EERI418 Lab Assignment 1: DC motor characterisation

March 10, 2017

Abstract

The goal of this lab assignment is to characterise a dc motor with a load. This entails taking measurements in order to parametrise a mathematical model of a dc motor. This model is of course required for control system design purposes. The armature voltage $v_a(t)$ is considered as the input of the motor model and the motor speed $\omega(t) = \dot{\theta}(t)$ the output. As described in the practical document set up by Prof. van Schoor the final goal is to control the motor speed according to pre-defined specifications.

1 Background

In general a dc motor can be considered as a power actuating device that delivers energy to a load, as shown in Figure 1. In Figure 1(a) an electrical and mechanical circuit diagram is shown and in Figure 1(b) a sketch of a dc motor is shown.

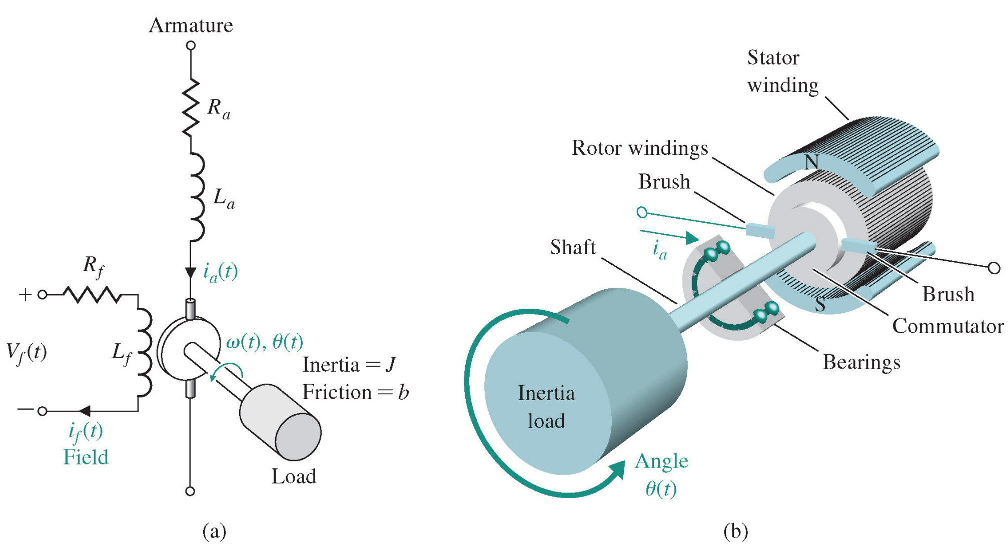

Figure 1: A DC motor, (a) electrical diagram, (b) sketch. [1]

The dc motor converts direct current electrical energy into rotational mechanical energy. A major fraction of the torque generated in the rotor (armature) of the motor is available to drive an external load.

Due to features such as high torque, speed controllability over a wide range, portability, well-behaved speed-torque characteristics, and adaptability to various types of control methods, dc motors are widely used in numerous control applications, including robotic manipulators, tape transport mechanisms, disk drives, machine tools, and servo-valve actuators.

2 Theory

The theory described in section is taken from the textbook Dorf and Bishopp [1]. A transfer function model of the dc motor will be developed for a linear approximation to an actual motor, and second-order effects, such as hysteresis and the voltage drop across the brushes, will be neglected. A fixed field voltage $v_f(t)$ will be applied and the armature voltage $v_a(t)$ will be considered as an input that will be controlled. The air-gap flux ϕ of the motor is proportional to the field current i_f, provided the field is unsaturated, so that

$$\phi = K_f i_f. \tag{1}$$

The torque developed by the motor $T_m(t)$ is assumed to be related linearly to ϕ and the armature current is given as follows:

$$T_m(t) = K_1 \phi i_a(t) = K_1 K_f i_f(t) i_a(t). \tag{2}$$

The armature-controlled dc motor uses the armature current $i_a(t)$ as the control variable. The stator field is established by a field coil and current. When a constant field current is established in a field coil, (2) can be written in frequency domain as

$$T_m(s) = (K_1 K_f I_f) I_a(s) = K_m I_a(s). \tag{3}$$

The armature current is related to the input voltage applied to the armature by

$$V(s) = (R_a + L_a s) I_a(s) + V_b(s), \tag{4}$$

where $V_b(s)$ is the back electromotive-force voltage proportional to the motor speed, R_a is the armature resistance and L_a the armature inductance. Therefore,

$$V_b(s) = K_b \omega(s) \tag{5}$$

where $\omega(s) = s\theta(s)$ is the angular speed, K_b is the back-emf constant and the armature current is given by

$$I_a(s) = \frac{V_a(s) - K_b \omega(s)}{R_a + L_a s}. \tag{6}$$

The motor torque $T_m(s)$ is equal to the torque delivered to the load. This relation can be expressed as

$$T_m(s) = T_L(s) + T_d(s), \tag{7}$$

where $T_L(s)$ is the load torque and $T_d(s)$ is the disturbance torque, which is often negligible. The load torque for rotating inertia as shown in Figure 2, may be written as

$$T_L(s) = Js^2\theta(s) + bs\theta(s), \tag{8}$$

where J is the rotor inertia and b is the friction coefficient. By rearranging (7) and substituting it in (8) represent the load torque, so that

$$T_L(s) = Js^2\theta(s) + bs\theta(s) = T_m(s) - T_d(s). \tag{9}$$

The relations for the armature-controlled dc motor are shown schematically in Figure 2. Using equations (3), (6), (9) and the block diagram, letting $T_d(s) = 0$, one can solve to

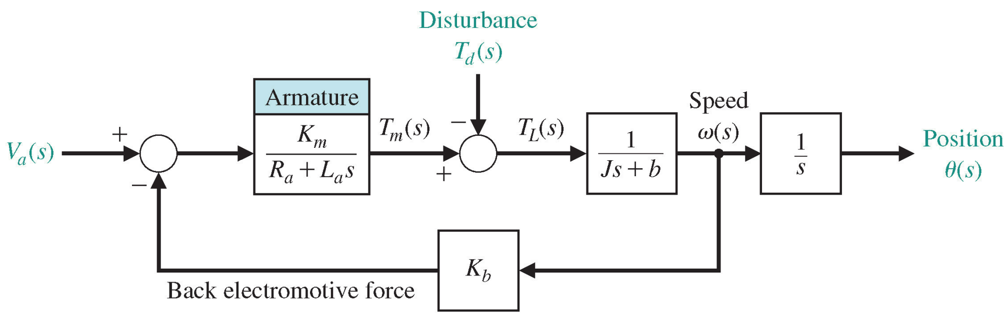

Figure 2: Armature-controlled dc motor.

obtain the transfer function

$$G(s) = \frac{\omega(s)}{V_a(s)} = \frac{K_m}{(R_a + L_a s)(Js + b) + K_b K_m}, \tag{10}$$

where $\omega(s)$ is the rotor speed and $V_a(s)$ the armature voltage. However, for many dc motors, the time constant of the armature, $\tau_a = L_a/R_a$, is negligible, which means (10) can be written as

$$G(s) = \frac{\omega(s)}{V_a(s)} = \frac{K_m}{R_a(Js + b) + K_b K_m} = \frac{K_m/(R_a b + K_b K_m)}{(\tau_l s + 1)}, \tag{11}$$

where the equivalent time constant is $\tau_l = R_a J/(R_a b + K_b K_m)$.

Note that K_m is equal to K_b. This equality may be shown by considering the steady-state motor operation and the power balance when the rotor resistance is neglected. The power input to the rotor is $(K_b\omega)i_a$, and the power delivered to the shaft is $T\omega$. In the steady-state condition, the power input is equalt to the power delivered to the shaft so that $(K_b\omega)i_a = T\omega$; since $T = K_m i_a$ is obtained from the equation $T_m(s) = K_m I_a(s)$, it is found that $K_b = K_m$.

3 Procedure

The variables and parameters described in Table 1 can be measured in the lab session. The other parameters need to be derived mathematically from the measurements obtained in the lab. The idea is to set up experiments in such a way to determine time constants τ_a and τ_l. Consult the document [2] appended or other sources you find in order to setup the experiments accordingly. The experiments are categorised in terms of the static characterisation and dynamic characterisation. From each of these characterisations it will be possible to determine the required parameters for the model. The interesting article [3] may also give you some more insight regarding the mathematical modelling of the dc motor.

Table 1: Measurable variables and parameters

Variables and parameters	Symbol and unit
Armature voltage	$v_a(t)$ $[V]$
Armature current	$i_a(t)$ [A]
Motor speed	$\omega(t) = \dot{\theta}(t)$ [rad/s]
Armature resistance	R_a [Ω]

4 Logistics

Each group needs to setup the experiments in the lab and take the required measurements for the mathematical model parametrisation. Please bring along a one pager on the different experiments you want to do and how you are going to do the measurements. This one pager will gain you access to do the lab work, otherwise you will not be allowed in the lab. A report will be required in the following week were the model is compared with the actual system data. The Lab session will be in groups, but each student will be required to write his/her own report. Also, please keep these lab reports, since it will be compiled in a final report document that will be submitted by each student. Each student will then be assessed also on the final report to determine certain ECSA level outcomes.

References

[1] R. Dorf and R. Bishop, *Modern Control Systems.* Pearson Prentice Hall, 2011.

[2] D. Vrančić, . Juričić, and T. Höfling, "Measurements and mathematical modeling of a dc motor for the purpose of fault diagnosis," *Institute of Automatic Control, Technical University of Darmstadt*, 1994.

[3] A. B. Yildiz, "Electrical equivalent circuit based modeling and analysis of direct current motors," *International Journal of Electrical Power & Energy Systems*, vol. 43, no. 1, pp. 1043–1047, 2012.

