

Proof of the Ridge Regression Formula

Description

In this exercise, you are tasked with proving the formula related to the Ridge Regression method. The goal of this exercise is to deepen your understanding of regression concepts as well as to apply mathematical optimization principles.

Formula:

$$\begin{aligned} w_{\lambda}^{\text{Ridge}} &= \underset{w}{\operatorname{argmin}} \mathcal{J}_{\text{reg}}(w) = \underset{w}{\operatorname{argmin}} \frac{1}{2N} \|Xw - t\|_2^2 + \frac{\lambda}{2} \|w\|_2^2 \\ &= (X^T X + \lambda N I)^{-1} X^T t \end{aligned}$$

Notes:

- Utilize linear algebra concepts such as matrix differentiation and matrix inversion operations.
- Provide clear explanations for each step of the proof.
- If necessary, refer to reputable sources and cite your references.

Your Task:

- 1 . Precisely define and explain the cost function $\mathcal{J}_{\text{reg}}(w)$.
- 2 . Use optimization principles (differentiation) to find the optimal point w .
- 3 . Step-by-step, demonstrate how the provided formula is derived from this method.