QMM\_Assign5

# #Problem 1 [DEA]

**Problem Statement:** The Hope Valley Health Care Association owns and operates six nursing homes in adjoining states. An evaluation of their efficiency has been undertaken using two inputs and two outputs. The inputs are staffing labor (measured in average hours per day) and the cost of supplies (in thousands of dollars per day). The outputs are the number of patient-days reimbursed by third-party sources and the number of patient-days reimbursed privately.

Formulating the problem as lp to get the weight for Facility 1

# LP formulation:

// Objective Function max: 14000 u1 + 3500 u2;

/\* Constraints \*/

14000 u1 + 3500 u2 - 150 v1 - 0.2 v2 <= 0;

14000 u1 + 21000 u2 - 400 v1 - 0.7 v2 <= 0;

42000 u1 + 10500 u2 - 320 v1 - 1.2 v2 <= 0;

28000 u1 + 42000 u2 - 520 v1 - 2.0 v2 <= 0;

19000 u1 + 25000 u2 - 350 v1 - 1.2 v2 <= 0;

14000 u1 + 15000 u2 - 320 v1 - 0.7 v2 <= 0;

150 v1 + 0.2 v2 = 1;

===================================================================

library(Benchmarking)

## Loading required package: lpSolveAPI

## Loading required package: ucminf

## Loading required package: quadprog

library(lpSolveAPI)  
Facility1 <- read.lp("Health.lp")

# **Questions**

1. Formulate and perform DEA analysis under all DEA assumptions of FDH, CRS, VRS, IRS, DRS, and FRH.
2. Determine the Peers and Lambdas under each of the above assumptions
3. Summarize your results in a tabular format
4. Compare and contrast the above results

solve(Facility1)

## [1] 0

get.objective(Facility1) #the lp was able to achieve the max efficiency for Facility 1

## [1] 1

get.variables(Facility1) #The proposed inputs and outputs weights for maximum efficiency

## [1] 7.142857e-05 0.000000e+00 5.172414e-03 1.120690e+00

First we type our inputs and outputs as vectors.

We have 2 inputs (Staff hours, Supplies) and 2 outputs (“Reimbursed Patient\_Days”,"Privately Paid Patient\_Day) .

x <- matrix(c(150, 400, 320, 520, 350, 320, 0.2, 0.7, 1.2, 2.0, 1.2, 0.7), ncol = 2)  
y <- matrix(c(14000,14000,42000,28000,19000,14000,3500,21000,10500,42000,25000,15000),ncol = 2)  
colnames(y) <- c("Reimbursed Patient\_Days","Privately Paid Patient\_Days")  
colnames(x) <- c("Staff\_Hours", "Supplies")  
print(x)

## Staff\_Hours Supplies  
## [1,] 150 0.2  
## [2,] 400 0.7  
## [3,] 320 1.2  
## [4,] 520 2.0  
## [5,] 350 1.2  
## [6,] 320 0.7

print(y)

## Reimbursed Patient\_Days Privately Paid Patient\_Days  
## [1,] 14000 3500  
## [2,] 14000 21000  
## [3,] 42000 10500  
## [4,] 28000 42000  
## [5,] 19000 25000  
## [6,] 14000 15000

Table<- cbind(x,y)  
row.names(Table) = c("Fac1", "Fac2", "Fac3", "Fac4", "Fac5", "Fac6")  
Table

## Staff\_Hours Supplies Reimbursed Patient\_Days Privately Paid Patient\_Days  
## Fac1 150 0.2 14000 3500  
## Fac2 400 0.7 14000 21000  
## Fac3 320 1.2 42000 10500  
## Fac4 520 2.0 28000 42000  
## Fac5 350 1.2 19000 25000  
## Fac6 320 0.7 14000 15000

Next we run DEA Analysis under all DEA assumptions (FDH, CRS, VRS, IRS, DRS, and FRH)

#Constant returns to scale, convexity and free disposability  
CRS <- dea(x,y, RTS = "crs") # provide the input and output . The results show that Facilities 1,2,3,4 are efficient whereas facilities 5,6 have efficiency rates of 98% and 87% respectively.   
print(CRS)

## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675

peers(CRS) # identify the peers. The peers units for for facilities 5,6 are 1,2,4

## peer1 peer2 peer3  
## [1,] 1 NA NA  
## [2,] 2 NA NA  
## [3,] 3 NA NA  
## [4,] 4 NA NA  
## [5,] 1 2 4  
## [6,] 1 2 4

CRS\_Weights <- lambda(CRS) # identify the relative weights given to the peers. The weights for facility 4 are 0.20, 0.08, 0.54. The weights for facility 6 are 0.34, 0.39, 0.13  
#Free disposability hull  
  