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Measurements and mathematical modeling of a DC motor for the purpose of fault diagnosis

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Table of contents

1. Introduction

Fault detection gains increasing attention in last two decades. Finding a process model turns to be still very important for implementing fault detection algorithms [1,2].

In this report a way of building up the grey-box model of DC motor is introduced. As a basis of building procedure were grey-box models made by Höfling [1] and Juričić [2]. The difference between those two models is that Höfling's approach is closer to white-box modelling and Juričić's model is closer to black-box one. This report tries to combine both approaches to achieve useful model of the DC motor that can be used in fault detection analysis. Main goal was to find a model which would satisfactorily describe behaviour of the DC motor through all of its working range.

DC motor consists of two sub-processes: electrical and mechanical. ***Electrical*** sub-process consists of armature inductance (L_a), armature resistance (R_a) and magnetic flux of the stator (Ψ). Armature current (I_a) is caused by armature voltage (U_a) on the coil (L_a). Because the motor is rotating, there is an opposite induced voltage on inductance proportional to the speed of the motor (ω) and magnetic flux (Ψ). Armature current through inductance is therefore:

$$I_a(t) = \frac{1}{L_a}\int_0^t \left(U_a(t) - R_a I_a(t) - \Psi\omega(t)\right)dt + I_a(0) \tag{1}$$

Second sub-process in motor is a ***mechanical*** one. It consists of inertia of the motor and a load (J). The difference in motor speed is caused by electromagnetic moment (M_{em}), load (M_l) and friction of the motor (M_f):

$$\omega(t) = \frac{1}{J}\int_0^t \left(M_{em} - M_l - M_f\right)dt + \omega(0), \tag{2}$$

where M_{em} is a function of armature current and M_f is a function of speed.

As a load (M_l), we used an electric brake. It has a non-linear characteristic between output torque (M_l) and input control voltage and is also speed-dependant. To have a constant torque through all the range of motor speed, we had to change input control voltage to electric brake during the experiment. All the measurements were taken in such a way that constant braking force is achieved.

2. Measurements and modeling

Static and dynamic measurements of the DC motor were performed. When measuring ***static*** characteristics of the motor, we changed armature voltage (U_a) from zero to full range (approx. 0V to 100V) in 200 seconds. Increasing of U_a is slow compared with electrical and mechanical[1] time constants, so measurement can be considered as static. Several measurements were taken, each with different load moment (M_l). It changed from 0 to 1.4 Nm in steps of 0.2 Nm. Maximum value of M_l=1.4 Nm is taken according to maximum power of the brake (400 W).

Measurements of armature current I_a and motor speed ω vs. armature voltage U_a are shown on Fig. 1 and 2.

Dynamic measurements were made by step changes of U_a through the period of 10s. There were measurements taken at the load moment of 0 Nm and 1.0 Nm. Fig. 3 and 4 show results.

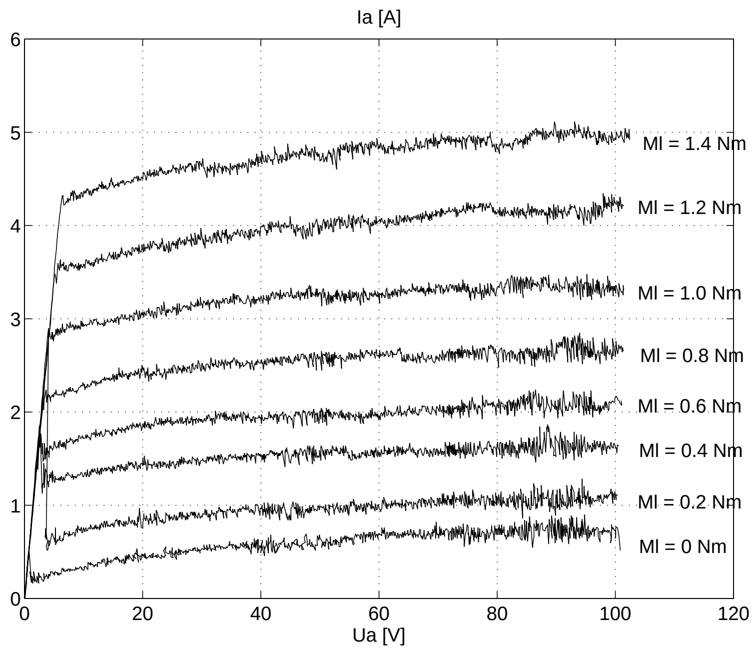

Fig. 1. Static characteristic of DC motor; $I_a = f(U_a, M_l)$

[1] Some corrections because of mechanical time constant were made when calculating M_{em} (see page 10).

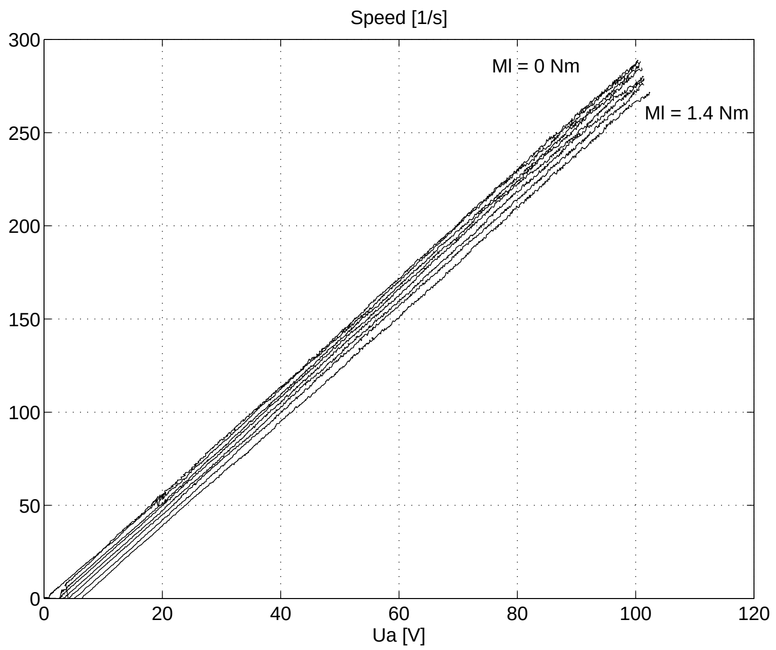

Fig. 2. Static characteristic of DC motor; $\omega = f(U_a, M_l)$

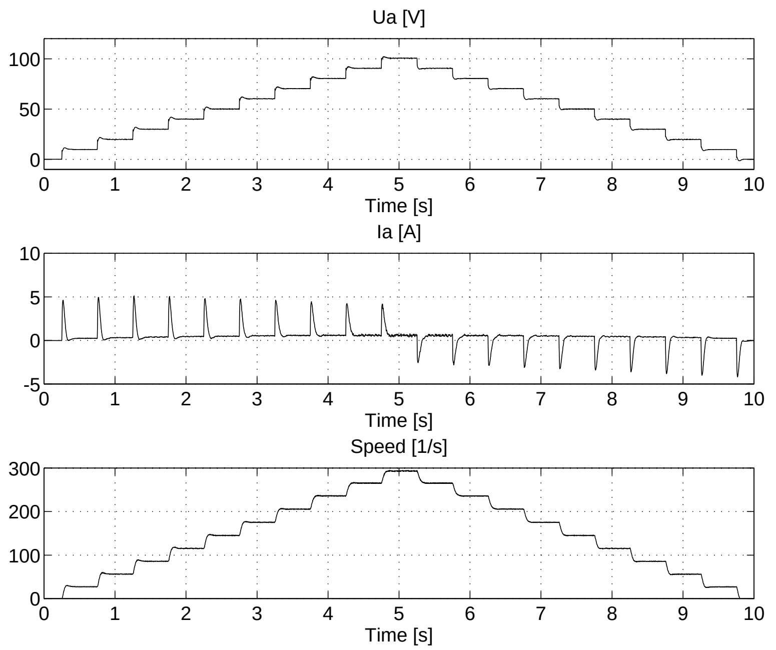

Fig. 3. Dynamic characteristic of DC motor @ $M_l = 0$ *Nm*

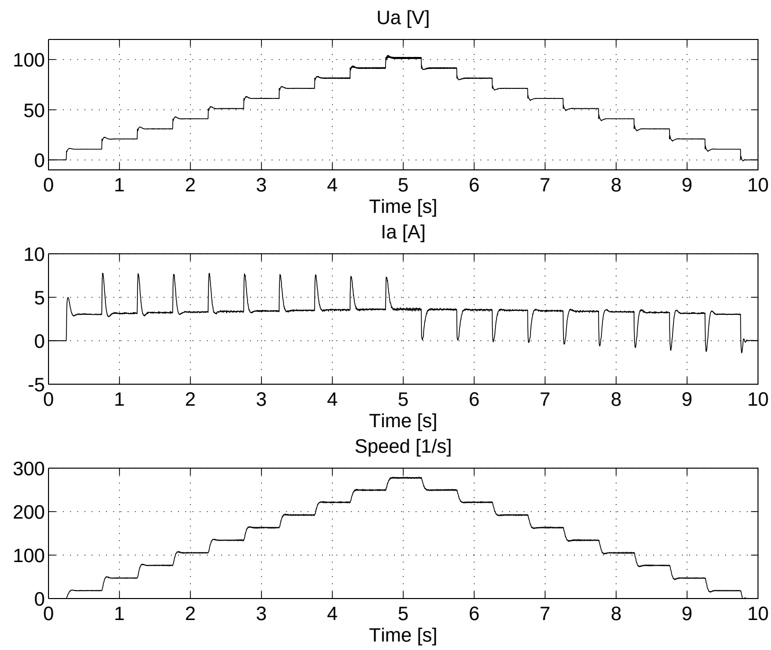

Fig. 4. Dynamic characteristic of DC motor @ M_l = 1.0 Nm

2.1 Static characteristics

From static measurements, the characteristics of $I_a = f(U_a, \omega)$, $M_f = f(\omega)$ and $M_{em} = f(I_a)$ have been calculated.

