QMM_Assign5

#Problem 1 [DEA]

Problem Statement: The Hope Valley Health Care Association owns and operates six nursing homes in adjoining states. An evaluation of their efficiency has been undertaken using two inputs and two outputs. The inputs are staffing labor (measured in average hours per day) and the cost of supplies (in thousands of dollars per day). The outputs are the number of patient-days reimbursed by third-party sources and the number of patient-days reimbursed privately.

Formulating the problem as lp to get the weight for Facility 1

LP formulation:

// Objective Function max: 14000 u1 + 3500 u2;

/* Constraints */

14000 u1 + 3500 u2 - 150 v1 - 0.2 v2 <= 0;

14000 u1 + 21000 u2 - 400 v1 - 0.7 v2 <= 0;

42000 u1 + 10500 u2 - 320 v1 - 1.2 v2 <= 0;

28000 u1 + 42000 u2 - 520 v1 - 2.0 v2 <= 0;

19000 u1 + 25000 u2 - 350 v1 - 1.2 v2 <= 0;

14000 u1 + 15000 u2 - 320 v1 - 0.7 v2 <= 0;

150 v1 + 0.2 v2 = 1;

==========

```
library(Benchmarking)
```

```
## Loading required package: lpSolveAPI
```

```
## Loading required package: ucminf
```

```
## Loading required package: quadprog
```

```
library(lpSolveAPI)
Facility1 <- read.lp("Health.lp")
```

Questions

1) Formulate and perform DEA analysis under all DEA assumptions of FDH, CRS, VRS, IRS, DRS, and FRH.
2) Determine the Peers and Lambdas under each of the above assumptions
3) Summarize your results in a tabular format
4) Compare and contrast the above results

```
solve(Facility1)

## [1] 0

get.objective(Facility1)     #the lp was able to achieve the max efficiency
for Facility 1

## [1] 1

get.variables(Facility1)    #The proposed inputs and outputs weights for
maximum efficiency

## [1] 7.142857e-05 0.000000e+00 5.172414e-03 1.120690e+00
```

First we type our inputs and outputs as vectors.

We have 2 inputs (Staff hours, Supplies) and 2 outputs ("Reimbursed Patient_Days","Privately Paid Patient_Day) .

```
x <- matrix(c(150, 400, 320, 520, 350, 320, 0.2, 0.7, 1.2, 2.0, 1.2, 0.7),
ncol = 2)
y <-
matrix(c(14000,14000,42000,28000,19000,14000,3500,21000,10500,42000,25000,150
00),ncol = 2)
colnames(y) <- c("Reimbursed Patient_Days","Privately Paid Patient_Days")
colnames(x) <- c("Staff_Hours", "Supplies")
print(x)

##      Staff_Hours Supplies
## [1,]         150      0.2
## [2,]         400      0.7
## [3,]         320      1.2
## [4,]         520      2.0
## [5,]         350      1.2
## [6,]         320      0.7

print(y)

##      Reimbursed Patient_Days Privately Paid Patient_Days
## [1,]                   14000                        3500
## [2,]                   14000                       21000
## [3,]                   42000                       10500
## [4,]                   28000                       42000
## [5,]                   19000                       25000
## [6,]                   14000                       15000
```

```
Table<- cbind(x,y)
row.names(Table) =  c("Fac1", "Fac2", "Fac3", "Fac4", "Fac5", "Fac6")
Table
```

```
##      Staff_Hours Supplies Reimbursed Patient_Days Privately Paid
Patient_Days
## Fac1         150      0.2                  14000
3500
## Fac2         400      0.7                  14000
21000
## Fac3         320      1.2                  42000
10500
## Fac4         520      2.0                  28000
42000
## Fac5         350      1.2                  19000
25000
## Fac6         320      0.7                  14000
15000
```

