HW 6 CprE 310 Jacob Boicken

1.) $h > -1$

$P(0) = 1 + 0 \cdot h \leq (1+h)^0 = 1 \leq 1$ = true

$P(n) = 1 + nh \leq (1+h)^n$ is assumed true

$$
\begin{aligned}
P(n+1) &= (1+h)^{n+1} \\
&= (1+h)^n \cdot (1+h) \geq (1+nh)(1+h) \\
&= 1 + nh + h + nh^2 \\
&= 1 + h(n+1) + nh^2 \\
&\geq 1 + h(n+1)
\end{aligned}
$$

$$\boxed{(1+h)^n(1+h) \geq 1 + h(n+1) + nh^2 \geq 1 + h(n+1) \quad \text{true}}$$

2.) $n > 0$ $P(1) = 4^2 + 5^1 = 16 + 5 = 21$ true (divisible by 21)

$P(n) = 4^{n+1} + 5^{2n-1}$ is divisible by 21

$$
\begin{aligned}
P(n+1) &= 4^{n+2} + 5^{2n+1} \\
&= 4 \cdot 4^{n-1} + 25 \cdot 5^{2n-1} \\
&= 4(4^{n-1} + 5^{2n-1}) + 21 \cdot 5^{2n-1}
\end{aligned}
$$

$$\boxed{\begin{aligned} &= 4 \cdot 21 \cdot a + 21 \cdot b \\ &= 21k \quad \text{so true} \end{aligned}}$$

3.) $G(1) = 0$ $G(2) = 1$ $G(3) = 3$ $G(4) = 4$ $G(5) = 6$ $G(6) = 8$

$G(4) = 4 \leq 8 - 4 = 4$ true

$G(n) \leq 2n - 4$ is assumed true

kind of told me w/ hint

$$\boxed{G(n+1) = G(n) + 2 \leq 2n - 4 + 2 = 2(n+1) - 4 \quad \text{true}}$$

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4.) $r = n - s \quad s = n - r$

$n = s + r$

Base

$n \geq 2$ only possibility

$P(2) = 1 \cdot 1 = \frac{2(1)}{2}$ ($r=1$, $s=1$)

$= 1 = 1$ true

$P(n) = rs + P(r) + P(s) = \frac{n(n-1)}{2} = \frac{(r+s)(r+s-1)}{2}$

$= rs + \binom{r}{2} + \binom{s}{2} = \binom{n}{2} = \binom{r+s}{2}$

$P(n+1) =$ case $r = r+1$

$= rs + s + \frac{(r+1)r}{2} + \frac{(s-1)s}{2} = \frac{(r+s+1)(r+s)}{2}$

$= \frac{2rs + 2s + r^2 + r + s^2 - s}{2} = \frac{r^2 + rs + r + rs + s^2 + s}{2}$

$\boxed{\text{True}}$

5.) a.) Base of 1 and every other $(P(n+2))$ is true so n is all odd numbers for $n \geq 1$.

b.) All $n \geq 1$ are true as bases are 1, 2 and $P(n)$ & $P(n+1)$ make $P(n+2)$ true. This would start at $(1,2) \to 3$ then $(2,3) \to 4$ and so on getting all positive ints.

c.) All n that are powers of 2 are true, as the base is 1 and induction is $2n$. So each value cause a 2 · self to be true. $1 \to 2 \to 4 \to 8 \to 16 \to 32 \ldots$

D.) All $n \geq 1$ are true, base 1 and induction $n+1$ so every value has to be met. $1 \to 2 \to 3 \to 4 \to 5 \to 6 \ldots$

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6.) (Psuedocode as algorithm)

```
power_mod(x, n, m):
    if x = 0: return 0;
    if n = 0: return 1;
    return ((x % m) · power_mod(x, n-1, m)) % m
```

Proof: $m > 0$

$P(0) = 1 = x^0 \% m$
$= 1$ true

$P(n) = power\_mod(x,n,m) = x^n \% m$

$P(n+1) = ((x \% m) \cdot power\_mod(x,n,m)) \% m$

$= (x \% m) \cdot (x^n \% m) \% m$

$= x^{n+1} \% m$

7.) $A = \{100 ... 999\}$

a.) $\frac{|A|}{7} = \frac{900}{7} = 128$ b.) $\frac{|A|}{2} = \frac{900}{2} = 450$

c.) $9 \cdot 1 \cdot 1 = 9$ D.) $|A| - \frac{|A|}{4} = 675$

First digit same second & third

e.) $\frac{|A|}{3} + \frac{|A|}{4} - \frac{|A|}{12} = 450$

f.) $|A| - \frac{|A|}{3} - \frac{|A|}{4} + \frac{|A|}{12} = 450$ g.) $\frac{|A|}{3} - \frac{|A|}{12} = 225$

h.) $\frac{|A|}{12} = 75$

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8.) a.) 2^{10}

b.) 3^{10}

c.) 4^{10}

d.) 5^{10}

A = set of 10
each must map
to one B
B is either 2, 3, 4, 5

So we have $A_1 \ldots A_{10}$ where $|A| = N$
so $|A_1| + \ldots |A_{10}| = |A|^{10} = N^{10}$

9.) 10 red 10 blue

a.) 5, at least ½ of the size of one set must be gone through. ie split the balls in half then at least half of one group is needed to get 3 of the same color.

b.) 13, Its possible for first 10 to be all reds so 13 are needed.

$A = \{1, 3, 5, 7, 9, 11, 13, 15\}$

10.) 5, $\rightarrow$ (1, 15), (3, 13), (5, 11), (7, 9)

We have 4 groups that combine to 16, so we will need to take 5 to ensure we get at least two in same group.