2.1.1 Characteristics of I_a

Several functions were used to describe armature current I_a:

a) $I_a' = k_1 U_a + k_2 \omega + k_3 U_a \omega$

b) $I_a' = k_1 U_a + k_2 \omega + k_3 U_a \omega + k_4 U_a{}^2 + k_5 \omega^2$

c) $I_a' = \dfrac{U_a + k_1 \omega}{k_2 + k_3 \omega}$

d) $I_a' = \dfrac{U_a + k_1 \omega + k_2 \omega^2}{k_3 + k_4 \omega}$

e) $I_a' = \dfrac{U_a + k_1 \omega}{k_2 + k_3 \omega + k_4 \omega^2}$

To find appropriate constants $\{k_i\}$, the minimum of criterion function, written in table 1, was used. Table 1 shows models and results of criterion function. We can see, the model *b* have the smallest value of criterion function, but it is much more complex and without direct physical background. We chose model *c* which have still a small value of criterion function, but model is simple and has physical background similar to mathematical model written in eq. (1) (see [1]).

$I_a = f(U_a, \omega)$	$Crit.function = \frac{1}{T_{max}} \int_0^{T_{max}} (I_a' - I_a)^2 \, dt$					
Function	k1	k2	k3	k4	k5	crit. f.
$I_a' = k_1 U_a + k_2\omega + k_3 U_a \omega$	0.6341	-0.2161	-1.938e-5			3.74e-2
$I_a' = k_1 U_a + k_2\omega + k_3 U_a \omega + k_4 U_a^2 + k_5 \omega^2$	0.6843	-0.2345	-1.371e-6	-5.710e-4	6.903e-5	2.93e-2
$I_a' = \frac{U_a + k_1\omega}{k_2 + k_3\omega}$	-0.3421	1.4723	9.602e-4			2.96e-2
$I_a' = \frac{U_a + k_1\omega + k_2\omega^2}{k_3 + k_4\omega}$	-0.3426	2.57e-6	1.4609	1.059e-3		2.96e-2
$I_a' = \frac{U_a + k_1\omega}{k_2 + k_3\omega + k_4\omega^2}$	-0.3421	1.467	1.06e-3	-2.483e-7		2.96e-2

Table 1: Mathematical models of armature current $I_a = f(U_a, \omega)$

Chosen mathematical model of I_a is therefore:

$$\boxed{I_a' = \frac{U_a - 0.3421\omega}{1.4723 + 9.602 \cdot 10^{-4}\omega}} \qquad (3)$$

Optimisation procedure based on criterion function was written in program package MATLAB and is printed in appendix. The name of used MATLAB program is *DETIA1.M*.

Fig. 5 shows how model *c* fits the actual measurements of I_a.

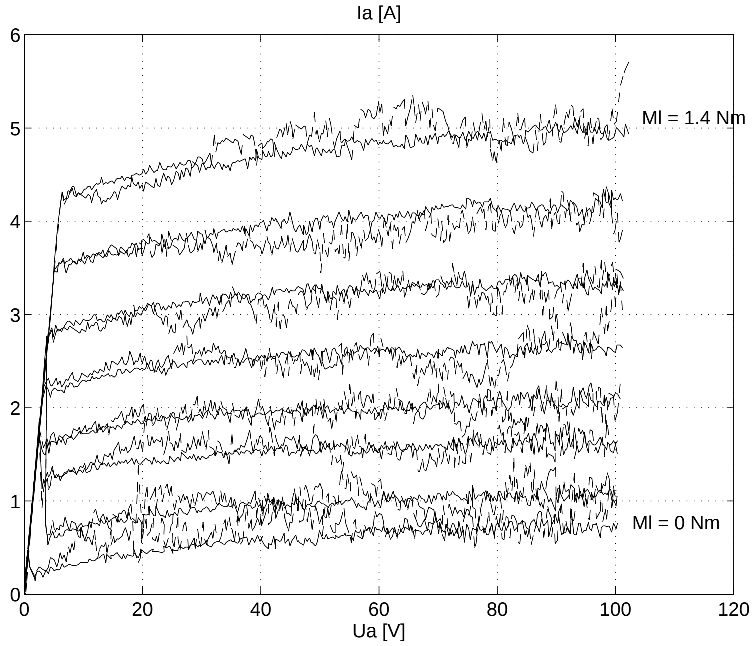

Fig. 5. Armature current (I_a); __ measurements, -- model

2.1.2 Characteristics of Mf

The second function which was determined was a moment of friction (M_f). To detect it, we added no load to the motor (M_l=0), connected a maximum voltage to the armature (U_a) and then disconnected it. That caused I_a=0 and M_{em}=0[2]. Eq. (2) changes:

$$\omega(t) = -\frac{1}{J}\int_0^t M_f(t)dt + \omega(0) \tag{4}$$

where $M_f = f(\omega)$. With a little modification of (4), we get:

$$J\big(\omega(t) - \omega(0)\big) = -\int_0^t M_f(t)dt \tag{5}$$

The measurement of speed (ω), when the armature current (I_a) was cut off is shown on Fig. 6.

[2] If there is no armature current, there should be no electromagnetic moment

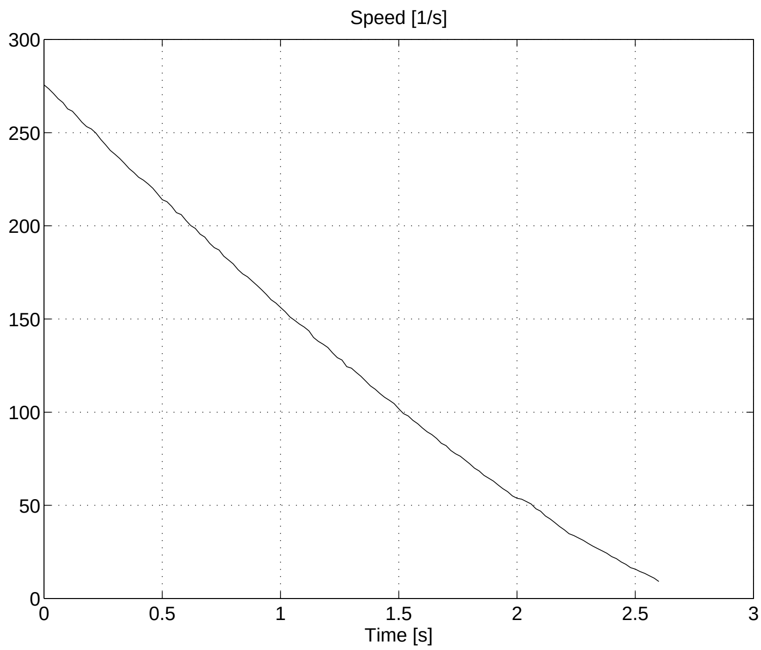

Fig. 6. Speed of the motor vs. time @ I_a=0

Three models were used to find friction moment M_f as a function of ω:

a) $M_f = k_1 + k_2\omega$

b) $M_f = k_1 + k_2\sqrt{\omega}$

c) $M_f = k_1 + k_2\omega + k_3\sqrt{\omega}$

$M_f = f(\omega)$	$Crit.function = \frac{1}{T_{max}} \int_0^{T_{max}} \left(J(\omega(t) - \omega(0)) + T\int_0^t M_f(t) \right)^2 dt$			
Function	k1	k2	k3	Crit. f.
$M_f = k_1 + k_2\omega$	0.1547	3.629e-4		1.16e-5
$M_f = k_1 + k_2\sqrt{\omega}$	0.1136	8.262e-3		4.58e-6
$M_f = k_1 + k_2\omega + k_3\sqrt{\omega}$	0.05304	-5.116e-4	0.01968	9.48e-7

Table 2: Mathematical models of the friction moment M_f=f(ω)

We can see, third model gives the best result. Fig. 7 to 9 show us measured speed of the DC motor and mathematical result when using all three models. Fig. 9 represents the result of the chosen model which obviously gives superior result.

The chosen model (*c*) of M_f is:

$$\boxed{M_f = 0.05304 - 5.116 \cdot 10^{-4}\omega + 0.01968\sqrt{\omega}} \tag{6}$$