Next we run DEA Analysis under all DEA assumptions (FDH, CRS, VRS, IRS, DRS, and FRH)

```
#Constant returns to scale, convexity and free disposability
CRS <- dea(x,y, RTS = "crs") # provide the input and output . The results
show that Facilities 1,2,3,4 are efficient whereas facilities 5,6 have
efficiency rates of 98% and 87% respectively.
print(CRS)
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675
```

```
peers(CRS)    # identify the peers. The peers units for for facilities 5,6
are 1,2,4
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     1     2     4
## [6,]     1     2     4
```

```
CRS_Weights <- lambda(CRS)    # identify the relative weights given to the
peers. The weights for facility 4 are 0.20, 0.08, 0.54. The weights for
facility 6 are 0.34, 0.39, 0.13
#Free disposability hull

FDH <- dea(x,y, RTS= "fdh")
FDH  #all facilities are efficient
```

```
## [1] 1 1 1 1 1 1
```

```
peers(FDH)  #the peer for each facility is itself
```

```
##      peer1
## [1,]     1
## [2,]     2
## [3,]     3
## [4,]     4
## [5,]     5
## [6,]     6
```

```
FDH_Weights <- lambda(FDH)
#Variable returns to scale, convexity and free disposability
VRS <- dea(x,y, RTS = "vrs")
VRS    #All facilities are efficient except for facility 6
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
```

```
peers(VRS)   #peers for facility 6 are 1,2,5
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     5    NA    NA
## [6,]     1     2     5
```

```
VRS_Weights <- lambda(VRS)
#Increasing returns to scale, (up-scaling, but not down-scaling), convexity
and free disposability
IRS <- dea(x,y, RTS= "irs")
IRS    #All facilities are efficient except for facilit
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
```

```
peers(IRS)    #peers for facility 6 are 1,2,5
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     5    NA    NA
## [6,]     1     2     5
```

```
IRS_Weights <- lambda(IRS)
#Decreasing returns to scale, convexity, down-scaling and free disposability
DRS <- dea(x,y, RTS= "drs")  #DRS gave same results as CRS
DRS     #All facilities are efficient except for facility 5,6
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675
```

```
peers(DRS)  # The peers units for for facilities 5,6 are 1,2,4
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     1     2     4
## [6,]     1     2     4
```

```
DRS_Weights <- lambda(DRS)
```

```
FRH <- dea(x,y, RTS= "add")
FRH  #all facilities are efficient
```

```
## [1] 1 1 1 1 1 1
```

```
peers(FRH)    #the peer unit for each facility is itself
```

```
##      peer1
## [1,]     1
## [2,]     2
## [3,]     3
## [4,]     4
## [5,]     5
## [6,]     6
```

```
FRH_Weights <- lambda(FRH)
```

```
as.data.frame(Table)
```

```
##      Staff_Hours Supplies Reimbursed Patient_Days Privately Paid
Patient_Days
## Fac1         150      0.2                  14000
3500
## Fac2         400      0.7                  14000
21000
## Fac3         320      1.2                  42000
10500
## Fac4         520      2.0                  28000
42000
## Fac5         350      1.2                  19000
25000
## Fac6         320      0.7                  14000
15000
```

```
Df <-data.frame (CRS = c(1.0000, 1.0000, 1.0000, 1.0000, 0.9775, 0.8675),
FDH= c(1,1,1,1,1,1), VRS= c(1.0000, 1.0000, 1.0000, 1.0000, 1.0000,
0.8963),IRS =c( 1.0000, 1.0000, 1.0000, 1.0000, 1.0000 ,0.8963), DRS=
c(1.0000 ,1.0000 ,1.0000, 1.0000, 0.9775, 0.8675), FRH= c(1,1,1,1,1,1))
Df
```

```
##      CRS FDH    VRS    IRS    DRS FRH
## 1 1.0000   1 1.0000 1.0000 1.0000   1
```

```
## 2 1.0000   1 1.0000 1.0000 1.0000   1
## 3 1.0000   1 1.0000 1.0000 1.0000   1
## 4 1.0000   1 1.0000 1.0000 1.0000   1
## 5 0.9775   1 1.0000 1.0000 0.9775   1
## 6 0.8675   1 0.8963 0.8963 0.8675   1
```

```
#Now we look at the efficiency results at each facility in every DEA
assumption. CRS and DRS give same results, FDH and FRH gave same results, and
finally both VRS and IRS gave same results as well.
```

```
Results <- cbind(Table,Df)
Results[,-c(1:4)]
```

```
##         CRS FDH    VRS    IRS    DRS FRH
## Fac1 1.0000   1 1.0000 1.0000 1.0000   1
## Fac2 1.0000   1 1.0000 1.0000 1.0000   1
## Fac3 1.0000   1 1.0000 1.0000 1.0000   1
## Fac4 1.0000   1 1.0000 1.0000 1.0000   1
## Fac5 0.9775   1 1.0000 1.0000 0.9775   1
## Fac6 0.8675   1 0.8963 0.8963 0.8675   1
```

```
#Summary of the weights assigned to each Facility in every DEA assumption
```

```
Weights_tbl <- cbind(FDH_Weights, CRS_Weights, VRS_Weights, IRS_Weights,
DRS_Weights, FRH_Weights)
```

```
row.names(Weights_tbl) =   c("Fac1", "Fac2", "Fac3", "Fac4", "Fac5", "Fac6")
```

```
colnames(Weights_tbl) <- c("FDH","FDH", "FDH", "FDH","FDH","FDH", "CRS",
"CRS", "CRS", "CRS", "VRS", "VRS", "VRS","VRS", "VRS", "IRS", "IRS", "IRS",
"IRS","IRS", "DRS", "DRS", "DRS", "DRS", "FRH", "FRH", "FRH", "FRH",
"FRH","FRH")
```

```
as.data.frame(Weights_tbl)   #the table summarizes  the weights for inputs
and outputs for  each facility under each DEA assumption.
```

```
##      FDH FDH FDH FDH FDH FDH       CRS        CRS CRS       CRS       VRS
## Fac1   1   0   0   0   0   0 1.0000000 0.00000000   0 0.0000000 1.0000000
## Fac2   0   1   0   0   0   0 0.0000000 1.00000000   0 0.0000000 0.0000000
## Fac3   0   0   1   0   0   0 0.0000000 0.00000000   1 0.0000000 0.0000000
## Fac4   0   0   0   1   0   0 0.0000000 0.00000000   0 1.0000000 0.0000000
## Fac5   0   0   0   0   1   0 0.2000000 0.08048142   0 0.5383307 0.0000000
## Fac6   0   0   0   0   0   1 0.3428571 0.39499264   0 0.1310751 0.4014399
##           VRS VRS VRS       VRS       IRS       IRS IRS IRS       IRS
## Fac1 0.0000000   0   0 0.0000000 1.0000000 0.0000000   0   0 0.0000000
## Fac2 1.0000000   0   0 0.0000000 0.0000000 1.0000000   0   0 0.0000000
## Fac3 0.0000000   1   0 0.0000000 0.0000000 0.0000000   1   0 0.0000000
## Fac4 0.0000000   0   1 0.0000000 0.0000000 0.0000000   0   1 0.0000000
## Fac5 0.0000000   0   0 1.0000000 0.0000000 0.0000000   0   0 1.0000000
## Fac6 0.3422606   0   0 0.2562995 0.4014399 0.3422606   0   0 0.2562995
##           DRS        DRS DRS       DRS FRH FRH FRH FRH FRH FRH
```

```
## Fac1 1.0000000 0.00000000   0 0.0000000   1   0   0   0   0   0
## Fac2 0.0000000 1.00000000   0 0.0000000   0   1   0   0   0   0
## Fac3 0.0000000 0.00000000   1 0.0000000   0   0   1   0   0   0
## Fac4 0.0000000 0.00000000   0 1.0000000   0   0   0   1   0   0
## Fac5 0.2000000 0.08048142   0 0.5383307   0   0   0   0   1   0
## Fac6 0.3428571 0.39499264   0 0.1310751   0   0   0   0   0   1
```

