BAX 443 | Analytical Decision Making | HW 2 | Portfolio Optimization

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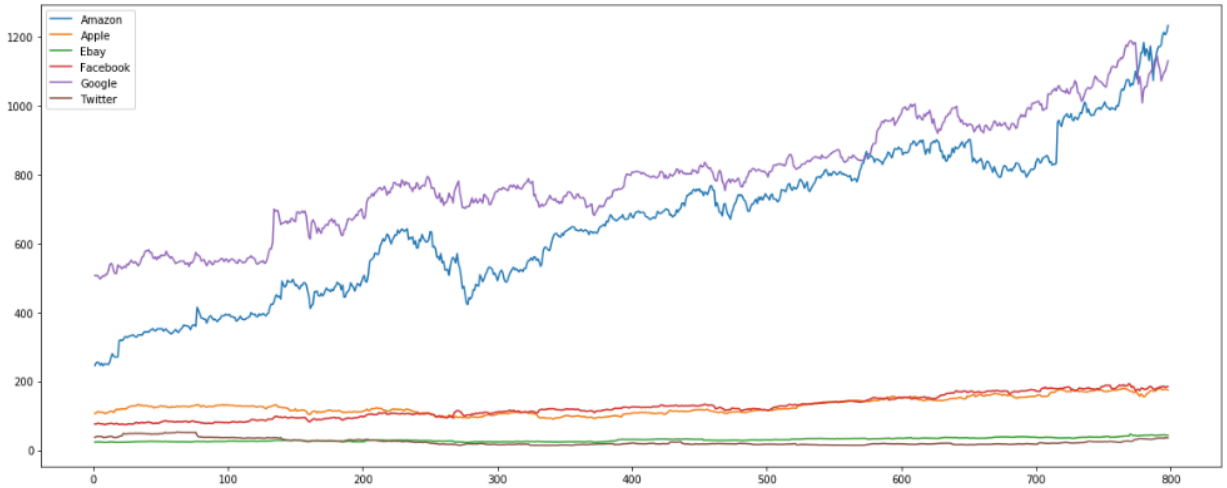
Overview

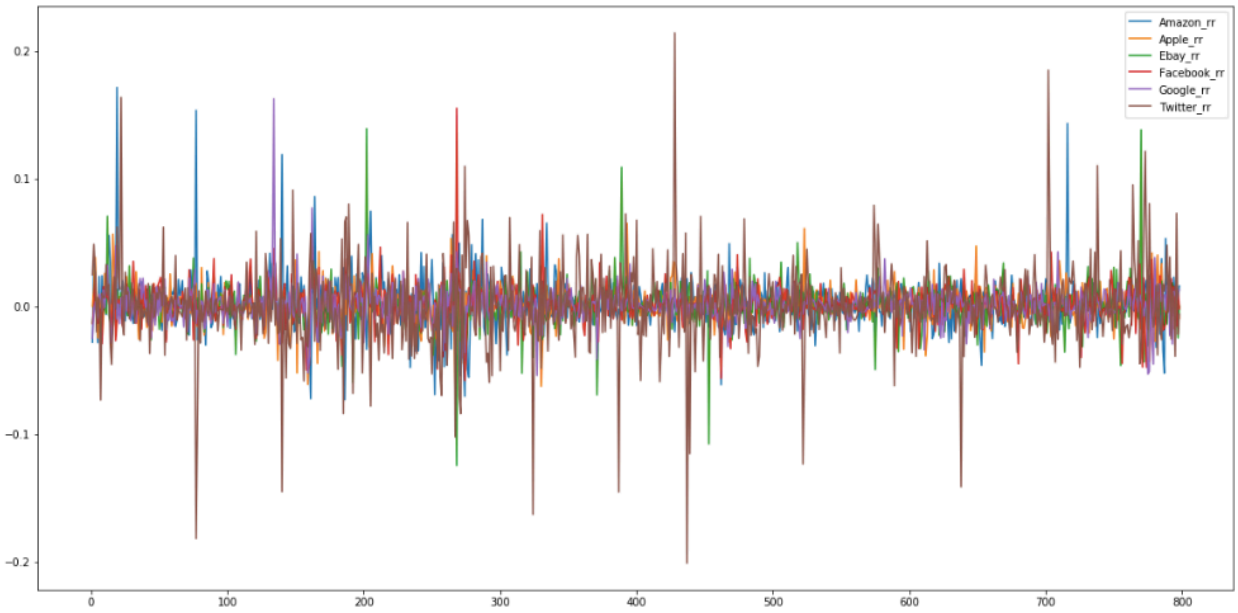
The objective of the homework is to consider different approaches with respect to portfolio optimization. Input data has trading information about 6 stocks. Based on the evolution of these stocks, recommend different portfolio allocations.