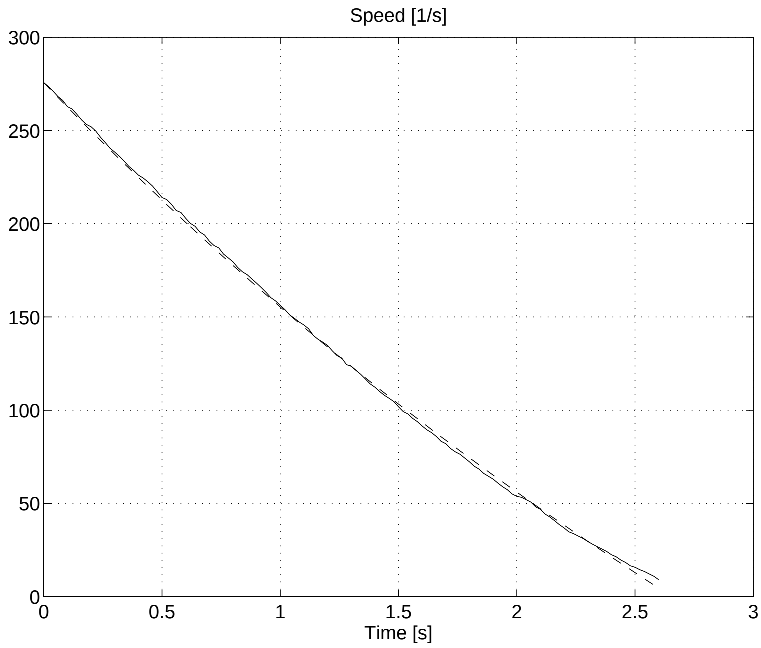

Fig. 7. Detection of M_f - model a; __ measurement of the speed, -- model

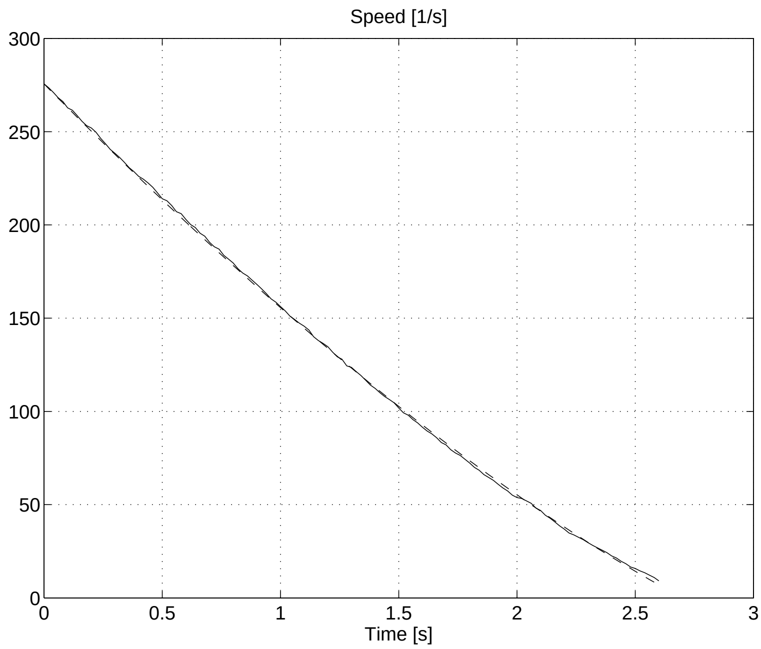

Fig. 8. Detection of M_f - model b; __ measurement of the speed, -- model

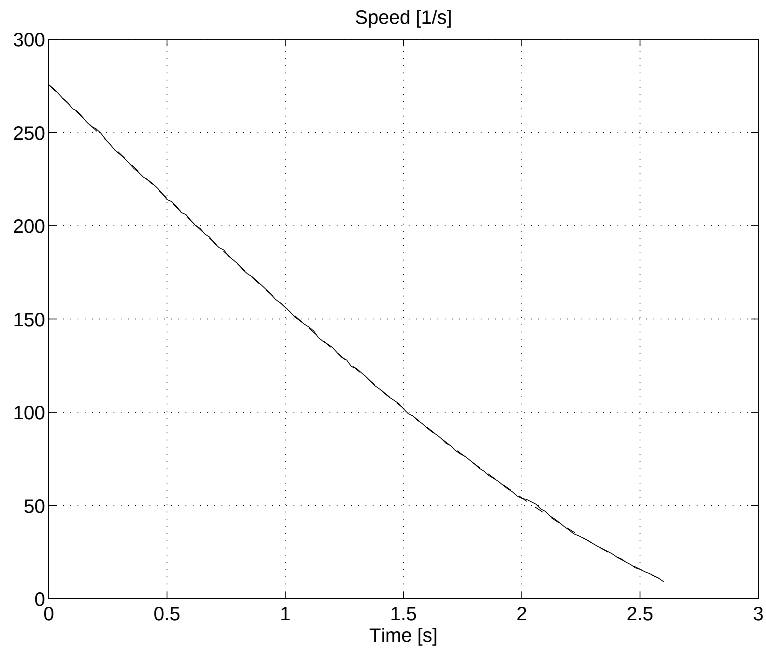

Fig. 9. Detection of M_f - model c; __ measurement of the speed, -- model

Optimisation procedure based on criterion function is given in appendix. The name of MATLAB program is *DETMF1.M*.

2.1.3 Characteristics of Mem

To made entire static model of the DC motor, we had to find an electromagnetic moment (M_{em}). It is caused by the current flowing through the coil of the rotor. We tried the following models:

a) $M_{em} = k_1 I_a$

b) $M_{em} = k_1 I_a + k_2 I_a^2$

c) $M_{em} = k_1 I_a + k_2 I_a^2 + k_3 I_a^3$

$M_{em} = f(I_a)$	$Crit.function = \frac{1}{T_{\max}} \int_0^{T_{\max}} \left(M_{em} - J\frac{d\omega}{dt} + M_l + M_f \right)^2 dt$			
Function	k1	k2	k3	Crit. f.
$M_{em} = k_1 I_a$	0.36132			5.68e-3
$M_{em} = k_1 I_a + k_2 I_a^2$	0.44737	-0.02314		8.35e-4
$M_{em} = k_1 I_a + k_2 I_a^2 + k_3 I_a^3$	0.41956	-4.1251e-3	-2.8642e-3	7.40e-4

Table 3: Mathematical models of electromagnetic moment $M_{em}=f(I_a)$

If eq. (2) is derived, we get:

$$J\frac{d\omega}{dt} = M_{em} - M_l - M_f \tag{7}$$

In a strictly static model, the left side of (7) is equal to 0. Because this is not quite true (see page 2), we add it to the criterion function in table 3 though its presence does not change the result. Because the value is so small, we used pre-defined value of J taken from Höfling [1]. We did not do it when calculating I_a (see chapter 2.1.1), because electrical time constant is much smaller compared with mechanical one.

Results obtained by three models can be seen on Fig. 10 to 12. The second model (*b*) gives the most satisfactory result. The mathematical model is therefore:

$$\boxed{M_{em} = 0.44737 I_a - 0.02314 I_a^{\ 2}} \tag{8}$$

Optimisation procedure based on criterion function is written in appendix. The name of MATLAB program is *DETMEM1.M*.

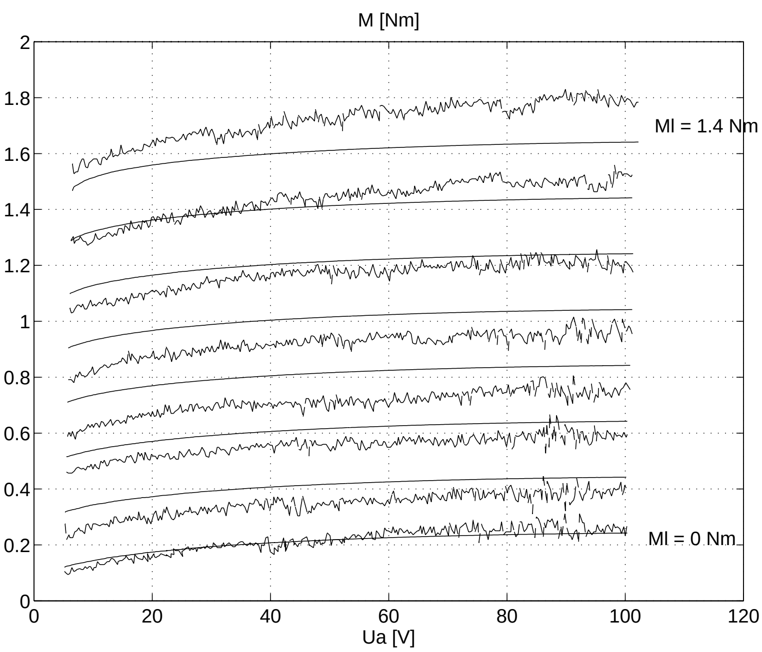

Fig. 10. Detection of M_{em} - model a; __ measurements -- model

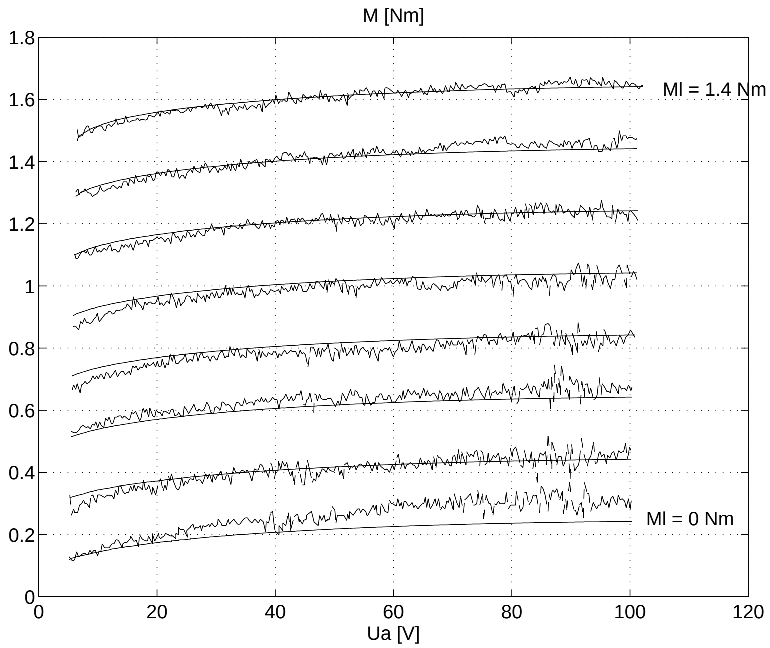

Fig. 11. Detection of M_{em} - model b; __ measurements, -- model

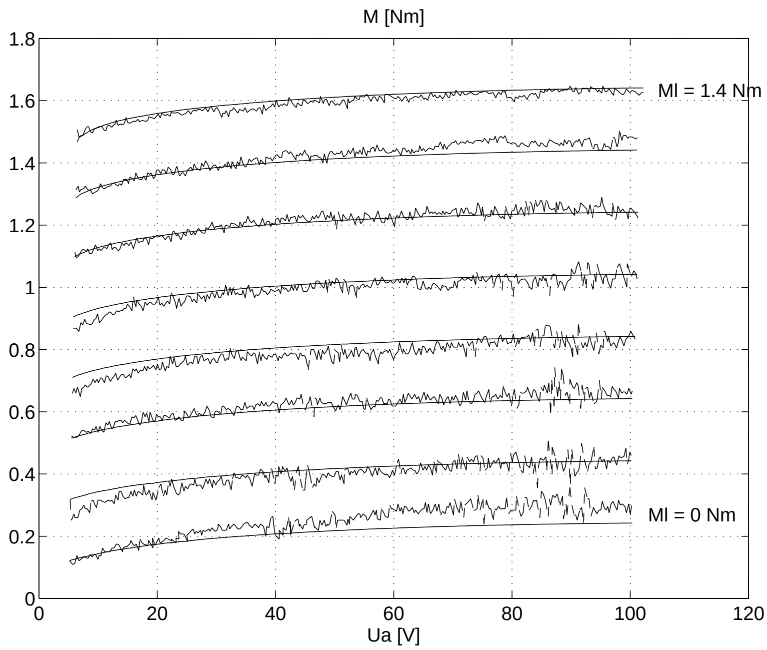

Fig. 12. Detection of M_{em} - model c; __ measurements, -- model

2.2 Dynamic characteristics

To detect dynamic constants we used dynamic measurements (see Fig. 3 and 4). There are two dynamic constants to be detected: armature inductance (L_a) and inertia (J). Eq. (1) and (2) describe system dynamics.

