1.) a.) $T(n) = 2T(n/2) + c$

$$\begin{array}{cccc} & & c & \\ & c & & c \\ c & c & c & c \end{array}$$

1	n	c
2	$n/2$	$2c$
3	$n/2^2$	2^2c
$k+1$	$n/2^k$	2^kc

$n/2^k = 1 \qquad k = \log n$

$\rightarrow c(2^0+2^1+2^2 \cdots + 2^k)$

$c(2^{k+1}-1)$

$c(2^{\log n + 1}-1)$

$c(2 \cdot 2^{\log n}-1)$

$c(2n-1)$

$\boxed{\in O(n)}$

---

b.) $T(n) = 2T(n/4) + cn$

$$\begin{array}{cccc} & cn & & \\ c\frac{n}{4} & & c\frac{n}{4} & \\ c\frac{n}{16} \; c\frac{n}{16} & & c\frac{n}{16} \; c\frac{n}{16} & \\ & 8 \cdot \frac{n}{64} & & \end{array}$$

1	n	cn
2	$n/4$	$c\frac{n}{2}$
3	$n/4^2$	$c\frac{n}{4}$
4	$n/4^3$	$c\frac{n}{8}$
$k+1$	$n/4^k$	$c\frac{n}{2^k}$

$n/4^k = 1 \qquad k = \log_4 n$

$\rightarrow cn\left(\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2} \cdots \frac{1}{2^k}\right)$

$cn\left(\left(\frac{1}{2}\right)^0+\left(\frac{1}{2}\right)^1+\left(\frac{1}{2}\right)^2 \cdots \left(\frac{1}{2}\right)^k\right)$

$cn\dfrac{\left(\left(\frac{1}{2}\right)^{k+1}-1\right)}{-\frac{1}{2}}$

$2cn\left(1-\left(\frac{1}{2}\right)^{k+1}\right)$

$2cn\left(1-\left(\frac{1}{2}\right)^{\log_4 n+1}\right)$

$\leq 2cn$

$\boxed{\in O(n)}$

2.) Find_Majority(S): S is set of bank cards

```
if |S| is 1                                  | Base case
    return (true, S[0])

Divide S in half to C_1, C_2
(res_1, B_1) = Find_Majority(C_1)            | res is true if
(res_2, B_2) = Find_Majority(C_2)            |   B is majority
                                             |   2T(n/2)
if(res_1)
    num_1 = count equivalences w/ B_1 in S   | O(n)
    if(num_1 > |S|/2)  return (true, B_1)

if(res_2)
    num_2 = count eq w/ B_2 in S             | O(n)
    if(num_2 > |S|/2)  return (true, B_2)

return (false, {})                           | No majority so no card
```

Only possible for a card to be majority in S if it is a majority in one half of the cards, C_1 or C_2.

Break down until 1 card which has to be majority in S of 1 card.

Subsequent, higher levels check if majorities of halves are still majorities of larger S.

$T(n) = 2T(n/2) + Cn$

lvl	size	time
1	n	Cn
2	$n/2$	Cn
$k+1$	$n/2^k$	Cn

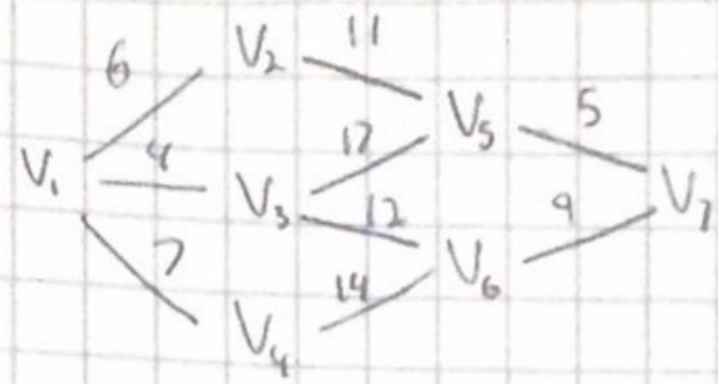
$\rightarrow (k+1)Cn$

$n/2^k = 1 \rightarrow k = \log n$

$\rightarrow Cn\log n + Cn$

$\rightarrow \in O(n\log n)$

3.)

