affects the settling time. Fig. 9 shows the closed loop time response when the values of PID controller parameters are increased by 50%. In this case integral term produces more overshoot and also affects settling time. It is easy to observe that in both the cases the closed loop time response is more or less unaffected by the change in value K_d.

IV. CONCLUSION

In the present paper we have developed an LQR based tuning method of PID controller for SOPTD process utilizing the pole placement techniques. To demonstrate the effectiveness of present method we have performed simulations considering the example of a speed control of DC motor with time delay. It is shown that present method gives much improved results than the previous LQR based PID tuning method where the derivative term is taken as one of the real pole of the system [5].

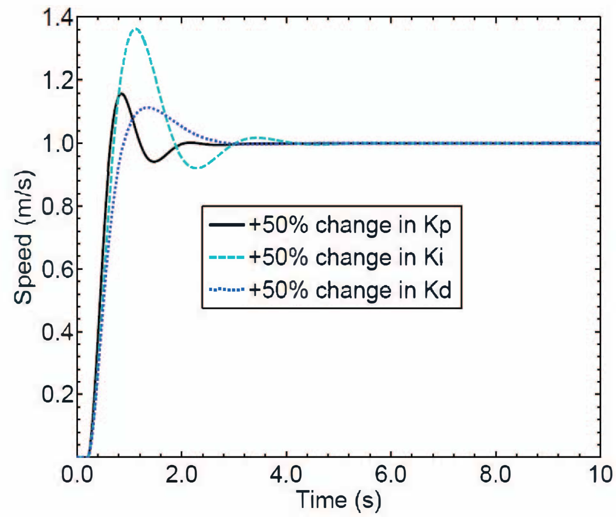

Fig. 9. Closed loop time response when the values of PID controller parameters are increased by 50%.

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