2.2.1 Estimation of La

Eq. (1) and (3) lead us to the next expression:

$$\begin{aligned} I_a'(t) = {} & \frac{1}{L_a}\int\left(U_a(t) - 0.34212\omega(t) - 1.4723 I_a'(t) - 9.6016\cdot 10^{-4} I_a'(t)\omega(t)\right)dt + \\ & + I_a'(0) \end{aligned} \tag{9}$$

Realised discrete algorithm to calculate I_a' can be as follows:

$$\begin{aligned} I_a'(i) = {} & I_a'(i-1) + \\ & + \frac{T_s}{L_a}\left(U_a(i) - 0.34212\omega(i) - 1.4723 I_a'(i-1) - 9.6016\cdot 10^{-4} I_a'(i-1)\omega(i)\right), \end{aligned} \tag{10}$$

where T_s is a sampling time. To find appropriate L_a, we were searching for such I_a' which would fit I_a. To diminish possible static errors, we used only dynamic measurements of the first current peak (see Fig. 3 and 13) as a used data.

Criterion function to find L_a was:

$$Crit.f. = \frac{1}{T_{\max}}\int_0^{T_{\max}}\left(I_a' - I_a\right)^2 dt \tag{11}$$

where we were searching for the minimum.

The results were L_a=11.71 mH for dynamic measurement at M_l=0Nm (see Fig. 3) and L_a=11.63mH when fitted measurements were those at M_l=1Nm (Fig. 4). How the model fits real measurement is shown on Fig. 13 and 14. As a resulting L_a, we took the mean value of the upper results and have

$$\boxed{L_a = 11.67mH} \tag{12}$$

Optimisation procedure based on criterion function is written in appendix. The name of MATLAB program is *DETLA1.M*.

Fig. 13 and 14 show us the difference between measurements and a model at M_l=0Nm and M_l=1.0Nm.

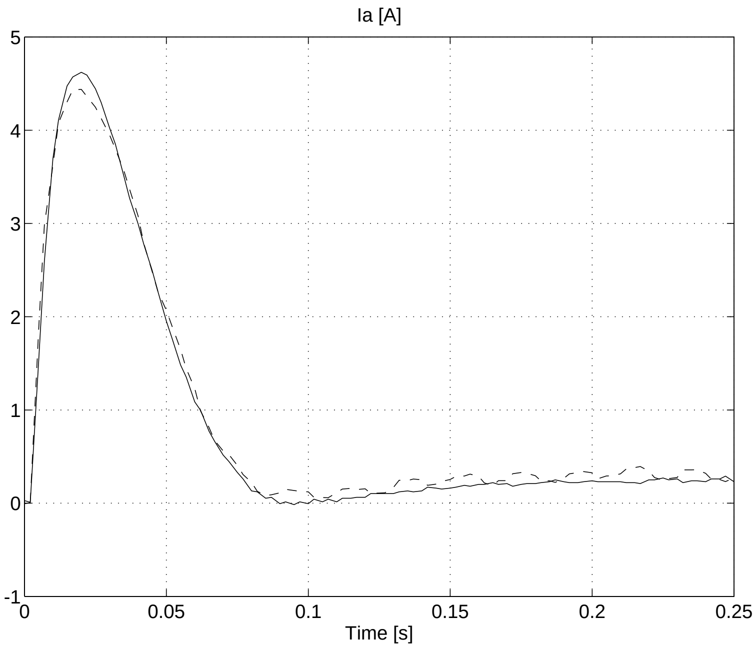

Fig. 13. Detection of L_a - L_a=11.71 mH @ M_l = 0 Nm; __ measurements, -- model

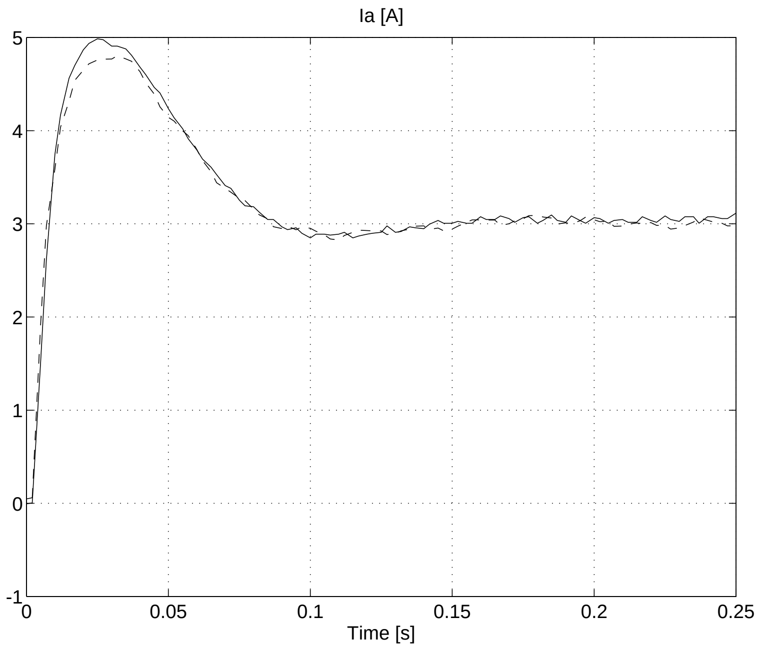

Fig. 14. Detection of L_a - L_a=11.63 mH @ M_l = 0 Nm; __ measurements, -- model

2.2.2 Detection of J

The same procedure was used to detect inertia moment (*J*). Instead of using eq. (1) and (3), we used (2), (6) and (8). We used only dynamic measurements at M_l=0Nm and we got:

$$\boxed{J = 1.889 \cdot 10^{-3}\, kgm^2} \tag{13}$$

Optimisation procedure based on criterion function is written in appendix. The name of MATLAB program is *DETJ1.M*.

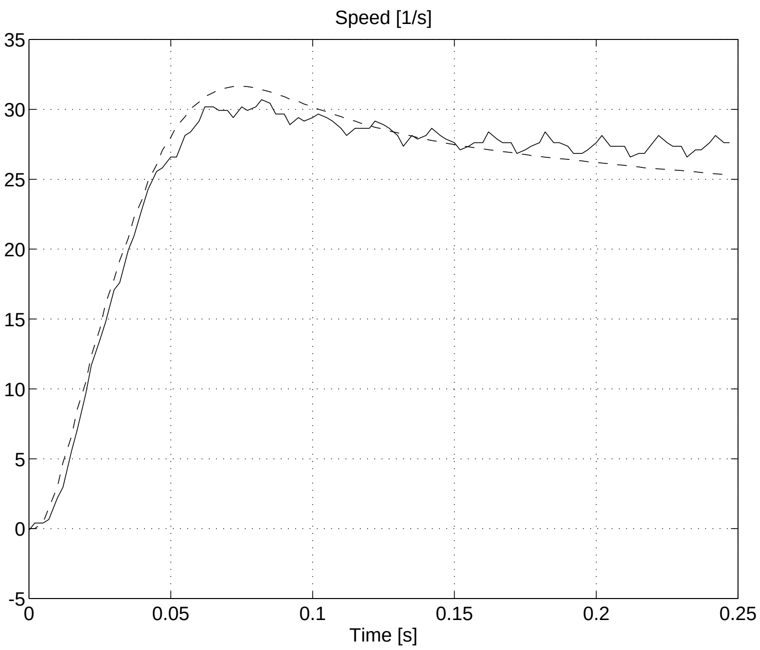

Fig. 15. Dynamic response; J=1.889 mH, M_l=0 Nm; __ measurements, -- model

2.3 Model of DC motor

The final model of the DC motor is:

$$\boxed{\begin{aligned}1.889e-3*\frac{d\omega}{dt} &= 0.44737I_a - 0.023142{I_a}^2 - \\ &\quad -0.05304 + 5.116e-4\omega - 1.968e-2\sqrt{\omega} - M_l\end{aligned}} \tag{14}$$

$$\boxed{1.167e-2\frac{dI_a}{dt} = U_a - 0.34212\omega - 1.4723I_a - 9.6016e-4I_a\omega} \tag{15}$$

The model was built and simulated in program package SIMULINK. The results, how model fits static and dynamic measurements, are shown on Fig. 15 to 27.

