EENG307: Motors and Hydraulic Actuators*

Lecture 11

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Contents

1 Pre-requisite Material

This lecture assumes that the reader is familiar with the following material:

- Lecture 1: Modeling Mechanical Systems
- Lecture 2: Modeling Electrical Systems
- Lecture 7: Fluid Systems and System Analogies

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2 Actuators

3 Electric Motors

Electric Motors

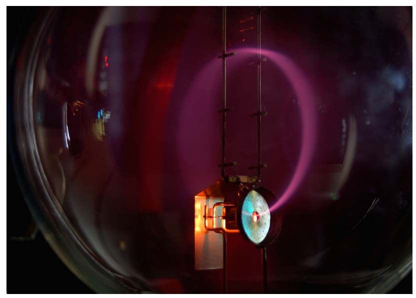
- Motors are *transducers* which convert electrical energy to mechanical energy.
- We will look at models for brushed DC motors.

3.1 Key Ideas

Key Idea 1: Lorentz Force

- An electron moving in a magnetic field experiences a force perpendicular to its motion and the magnetic field.

$$F = q(v \times B)$$



(http://commons.wikimedia.org/wiki/File:Draaibank.png)

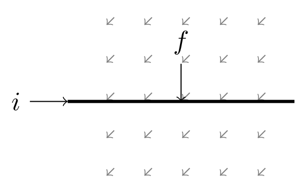
- Electrons move in a circle when exposed to a constant magnetic field.

In the figure, electrons are ejected by the hot-cathode electron gun. Purple light is emitted along the electron path, as gas molecules are excited when collisions with electrons occur.

Key Idea 1: Lorentz Force

- Electrons in a wire are constrained to move in a straight line. When a wire is placed in a magnetic field, and a current is set up through the wire, the Lorentz force creates a force perpendicular to the current and the magnetic field.

$$f = Ki$$



- This is the basis for a motor - the force on the wire can drive a mechanical load.

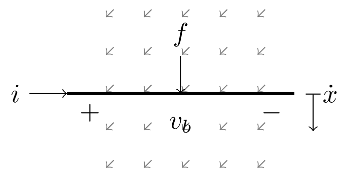
Key Idea 2: Conservation of Energy

- A force on a wire will cause it to accelerate.
- Work done
  - on the wire: $\dot{x}f = \dot{x}Ki$.
  - to create the current: iv_b.

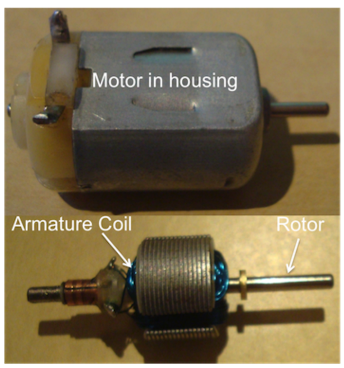
- Conservation of energy suggests these are equal

$$iv_b = \dot{x}Ki,$$
$$v_b = \dot{x}K.$$

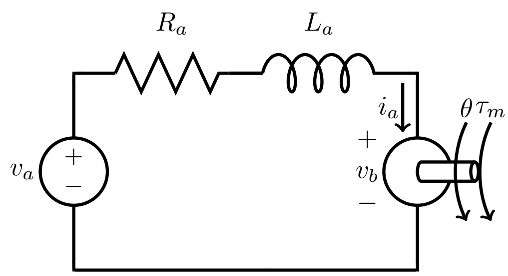
- voltage (back electro-motive force) v_b is proportional to the wire's velocity



DC motor components



http://en.wikipedia.org/wiki/File:Motor_internals.JPG

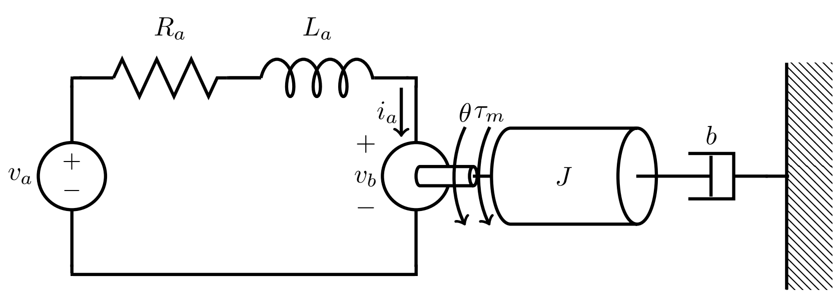


Motor Constants: K_t, K_e

Transducer Relations: $\boxed{v_b = K_e\dot{\theta}}$ $\boxed{\tau_m = K_t i_a}$

3.2 Motor Analysis

DC motor diagram



This system can be analyzed by finding the equations for the circuit, the equations for the mechanical system, and adding the motor transducer relationships.

