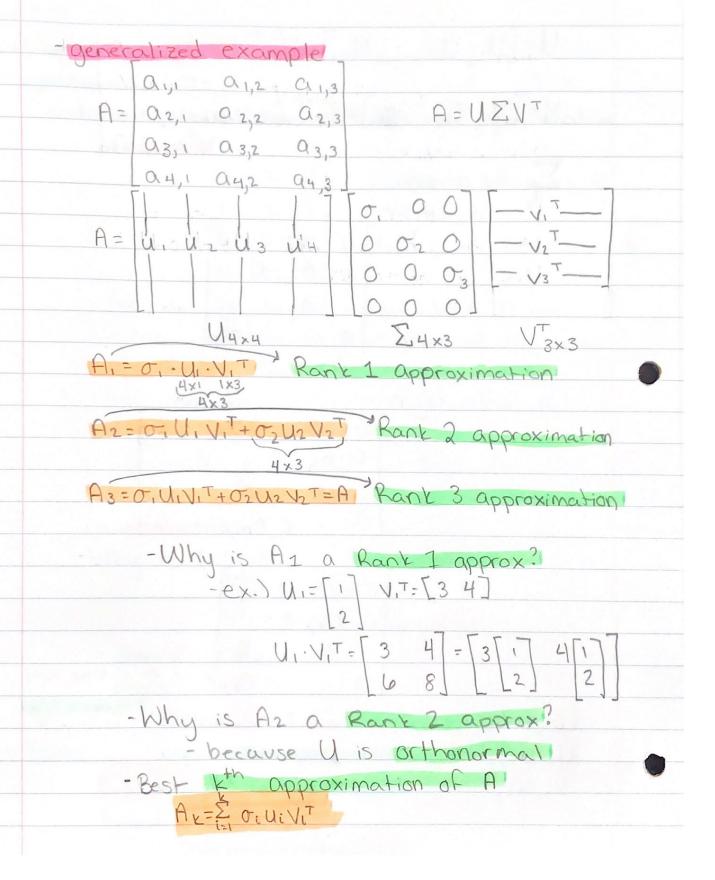
01/17/2023	SVD
	Singular Value Decomposition
_	rank of a matrix
	-number of independent columns/rows in a matrix
	$\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1 \cdot \text{col}_{1} + 1 \cdot \text{col}_{2} - 1 \cdot \text{col}_{3} = 0}{1 \cdot 1}$ $\frac{1}{2} \frac{1}{2} 1$
	-a column is not independent if you
	can find scalar multiples of the other columns that add to that column
	-a matrix is not full rank if
	there are scalar multiples of the columns that will result in the O
	vector when the columns are added - example:
	1 2 rank is 2 breause 1 1 there are 2 independent 1 1 rows/columns
	- Dim(A) → mxn
	rant: r
_	SVD A E R m x n A m x n = U \(\subseteq \subseteq \subseteq \)

U=[U, U2, ..., Um] ER (orthonormal matrix) V=[V1, V2,...Vn] EROXO (orthonormal matrix VTV=Inxn if mon mLn 0 - - - . 0 · 0 m, m 0 - · · · 0] mxn Z= 0, 0. 1. 0 0 02,0 0 : 6 Omm= or {0,1702,2303,32...20min(m,n)≥0} U, Uz, ... Um: left singular values Vi, Vz, ... Vn: right singular values



rank: r &r = min(m,n)} Ax = \ oiuiviT - if ker Ax=Ar=A because there is no improvement after Ar as there are no or after or - Furthermore, there is the most change in the smaller A approximations because they have the largest o's - Storage savings with SVD - Amon where m and n are both large - mxn entries need to be sowed - Rank 1 good enough - Save o, u, V, T 2m+n+13 - Rank 2 good enough - save 0, 02, U1, U2, V,T, V2T {2(m+n+1)} - Rank k good enough - Save o ... o k, U ... U k, VIT ... V x & K (m+n+1)} - example M=10,000 n=10000 rank=100 mxn=108 m+nH= 20001 Savings for rank 1 approx: 20001 = 0.5×104

Amxn

savings for rank & approximation F×50,000 K=100... Savings 50 times while still 1=100 obtaining full A matrix A100 = A ... instead of Saving 108 values, you are saving 100*(m+n+1) ~ 100*(20000) ~ 2×10°