Linear Regression

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1 Linear Regression

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Note: Based on the data provided, an accurate hypothesis should be $h_\theta(x^{(i)}) = X\theta = 1.5 + 2x_1^{(i)} + x_2^{(i)}$

$$\theta = \begin{pmatrix} 1.5 \\ 2 \\ 1 \end{pmatrix}$$

[1]:
```
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt

%matplotlib inline
```

1.1 Finding the parameters of hypothesis using gradient descent

1.1.1 Handling the data from the csv file into arrays I can work with

[2]:
```
# loading the data from the csv file
raw_data = pd.read_csv('data.csv')

m = len(raw_data) # number of training examples
n = 2 # number of features

# assuming the hypothesis is of the form X*theta where X is an m by n matrix␣
↪and theta, an n by 1 matrix
feature1 = np.array(raw_data['X']) #x1
feature2 = np.array(raw_data['Y']) #x2

X = np.array([
    np.ones(m),     # x0 is 1
    feature1,
    feature2
]).transpose()

Y = np.array([[z] for z in np.array(raw_data['Z'])]) # correct results
```

```python
# initializing theta as an n by 1 matrix made up of only ones
theta = np.array([[1] for _ in range(3)])
```

What is X? X is the design matrix of m rows and $(n+1)$ columns. An $m \times (n+1)$ matrix with each row representing a training example of the dataset.

[3]:
```python
X[:5] # first 5 rows of X
```

[3]:
```
array([[1., 2., 3.],
       [1., 1., 2.],
       [1., 4., 1.],
       [1., 3., 2.],
       [1., 2., 1.]])
```

What is Y? Y is an m dimensional vector. An $m \times 1$ matrix with each element corresponding to the expected result of a specific training example of the dataset.

[4]:
```python
Y[:5] # first 5 elements of Y
```

[4]:
```
array([[ 8.5],
       [ 5.5],
       [10.5],
       [ 9.5],
       [ 6.5]])
```

1.1.2 The Hypothesis - $h_\theta(x^{(i)})$

Instead of taking each feature, $x_n^{(i)}$, one at a time in a training example to multiply with its corresponding θ_j, a more conservative way to do this is to vectorize the θ. Making θ an $(n+1)$ dimensional vector, with the first element being the constant term in the expression $h_\theta(x^{(i)}) = \theta_0 + \theta_1 x_1^{(i)} + \theta_2 x_2^{(i)} + ... + \theta_n x_n^{(i)}$ and the rest of the elements being the coefficients of $x_1^{(i)}, x_2^{(i)}, ..., x_n^{(i)}$ where n is the number of features of each training example. Multiplying the design matrix, X (an $m \times (n+1)$ matrix), by θ will produce an m dimensional vector(an $m \times 1$ matrix). This vector contains the predictions of the hypothesis.

$$h_\theta(x^{(i)}) = X\theta$$

[5]:
```python
def hypothesis():
    return X @ theta # matrix product of X and theta
```

After some trials, a learning rate of 0.01 is enough for the gradient descent.

[6]:
```python
# the learning rate
alpha = 0.01
```

1.1.3 The Cost Function for Linear Regression

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$

[7]:
```
def cost():
    return (1/(2*m))*(((hypothesis() - Y) ** 2).sum())
```

1.1.4 The Gradient Descent

$$\theta_j := \theta_j - \alpha\frac{\partial}{\partial\theta_j}J(\theta)$$

But since I have decided to vectorize the θ, I can think of the gradient descent as:

$$\theta := \theta - \alpha\delta$$

where δ is representing the partial derivative of each of elements of θ. δ will be an $(n+1)$ dimensional vector given by:

$$\delta = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x^{(i)}$$

The function in the cell below does the update of the theta as it should be done - simultaneously.

[8]:
```
def new_theta():
    x0 = np.array([[u] for u in X.transpose()[0]])
    x1 = np.array([[u] for u in X.transpose()[1]])
    x2 = np.array([[u] for u in X.transpose()[2]])
    # computing the elements of the delta vector
    delta0 = (1/m)*(((hypothesis() - Y)*x0).sum())
    delta1 = (1/m)*(((hypothesis() - Y)*x1).sum())
    delta2 = (1/m)*(((hypothesis() - Y)*x2).sum())
    # computing the new values for the elements of theta
    return np.array([
        [theta[0, 0] - (alpha*delta0)],
        [theta[1, 0] - (alpha*delta1)],
        [theta[2, 0] - (alpha*delta2)]
    ])
```

A while loop to update the theta and keep the gradient descent going until the cost is minimal

[9]:
```
costs = np.array([])
number_of_steps = 0
while True:
    # keeping track of the number of steps and the gradient descent at each step
    # this is to help plot a graph of cost against number of steps
    # to show that the gradient descent actually worked
    number_of_steps += 1
    costs = np.append(costs, cost())
```

```
    # calculation of accuracy of model in terms of MSE
    mean_sq_error = 2*cost()
    # making sure the cost is minimun
    if cost() < 10**(-20):
        print("Success.")
        break
    # updating the theta simultaneously
    theta = new_theta()
```

