Exercise 5

Monday, November 13, 2023 5:12 PM

5. $x = 3.5467$ $f(x) = \sin(x)$

$f'(x) = \cos(x)$ $\dfrac{f(x+h) - f(x-h)}{2h}$

$h = 0.0001$ $f'(3.5467) = -0.9190600954$

Iter #1 $\dfrac{f(3.5468) - f(3.5466)}{2(0.0001)} = -0.9190600939$

Iter #2 $h = \frac{h}{2} = 0.00005$

$\dfrac{f(3.54675) - f(3.54665)}{2(0.00005)} = -0.919060095$

Iter #3 $h = \frac{h}{2} = 0.000025$

$\dfrac{f(3.546725) - f(3.546675)}{2(0.000025)} = -0.919060096$

Iter #4 $h = \frac{h}{2} = 0.0000125$

$\dfrac{f(3.5467125) - f(3.5466875)}{2(0.0000125)} = -0.919060096$

Iter #5 $h = \frac{h}{2} = 0.00000625$

$\dfrac{f(3.54670625) - f(3.54669375)}{2(0.00000625)} = -0.919060096$

Richardson's interpolation: $N = 5$ $x = 3.5467$

$D(i,0) = \dfrac{f(x+h) - f(x-h)}{2(h_i)}$ $h(i) = \dfrac{h}{2^i}$

$D(i,j) = D(i,j-1) + \dfrac{D(i,j-1) - D(i-1,j-1)}{4^j - 1}$

$N = 0$

$D(0,0) = \dfrac{f(3.5468) - f(3.5466)}{2(0.0001)} = -0.9190600939$

$D(1,0) = \dfrac{f(3.54675) - f(3.54665)}{\ldots} = -0.919060095$

$$D(1,0) = \frac{f(3.54675)-f(3.54665)}{0.0001} = -0.919060095$$

$$D(1,1) = D(1,0) + \frac{D(1,0)-D(0,0)}{4-1} = -0.9190600954$$

$$D(2,0) = \frac{f(3.546725)-f(3.546675)}{0.00005} = -0.919060096$$

$$D(2,1) = D(2,0) + \frac{D(2,0)-D(1,0)}{4-1} = -0.9190600963$$

$$D(2,2) = D(2,1) + \frac{D(2,1)-D(1,1)}{4^2-1} = -0.9190600964$$

$\frac{h}{8}$

$$D(3,0) = \frac{f(3.5467125)-f(3.5466875)}{0.000025} = -0.919060096$$

$$D(3,1) = D(3,0) + \frac{D(3,0)-D(2,0)}{4-1} = -0.919060096$$

$$D(3,2) = D(3,1) + \frac{D(3,1)-D(2,1)}{4^2-1} = -0.919060096$$

$$D(3,3) = D(3,2) + \frac{D(3,2)-D(2,2)}{4^3-1} = -0.919060096$$

$$D(4,0) = \frac{f(3.54670625)-f(3.54669375)}{0.0000125} = -0.919060096$$

$$D(4,1) = D(4,0) + \frac{D(4,0)-D(3,0)}{4-1} = -0.919060096$$

$$D(4,2) = D(4,1) + \frac{D(4,1)-D(3,1)}{4^2-1} = -0.919060096$$

$$D(4,3) = D(4,2) + \frac{D(4,2)-D(3,2)}{4^3-1} = -0.919060096$$

$$D(4,4) = D(4,3) + \frac{D(4,3)-D(3,3)}{4^4-1} = -0.919060096$$

Richardson's interpolation is more accurate because it got the derivative in 1 iteration

5b. $f(x) = 1 + \ln x$ $\quad x = 3.5467$

$f'(x) = \frac{1}{x}$

$f'(3.5467) = 0.2819522373$

$h = 0.0001$

#1 $$\frac{f(3.5468)-f(3.5466)}{2(0.0001)} = 0.2819522375$$

Iter #2 $h = \frac{h}{2} = 0.00005$

Iter #2 $h = \frac{h}{2} = 0.00005$

$$\frac{f(3.54675) - f(3.54665)}{2(0.00005)} = 0.28195223 8$$

Iter #3 $h = \frac{h}{2} = 0.000025$

$$\frac{f(3.546725) - f(3.546675)}{2(0.000025)} = 0.281952236$$

Iter #4 $h = \frac{h}{2} = 0.0000125$

$$\frac{f(3.5467125) - f(3.5466875)}{2(0.0000125)} = 0.281952236$$

Iter #5 $h = \frac{h}{2} = 0.00000625$

$$\frac{f(3.54670625) - f(3.54669375)}{2(0.00000625)} = 0.28195224$$

$N = 5 \quad h = 0.0001$

$$D(0,0) = \frac{f(3.5468) - f(3.5466)}{0.0002} \quad 0.2819522375$$

$$D(1,0) = \frac{f(3.54675) - f(3.54665)}{0.0001} = 0.2819522373$$

$$D(1,1) = D(1,0) + \frac{D(1,0) - D(0,0)}{4-1} = 0.2819522373$$

Richardson's interpolation results in the derivative

5c, $f(x) = x^2 - 3x + 5 \qquad x = 3.5467$

$f'(x) = 2x - 3$

$f'(3.5467) = 4.0934$

$h = 0.0001$

#1. $\dfrac{f(3.5468) - f(3.5466)}{2(0.0001)} = 4.0934$

Both got the derivative in 1 iteration

$$D(0,0) = \frac{f(3.5468) - f(3.5466)}{2(0.0001)} = 4.0934$$