DEA Analysis Summary

For Hope Vally Health Care Association: Under FDH and FRH all facilities are efficient, under CRS and DRS all facilities were efficient except for Facility 5,6. Under VRS and IRS assumptions all except for facility 6 were efficient. The peer units for efficient facilities are themselves. Under VRS and IRS assumption the peers unit for inefficient facilities were 1,2 and 5. Under CRS and DRS, the peers unites were 1,2,and 4.

Problem 2 [Goal Programming]

Emax Corporation Problem:

Maximize Z = P - 6C - 3D, where P = total (discounted) profit over the life of the new products,

C = change (in either direction) in the current level of employment,

D = decrease (if any) in next year's earnings from the current year's level.

– P= 20 x1 + 15 x2 + 25 x3;

– y1= 6 x1 + 4 x2 + 5 x3 - 50;

– y2= 8 x1 + 7 x2 +5 x3 - 75

y1+ is going over the employment level goal and the weighted penality is 6

y1- is going under the employment level goal and the weighted penality is 6

y2+ is going over the earnings goal for next year- no penality

y2- is going under the earnings goal for next year and the peanlity is 3.

x1 is the quantity of product 1 to be produced

x2 is the quantity of product 2 to be produced

x3 is the quantity of product 3 to be produced

LP formulation:

// Objective function max: 20x1 + 15x2 + 25x3 - 6 y1p - 6 y1m - 3 y2m;

// Constraints

6x1 + 4x2 + 5x3 + y1p - y1m = 50;

8x1 + 7x2 + 5x3 + y2p - y2m = 75;

==========

```
library(lpSolveAPI)
Emax_GP <- read.lp("Emax.lp")
Emax_GP

## Model name:
##             x1    x2    x3   y1p   y1m   y2m   y2p
## Maximize    20    15    25    -6    -6    -3     0
## R1           6     4     5    -1     1     0     0  =  50
## R2           8     7     5     0     0     1    -1  =  75
## Kind       Std   Std   Std   Std   Std   Std   Std
## Type      Real  Real  Real  Real  Real  Real  Real
## Upper      Inf   Inf   Inf   Inf   Inf   Inf   Inf
## Lower        0     0     0     0     0     0     0
```

Solving the problem

```
solve (Emax_GP)

## [1] 0

get.objective(Emax_GP)

## [1] 225
```

Emax need to produce 15 units of product 3 and none of product 1 and 2 to achieve 225 millions in profit. The employment level will go over the goal by 2500

```
get.variables(Emax_GP)

## [1]  0  0 15 25  0  0  0
```