Circuit:

$$V_a(s) = I_a(s)R_a + I_a(s)L_a s + V_b(s).$$

Mechanical System:

$$Js^2\theta(s) = \tau_m(s) - bs\theta(s).$$

Transducer relationships:

$$\tau_m = K_t I_a(s),$$
$$V_b(s) = K_e s\theta(s).$$

A convenient way of viewing these equations is by using a block diagram.

3.3 Block Diagram

The block diagram will relate the variables $V_a(s)$, $I_a(s)$, $V_b(s)$, $\tau_m(s)$, and $\theta(s)$. Since $V_a(s)$ is an input, we should put it at the left of the block diagram. Note that the circuit equation implies

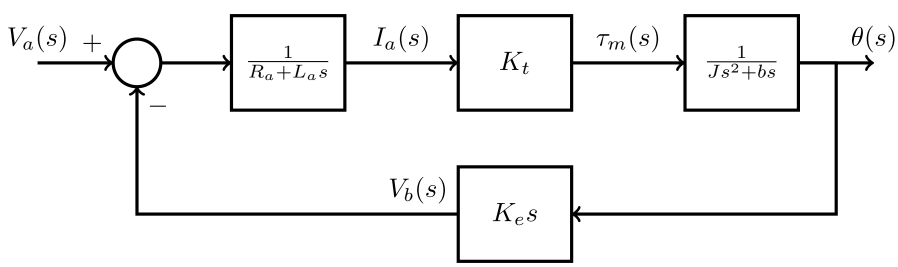
$$I_a(s) = \frac{1}{R_a + L_a s}\left(V_a(s) - V_b(s)\right),$$

and the mechanical equation implies

$$\theta(s) = \frac{1}{Js^2 + bs}\tau_m(s).$$

Using blocks that define the relationships between the variables, we get the following block diagram.

DC motor block diagram

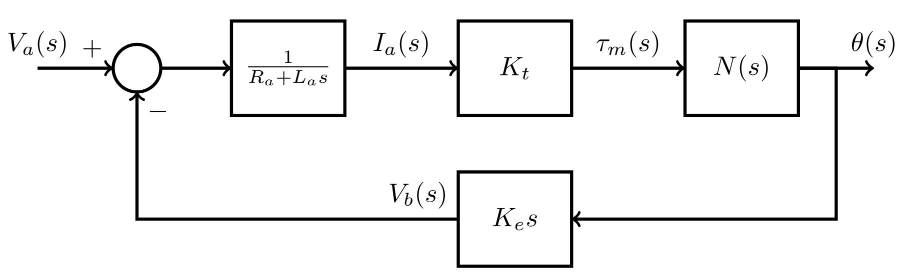


Using this block diagram, you can find the transfer function from $V_a(s)$ to any of the other variables.

3.4 Arbitrary Load

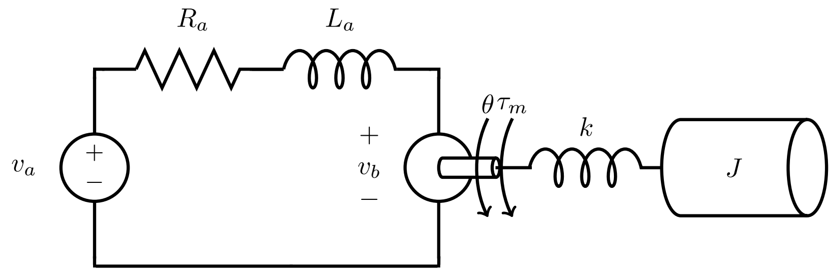
In the block diagram above, the load was specified to be an inertial with viscous damping. The load came into the block diagram by specifying the mapping from the motor torque $\tau_m(s)$ to the rotation of the motor shaft, $\theta(s)$. Note that $\frac{\theta(s)}{\tau_m(s)} = N(s)$ defines a mechanical impedance. Thus, we can easily accommodate other loads by substituting in the specific mechanical impedance of the load.