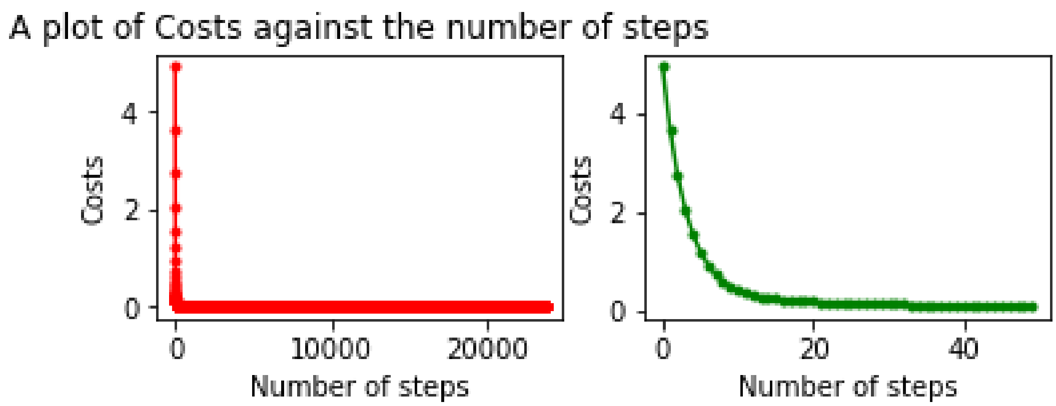
```
Success.
```

[10]:
```
# printing out results on the screen
print("Results\n----------------")
print(f"Theta:\n {theta}\nMean Squared Error(MSE): {mean_sq_error: .15f}")
```

```
Results
----------------
Theta:
 [[1.5]
 [2. ]
 [1. ]]
Mean Squared Error(MSE):  0.000000000000000
```

[11]:
```
# This is a plot of costs against number of steps taken in gradient descent.
# The nature of the graph determines the success of the gradient descent.
plt.subplot(221)
plt.plot(range(number_of_steps), costs, '.-r')
plt.xlabel('Number of steps')
plt.ylabel('Costs')
plt.title('A plot of Costs against the number of steps')
plt.subplot(222)
plt.plot(range(50), costs[:50], '.-g')
plt.xlabel('Number of steps')
plt.ylabel('Costs')
```

[11]:
```
Text(0, 0.5, 'Costs')
```



1.2 Finding the parameters of hypothesis using the Normal Equation

1.2.1 Handling the data from the csv file into arrays I can work with

[12]:
```
data = pd.read_csv('data.csv')
x_feature = np.array(data['X'])
y_feature = np.array(data['Y'])

Y = np.array([[z] for z in data['Z']])

X = np.array([
    np.ones(len(x_feature)),
    x_feature,
    y_feature
]).transpose()
```

What is X? X is the design matrix of m rows and $(n+1)$ columns. An $m \times (n+1)$ matrix with each row representing a training example of the dataset.

[13]:
```
X[:3] # first 3 rows of X
```

[13]:
```
array([[1., 2., 3.],
       [1., 1., 2.],
       [1., 4., 1.]])
```

What is Y? Y is an m dimensional vector. An $m \times 1$ matrix with each element corresponding to the expected result of a specific training example of the dataset.

[14]:
```
Y[:3] # first 3 elements of Y
```

[14]:
```
array([[ 8.5],
       [ 5.5],
```

```
        [10.5]])
```

1.2.2 The normal equation

The normal equation is given by:
$$\theta = (X^T X)^{-1} X^T Y$$

[15]:
```
theta = np.linalg.inv(X.transpose() @ X) @ X.transpose() @ Y
```

[16]:
```
# printing out results on the screen
print("Results\n----------------")
print(f"Theta:\n {theta}")
```

```
Results
----------------
Theta:
 [[1.5]
 [2. ]
 [1. ]]
```

1.3 The Data used for the aboves codes

[17]:
```
Data = pd.read_csv('data.csv')
```

[18]:
```
Data.shape
```

[18]:
```
(15, 3)
```

[19]:
```
Data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 15 entries, 0 to 14
Data columns (total 3 columns):
 #   Column  Non-Null Count  Dtype
---  ------  --------------  -----
 0   X       15 non-null     int64
 1   Y       15 non-null     int64
 2   Z       15 non-null     float64
dtypes: float64(1), int64(2)
memory usage: 488.0 bytes
```

[20]:
```
Data.describe()
```

[20]:

	X	Y	Z
count	15.000000	15.000000	15.000000
mean	2.466667	2.533333	8.966667
std	1.060099	1.187234	2.503331

```
min     1.000000   1.000000   4.500000
25%     2.000000   1.500000   7.500000
50%     2.000000   3.000000   8.500000
75%     3.000000   3.500000  10.500000
max     4.000000   4.000000  13.500000
```

1.3.1 The full data

[21]:
```
Data
```

[21]:
```
    X  Y     Z
0   2  3   8.5
1   1  2   5.5
2   4  1  10.5
3   3  2   9.5
4   2  1   6.5
5   2  2   7.5
6   1  4   7.5
7   3  3  10.5
8   1  1   4.5
9   3  4  11.5
10  4  4  13.5
11  3  1   8.5
12  2  3   8.5
13  2  4   9.5
14  4  3  12.5
```