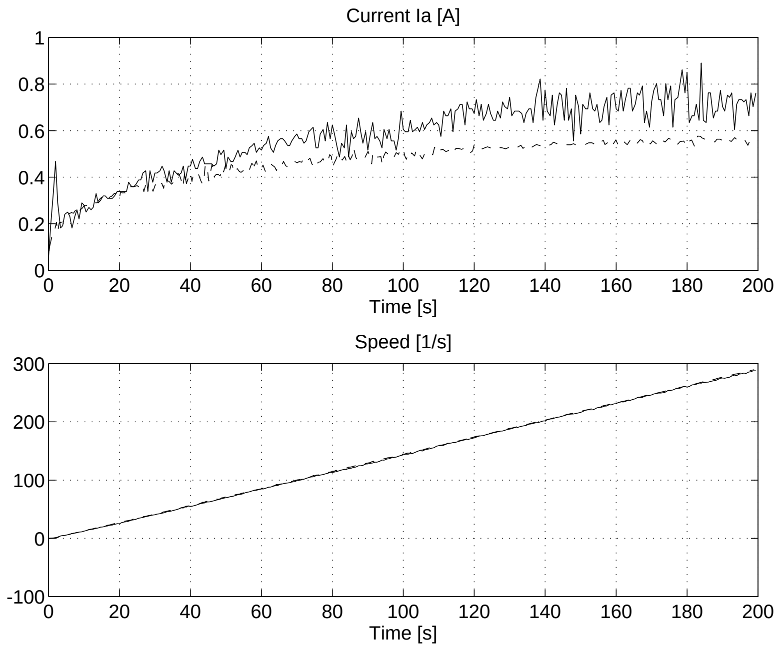

Fig. 16. Static data @ M_l = 0 Nm; __ measurements, -- model

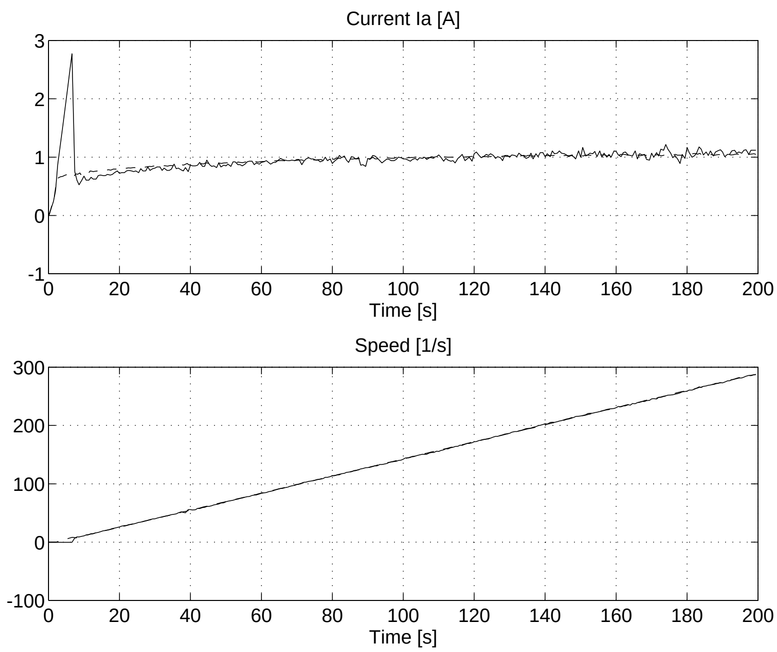

Fig. 17. Static data @ M_l = 0.2 Nm; __ measurements, -- model

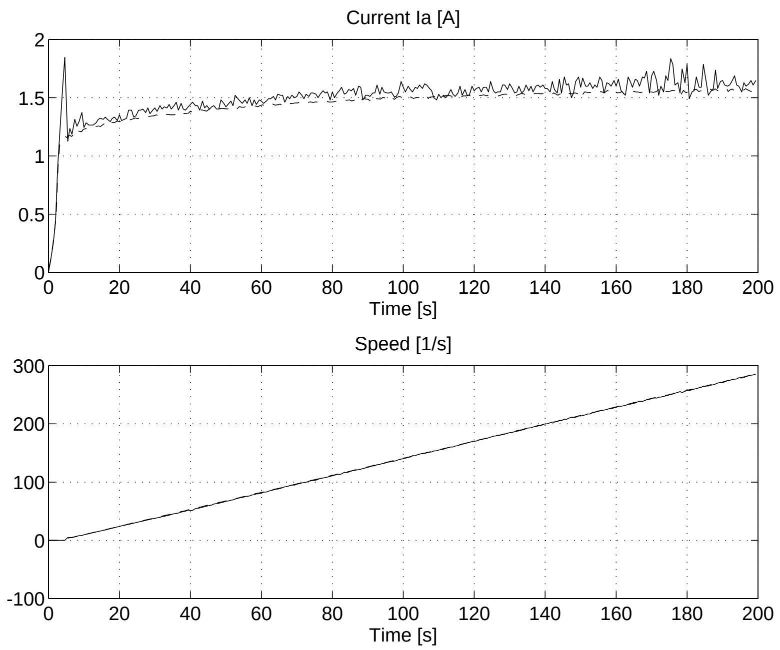

Fig. 18. Static data @ M_l = 0.4 Nm; __ measurements, -- model

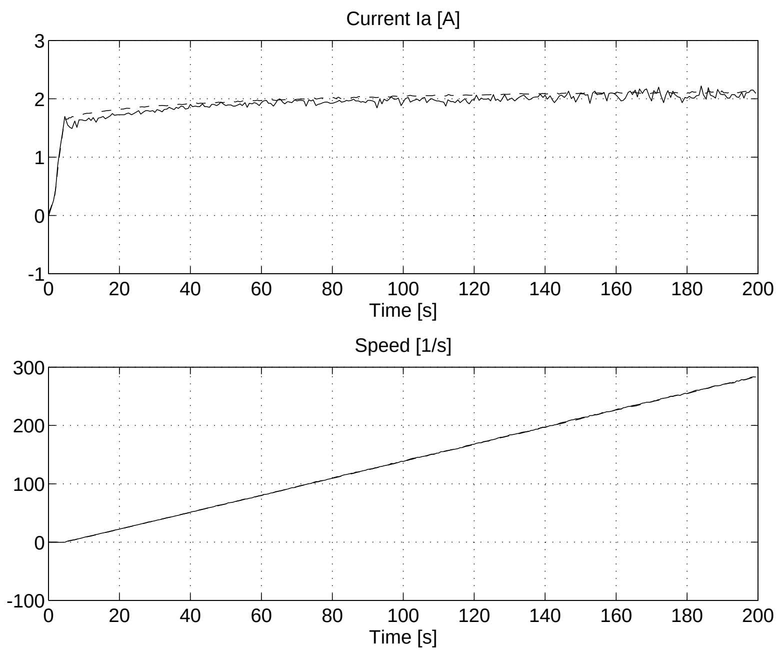

Fig. 19. Static data @ M_l = 0.6 Nm; __ measurements, -- model

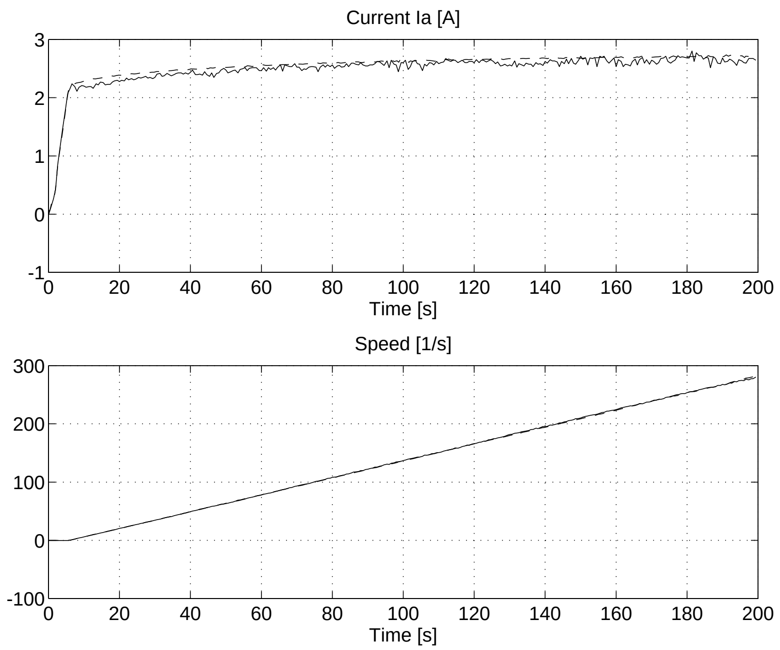

Fig. 20. Static data @ M_l = 0.8 Nm; __ measurements, -- model

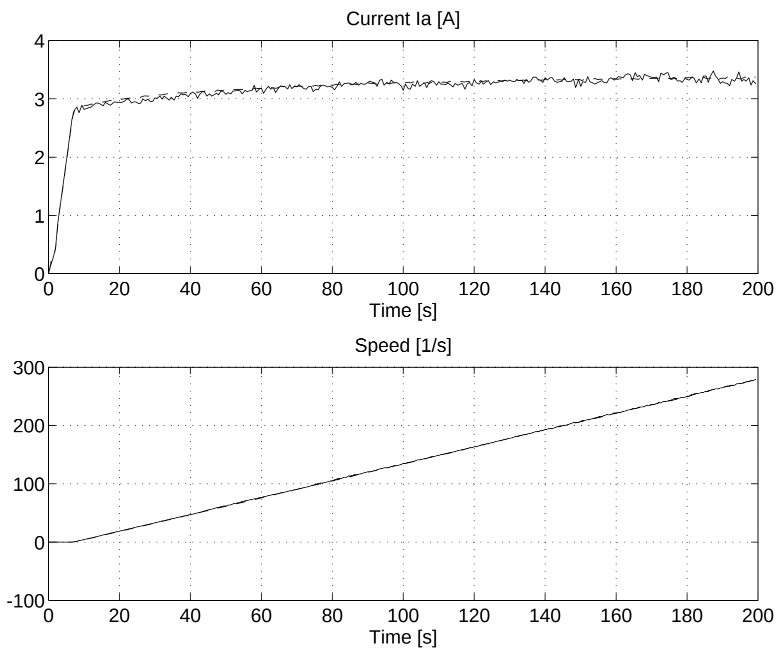

Fig. 21. Static data @ M_l = 1.0 Nm; __ measurements, -- model

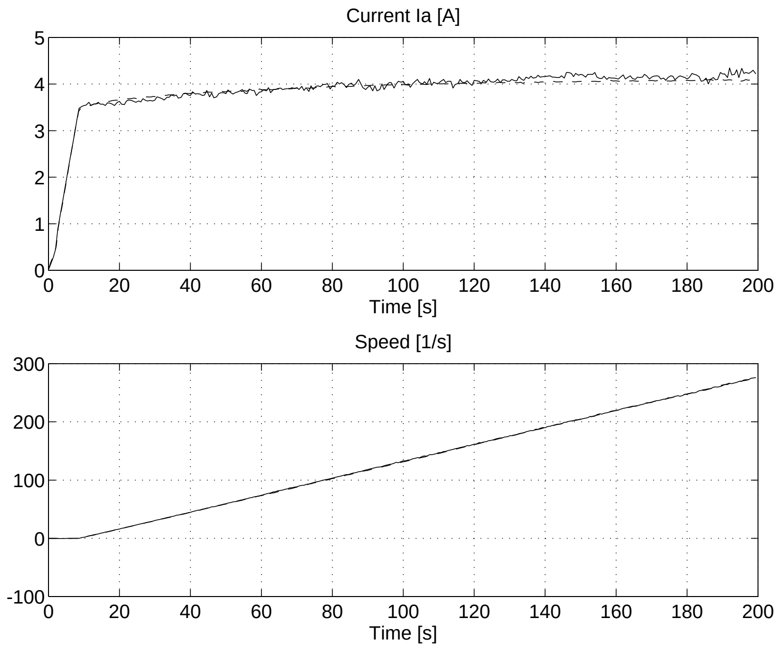

Fig. 22. Static data @ M_l = 1.2 Nm; __ measurements, -- model

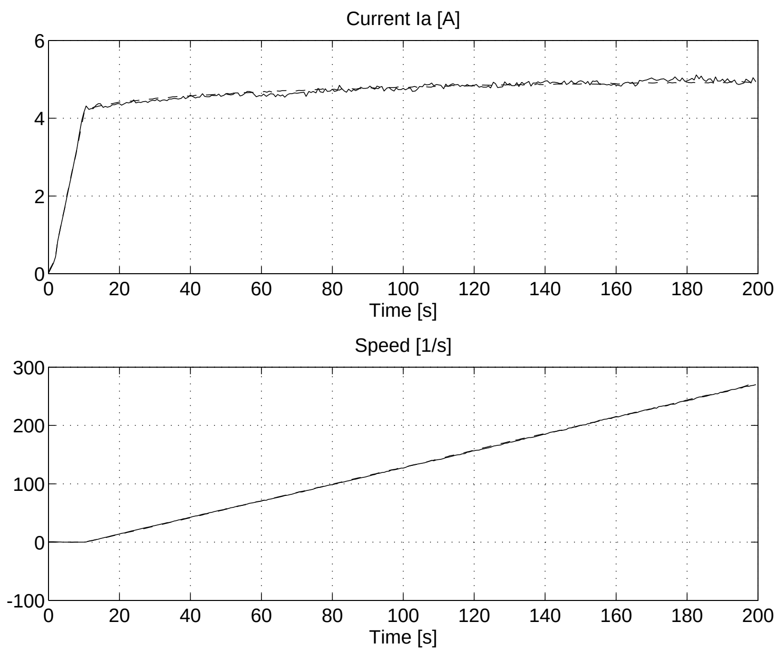

