# Homework 1

## MA 506 Probability and Statistical Inference: Fall 2021

### Due: September 17 (Friday), 11:59pm

### Question 1: (20 points)

Fibonacci sequence $(F_n)$, is defined as a sequence of numbers such that each number is the sum of two preceding ones starting from 0 and 1 ($F_0 = 1$ and $F_1 = 1$). Hence, we have the following value at $n^{th}$ position

$$F_n = F_{n-1} + F_{n-2}$$

Write a python function that takes the number of elements the user wants in the generated Fibonacci sequence as a parameter. The function should return the generated Fibonacci sequence when called.

### Question 2: (20 points)

Write a function in python that takes a list of numbers from the user and returns a different list which only contains those numbers from the original list that are within the range $[\mu - \sigma, \mu + \sigma]$. Here $\mu$ and $\sigma$ are the mean and standard deviation of numbers in the original list.

### Question 3: (30 points)

Write a function in python that takes a list (or an 1 dimension array) from the user as a parameter and

1. Inserts a 0 at the second last position. For example if list was [0,1,2,3], then the modified list would be [0,1,2,0,3]

2. Then checks for all the duplicates in the list and removes them. For example if the list was [0,1,2,0,3], then after removal it will become [0,1,2,3], i.e. always keeps the first instance of the number to be removed.

3. Then finds the location of the 3rd smallest number in the list. For example if the list was [0,1,2,3], the location would be 2.

The function should return the final modified list and the location found in part 3.

# Question 4: (30 points)

Write a python function that takes a numpy array A (square of size n) and a positive integer (k) from the user. The function should return the rank-k approximation $A^{'}$ of the received array. The functions should also plot the decay of the 2-norm error $||A - A^{'}||_2$ versus the approximation order (rank) for $A^{'}$ and clearly indicate the position of rank-k approximation on the plot.