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1.)

a.) All comedians are funny.

b.) All people are comedians and funny.

c.) Some people are funny only if they are comedians.

d.) Some comedians are funny.

2.)

a.) $\exists x(P(x) \land Q(x))$

b.) $\exists x(P(x) \land \neg Q(x))$

c.) $\forall x(P(x) \lor Q(x))$

d.) $\neg\exists x(P(x) \lor Q(x))$

3.) $\exists x \forall y P(x \leq y^2)$

means for a specific x value, all possible y values satisfy $x \leq y^2$

$x = 2$, $y = 1$ then $2 \leq 1 \equiv F$ so $\exists x \forall y P(x,y)$ is F
since not every y makes $x \leq y^2$ true

This above example works for all three domains, positive reals, integers, and non-zero reals so...

a.) F b.) F c.) F

4.) a.) $P(x)$: x is positive $Q(x)$: x is real

$$\begin{array}{l} P(x) \\ Q(x) \\ \hline \therefore P(x) \land Q(x) \end{array}$$

Take as truth

$$\begin{array}{l} P(a^2) \\ Q(a) \\ (P(a) \land Q(a)) \to (P(a^2) \land Q(a^2)) \\ \hline \therefore P(a) \land Q(a) \end{array} \equiv \begin{array}{l} P(a^2) \\ Q(a) \\ \neg(P(a) \land Q(a)) \lor (P(a^2) \land Q(a^2)) \\ \hline \therefore P(a) \land Q(a) \end{array}$$

There is no rule of inference that we know that can solve this. This conclusion is also false/argument is invalid.

$P(a) = F$ $Q(a) = T$ $P(a^2) = T$ $Q(a^2) = T$

$T \land T \land (F \to T) \to F$

$T \land T \land T \to F$

The logical error here is that there is an assumption that since a^2 is positive, then a is positive. Which isn't true i.e. $(-2)^2 = 4$.

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4.) b.) $P(x): x \neq 0$ $Q(x)$: x is real

given — $\dfrac{\begin{array}{c}P(x^2)\\ Q(x)\end{array}}{P(x)} \equiv P(x^2) \wedge Q(x) \rightarrow P(x)$

$$\begin{array}{l} Q(a) \\ P(a^2) \\ P(a^2) \wedge Q(a) \rightarrow P(a) \\ \hline \therefore P(a) \end{array} \qquad \begin{array}{l} Q(a) \\ P(a^2) \\ \hline \therefore P(a^2) \wedge Q(a) \end{array}$$

Conjunction Modus Ponens

$$\equiv \begin{array}{l} P(a^2) \wedge Q(a) \\ P(a^2) \wedge Q(a) \rightarrow P(a) \\ \hline \therefore P(a) \end{array} \equiv \begin{array}{l} P(a) \\ \hline \therefore P(a) \end{array}$$

This arguement is valid and it uses conjunction in combination w/ modus ponens.

5.) a.)

$$\begin{array}{l} p \vee q \\ q \rightarrow r \\ p \wedge s \rightarrow t \\ \neg r \\ \neg q \rightarrow u \wedge s \\ \hline \therefore t \end{array}$$

$p \vee q \equiv \neg p \rightarrow q \equiv \neg q \rightarrow p$

$q \rightarrow r \equiv \neg r \rightarrow \neg q$

modus ponens modus ponens simplification

$$\begin{array}{l} \neg r \\ \neg r \rightarrow \neg q \\ \neg q \rightarrow p \\ \neg q \rightarrow u \wedge s \\ p \wedge s \rightarrow t \\ \hline \therefore t \end{array} \equiv \begin{array}{l} \neg q \\ \neg q \rightarrow p \\ \neg q \rightarrow u \wedge s \\ p \wedge s \rightarrow t \\ \hline \therefore t \end{array} \equiv \begin{array}{l} p \\ u \wedge s \\ p \wedge s \rightarrow t \\ \hline \therefore t \end{array} \equiv \begin{array}{l} p \\ s \\ p \wedge s \rightarrow t \\ \hline \therefore t \end{array}$$

conjunction modus ponens

$$\equiv \begin{array}{l} p \wedge s \\ p \wedge s \rightarrow t \\ \hline \therefore t \end{array} \equiv \begin{array}{l} t \\ \hline \therefore t \end{array}$$

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5.) b.)
$$\begin{array}{l} p \to q \\ r \vee s \\ \neg s \to \neg t \\ \neg q \vee s \\ \neg s \\ \neg p \wedge r \to u \\ \underline{w \vee t} \\ \therefore u \wedge w \end{array}$$