Part A: Portfolio Optimization based on the Full Data Set

Below is the stock growth for each company –



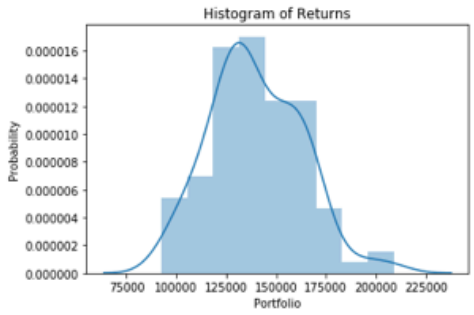
We can see that Google and Amazon have the highest valued stocks, however there are certain fluctuations, so we look at the daily return rates –



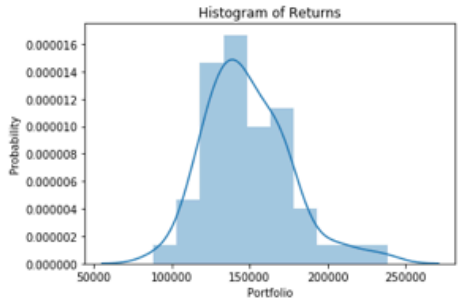
Based on the above we can see that Twitter seems to be a risky stock.

There were two parts of the problem – first to minimize risk and second to maximize utility (maximize returns and penalize volatility) with different risk aversion factors. We solved the first problem using cvxpy and second with scipy packages in Python. After optimization, we run the simulation of returns for 300 trading days. The results are summarized below –

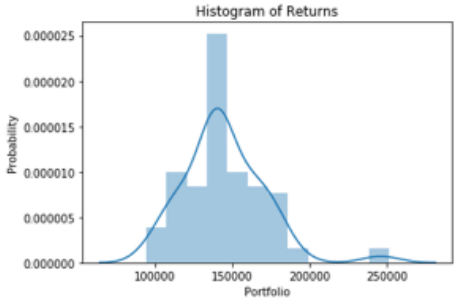
Minimize risk



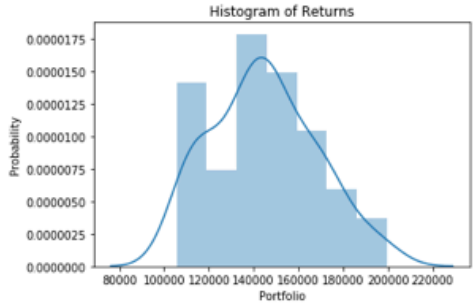
Maximize utility, risk aversion = 0.1



Maximize utility, risk aversion = 0.15



Maximize utility, risk aversion = 0.2



Results Summary / Recommendations

Background - Portfolio optimization is the process of selecting the best portfolio allocations for the given set of stocks according to some objective. The objective typically maximizes factors such as expected return, and minimizes costs like financial risk. Following are the building blocks of the two optimization problems -

- The decision variable is the portfolio allocation for each stock - x
- The objective function in the first problem is to minimize the risk function, and in the second problem is to maximize the returns/utility function
- The constraints are sum of all portfolio allocations should be 100% and non-negativity for each allocation. For the first problem while minimizing risk, additional constraint is on minimum returns for a threshold

We look at 799 days of portfolio stock values and calculate the expected returns on the 800th day.

Explain the different allocations, i.e., why they are different

The expected returns from the 6 stocks –

Amazon	Apple	Ebay	Facebook	Google	Twitter
0.199%	0.063%	0.075%	0.109%	0.097%	-0.003%

The allocations for each stock and (risk and) expected returns from each scenario –

Scenario	Amazon allocation	Apple allocation	Ebay allocation	Facebook allocation	Google allocation	Twitter allocation	Objective fn value
Objective - Minimize risk	20.4%	19.5%	16.2%	17.4%	19.0%	7.5%	0.004%
Objective - Maximize utility, risk aversion = 0.1	29.7%	15.5%	16.3%	19.7%	18.9%	0.0%	0.026%
Objective - Maximize utility, risk aversion = 0.15	28.0%	16.3%	16.7%	19.5%	19.0%	0.5%	-0.022%
Objective - Maximize utility, risk aversion = 0.2	25.9%	16.8%	16.9%	19.1%	18.8%	2.4%	-0.068%

The mean and variance from the simulations for each scenario *(Please note the simulation results will change slightly every time it's run again)* –