DC motor block diagram with arbitrary load

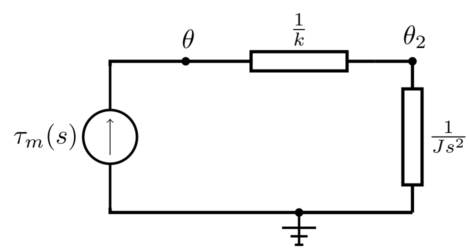


For example, suppose we were given a problem with a different load, say the following:

DC motor with different load



All we need to do is mentally disconnect the motor, and find the transfer function $\frac{\theta(s)}{\tau_m(s)}$. In this case, the load impedance would be found from the network:



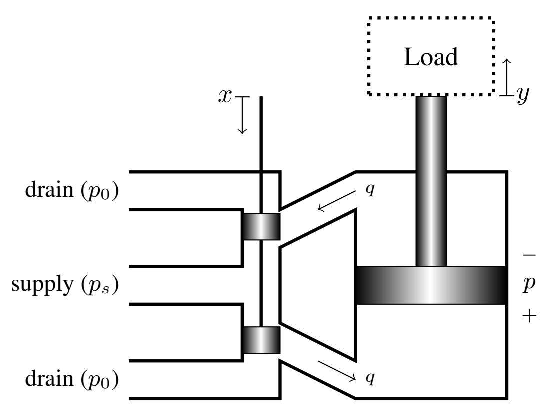
And the transfer function from motor torque to motor axis rotation would be

$$N(s) = \frac{\theta(s)}{\tau_m(s)} = \frac{1}{k} + \frac{1}{Js^2} = \frac{Js^2 + k}{Jks^2}.$$

This is then plugged into the block diagram above.

4 Hydraulic Actuators

Diagram of Hydraulic Actuator: Servo Valve and Hydraulic Piston



Piston area: A

4.1 Flow behavior

- When the valve is moved down, high pressure liquid is allowed into the bottom chamber with flow q.
- Since the liquid is incompressible, the same amount of flow will enter the drain.

- The amount of flow is regulated by the pressure drop p. The higher the pressure, the lower the flow.
- The flow thus follows a general nonlinear relationship:

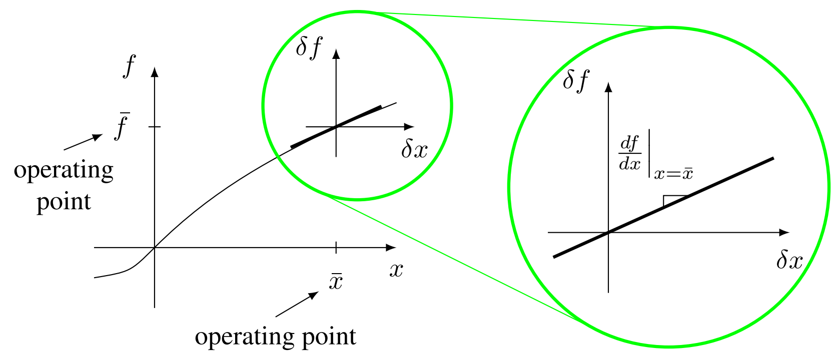
$$\boxed{q = f(x, p)}$$

This nonlinear relationship between p, x, and q is problematic. However, as you recall from calculus, a sufficiently smooth nonlinear function can be locally approximated by a linear function.

