Exercise 6

Tuesday, November 14, 2023 2:40 PM

6a. $f(x) = \sin(x)$ $a = 1$ $b = 5$

$$\int_1^5 (\sin(x))dx = \Big|_1^5 -\cos(5) + \cos(1) = 0.2566401204$$

#1 $x_0 = 1$ $x_1 = 5$ $h = 4$

$$A = \frac{f(1) + f(5)}{2}(4) = -0.2349065797$$

#2 $x_0 = 1$ $h = \frac{4}{2} = 2$

$x_1 = x_0 + h = 1 + 2 = 3$

$$A_1 = \frac{f(1) + f(3)}{2}(2) = 0.9825909929$$

$x_2 = x_1 + h = 3 + 2 = 5$

$$A_2 = \frac{f(3) + f(5)}{2}(2) = -0.8178042666$$

$A = A_1 + A_2 = 0.1647867263$

$\Delta = -0.399693306$

#3. $h = \frac{2}{2} = 1$

$x_0 = 1$

$x_1 = 1 + 1 = 2$

$$A_1 = \frac{f(1) + f(2)}{2}(1) = 0.8753842058$$

$x_2 + x_1 + h = 2 + 1 = 3$

$$A_2 = \frac{f(2) + f(3)}{2} = 0.5252087174$$

$x_3 = x_2 + h = 3 + 1 = 4$

$$A_3 = \frac{f(3) + f(4)}{2} = -0.3078412436$$

$x_4 = x_3 + h = 4 + 1 = 5$

$$A_4 = \frac{f(4) + f(5)}{2} = -0.857863385$$

$$\frac{\qquad}{2} = 0.857805983$$

$$A = A_1 + A_2 + A_3 + A_4 = 0.2348882946$$

$$\Delta = -0.28224 - 0.23488 = -0.517128$$

#4 $h = \frac{1}{2}$

$x_0 = 1$

$x_1 = 1 + 0.5 = 1.5$

$$A_1 = \frac{f(1) + f(1.5)}{2}(0.5) = 0.4597414929$$

$x_2 = x_1 + h = 1.5 + 0.5 = 2$

$$A_2 = \frac{f(1.5) + f(2)}{2}(0.5) = 0.4766981034$$

$x_3 = x_2 + h = 2 + 0.5 = 2.5$

$$A_3 = \frac{f(2) + f(2.5)}{2}(0.5) = 0.3769423927$$

$x_4 = x_3 + h = 2.5 + 0.5 = 3$

$$A_4 = \frac{f(2.5) + f(3)}{2}(0.5) = 0.184898038$$

$x_5 = 3 + 0.5 = 3.5$

$$A_5 = \frac{f(3) + f(3.5)}{2}(0.5) = -0.0524158049$$

$x_6 = 3.5 + 0.5 = 4$

$$A_6 = \frac{f(3.5) + f(4)}{2}(0.5) = -0.2768964307$$

$x_7 = 4.5$

$$A_7 = \frac{f(4) + f(4.5)}{2}(0.5) = -0.4335831532$$

$x_8 = 5$

$$A_8 = \frac{f(4.5) + f(5)}{2}(0.5) = -0.4841135981$$

$$A = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 = \boxed{0.2512710401}$$