Scenario	Portfolio Mean	Portfolio Variance
Objective - Minimize risk	$139,743	$515,788,280
Objective - Maximize utility, risk aversion = 0.1	$148,530	$724,035,116
Objective - Maximize utility, risk aversion = 0.15	$144,832	$698,230,568
Objective - Maximize utility, risk aversion = 0.2	$144,692	$536,847,504

The investment strategies are different for the two problems we are trying to solve above. In one of the problems, we focus on minimizing the downside risk and portfolio diversification seems to be the best option. However, the optimization problem for risk minimization doesn't distinguish between positive and negative returns and that is why still assigns ~8% to the Twitter portfolio whose expected returns are negative. But overall downside risk is minimized to a 0.004% level. In the second problem, objective is to maximize the expected returns penalizing it for volatility. Since, the returns are highest from the first stock, Amazon, that is why the allocation to that one is the highest (25-30%). As the risk aversion increases between 0.1 to 0.2, the allocations start changing - some part from amazon is distributed among the other stocks.

What should an investor with risk aversion $\gamma = 0.1$ do? What about investors with $\gamma = 0.15$ and $\gamma = 0.2$?

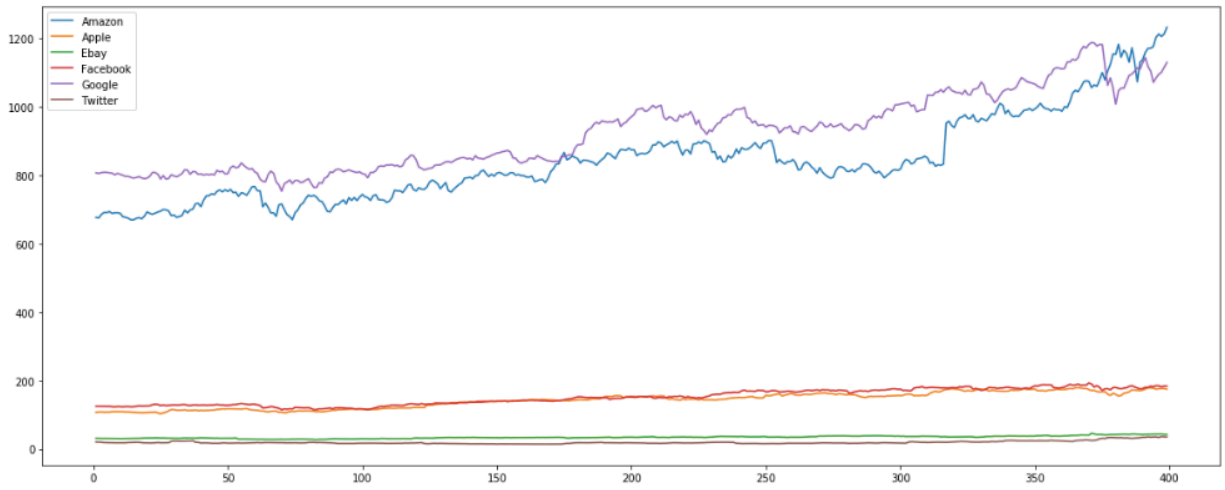
As we know, when investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will be the same for all investors, but different investors will evaluate the trade-off differently based on individual risk aversion characteristics. The implication is that a rational investor will not

invest in a portfolio if a second portfolio exists with a more favorable risk-expected return profile – i.e., if for that level of risk an alternative portfolio exists that has better expected returns.