Linear Approximation

- The linear approximation defines a *linear* relationship between *deviations* from the operating point.
- Notation: a deviation of variable x from operating point $\bar{x}$ is denoted δx

$$x = \bar{x} + \delta x$$



The nonlinear flow behavior can be linearized at operating point $(\bar{x}, \bar{p})$, giving flow $q = \bar{q} + \delta q$. Since there are two inputs, the small signal relationship depends on two slopes:

$$\delta q = \frac{\partial f}{\partial x}\bigg|_{x=\bar{x}, p=\bar{p}} \delta x + \frac{\partial f}{\partial p}\bigg|_{x=\bar{x}, p=\bar{p}} \delta p.$$

For convenience, define

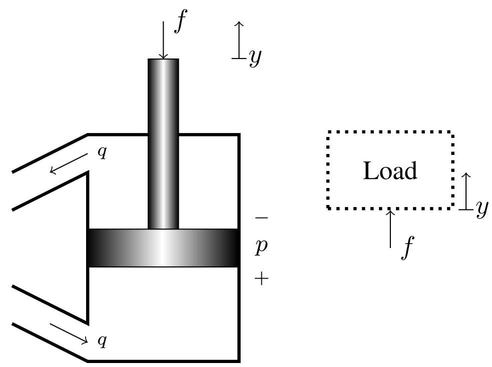
$$k_x = \frac{\partial f}{\partial x}\bigg|_{x=\bar{x}, p=\bar{p}} \qquad k_p = -\frac{\partial f}{\partial p}\bigg|_{x=\bar{x}, p=\bar{p}}.$$

We put a negative sign in the second equation because the slope with respect to pressure is usually negative, and this will ensure that k_p is positive. Then we can write

$$\boxed{\delta q = k_x \delta x - k_p \delta p}.$$

4.2 Transducer relationships between fluid and translational systems

Load and piston interaction



Piston area: A

- Positive flow q implies that y changes

  Net volume into bottom of chamber in time Δt due to flow:

  $$V = q\Delta t.$$

  Net change in volume due to movement of piston:

  $$V = (y(t + \Delta t) - y(t))A.$$

  Thus

  $$q = \frac{(y(t + \Delta t) - y(t))}{\Delta t} A,$$

  and in the limit as Δt goes to zero,

  $$\boxed{q = \dot{y}A}$$

- Pressure across piston gives rise to force on load.

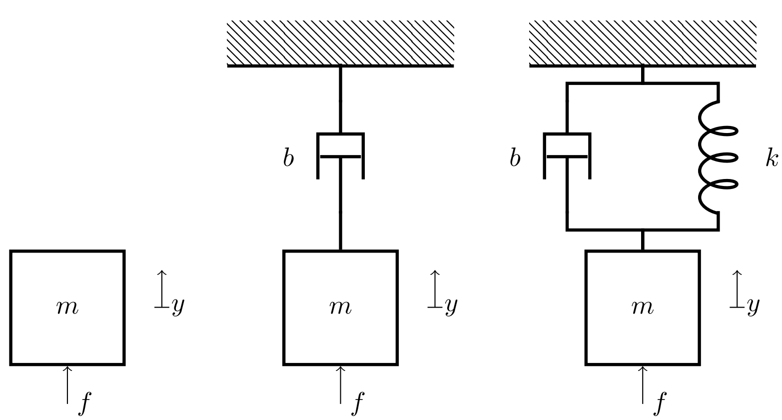
  $$\boxed{f = pA}$$

Do we have enough equations?

- 5 Variables: q,x,p,y,f.
- Only 3 equations - we need one more! We are missing an expression for how the load responds to an applied force!

4.3 Load Impedance

Example Possible Loads



We don't know what the load is, but whatever it is, it will be characterized by the transfer function from force f to position y.

$$\boxed{\frac{Y(s)}{F(s)} = N(s)}$$

Note that $N(s)$ is mechanical impedance.

4.4 Block Diagram

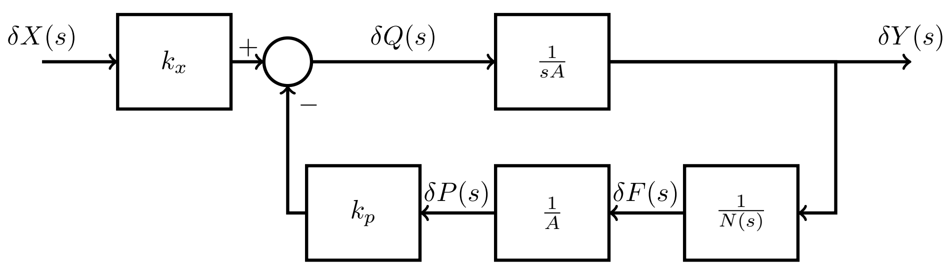
4.4.1 Block Diagram Representation #1

To draw a block diagram that represents the system, we take the Laplace Transform of all equations, and replace variables with their δ versions for all linear equations.