$$\Delta = 0.0163827455$$

$f(x) = \sin(x)$ $a = 1$ $b = 5$ $h = 4$

$$R(0,0) = \frac{f(1) + f(5)}{2}(4) = -0.2349065797$$

$$R(1,0) = \frac{R(0,0)}{2} + 2\sum_{k=1}^{1} f(1+2) = 0.1647867262$$

$$R(1,1) = R(1,0) + \frac{R(1,0) - R(0,0)}{3} = 0.2980178283$$

$$B(1,1) = B(1,0) + \frac{B(1,0) - B(0,0)}{3} = 0.2480176283$$

$$B(2,0) = \frac{B(1,0)}{2} + 1\sum_{k=1}^{2} f(1+(2k-1)(1))$$

$$= 0.0823933631 + 1(f(2)+f(4)) = 0.2348882946$$

$$B(2,1) = B(2,0) + \frac{B(2,0) - B(1,0)}{3} = 0.2582554841$$

$$B(2,2) = B(2,1) + \frac{B(2,1) - B(1,1)}{15} = 0.2556046612$$

$$B(3,0) = \frac{B(2,0)}{2} + 0.5\sum_{k=1}^{4} f(1+(2k-1)(0.5)) =$$

$$= 0.1174441473 + 0.5(f(1.5)+f(2.5)+f(3.5)+f(4.5))$$

$$= 0.25127104$$

$$B(3,1) = B(3,0) + \frac{B(3,0) - B(2,0)}{3} = 0.2567319551$$

$$B(3,2) = B(3,1) + \frac{B(3,1) - B(2,1)}{15} = 0.2566303865$$

$$B(3,3) = B(3,2) + \frac{B(3,2) - B(2,2)}{63} = 0.2566466679$$

$$B(4,0) = \frac{B(3,0)}{2} + 0.25\sum_{k=1}^{8} f(1.25)+f(1.75)+\dots+f(4.75)$$

$$= 0.2553020587$$

$$B(4,1) = B(4,0) + \frac{B(4,0) - B(3,0)}{3} = 0.2566457366$$

$$B(4,2) = B(4,1) + \frac{B(4,1) - B(3,1)}{15} = 0.2566399834$$

$$B(4,3) = B(4,2) + \frac{B(4,2) - B(3,2)}{63} = 0.2566401357$$

$$B(4,4) = B(4,3) + \frac{B(4,3) - B(3,3)}{255} = 0.2566401101$$

$$B(5,0) = \frac{B(4,0)}{2} + (0.125)\sum_{k=1}^{16} f(1.125)+f(1.375)+f(1.625)+\dots+f(4.875)$$

$$= 0.2563058667$$

$$B(5,1) = B(5,0) + \frac{B(5,0) - B(4,0)}{3} = 0.2566404694$$

$$B(5,2) = B(5,1) + \frac{B(5,1) - B(4,1)}{15} = 0.256640186$$

$$B(5,3) = B(5,2) + \frac{B(5,2) - B(4,2)}{63} = 0.2566401207$$

$$B(5,4) = B(5,3) + \frac{B(5,3) - B(4,3)}{255} = 0.2566401206$$

$$B(5,5) = B(5,4) + \frac{B(5,4) - B(4,4)}{1023} = 0.2566401206$$

Richardson's is more accurate while the other is faster

5b. $f(x) = 1 + \ln x \quad a = 1 \quad b = 5 \quad h = 4$

$$\int_1^5 1+\ln x = x\ln(5)\Big|_1^5 = 8.047189562$$

#1 $x_0 = 1 \quad x_1 = 5 \quad h = 4$

$$A_1 = \frac{f(1)+f(5)}{2}(4) = 7.218875824$$

#2 $x_0 = 1 \quad x_1 = 3 \quad h = 2$

$$A_1 = \frac{f(1)+f(3)}{2}(2) = 3.09861229$$

$x_2 = 5$

$$A_2 = \frac{f(3)+f(5)}{2}(2) = 4.70805202$$

$$x_2 = 5$$

$$A_2 = \frac{f(3)+f(5)}{2}(2) = 4.70805202$$

$$A = A_1 + A_2 = 7.806662492$$

\# 3 $h = 1$ $x_0 = 1$ $x_1 = 2$

$$A_1 = \frac{f(1)+f(2)}{2} = 1.348573591$$

$$x_2 = 3$$

$$A_2 = \frac{f(2)+f(3)}{2} = 1.845879735$$

$$x_3 = 4$$

$$A_3 = \frac{f(3)+f(4)}{2} = 2.242453325$$

$$x_4 = 5$$

$$A_4 = \frac{f(4)+f(5)}{2} = 2.497866137$$

$$A = A_1 + A_2 + A_3 + A_4 = 7.982772788$$

\# 4

$h = 0.5$ $x_0 = 1$ $x_1 = 1.5$

$$A_1 = \frac{f(1)+f(1.5)}{2}(0.5) = 0.801366277$$

$$A_2 = \frac{f(1.5)+f(2)}{2}(0.5) = 0.7746530725$$

$$A_3 = \frac{f(2)+f(2.5)}{2}(0.5) = 0.9023549785$$

$$A_4 = \frac{f(2.5)+f(3)}{2}(0.5) = 1.003725756$$

$$A_5 = \frac{f(3)+f(3.5)}{2}(0.5) = 1.087843815$$

$$A_6 = \frac{f(3.5)+f(4)}{2}(0.5) = 1.159764333$$

$$A_7 = \frac{f(4)+f(4.5)}{2}(0.5) = 1.22259294$$

$$A_8 = \frac{f(4.5)+f(5)}{2}(0.5) = 1.278378828$$

$$A = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 = 8.0306895$$

$f(x) = 1 + \ln(x)$ $a = 1$ $b = 5$ $h = 4$

$$B(0,0) = \frac{f(1)+f(5)}{2}(4) = 7.218875829$$

$$B(1,0) = \frac{B(0,0)}{2} + h\sum_{k=1}^{1} f(1+2) = 7.806662419$$

$$B(1,1) = B(1,0) + \frac{B(1,0) - B(0,0)}{3} = 8.002591379$$

$$B(2,0) = \frac{B(1,0)}{2} + 1\sum_{k=1}^{2} f(2) + f(4) = 7.982772787$$

$$B(2,1) = B(2,0) + \frac{B(2,0) - B(1,0)}{3} = 8.041176219$$

$$B(2,2) = B(2,1) + \frac{B(2,1) - B(1,1)}{15} = 8.043906522$$

[illegible] $\frac{\ldots}{3}$ [illegible]