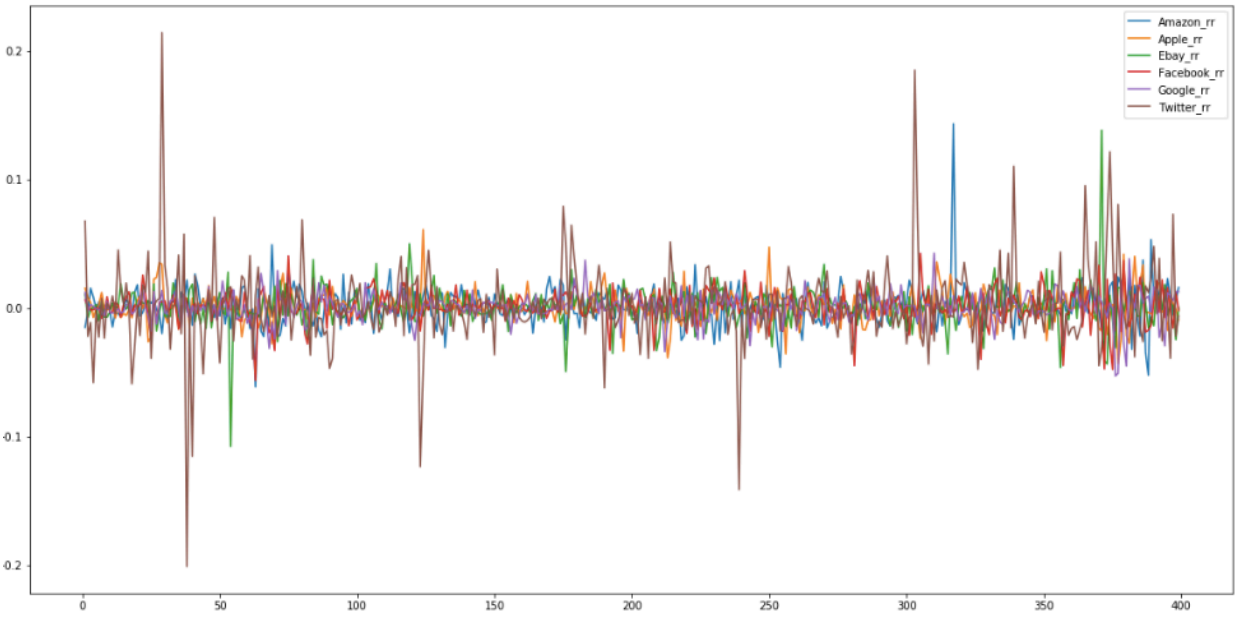
The difference between the allocations of the portfolio stocks among the three risk aversion choices is not very high, but we do see that the expected returns start decreasing with increase in the risk aversion factor. In fact, for risk aversion=0.15 and 0.2, the expected returns are negative whereas for risk aversion factor=0.1, the returns are the highest even higher than the first problem where we minimize risk. This is intuitive as we are trying to maximize is the expected returns in the second problem. And that is why minimum% is allocated to twitter stocks as the returns are negative. So, an investor with risk $\gamma = 0.1$ should not invest in twitter at all and invest maximum in Amazon to get the highest returns. This portfolio is high return but also high risk given the high variances as evident from the mean-variances tabulated from the simulation results. For investors with $\gamma = 0.15$, the medium return-medium risk portfolio is suited so that at least some amount if invested in each stock even if it is negative returns. For investors with $\gamma = 0.2$, since they are more risk averse, the rate of diversification should be even higher and hence similar to investors who want to minimize risk they should look at low return-low risk portfolio and put their eggs in different baskets. However, the last two sets of investors should keep in mind that the overall expected returns are negative.

Part B: Portfolio Optimization based on the last 400 trading days.

Below is the stock growth for each company based on last 400 days –



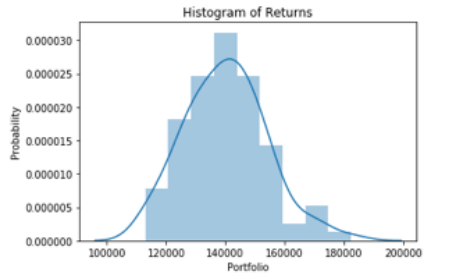
We can see that Google and Amazon still have the highest valued stocks, however there are certain fluctuations and Amazon grows rapidly overtaking Google in the last 20 days or so. We again look at the daily return rates –



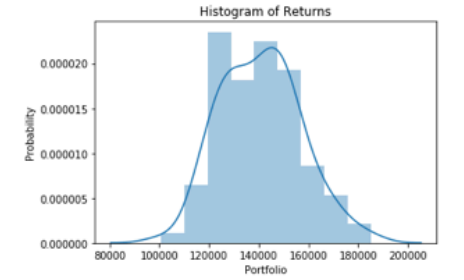
Based on the above we can see that Twitter still seems to be a risky stock and after 300 days, and as evident from the first chart, we can see a clear peak in Amazon between 310-320 days.

