Assignment 3

Thursday, November 9, 2023 2:40 PM

$x_0\ y_0$ $x_1\ y_1$ $x_2\ y_2$ $x_3\ y_3$ $x_4\ y_4$

$(3,1), (1, 0.12), (0, -0.3), (4, 2), (7, 2.5)$

1. $x = 2$ $\qquad l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n-1} \frac{x - x_j}{x_i - x_j}$

$$l_0(x) = \frac{(x-1)}{(3-1)} \cdot \frac{(x-0)}{(3-0)} \cdot \frac{(x-4)}{(3-4)} \cdot \frac{(x-7)}{(3-7)}$$

$$= \frac{1}{2}(x-1) \cdot \frac{1}{3}x \cdot \frac{(x-4)}{-1} \cdot \frac{(x-7)}{-4} = \frac{5}{6}$$

$$l_1(x) = \frac{(x-3)}{(1-3)} \cdot \frac{(x-0)}{(1-0)} \cdot \frac{(x-4)}{(1-4)} \cdot \frac{(x-7)}{(1-7)}$$

$$= \frac{(x-3)}{-2} \cdot (x-0) \cdot \frac{(x-4)}{-3} \cdot \frac{(x-7)}{-6}$$

$$= \frac{5}{9}$$

$$l_2(x) = \frac{(x-3)}{(0-3)} \cdot \frac{(x-1)}{(0-1)} \cdot \frac{(x-4)}{(0-4)} \cdot \frac{(x-7)}{(0-7)}$$

$$= \frac{(x-3)}{-3} \cdot \frac{(x-1)}{-1} \cdot \frac{(x-4)}{-4} \cdot \frac{(x-7)}{-7}$$

$$= -\frac{5}{42}$$

$$l_3(x) = \frac{(x-3)}{(4-3)} \cdot \frac{(x-1)}{(4-1)} \cdot \frac{(x-0)}{(4-0)} \cdot \frac{(x-7)}{(4-7)}$$

$$= (x-3) \cdot \frac{(x-1)}{3} \cdot \frac{(x-0)}{4} \cdot \frac{(x-7)}{-3}$$

$$= -\frac{5}{18}$$

$$l_4(x) = \frac{(x-3)}{(7-3)} \cdot \frac{(x-1)}{(7-1)} \cdot \frac{(x-0)}{(7-0)} \cdot \frac{(x-4)}{(7-4)}$$

$$= \frac{(x-3)}{4} \cdot \frac{(x-1)}{6} \cdot \frac{(x-0)}{7} \cdot \frac{(x-4)}{3}$$

$$= \frac{1}{126}$$

$$L(x) = \left[\frac{1}{2}(x-1) \cdot \frac{1}{3}x \cdot \frac{(x-4)}{-1} \cdot \frac{(x-7)}{-4}\right] \cdot$$

$$+ \left[(x-3) \times (x-4)(x-7)\right](0.12)$$

$$L(x) = \left[\frac{1}{2}(x-1)\cdot\frac{1}{3}x\cdot\frac{(x-4)}{-1}\cdot\frac{(x-7)}{-4}\right]$$
$$+\left[\frac{(x-3)}{-2}\times\frac{(x-4)}{-3}\frac{(x-7)}{-6}\right](0.12)$$
$$+\left[\frac{(x-3)}{-3}\frac{(x-1)}{-1}\frac{(x-4)}{-4}\frac{(x-7)}{-7}\right](-0.3)$$
$$+\left[(x-3)\cdot\frac{(x-1)}{3}\cdot\frac{(x-0)}{4}\cdot\frac{(x-7)}{-3}\right](2)$$
$$+\left[\frac{(x-3)}{4}\frac{(x-1)}{6}\frac{x}{7}\frac{(x-4)}{3}\right](2.5)$$

$$L(2) = \frac{8}{6} + \frac{5}{9}(0.12) - \frac{5}{42}(-0.3) - \frac{5}{18}(2) + \frac{1}{126}(2.5) = \boxed{0.4}$$

2. $x = 2$ $\quad (x_0, y_0)\ (x_1, y_1)\ (x_2, y_2)\ (x_3, y_3)\ (x_4, y_4)$

$(3,1)\ (1, 0.12)\ (0, -0.3)\ (4, 2)\ (7, 2.5)$

x_i	c_0	c_1	c_2	c_3	c_4
x_0 3	1	$\frac{11}{25}$	$\frac{1}{150}$	$\frac{9}{200}$	$-\frac{19}{1200}$
x_1 1	0.12	$\frac{21}{50}$	$\frac{31}{600}$	$\frac{-11}{600}$	
x_2 0	-0.3	$\frac{23}{40}$	$\frac{-7}{120}$		
x_3 4	2	$\frac{1}{6}$			
x_4 7	2.5				

$$P_0(x) = 1$$

$$P_1(x) = P_0 + c_1(x - x_0)$$

$$c_1 = \frac{P_1(x) - P_0(x)}{x - x_0} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{0.12 - 1}{1 - 3} = \frac{-0.88}{-2} = 0.44$$

$$f[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0} = \frac{11}{25} \qquad f[x_1, x_2] = \frac{y_2 - y_1}{x_2 - x_1} = \frac{21}{50}$$

$$f[x_2, x_3] = \frac{y_3 - y_2}{x_3 - x_2} = \frac{23}{40} \qquad f[x_3, x_4] = \frac{y_4 - y_3}{x_4 - x_3} = \frac{1}{6}$$

$$f[x_2, x_3, x_4] = \frac{\frac{1}{6} - \frac{23}{40}}{7} = \frac{-7}{120} \qquad f[x_1, x_2, x_3] = \frac{\frac{23}{40} - \frac{21}{50}}{4 - 1} = \frac{31}{600}$$

$$f[x_0, x_1, x_2] = \frac{\frac{21}{50} - \frac{11}{25}}{-3} = \frac{1}{150}$$

$$f[x_0, x_1, x_2, x_3] = \frac{\frac{31}{600} - \frac{1}{150}}{4 - 3} = \frac{9}{200} \qquad f[x_1, x_2, x_3, x_4] = \frac{\frac{-7}{120} - \frac{31}{600}}{7 - 1} = \frac{-11}{600}$$

$$f[x_0, x_1, x_2, x_3, x_4] = \frac{\frac{-11}{600} - \frac{9}{200}}{7 - 3} = \frac{-19}{1200}$$

$$P(x) = 1 + \frac{11}{25}(x-3) + \frac{1}{150}(x-3)(x-1) + \frac{9}{200}(x-3)(x-1)x + \frac{-19}{1200}(x-3)(x-1)(x)(x-4)$$

$$= \frac{2}{5} = 0.4$$