$p \to q \equiv \neg q \to \neg p$

$\neg q \vee s \equiv q \to s \equiv \neg s \to \neg q$

$$\equiv \begin{array}{l} \neg s \\ r \vee s \\ \neg s \to \neg t \\ \neg s \to \neg q \\ \neg q \to \neg p \\ \neg p \wedge r \to u \\ \underline{w \vee t} \\ \therefore u \wedge w \end{array}$$

$$\begin{array}{l} \neg s \\ \underline{r \vee s} \\ \therefore r \end{array}$$
disjunctive syllogism

$$\begin{array}{l} \neg s \\ \underline{\neg s \to \neg t} \\ \therefore \neg t \end{array}$$
modus pones

$$\begin{array}{l} \neg s \\ \underline{\neg s \to \neg q} \\ \therefore \neg q \end{array}$$
modus ponens

$$\equiv \begin{array}{l} r \\ \neg t \\ \neg q \\ \neg q \to \neg p \\ \neg p \wedge r \to u \\ \underline{w \vee t} \\ \therefore u \wedge w \end{array}$$

$$\begin{array}{l} \neg q \\ \underline{\neg q \to \neg p} \\ \therefore \neg p \end{array}$$
Modus ponens

$$\begin{array}{l} \neg t \\ \underline{w \vee t} \\ \therefore w \end{array}$$
disjunctive syllogism

$$\begin{array}{l} \neg p \\ r \\ \underline{\neg p \wedge r \to u} \\ \therefore u \end{array}$$
conjunction

$$= \begin{array}{l} \neg p \wedge r \\ \underline{\neg p \wedge r \to u} \\ \therefore u \end{array}$$

$$\equiv \begin{array}{l} u \\ \underline{w} \\ \therefore u \wedge w \end{array} = \begin{array}{l} \underline{u \wedge w} \\ \therefore u \wedge w \end{array}$$
conjunction

6.) If r is irrational, then $\sqrt{r}$ is irrational $(p \to q)$

$\equiv$ If $\sqrt{r}$ is rational, then r is rational. $(\neg q \to \neg p)$

This means $\sqrt{r} = x = \frac{a}{b}$ thus $(\sqrt{r})^2 = r = x^2 = \frac{a^2}{b^2} = \frac{c}{d}$

where a, b, c and d are integers

so $\sqrt{r}$ and r can be represented as a ratio of integers

so $\neg q \to \neg p$ is true thus $p \to q$ is true.

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$P(x)$: x is frampy $Q(x)$: x is odd $\boxed{\text{by Contra-position}}$

7.) $\forall n(P(n) \to Q(n))$

this means $\forall n(\neg Q \to \neg P)$
so if n is even, then n is not frampy (n^2+2n is even)

$$n = 2k \rightarrow n^2+2n = 2x$$
$$(2k)^2+4k = 4k^2+4k = 2(2k^2+2k) = 2x$$

so if n is even, then n is not frampy
proving that all frampy numbers are odd

8.) $\exists x(a_x \geq A)$ $x \in \{1 \ldots, n\}$ $A = \frac{a_1 \ldots + a_n}{n}$ $\boxed{\text{by Contradiction}}$

contradiction
$\neg\exists x(a_x \geq A)$ or $\forall x(a_x < A)$ is true

Which implies $a_1 < A$, $a_2 < A$ $\ldots$, $a_n < A$

thus $a_1 + a_2 \cdots + a_n < n \cdot A \equiv \frac{a_1 \cdots + a_n}{n} < A$

$\equiv A < A \equiv F$

so $\forall x(a_x < A) \to F$ meaning at least one of $a_1 \ldots a_n$ is $\geq A$.

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$P(x)$: x is odd $Q(x)$: $5x+6$ is odd

9.) $\forall n(P(n) \leftrightarrow Q(n))$

$P(n) \leftrightarrow Q(n) \equiv (P \to Q) \wedge (Q \to P)$

$P(n)$ means $n = 2k+1$
so $5(2k+1)+6 = 10k+11 = 2(5k+5)+1 = 2x+1$

so $5n+6$ is odd when n is odd meaning $P(n) \to Q(n)$

$Q \to P \equiv \neg P \to \neg Q$

$\neg P(n)$ means $n = 2k$
so $5(2k)+6 = 10k+6 = 2(5k+3) = 2x$

so $5n+6$ is even when n is even making

$\neg P(n) \to \neg Q(n)$ thus $Q(n) \to P(n)$

10.) X is one of the six students

Y is the number of students within S who shook hands with X
meaning Y is one of $0-5$

Z is the number of students in S who haven't met X
meaning $Z = 5 - Y$

so when $Y \geq 3$, $Z < 3$ and when $Y < 3$, $Z \geq 3$

so $Y \geq 3$ or $Z \geq 3$ at one time

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when $y=3$, X shook hands A, B, & C

this makes it so that there is always a clique of as long as A B or C has shook hands with at least one of the others.
This is shown by say when A has shook hads with B then X, A, B are a clique of 3 or more. That goes for if B shook hands with C or C shook hands with A.
Thus there is always a clique 3 or more unless A, B and C have never met. However, this then makes A, B, C a cabal of 3, proving the original assertation when $y \geq 3$.

when $z \geq 3$, X hasn't met A, B, C

There is always a cabal of 3 or more as long as A, B, or C haven't at least met one of the others.
So if A hasn't met B, then X, A, B are a cabal of ≥ 3. Then, it also means a cabal of ≥ 3 exist if B doesn't know C or C doesn't know A.
So there is always a cabal unless A, B, C have shook hands, which makes a clique of 3 proving the original assertation when $z \geq 3$.