FDH <- dea(x,y, RTS= "fdh")  
FDH #all facilities are efficient

## [1] 1 1 1 1 1 1

peers(FDH) #the peer for each facility is itself

## peer1  
## [1,] 1  
## [2,] 2  
## [3,] 3  
## [4,] 4  
## [5,] 5  
## [6,] 6

FDH\_Weights <- lambda(FDH)  
#Variable returns to scale, convexity and free disposability  
VRS <- dea(x,y, RTS = "vrs")  
VRS #All facilities are efficient except for facility 6

## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963

peers(VRS) #peers for facility 6 are 1,2,5

## peer1 peer2 peer3  
## [1,] 1 NA NA  
## [2,] 2 NA NA  
## [3,] 3 NA NA  
## [4,] 4 NA NA  
## [5,] 5 NA NA  
## [6,] 1 2 5

VRS\_Weights <- lambda(VRS)  
#Increasing returns to scale, (up-scaling, but not down-scaling), convexity and free disposability  
IRS <- dea(x,y, RTS= "irs")  
IRS #All facilities are efficient except for facilit

## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963

peers(IRS) #peers for facility 6 are 1,2,5

## peer1 peer2 peer3  
## [1,] 1 NA NA  
## [2,] 2 NA NA  
## [3,] 3 NA NA  
## [4,] 4 NA NA  
## [5,] 5 NA NA  
## [6,] 1 2 5

IRS\_Weights <- lambda(IRS)  
#Decreasing returns to scale, convexity, down-scaling and free disposability  
DRS <- dea(x,y, RTS= "drs") #DRS gave same results as CRS  
DRS #All facilities are efficient except for facility 5,6

## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675

peers(DRS) # The peers units for for facilities 5,6 are 1,2,4

## peer1 peer2 peer3  
## [1,] 1 NA NA  
## [2,] 2 NA NA  
## [3,] 3 NA NA  
## [4,] 4 NA NA  
## [5,] 1 2 4  
## [6,] 1 2 4

DRS\_Weights <- lambda(DRS)  
   
FRH <- dea(x,y, RTS= "add")  
FRH #all facilities are efficient

## [1] 1 1 1 1 1 1

peers(FRH) #the peer unit for each facility is itself

## peer1  
## [1,] 1  
## [2,] 2  
## [3,] 3  
## [4,] 4  
## [5,] 5  
## [6,] 6

FRH\_Weights <- lambda(FRH)

as.data.frame(Table)

## Staff\_Hours Supplies Reimbursed Patient\_Days Privately Paid Patient\_Days  
## Fac1 150 0.2 14000 3500  
## Fac2 400 0.7 14000 21000  
## Fac3 320 1.2 42000 10500  
## Fac4 520 2.0 28000 42000  
## Fac5 350 1.2 19000 25000  
## Fac6 320 0.7 14000 15000

Df <-data.frame (CRS = c(1.0000, 1.0000, 1.0000, 1.0000, 0.9775, 0.8675),  
FDH= c(1,1,1,1,1,1), VRS= c(1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 0.8963),IRS =c( 1.0000, 1.0000, 1.0000, 1.0000, 1.0000 ,0.8963), DRS= c(1.0000 ,1.0000 ,1.0000, 1.0000, 0.9775, 0.8675), FRH= c(1,1,1,1,1,1))  
Df

## CRS FDH VRS IRS DRS FRH  
## 1 1.0000 1 1.0000 1.0000 1.0000 1  
## 2 1.0000 1 1.0000 1.0000 1.0000 1  
## 3 1.0000 1 1.0000 1.0000 1.0000 1  
## 4 1.0000 1 1.0000 1.0000 1.0000 1  
## 5 0.9775 1 1.0000 1.0000 0.9775 1  
## 6 0.8675 1 0.8963 0.8963 0.8675 1

#Now we look at the efficiency results at each facility in every DEA assumption. CRS and DRS give same results, FDH and FRH gave same results, and finally both VRS and IRS gave same results as well.

Results <- cbind(Table,Df)  
Results[,-c(1:4)]

## CRS FDH VRS IRS DRS FRH  
## Fac1 1.0000 1 1.0000 1.0000 1.0000 1  
## Fac2 1.0000 1 1.0000 1.0000 1.0000 1  
## Fac3 1.0000 1 1.0000 1.0000 1.0000 1  
## Fac4 1.0000 1 1.0000 1.0000 1.0000 1  
## Fac5 0.9775 1 1.0000 1.0000 0.9775 1  
## Fac6 0.8675 1 0.8963 0.8963 0.8675 1

#Summary of the weights assigned to each Facility in every DEA assumption  
  
Weights\_tbl <- cbind(FDH\_Weights, CRS\_Weights, VRS\_Weights, IRS\_Weights, DRS\_Weights, FRH\_Weights)  
  
row.names(Weights\_tbl) = c("Fac1", "Fac2", "Fac3", "Fac4", "Fac5", "Fac6")  
  
colnames(Weights\_tbl) <- c("FDH","FDH", "FDH", "FDH","FDH","FDH", "CRS", "CRS", "CRS", "CRS", "VRS", "VRS", "VRS","VRS", "VRS", "IRS", "IRS", "IRS", "IRS","IRS", "DRS", "DRS", "DRS", "DRS", "FRH", "FRH", "FRH", "FRH", "FRH","FRH")   
  
as.data.frame(Weights\_tbl) #the table summarizes the weights for inputs and outputs for each facility under each DEA assumption.