Fig. 23. Static data @ M_l = 1.4 Nm; __ measurements, -- model

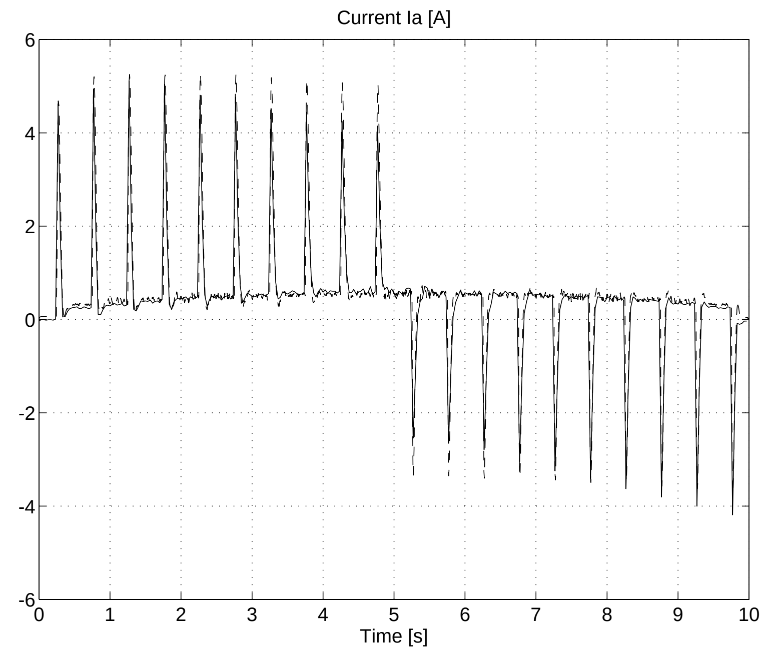

Fig. 24. Dynamic data - I_a @ M_l = 0 Nm; __ measurements, -- model

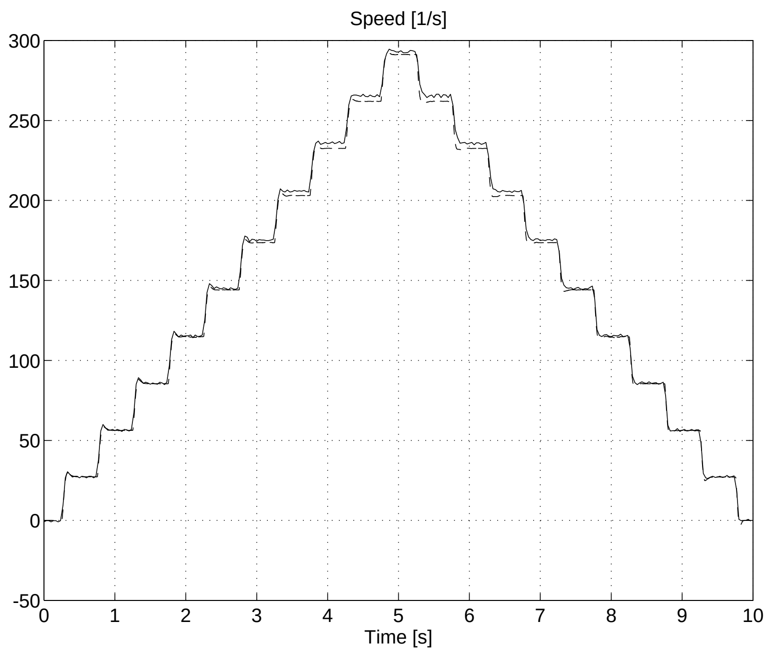

Fig. 25. Dynamic data - ω@ M_l = 0 Nm; __ measurements, -- model

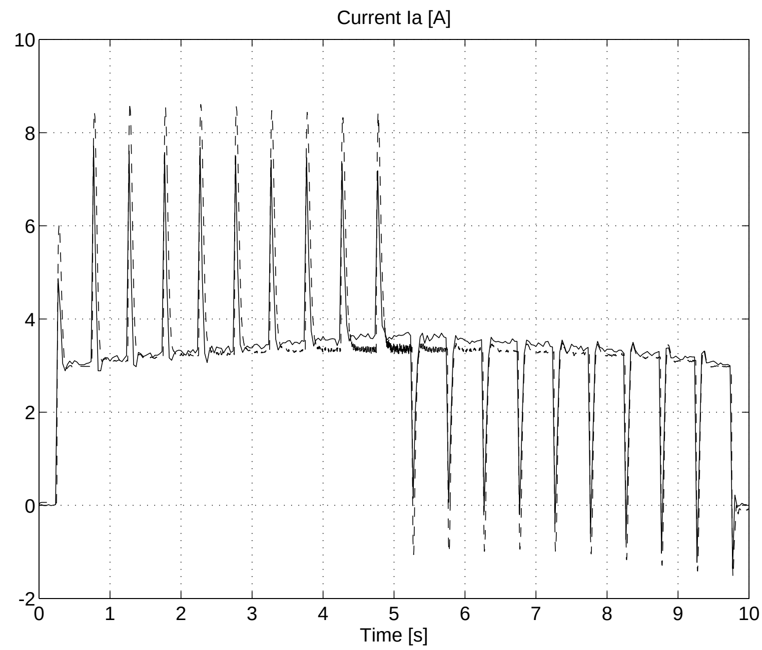

Fig. 26. Dynamic data - I_a @ M_l = 1.0 Nm; __ measurements, -- model

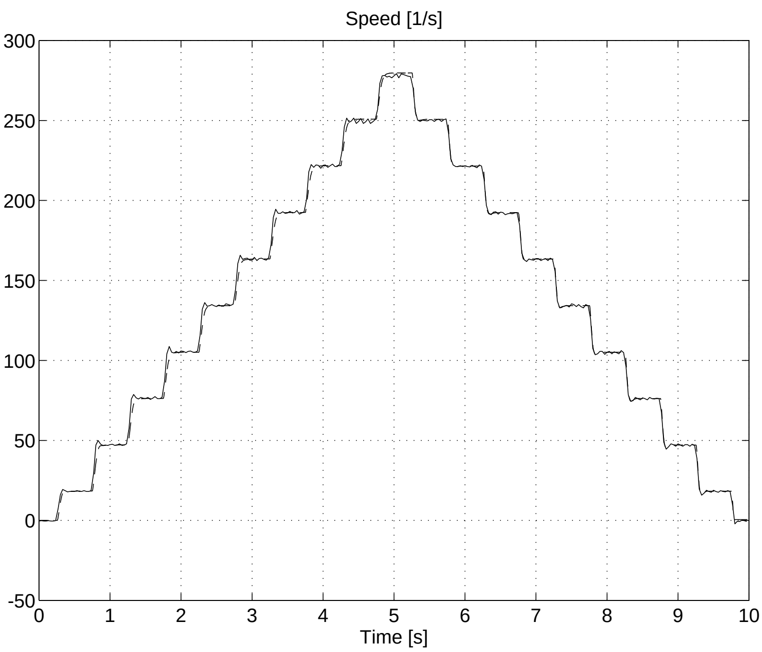

Fig. 27. Dynamic data - ω @ M_l = 1.0 Nm; __ measurements, -- model

3. Conclusions

Simulation results show that the mathematical model fits a real DC motor quite well. Results are very good when simulating static behaviour. Dynamic results are not as good as static, so more attention should be paid to build-up better dynamic model[3].