$$\begin{aligned}
\delta Q(s) &= k_x \delta X(s) - k_p \delta P(s), \\
\delta Y(s) &= \frac{1}{sA} \delta Q(s), \\
\delta P(s) &= \frac{1}{A} \delta F(s), \\
\delta F(s) &= \frac{1}{N(s)} \delta Y(s).
\end{aligned}$$

Using blocks that define the relationships between the variables, we get the following block diagram.

Hydraulic system block diagram #1



Using this block diagram, you can find the transfer function from $\delta X(s)$ to any of the other variables.

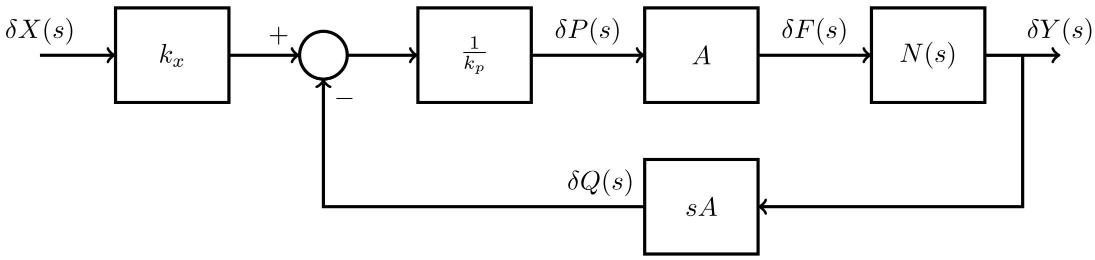
4.4.2 Block Diagram Representation #2

There is an alternate, equivalent block diagram that can be found by solving for a different set of variables in the governing equations. Specifically, note that the following set of equations are equivalent to what was written above, but with different variables on the left hand side of the equal sign

$$\begin{aligned}
\delta P(s) &= \frac{1}{k_p} \left(k_x \delta X(s) - \delta Q(s) \right), \\
\delta Q(s) &= sA \delta Y(s) \\
\delta F(s) &= A \delta P(s), \\
\delta Y(s) &= N(s) \delta F(s).
\end{aligned}$$

Using blocks that define the relationships between the variables, we get the following block diagram.

Hydraulic system block diagram #2



5 Lecture Highlights

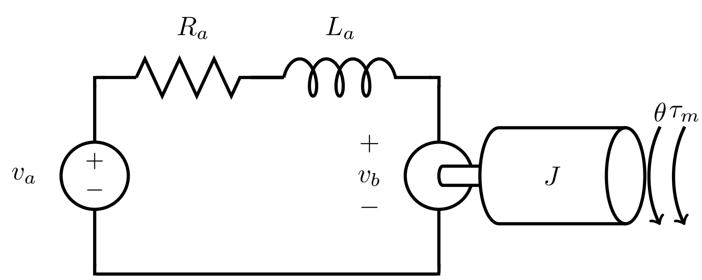
The primary takeaways from this article include

1. Motors and hydraulic actuators combine previously-studied types of systems (electrical and rotational systems for motors, fluid and translational mechanical systems for hydraulic actuators).
2. For the purposes of this class, we assume linear DC motors and we perform linearization to obtain simplified models of hydraulic actuators.
3. In general, you can use the structure shown in the "DC motor block diagram with arbitrary load" and "Hydraulic system block diagram #1" (or $\#2$) figures to solve problems in this class. You will need to solve for $N(s)$ in each case and may need to slightly modify other blocks (such as setting $L_a = 0$ if the inductance is assumed to be negligible).
4. The apparent "feedback" path in the block diagrams for motors and hydraulic actuators should not be confused with feedback *control* in future lectures. Rather, it is part of the inherent system (plant) property for the motor or hydraulic actuator.

