HW 3 CprE 310 Jacob Boicken

1.) a.) $B \subseteq \bar{A} \rightarrow A \cap B = \emptyset$ is true.

So if $B \subseteq \bar{A} \equiv T$, then any set B has no element that is in A.
This means $B \cap \bar{A} = B$.
We can then substitute: $A \cap B \cap \bar{A} = \emptyset$
$$B \cap \emptyset = \emptyset$$
$$\emptyset = \emptyset$$

b.) $B \subseteq C \land A \cap C = \emptyset \rightarrow A \cap B = \emptyset$ is true.

If $A \cap C = \emptyset$, then $C \subseteq \bar{A} \equiv T$.
Meaning $B \subseteq C \land C \subseteq \bar{A} \equiv T$ so $B \subseteq \bar{A} \equiv T$.
Now we have part a again making the statement true.

2.) a.) $Z^+ \subseteq Q \equiv T$, positive ints are a subset of rational numbers can be writen as any number over 1.

b.) $Q \subseteq Z \equiv F$, $\frac{1}{2} \in Q$ but $\frac{1}{2} \notin Z$

c.) $Q \cap R = Q \equiv T$, $Q \subset R$ so the matching elements are Q

d.) $Z^+ \cap R = Z^+ \equiv T$, same as part c $Z^+ \subset R$

e.) $\emptyset \subseteq N \equiv T$, an empty set is a subset of all sets

3.) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$x \in A \cup (B \cap C) = \{x \mid (x \in A) \lor [(x \in B) \land (x \in C)]\}$

By distribution $= \{x \mid [(x \in A) \lor (x \in B)] \land [(x \in A) \lor (x \in C)]\}$

$= (A \cup B) \cap (A \cup C)$

HW 3 CprE 310 Jacob Baicken

4.) $A = \{a, b, c, d\}$ $B = \{y, z\}$

$A \times B \neq B \times A$

$A \times B$ creates a set of points $\{(a,b) \mid a \in A \wedge b \in B\}$
$B \times A$ creates $\{(b,a) \mid b \in B \wedge a \in A\}$

so $A \times B = \{(a,y), (b,y), (c,y), (d,y), (a,z), (b,z), (c,z), (d,z)\}$
but $B \times A = \{(y,a), (y,b), (y,c), (y,d), (z,a), (z,b), (z,c), (z,d)\}$

5.) i.) $A \times (B - C) = \{(x,y) \mid (x \in A) \wedge (y \in B) \wedge (y \in \bar{C})\}$

$= \{(x,y) \mid [(x \in A) \wedge (y \in B)] \wedge [(x \in A) \wedge (y \in \bar{C})]\}$

$= \{(x,y) \mid (x,y) \in A \times B \wedge (x,y) \in A \times \bar{C}\}$

$= (A \times B) \cap (A \times \bar{C})$

$= (A \times B) \cap [(A \times \bar{C}) \cup (\bar{A} \times \bar{C}) \cup (\bar{A} \times C)]$

$= (A \times B) - (A \times C)$

Can use addition to make $\overline{A \times C}$ since only case that matters is $A \times \bar{C}$ when intersecting $A \times B$

side work

$\overline{A - C} = \{(x,y) \mid x \in \bar{A} \vee y \in \bar{C}\}$

This creates 3 sets that are possible where $x \in \bar{A}$, $y \in \bar{C}$ or both. So we have:

$= (\bar{A} \times C) \cup (A \times \bar{C}) \cup (\bar{A} \times \bar{C})$

ii.) $\bar{A} \times \overline{(B \cup C)} = \overline{A \times (B \cup C)}$

$\bar{A} \times \overline{(B \cup C)} \neq (\bar{A} \times (B \cup C)) \cup (A \times \overline{(B \cup C)}) \cup (\bar{A} \times \overline{(B \cup C)})$

by side work above

which are not equal
but $\bar{A} \times \overline{(B \cup C)} \subseteq \overline{A \times (B \cup C)}$

6.) $f(x,y): E \times O \to \mathbb{Z}$ $f(x,y) = x \cdot y$ $x \in E, y \in O$

One to One: no, $f(2,3) = 6$ $f(-2,-3) = 6$

Onto: no, any prime, p, other than 2.
Any p has no factors other than 1 and itself but also falls in the co-domain of $\mathbb{Z}$.

(Also all odds don't work.)

HW 3 CprE 310 Jacob Boicken

7.) $F: A \to B$ $g: B \to C$ $h: A \to C$

where $h(a) = g(f(a))$

a.) if f is onto and g is onto, then h is onto

So if we pass $\forall a \in A$ into $h(a)$, we are then passing $\forall a \in A$ into $f(a)$. Being onto, $f(a)$ returns $\forall b \in B$ at least once into $g(b)$. Being onto, $g(b)$ then would return $\forall c \in C$ at least once making $h(a)$ onto.

b.) if f and g are both onto and one to one, then h is both.

This work in the same way as part a. Passing $\forall a \in A$ in $h(a)$ & subsequently $f(a)$. Being both one to one & onto, $f(a)$ returns $\forall b \in B$ exactly once, and pass them into $g(b)$. The same happens for $g(b)$ being both one to one & onto returning $\forall c \in C$ exactly once making $h(a)$ both as well.