There were two parts of the problem – first to minimize risk and second to maximize utility (maximize returns and penalize volatility) with different risk aversion factors. We solved the first problem using cvxpy and second with scipy packages in Python. After optimization, we run the simulation of returns for 300 trading days. The results are summarized below –

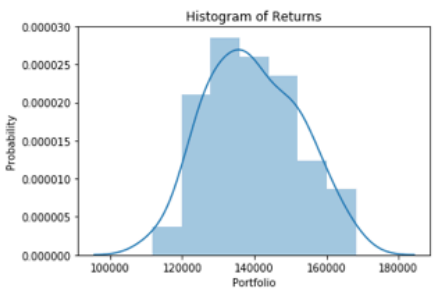
Minimize risk



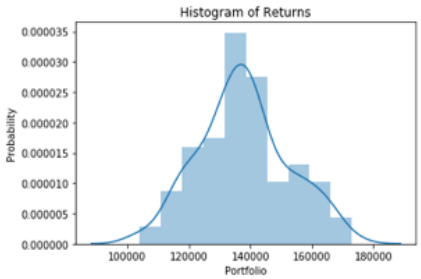
Maximize utility, risk aversion = 0.1



Maximize utility, risk aversion = 0.15



Maximize utility, risk aversion = 0.2



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Background - Portfolio optimization is the process of selecting the best portfolio allocations for the given set of stocks according to some objective. The objective typically maximizes factors such as expected return, and minimizes costs like financial risk. Following are the building blocks of the two optimization problems -

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We look at the last 400 days of portfolio stock values and calculate the expected returns on the 800th day.

Explain the different allocations, i.e., why they are different

The expected returns from the 6 stocks –

Amazon	Apple	Ebay	Facebook	Google	Twitter
0.146%	0.126%	0.081%	0.098%	0.087%	0.150%

The allocations for each stock and (risk and) expected returns from each scenario –

Scenario	Amazon allocation	Apple allocation	Ebay allocation	Facebook allocation	Google allocation	Twitter allocation	Objective fn value
Objective - Minimize risk	16.3%	23.0%	14.7%	18.9%	22.5%	4.6%	0.002%
Objective - Maximize utility, risk aversion = 0.1	22.0%	21.2%	15.3%	17.9%	17.3%	6.2%	0.05%
Objective - Maximize utility, risk aversion = 0.15	20.7%	20.6%	16.3%	18.3%	18.1%	5.9%	0.02%
Objective - Maximize utility, risk aversion = 0.2	20.2%	20.4%	16.8%	18.5%	18.5%	5.7%	-0.005%

The mean and variance from the simulations for each scenario *(Please note the simulation results will change slightly every time it's run again)* –

Scenario	Portfolio Mean	Portfolio Variance
Objective - Minimize risk	$140,072	$187,898,146
Objective - Maximize utility, risk aversion = 0.1	$140,726	$255,260,041
Objective - Maximize utility, risk aversion = 0.15	$139,578	$161,210,597
Objective - Maximize utility, risk aversion = 0.2	$137,762	$209,565,433

The investment strategies are different for the two problems we are trying to solve above. In one of the problems, we focus on minimizing the downside risk and portfolio diversification seems to be the best option. Overall downside risk is minimized to a 0.002% level and maximum allocation is to Apple and Google since they seem to be less volatile. In the second problem, objective is to maximize the expected returns penalizing it for volatility. Since, the

returns are high from the first two stocks, Amazon and Apple, that is why the total allocation to them is the highest (>40%). Even though Twitter's return rate is high, yet the overall dollar values are the lowest and hence it ends up with the least allocation. As the risk aversion increases between 0.1 to 0.2, the allocations start changing - Amazon, Apple and Twitter with high return rates allocations decrease and Ebay, facebook and Google allocations increase.

What should an investor with risk aversion γ = 0.1 do? What about investors with γ = 0.15 and γ = 0.2?

As we know, when investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will be the same for all investors, but different investors will evaluate the trade-off differently based on individual risk aversion characteristics. The implication is that a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk-expected return profile – i.e., if for that level of risk an alternative portfolio exists that has better expected returns.

The difference between the allocations of the portfolio stocks among the three risk aversion choices is not very high, but we do see that the expected returns start decreasing with increase in the risk aversion factor. In fact, for risk aversion=0.2, the expected returns are negative whereas for risk aversion factor=0.1, the returns are the highest. This is intuitive as more emphasis is on returns than the risk for the latter. Investors with risk γ = 0.1 should invest maximum in Amazon given they want high returns followed by Apple. The least allocation is to Twitter and the rest are in the range of 15-17%. Investors with γ = 0.15 and γ = 0.2 should invest similar percentages in most of the stocks but being more risk averse, they should take out some allocations from Amazon, Apple and twitter and invest a little more in Ebay, Facebook and Google.

Why are the allocations different?

The allocations are different across Part 1 and Part 2 because the timeframe considered has a significant impact on the overall returns. From the mean-variance tabulation of the simulation results, we can clearly see that the stocks and overall portfolio have become less volatile (lower variance) in the last 400 days. The twitter returns change from negative to positive if we consider only the last 400 days. In fact, the expected returns from twitter are the highest among all six stocks.