## FDH FDH FDH FDH FDH FDH CRS CRS CRS CRS VRS  
## Fac1 1 0 0 0 0 0 1.0000000 0.00000000 0 0.0000000 1.0000000  
## Fac2 0 1 0 0 0 0 0.0000000 1.00000000 0 0.0000000 0.0000000  
## Fac3 0 0 1 0 0 0 0.0000000 0.00000000 1 0.0000000 0.0000000  
## Fac4 0 0 0 1 0 0 0.0000000 0.00000000 0 1.0000000 0.0000000  
## Fac5 0 0 0 0 1 0 0.2000000 0.08048142 0 0.5383307 0.0000000  
## Fac6 0 0 0 0 0 1 0.3428571 0.39499264 0 0.1310751 0.4014399  
## VRS VRS VRS VRS IRS IRS IRS IRS IRS  
## Fac1 0.0000000 0 0 0.0000000 1.0000000 0.0000000 0 0 0.0000000  
## Fac2 1.0000000 0 0 0.0000000 0.0000000 1.0000000 0 0 0.0000000  
## Fac3 0.0000000 1 0 0.0000000 0.0000000 0.0000000 1 0 0.0000000  
## Fac4 0.0000000 0 1 0.0000000 0.0000000 0.0000000 0 1 0.0000000  
## Fac5 0.0000000 0 0 1.0000000 0.0000000 0.0000000 0 0 1.0000000  
## Fac6 0.3422606 0 0 0.2562995 0.4014399 0.3422606 0 0 0.2562995  
## DRS DRS DRS DRS FRH FRH FRH FRH FRH FRH  
## Fac1 1.0000000 0.00000000 0 0.0000000 1 0 0 0 0 0  
## Fac2 0.0000000 1.00000000 0 0.0000000 0 1 0 0 0 0  
## Fac3 0.0000000 0.00000000 1 0.0000000 0 0 1 0 0 0  
## Fac4 0.0000000 0.00000000 0 1.0000000 0 0 0 1 0 0  
## Fac5 0.2000000 0.08048142 0 0.5383307 0 0 0 0 1 0  
## Fac6 0.3428571 0.39499264 0 0.1310751 0 0 0 0 0 1

# **DEA Analysis Summary**

For Hope Vally Health Care Association: Under FDH and FRH all facilities are efficient, under CRS and DRS all facilities were efficient except for Facility 5,6. Under VRS and IRS assumptions all except for facility 6 were efficient. The peer units for efficient facilities are themselves. Under VRS and IRS assumption the peers unit for inefficient facilities were 1,2 and 5. Under CRS and DRS, the peers unites were 1,2,and 4.

# ## *Problem 2* [Goal Programming]

Emax Corporation Problem:

Maximize Z = P - 6C - 3D, where P = total (discounted) profit over the life of the new products,

C = change (in either direction) in the current level of employment,

D = decrease (if any) in next year’s earnings from the current year’s level.

– P= 20 x1 + 15 x2 + 25 x3;

– y1= 6 x1 + 4 x2 + 5 x3 - 50;

– y2= 8 x1 + 7 x2 +5 x3 - 75

y1+ is going over the employment level goal and the weighted penality is 6

y1- is going under the employment level goal and the weighted penality is 6

y2+ is going over the earnings goal for next year- no penality

y2- is going under the earnings goal for next year and the peanlity is 3.

x1 is the quantity of product 1 to be produced

x2 is the quantity of product 2 to be produced

x3 is the quantity of product 3 to be produced

# LP formulation:

// Objective function max: 20x1 + 15x2 + 25x3 - 6 y1p - 6 y1m - 3 y2m;

// Constraints

6x1 + 4x2 + 5x3 + y1p - y1m = 50;

8x1 + 7x2 + 5x3 + y2p - y2m = 75;

================================================

library(lpSolveAPI)  
Emax\_GP <- read.lp("Emax.lp")  
Emax\_GP

## Model name:   
## x1 x2 x3 y1p y1m y2m y2p   
## Maximize 20 15 25 -6 -6 -3 0   
## R1 6 4 5 -1 1 0 0 = 50  
## R2 8 7 5 0 0 1 -1 = 75  
## Kind Std Std Std Std Std Std Std   
## Type Real Real Real Real Real Real Real   
## Upper Inf Inf Inf Inf Inf Inf Inf   
## Lower 0 0 0 0 0 0 0

Solving the problem

solve (Emax\_GP)

## [1] 0

get.objective(Emax\_GP)

## [1] 225

Emax need to produce 15 units of product 3 and none of product 1 and 2 to achieve 225 millions in profit. The employment level will go over the goal by 2500

get.variables(Emax\_GP)

## [1] 0 0 15 25 0 0 0