$$B(2,2) = B(2,1) + \frac{B(2,1) - B(1,1)}{16} = 8.043906522$$

$$B(3,0) = \frac{B(2,0)}{2} + 0.5(f(1.5) + f(2.5) + f(3.5) + f(4.5))$$

$$= 8.030684497$$

$$B(3,1) = B(3,0) + \frac{B(3,0) - B(2,0)}{3} = 8.041655067$$

$$B(3,2) = B(3,1) + \frac{B(3,1) - B(2,1)}{15} = 8.047000324$$

$$B(3,3) = B(3,2) + \frac{B(3,2) - B(2,2)}{63} = 8.047049432$$

$$B(4,0) = \frac{B(3,0)}{2} + (0.25)(f(1.25) + f(1.75) + f(2.25) + f(2.75) + \ldots + f(4.75))$$

$$B(4,1) = B(4,0) + \frac{B(4,0) - B(3,0)}{3} = 8.0471498$$

$$B(4,2) = B(4,1) + \frac{B(4,1) - B(3,1)}{15} = 8.049182782$$

$$B(4,3) = B(4,2) + \frac{B(4,2) - B(3,2)}{63} = 8.047185678$$

$$B(4,4) = B(4,3) + \frac{B(4,3) - B(3,3)}{255} = 8.047186212$$

$$B(5,0) = \frac{B(4,0)}{2} + (0.125)(f(1.125) + f(1.375) + f(1.625) + f(1.875) + \ldots + f(4.875))$$

$$= 8.046148565$$

$$B(5,1) = B(5,0) + \frac{B(5,0) - B(4,0)}{3} = 8.047186971$$

$$B(5,2) = B(5,1) + \frac{B(5,1) - B(4,1)}{15} = 8.047189404$$

$$B(5,3) = B(5,2) + \frac{B(5,2) - B(4,2)}{63} = 8.047189509$$

$$B(5,4) = B(5,3) + \frac{B(5,3) - B(4,3)}{255} = 8.047189524$$

$$B(5,5) = B(5,4) + \frac{B(5,4) - B(4,4)}{1023} = 8.047189527$$

Neither hits the value

6c. $f(x) = x^2 - 3x + 5 \qquad a = 1 \quad b = 5$

$$\int_1^5 (x^2 - 3x + 5)\,dx = \frac{x^3}{3} - \frac{3x^2}{2} + 5x \Big|_1^5 = \frac{76}{3} = 25.3333$$

#1. $x_0 = 1 \quad x_1 = 5 \qquad h = 4$

$$A = \frac{f(1) + f(5)}{2}(4) = 36$$

#2. $h=2 \quad x_0=1 \quad x_1=3$

$$A_1 = \frac{f(1)+f(3)}{2}(2) = 8$$

$x_2 = 3+2 = 5$

$$A_2 = \frac{f(3)+f(5)}{2}(2) = 20$$

$A = 8+20 = 28$
$\Delta = 36-28 = 8$

#3 $h=1 \quad x_0=1 \quad x_1=2$

$$A_1 = \frac{f(1)+f(2)}{2} = 3$$

$x_2 = 3$

$$A_2 = \frac{f(2)+f(3)}{2} = 4$$

$x_3 = 4$

$$A_3 = \frac{f(3)+f(4)}{2} = 7$$

$x_4 = 5$

$$A_4 = \frac{f(4)+f(5)}{2} = 12$$

$A = 3+4+7+12 = 26$
$\Delta = 28-26 = 2$

#4 $h=0.5 \quad x_0=1 \quad x_1=1.5$

$$A_1 = \frac{f(1)+f(1.5)}{2}(0.5) = 1.4375$$

$x_2 = 2$

$$A_2 = \frac{f(1.5)+f(2)}{2}(0.5) = 1.4375$$

$x_3 = 2.5$

$A_2 = \frac{[illegible]}{2}(0.5) = 1.[illegible]375$

$x_3 = 2.5$

$A_3 = \frac{f(2) + f(2.5)}{2}(0.5) = 1.6875$

$x_4 = 3$

$A_4 = \frac{f(2.5) + f(3)}{2}(0.5) = 2.1875$

$x_5 = 3.5$

$A_5 = \frac{f(3) + f(3.5)}{2}(0.5) = 2.9375$

$x_6 = 4$

$A_6 = \frac{f(3.5) + f(4)}{2}(0.5) = 3.9375$

$x_7 = 4.5$

$A_7 = \frac{f(4) + f(4.5)}{2}(0.5) = 5.1875$

$x_8 = 5$

$A_8 = \frac{f(4.5) + f(5)}{2}(0.5) = 6.6875$

$A = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 = \boxed{25.5}$

$\Delta = 0.5$

$h = 0.0001$ $\quad a = 1 \quad b = 5$ $\quad h_0 = 4 \quad h_1 = \frac{4}{2^1}$

$$B(0,0) = \frac{f(1) + f(5)}{2} \cdot 4 = 36$$

$$B(1,0) = \frac{B(0,0)}{2} + 2 \sum_{k=1}^{1} f(1 + (2(1) - 1)\, 2) = 28$$

$$B(1,1) = B(1,0) + \frac{B(1,0) - B(0,0)}{4 - 1} = 25.3333$$