6 Quiz Yourself

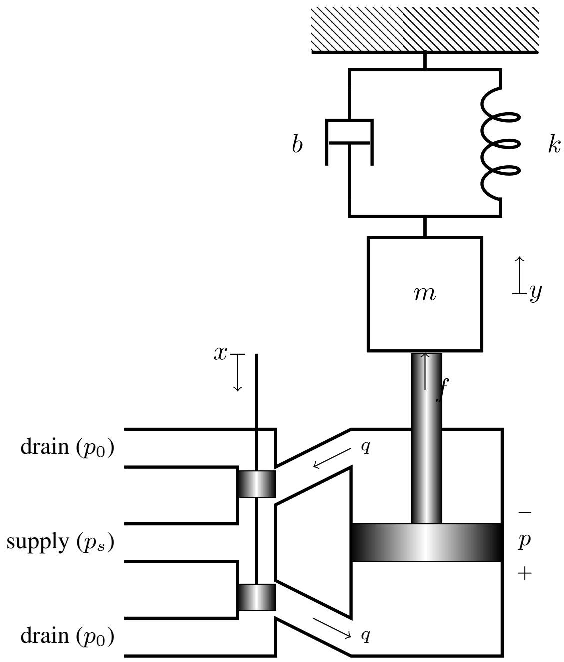
6.1 Questions

1. Consider the following motor system

   You are told that $J = 1$ kgm^2, the internal motor resistance $R_a = 1/4\ \Omega$ and the internal motor inductance can be neglected so that $L_a = 0$. The back emf constant $K_e = 2$ and the motor constant $K_t = 4$. Find the transfer function $\tau_m(s)/V_a(s)$.

2. The following system models the actuation system for one of the control surfaces of an airplane at cruising velocity. Using the linearized flow equation $\delta q = k_x \delta x - k_p \delta p$, find the transfer function $\delta Y(s)/\delta X(s)$.

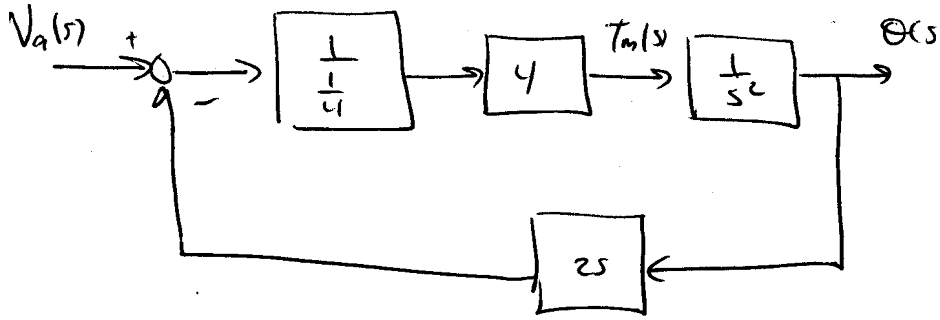


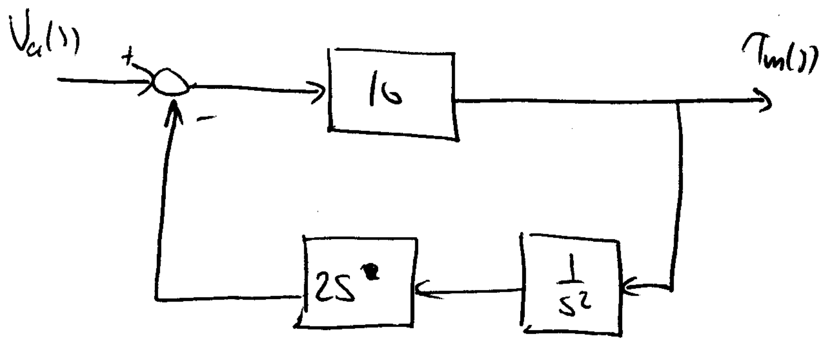
Piston area: A

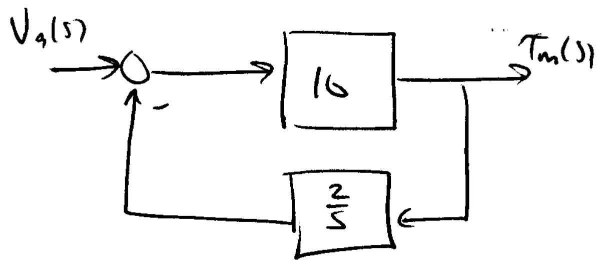
6.2 Solutions

1.

