# Assignment 1

Friday, September 29, 2023 3:46 PM

$$3x_1 + 4x_2 + 3x_3 = 10$$
$$x_1 + 5x_2 - x_3 = 7$$
$$6x_1 + 3x_2 + 7x_3 = 15$$

## 1. Naive Gaussian Elimination

$$-\tfrac{1}{3}\begin{bmatrix} 3 & 4 & 3 \\ 1 & 5 & -1 \\ 6 & 3 & 7 \end{bmatrix}\begin{bmatrix} 10 \\ 7 \\ 15 \end{bmatrix}$$

$-1 \quad -\tfrac{4}{3} \quad -1 \qquad -\tfrac{10}{3}$

$x_1 = 2$
$x_2 = 1$
$x_3 = 0$

$3(2) + 4(1) + 3(0) = 10$
$10 = 10$ ✓

$$-2\begin{bmatrix} 3 & 4 & 3 \\ 0 & \tfrac{11}{3} & -2 \\ 6 & 3 & 7 \end{bmatrix}\begin{bmatrix} 10 \\ \tfrac{11}{3} \\ 15 \end{bmatrix}$$

$-6 \quad -8 \quad -6 \qquad -20$

$$\tfrac{15}{11}\begin{bmatrix} 3 & 4 & 3 \\ 0 & \tfrac{11}{3} & -2 \\ 0 & -5 & 1 \end{bmatrix}\begin{bmatrix} 10 \\ \tfrac{11}{3} \\ -5 \end{bmatrix}$$

$5 \quad -\tfrac{30}{11} \qquad 5$

$$\begin{bmatrix} 3 & 4 & 3 \\ 0 & \tfrac{11}{3} & -2 \\ 0 & 0 & -\tfrac{19}{11} \end{bmatrix}\begin{bmatrix} 10 \\ \tfrac{11}{3} \\ 0 \end{bmatrix}$$

$x_3 = 0$

$\frac{11}{3} = \frac{11}{3}x_2 - 2(0)$

$x_2 = 1$

$10 = 3x_1 + 4(1) + 3(0)$

$x_1 = 2$

## 2. Scaled Partial Pivoting

$$3x_1 + 4x_2 + 3x_3 = 10$$
$$x_1 + 5x_2 - x_3 = 7$$
$$6x_1 + 3x_2 + 7x_3 = 15$$

Pivot ↓

$$\begin{matrix} 1 \\ \frac{1}{5} \\ \frac{6}{7} \end{matrix}\begin{bmatrix} \textcircled{3} & 4 & 3 \\ 1 & \textcircled{5} & -1 \\ 6 & 3 & \textcircled{7} \end{bmatrix}\begin{bmatrix} 10 \\ 7 \\ 15 \end{bmatrix}$$

$st = [3, 5, 7]$

$-6 \quad -8 \quad -6 \qquad -20$
$-1 \quad -\tfrac{4}{5} \quad -1 \qquad -\tfrac{10}{3}$

$\frac{5}{7}$ $\begin{bmatrix} 6 & 5 & 1 \end{bmatrix}$ $\begin{bmatrix} 15 \end{bmatrix}$

$-6$ $-8$ $-6$ $-20$

$-2$ $-\frac{1}{3}$ $\begin{bmatrix} 3 & 4 & 3 \\ 1 & 5 & -1 \\ 6 & 3 & 7 \end{bmatrix}$ $\begin{bmatrix} 10 \\ 7 \\ 15 \end{bmatrix}$

$-1$ $-\frac{4}{3}$ $-1$ $\frac{-10}{3}$

Pivot

$\frac{15}{7}$ $\frac{11}{3}$ $\begin{bmatrix} 3 & 4 & 3 \\ 0 & \frac{11}{3} & -2 \\ 0 & -5 & 1 \end{bmatrix}$ $\begin{bmatrix} 10 \\ \frac{11}{3} \\ -5 \end{bmatrix}$ $x_1 = 2$

st=[3,5,7]

Index=[1,2,3]

$\frac{15}{11}$ $\begin{bmatrix} 5 & \frac{-30}{11} \\ 0 & \frac{11}{3} & -2 \\ 0 & -5 & 1 \end{bmatrix}$ $\begin{bmatrix} 5 \\ \frac{11}{3} \\ -5 \end{bmatrix}$

$\begin{bmatrix} 0 & \frac{11}{3} & -2 \\ 0 & 0 & \frac{-19}{11} \end{bmatrix}$ $\begin{bmatrix} \frac{11}{3} \\ 0 \end{bmatrix}$ $x_2 = 1$

$x_3 = 0$