1 STUDY UNIT 1: State variable feedback systems

1.0.1 Sums

E11.3; E11.5; P11.1; P11.14; P11.16; AP11.1; AP11.2

1.1 Controllability

Formal Defenition of controllability: *A system is completely controllable if there exists an unconstrained control u(t) that can transfer any initial state $x(t_0)$ To any other desired location $x(t)$ in a finite time $t_0 \leq t \leq T$*
Controllability and observability are requirements for a system, so that all the poles of the colesd loop system can be arbitrarily placed in the complex plane. For the system:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

we can determine the controllability using the algebraic condition:

$$rank[\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A^2B} \ldots \mathbf{A^{n-1}B}] = n$$

Where $\mathbf{A}$ is a $n \times n$ matrix and $\mathbf{B}$ is an $n \times 1$ matrix for single input systems, or $n \times m$ for multi input systems. For the case of a single input single output system: define

$$\mathbf{P_c} = [\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A^2B} \ldots \mathbf{A^{n-1}B}]$$

Which is an $n \times n$ matrix. If the determinanat of $\mathbf{P_c}$ is nonzero, the system is controllable

1.2 Observability

Formal Defenition of observability: *A system is completely observable if and only if there exists a finite time T such that the intial state $x(0)$ can be determined from the observation history $y(t)$ given the control $u(t), 0 \leq t \leq T$*
Considder the single-input, single-output system:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad and \quad y = \mathbf{C}\mathbf{x}$$

Where $\mathbf{C}$ is a $1 \times n$ row vector and $\mathbf{x}$ is a $n \times 1$ column vector. The system is completely observable when the determinant of the **observability matrix**$\mathbf{P_O}$ is nonzero when:

$$\mathbf{P_O} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA^{n-1}} \end{bmatrix}$$

Which is an $n \times n$ matrix

1.3 Pole Placement

Remember from algerbra 2 that if a set of differential equations in the form $\mathbf{x}' = A\mathbf{x}$ then the time response of the system is given by $\mathbf{x}(t) = \mathbf{v}e^{\lambda t}$

1.4 Ackerman's equation

Way to calculate K with less variables. Set $u = -k\mathbf{x}$ and let $q(\lambda)$ be the desired charachteristic eqn. (as dictated by P.O ans Ts) then:

$$\mathbf{k} = [01\ldots1]\mathbf{P}^{-}1q(A);$$

Type A into calculator and apply q(A). Multiply [0 1] first since this is easier

2 STUDY UNIT 2: Mathemetical models of simple linear systems

2.1 sums

1-1; 1-2; 1-3; 1-4; 1-6; 1-7; 1-12

3 STUDY UNIT 3: The Z transform

3.1 sums

2-1; 2-2; 2-3; 2-4 ; 2-6; 2-7; 2-9a en c; 2-11; 2-12; 2-14; 2-16; 2-17; 2-18; 2-19; 2-23; 2-24; 2-25; 2-28; 2-30; 2-31; 2-32

3.2 Refresher on Mason's rule

`https://en.wikipedia.org/wiki/Mason's_gain_formula}` Type this out if you have some time later...

3.3 Discrete Time

3.4 refresher on partial fractions

$$
\begin{aligned}
T(z) &= \frac{z}{(z+a)(z+b)(z+c)} \\
T(z) &= \frac{A}{z+a} + \frac{B}{z+b} + \frac{C}{z+c} \\
A &= \left.\frac{z}{(z+b)(z+c)}\right|_{z=-a} \\
B &= \left.\frac{z}{(z+a)(z+c)}\right|_{z=-b} \\
C &= \left.\frac{z}{(z+a)(z+b)}\right|_{z=-c}
\end{aligned}
$$

3.5 Properties of the Z transform

3.5.1 Additition and subtraction

$$\mathfrak{z}[e_1(k) \pm e_2(k)] = E_1(z) \pm E_2(Z)$$

3.5.2 Multiplication by a constant

$$\mathfrak{z}[ae(k)] = a\zeta[e(k)] = aE(z)$$

3.5.3 Real translation

$$\mathfrak{z}[e(k-n)u(k-n)] = z^{-n}E(z)$$

3.5.4 Complex translation

$$\mathfrak{z}[\epsilon^{ak}e(k)] = E(z\epsilon^{-a})$$

3.5.5 Initial Value

$$e(0) = \lim_{z \to \infty} E(z)$$

3.5.6 Final Value

$$\lim_{n\to\infty} = \lim_{z\to 1}(z-1)E(z)$$

3.6 Difference equations

- The unit step function transforms to $\frac{z}{z-1}$
- The unit step function is delayed in one example in the textbook and transforms to $\frac{z}{z-1}$:

$$Z\{\delta(k-1)\} = z^{-1}\frac{z}{z-1}$$
$$Z\{\delta(k-1)\} = \frac{1}{z-1}$$

- most other transforms are in the form $c\frac{z}{z-a}$ and they transform to ca^k
- The transform of a shifted series is in the form:

$$Z\{e(k+n)u(k)\} = z^n\left[E(Z) - \sum_{k=0}^{n-1} e(k)z^{-k}\right]$$
$$e.g.$$
$$Z\{e(k-2)u(k)\} = z^{-2}E(z) - ze(1) - ze(0)$$

- sometimes the initial conditions are made zero, which makes:

$$Z\{e(k+n)u(k)\} = z^n E(z)$$
$$also:$$
$$Z\{e(k+n)u(k+n)\} = z^n E(z)$$
Regardless of initial conditions

- In some cases zero initial conditions are assuumed, but I may be confused
- So:

  Compute Z transforms

  Factorize and isolate the appropriate function (usually X(z) or Y(z))

  Apply partial fractions

  Compute inverse Z transforms to get a function of k

3.7 Simulation diagram, signal flow diagrams & State models

Basically the same as in control 1. The symbol ‘T‘ means a delay: $x(k) \rightarrow [T] \rightarrow x(k-1)$ Mason’s rule can be applied and state space models are derived in the same way.

3.8 Transfer Functions

Trasfer functions are a function of Z, and can be derived from the difference equation using the Z transform. In signal flow diagrams, [T] besomes $\frac{1}{z}$