Motor block diagram:



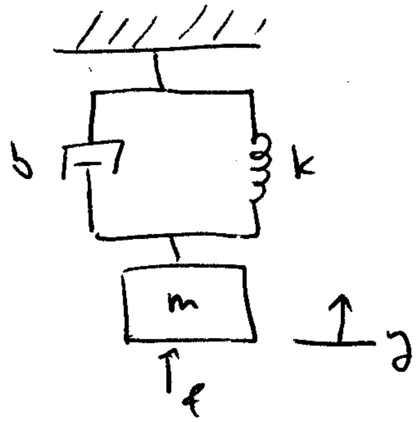




$$\frac{T_m(s)}{V_a(s)} = \frac{16}{1 + 16\frac{2}{s}} = \frac{16s}{s+32}$$

2.

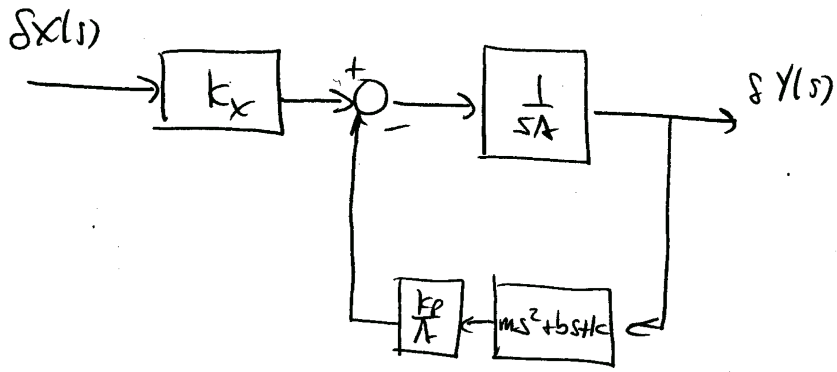
Load:

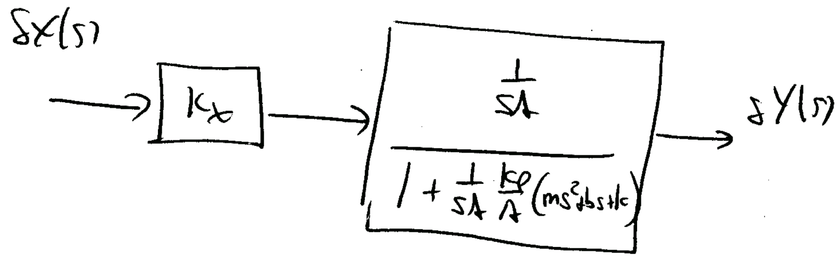


$$F(s) = \left(ms^2 + bs + k \right) Y(s)$$

Impedance: $\dfrac{Y(s)}{F(s)} = \dfrac{1}{ms^2 + bs + k}$

Hydraulic Block Diagram





$$\frac{\delta Y(s)}{\delta X(s)} = \frac{A K_x}{A^2 s + K_p(ms^2+bs+k)} = \frac{A K_x}{K_p m s^2 + (A^2+K_p b)s + K_p k}$$

7 Resources

7.1 Books

- Norman S. Nise, *Control Systems Engineering*, Wiley
  - 7th edition: Section 2.8 covers motors
- Gene F. Franklin, J. David Powell and Abbas Emami-Naeini, *Feedback Control of Dynamic Systems*, Pearson
  - 6th and 7th edition: Section 2.3 covers motors
  - 6th and 7th edition: Section 2.4 has a hydrualics example

7.2 Web resources

There are also some web resources that cover motors and hydraulic actuators. If you find something useful, or if you find a link that no longer works, please inform your instructor!

- `https://www.youtube.com/watch?v=GSvoQ4p3qV0`: A 12 minute lecture on DC motors