

## 2nd order ODE solver

Oscillatory motions are represented by 2nd order differential equations,

$$\begin{aligned}\frac{dx}{dt} &= f(t, x, y) \\ \frac{dy}{dt} &= g(t, x, y).\end{aligned}\tag{1}$$

To solve the 2-dimensional equation, we need to implement the Runge-Kutta Method for 2-dimensional systems. Here is the Runge-Kutta iteration for 2-dimensional system,

$$\begin{aligned}k_1 &= f(t_n, x_n, y_n) \\ l_1 &= g(t_n, x_n, y_n) \\ k_2 &= f(t_n + \frac{1}{2}h, x_n + \frac{1}{2}hk_1, y_n + \frac{1}{2}hl_1) \\ l_2 &= g(t_n + \frac{1}{2}h, x_n + \frac{1}{2}hk_1, y_n + \frac{1}{2}hl_1) \\ k_3 &= f(t_n + \frac{1}{2}h, x_n + \frac{1}{2}hk_2, y_n + \frac{1}{2}hl_2) \\ l_3 &= g(t_n + \frac{1}{2}h, x_n + \frac{1}{2}hk_2, y_n + \frac{1}{2}hl_2) \\ k_4 &= f(t_n + h, x_n + hk_3, y_n + hl_3) \\ l_4 &= g(t_n + h, x_n + hk_3, y_n + hl_3) \\ k &= \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\ l &= \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) \\ x_{n+1} &= x_n + hk \\ y_{n+1} &= y_n + hl \\ t_{n+1} &= t_n + h.\end{aligned}\tag{2}$$

Based on the above Runge-Kutta method, solve

$$\begin{aligned}\dot{x} &= \mu x - y + xy^2 \\ \dot{y} &= x + \mu y + y^3\end{aligned}\tag{3}$$

with  $\mu = -1.0, -0.5, -0.2, 0.0, 0.1, 0.5, 1.0$  and your initial conditions (choose several). Draw your solutions in (x,y) domain with several paths from the above initial conditions. Explain how your solutions change with various  $\mu$ s. Discuss about Hopf bifurcation.