Table 2.2 Sumn	nar	y of Governing Differen	tial Equations fo	r Ideal Element	S
Type of Element		Physical Element	Governing Equation	Energy <i>E</i> or Power <i></i>	Symbol
	ſ	Electrical inductance	$v_{21} = L \frac{di}{dt}$	$E = \frac{1}{2}Li^2$	$v_2 \circ \overbrace{\qquad \qquad }^L i \circ \tau$
		Translational spring	$v_{21} = \frac{1}{k} \frac{dF}{dt}$	$E = \frac{1}{2} \frac{F^2}{k}$	$v_2 \circ \phantom{aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa$
Inductive storage		Translational spring Rotational spring	$\omega_{21} = \frac{1}{k} \frac{dT}{dt}$	$E = \frac{1}{2} \frac{T^2}{k}$	$\omega_2 \circ \bigcap^k \circ^{\omega_1}$
		Fluid inertia	$P_{21} = I \frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$	$P_2 \circ \bigcap_{i=1}^{I} Q_{i}$
		Copyright © 2011 Pearson	Education, Inc. publishing as Prenti	ce Hall	
	ſ	Electrical capacitance	$i = C \frac{dv_{21}}{dt}$	$E = \frac{1}{2}Cv_{21}^2$	$v_2 \circ \qquad \qquad i \qquad \qquad i$
	}	Translational mass	$F = M \frac{dv_2}{dt}$	2	$F \longrightarrow \underbrace{\overline{M}}_{v_1} \longrightarrow \underbrace{v_1}_{constan}$
Capacitive storage		Rotational mass			$T \longrightarrow \omega_1$ ω_1 constan
		Fluid capacitance	$Q = C_f \frac{dP_{21}}{dt}$	$E = \frac{1}{2} C_f P_{21}^2$	$Q \xrightarrow{P_2} C_f \longrightarrow I$
	l	Thermal capacitance	$q = C_t \frac{d\mathcal{I}_2}{dt}$	$E=C_t\mathcal{I}_2$	$q \xrightarrow{\mathcal{T}_2} C_1 \xrightarrow{\mathcal{T}_1} =$ constan
	ſ	Electrical resistance	$i = \frac{1}{R}v_{21}$	$\mathcal{P} = \frac{1}{R} v_{21}^2$	$v_2 \circ \xrightarrow{R} i \circ i$
		Translational damper	$F = bv_{21}$	$\mathcal{P}=bv_{21}^2$	$F \longrightarrow v_2 \longrightarrow b$
Energy dissipators	{	Rotational damper	$T = b\omega_{21}$	$\mathcal{P}=b\omega_{21}^{2}$	$T \xrightarrow{\omega_2} b$
		Fluid resistance	$Q = \frac{1}{R_f} P_{21}$	$\mathcal{P} = \frac{1}{R_f} P_{21}^2$	$P_2 \circ \longrightarrow Q$
		Thermal resistance	$q = \frac{1}{R_t} \mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t} \mathcal{T}_{21}$	$\mathcal{T}_2 \circ \overset{R_t}{\longrightarrow} \overset{q}{\circ}$

Copyright © 2011 Pearson Education, Inc. publishing as Prentice Hall

Percent Peak Overshoot Versus Damping Ratio for Second-Order System							
Damping	0.9	0.8	0.7	0.6	0.5	0.4	0.3
ratio							
Percent	0.2	1.5	4.6	9.5	16.3	25.4	37.2
overshoot							

$$T_S = 4\tau = \frac{4}{\zeta \omega_n}$$
 $T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$ $P.O. = 100e^{-\zeta \pi/\sqrt{1-\zeta^2}}$

Table 2.3 Important Laplace Transform Pairs

f(t)	F(s)
Step function, $u(t)$	$\frac{1}{s}$
step function, $u(t)$	
-at	$\frac{1}{s+a}$
in ωt	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
	5
n	$\frac{n!}{\varsigma^{n+1}}$
$d^k f(t)$	<i>S</i>
$f^{(k)}(t) = \frac{d^k f(t)}{dt^k}$	$s^k F(s) - s^{k-1} f(0^-) - s^{k-2} f'(0^-)$
dt^{κ}	$-\ldots - f^{(k-1)}(0^-)$
0	
Copyright © 2011 Pearso	n Education, Inc. publishing as Prentice Hall
$\int_{0}^{t} f(t) dt$	$\frac{F(s)}{s} + \frac{1}{s} \int_{0}^{0} f(t) dt$
$I-\infty$	$s s \int_{-\infty}^{\infty} f(s) ds$
mpulse function $\delta(t)$	1
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
	(3 , 2)
$\frac{1}{\alpha}[(\alpha-a)^2+\omega^2]^{1/2}e^{-at}\sin(\omega t+\phi),$	$\frac{s+\alpha}{(s+a)^2+\omega^2}$
0	$(s+a)^2+\omega^2$
$\phi = \tan^{-1} \frac{\omega}{\alpha - a}$	
a u	.,2
$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t,\zeta<1$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_r^2}$
V 1 5	1
$\frac{1}{t^2 + \omega^2} + \frac{1}{\omega \sqrt{a^2 + \omega^2}} e^{-at} \sin(\omega t - \phi),$	$\frac{1}{s\lceil (s+a)^2+\omega^2\rceil}$
	$s[(s+a)^2+\omega^2]$
$\phi = \tan^{-1} \frac{\omega}{a}$	
-u	ω^2
$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t+\phi),$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_ns+\omega_n^2)}$
v 1 5	$s(s + 2\zeta \omega_n s + \omega_n)$
$\phi = \cos^{-1}\zeta, \zeta < 1$	
$\frac{\alpha}{a^2+\omega^2}+\frac{1}{\omega}\left[\frac{(\alpha-a)^2+\omega^2}{a^2+\omega^2}\right]^{1/2}e^{-at}\sin(\omega t+\phi).$	$\frac{s+\alpha}{s\lceil (s+a)^2+\omega^2\rceil}$
$a^2 + \omega^2 \omega \Big[a^2 + \omega^2 \Big]$	$s[(s+a)^2+\omega^2]$

Copyright © 2011 Pearson Education, Inc. publishing as Prentice Hall