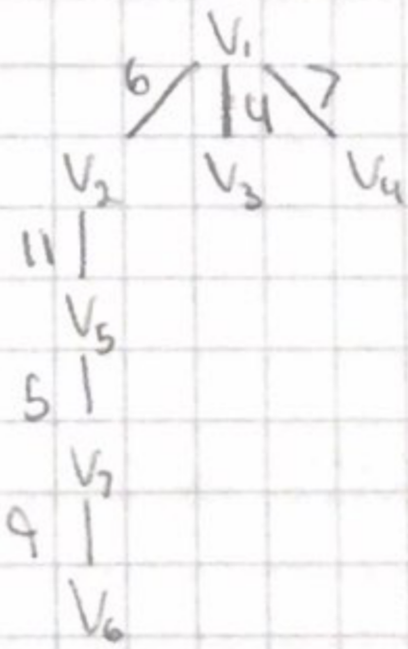


a.) Dijkstra's

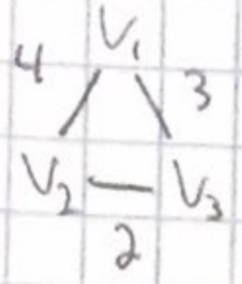
Iter	V_1	V_2	V_3	V_4	V_5	V_6	V_7
0	0	∞	∞	∞	∞	∞	∞
1	$\boxed{0}$	6	4	7	∞	∞	∞
2	$\boxed{0}$	6	$\boxed{4}$	7	21	16	∞
3	$\boxed{0}$	$\boxed{6}$	$\boxed{4}$	7	17	16	∞
4	$\boxed{0}$	$\boxed{6}$	$\boxed{4}$	$\boxed{7}$	17	16	∞
5	$\boxed{0}$	$\boxed{6}$	$\boxed{4}$	$\boxed{7}$	17	$\boxed{16}$	25
6	$\boxed{0}$	$\boxed{6}$	$\boxed{4}$	$\boxed{7}$	$\boxed{17}$	$\boxed{16}$	22
7	$\boxed{0}$	$\boxed{6}$	$\boxed{4}$	$\boxed{7}$	$\boxed{17}$	$\boxed{16}$	$\boxed{22}$

b.) Prim's

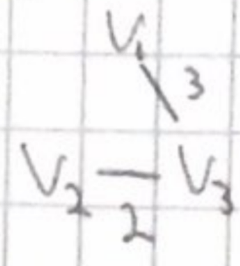
Iter	V_1	V_2	V_3	V_4	V_5	V_6	V_7
0	0	∞	∞	∞	∞	∞	∞
1	$\boxed{0}$	6^{V_1}	4^{V_1}	7^{V_1}			
2	$\boxed{0}$	6^{V_1}	$\boxed{4^{V_1}}$	7^{V_1}	17^{V_3}	12^{V_3}	
3	$\boxed{0}$	$\boxed{6^{V_1}}$	$\boxed{4^{V_1}}$	7^{V_1}	17^{V_3}	12^{V_3}	
4	$\boxed{0}$	$\boxed{6^{V_1}}$	$\boxed{4^{V_1}}$	$\boxed{7^{V_1}}$	11^{V_2}	12^{V_3}	
5	$\boxed{0}$	$\boxed{6^{V_1}}$	$\boxed{4^{V_1}}$	$\boxed{7^{V_1}}$	$\boxed{11^{V_2}}$	12^{V_3}	5^{V_5}
6	$\boxed{0}$	$\boxed{6^{V_1}}$	$\boxed{4^{V_1}}$	$\boxed{7^{V_1}}$	$\boxed{11^{V_2}}$	9^{V_7}	$\boxed{5^{V_5}}$
7	$\boxed{0}$	$\boxed{6^{V_1}}$	$\boxed{4^{V_1}}$	$\boxed{7^{V_1}}$	$\boxed{11^{V_2}}$	$\boxed{9^{V_7}}$	$\boxed{5^{V_5}}$



4.) Graph



Only MST using Prim's



Shortest Paths from V_1

V_2 – 4
V_3 – 3

a.) Disprove by counterexample

Apply Prim's on V_1 in the graph above gives us the MST $V_1 \overset{3}{-} V_3 \overset{2}{-} V_2$

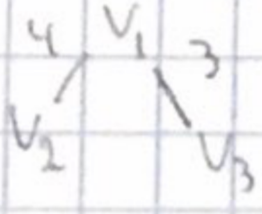
This does not contain the shortest path from $V_1 - V_2$ in the original graph.

In the original V_1 and V_2 had a weight of 4 apart in the only MST using Prim's the weight is 5.

b.) Disprove by Counterexample

If we apply Dijkstra's on V_1 in the graph above we get the following table & graph

	V_1	V_2	V_3
1	$\boxed{0}$	4^{V_1}	3^{V_1}
2	0	4^{V_1}	$\boxed{3^{V_1}}$
3	0	4^{V_1}	3^{V_1}



The graph doesn't match the only possible MST for the original graph.