[3] The problem is in fact that the static Ia-Ua function was modeled with the quadratic function instead of the square/root function, which gives *almost perfect static and dynamic matching of the model and the measurements*. The reason is that during transition of speed, we are moving quite beyond the static identified region, where quadratic characteristic does not describe well the real situation. [This remark was written after the report was already finished]

4. Literature

[1] Höfling, "*Fault detection of a D.C. motor using grey box modelling and continuous-time parity equations*", Electrotechnical and Computer Science Conference ERK'94, Portorož, Slovenia, Vol. A, pp. 175-178, 1994.

[2] Juričić, *"Grey box identification for diagnostic purposes: An example of a DC motor*", Internal report, J. Stefan Institute, Ljubljana, Slovenia, 1994.

5. Appendix

Program REDUC.M:

```
% res = reduc(input,number);
% function reduc reduces a number of points in vector (matrix)
%
% input - input vector or matrix
% number - number of desired legth of vector (matrix)

function res = reduc(input,number);

a = size(input);
ratio = a(1)/number;
vect = [1:ratio:a(1)];
res = input(vect,:);
```

Program DETIA1.M:

```
ia = [motpc00(:,4) motpc02(:,4) motpc04(:,4) motpc06(:,4) motpc08(:,4) motpc10(:,4) motpc12(:,4) motpc14(:,4)];
ua = [motpc00(:,5) motpc02(:,5) motpc04(:,5) motpc06(:,5) motpc08(:,5) motpc10(:,5) motpc12(:,5) motpc14(:,5)];
w = [motpc00(:,6) motpc02(:,6) motpc04(:,6) motpc06(:,6) motpc08(:,6) motpc10(:,6) motpc12(:,6) motpc14(:,6)];

k = fmins('detia',[-0.34212 1.4723 9.6016e-4],[1 1e-6 1e-10 0 0 0 0 0 0 0 0 0 0 800],[],t,ia,ua,w);

ian=reduc(ia,300);
uan=reduc(ua,300);
wn=reduc(w,300);

plot(uan,ian,uan,(uan+k(1)*wn)./(k(2)+k(3)*wn),'--');
title('Ia [A]')
grid
xlabel('Ua [V]')
gtext('Ml = 0 Nm')
gtext('Ml = 1.4 Nm')
axis([0 120 0 6])
```

Program DETIA.M:

```
% function a=detia(x,t,ia,ua,w);
%
% function detia is a criterion function to determine armature current
% of DC motor.

function a=detia(x,t,ia,ua,w);

b = (ua+x(1)*w)./(x(2)+x(3)*w);

c = (b-ia).*(b-ia);
a = mean(mean(c));
```

Program DETMF1.M:

```
t=iaoff(40:170,1)-iaoff(40,1);
w=iaoff(40:170,6);
dw=w-w(1);
wint=0.02*cumsum(w);
wint=wint-wint(1);
```

```
sqw=sqrt(w);
wsqint=0.02*cumsum(sqw);
wsqint=wsqint-wsqint(1);
w2=w.*w;
w2int=0.02*cumsum(w2);
w2int=w2int-w2int(1);

k = fmins('detmf',[0.15 0 0],[1 1e-6 1e-10 0 0 0 0 0 0 0 0 0 0 500],[],dw,t,wint,w2int,wsqint);

plot(t,w,t,(-k(1)*t-k(2)*wint-k(3)*wsqint)/1.932e-3+w(1),'--');
title('Speed [1/s]')
grid
xlabel('Time [s]')
```

Program DETMF.M:

```
% function a=detmf(x,ynorm,t,wint,w2int,wsqint);
%
% function detmf is criterion function to find friction Mf of DC motor

function a=detmf(x,ynorm,t,wint,w2int,wsqint);

b = 1.932e-3*ynorm+x(1)*t+x(2)*wint+x(3)*wsqint
b = b.*b;
a = mean(b);
```

Program DETMEM1.M:

```
t = motpc00(55:1000,1);
w = [motpc00(55:1000,6) motpc02(55:1000,6) motpc04(55:1000,6) motpc06(55:1000,6) motpc08(55:1000,6)
motpc10(55:1000,6) motpc12(55:1000,6) motpc14(55:1000,6)];
ia = [motpc00(55:1000,4) motpc02(55:1000,4) motpc04(55:1000,4) motpc06(55:1000,4) motpc08(55:1000,4)
motpc10(55:1000,4) motpc12(55:1000,4) motpc14(55:1000,4)];
ua = [motpc00(55:1000,5) motpc02(55:1000,5) motpc04(55:1000,5) motpc06(55:1000,5) motpc08(55:1000,5)
motpc10(55:1000,5) motpc12(55:1000,5) motpc14(55:1000,5)];
ml = 1.0*ones(size(w));
ml = [0.0*ml(:,1) 0.2*ml(:,1) 0.4*ml(:,1) 0.6*ml(:,1) 0.8*ml(:,1) 1.0*ml(:,1) 1.2*ml(:,1) 1.4*ml(:,1)];

k = fmins('detmem',[0.36 0],[1 1e-6 1e-10 0 0 0 0 0 0 0 0 0 0 300],[],w,ia,ml,ua);

y1 = 1.932e-3*288/200+5.3041e-2-5.116e-4*w+1.968e-2*sign(w).*sqrt(abs(w))+ml;
y2 = k(1)*ia+k(2)*ia.*ia;

y1 = reduc(y1,300);
y2 = reduc(y2,300);

plot(reduc(ua,300),y1,reduc(ua,300),y2,'--')

title('M [Nm]')
grid
xlabel('Ua [V]')
gtext('Ml = 0 Nm')
gtext('Ml = 1.4 Nm')
```

Program DETMEM.M:

```
% function a=detmem(x,w,ia,ml,ua);
%
% function detmem is a criterion function to find a value of electomagnetic
% moment Mem of a DC motor
```

```
function a=detmem(x,w,ia,ml,ua);

b = 1.932e-3*288/200+5.3041e-2-5.116e-4*w+1.968e-2*sign(w).*sqrt(abs(w))+ml-x(1)*ia-x(2)*ia.*ia;
b = b.*b;
a = mean(mean(b));
```

Program DETLA1.M:

```
t = motvar00(100:200,1)-motvar00(100,1);
ia = motvar00(100:200,4);
ua = motvar00(100:200,5);
w = motvar00(100:200,6);

global ias;

k = fmins('detla',[6.963e-3],[1 1e-6 1e-10 0 0 0 0 0 0 0 0 0 0 300],[],ia,ua,w);

plot(t,ia,t,ias,'--')
```

Program DETLA.M:

```
% function a = detla(x,ia,ua,w);
%
% function detla is criterion function to find armature inductance
% of DC motor

function a = detla(x,ia,ua,w);

global ias;

ias=[];
d = size(ia);

for i = 1:d(1)
  if (i == 1)
    di = 2.5e-3*(ua(i)-0.34212*w(i));
    ias(i) = di/x(1);
  else
    di = 2.5e-3*(ua(i)-0.34212*w(i)-1.4723*ias(i-1)-9.6016e-4*w(i).*ias(i-1));
    ias(i) = ias(i-1)+di/x(1);
  end
end

b = ia-ias';
c = b.^2;

a = mean(mean(c));
```

Program DETJ1.M:

```
t = motvar00(100:200,1)-motvar00(100,1);
ia = motvar00(100:200,4);
ua = motvar00(100:200,5);
w = motvar00(100:200,6);
ml = 0.0*ones(size(t));

t = reduc(t,500);
ia = reduc(ia,500);
w = reduc(w,500);
ml = reduc(ml,500);
```

```
global ws;

k = fmins('detj',[J],[1 1e-6 1e-10 0 0 0 0 0 0 0 0 0 0 300],[],ia,ua,w,ml);

plot(t,w,t,ws,'--')
```

Program DETJ.M:

```
% function a = detj(x,ia,ua,w,ml);
%
% function detj is a criterion function what serves to find inertia
% constant of DC motor

function a = detj(x,ia,ua,w,ml);

global ws;

ws=[];
d = size(ia);

for i = 1:d(1)
  if (i == 1)
    ws(i) = 0;
  else
    mem = 0.44737*ia(i)-0.023142*ia(i)*ia(i);
    mminus = ml(i)+0.053011-5.116e-4*w(i-1)+0.01968*sign(w(i-1))*sqrt(abs(w(i-1)));
    if ((abs(ws) < 0.1) & (abs(mem) < abs(mminus)))
      dw = 0;
    else
      dw = 0.2*2.5e-3*(mem-mminus);
    end
    ws(i) = ws(i-1)+dw/x(1);
  end
end

b = w-ws';
c = b.^2;

a = mean(mean(c));
```

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Electrical equivalent circuit based modeling and analysis of direct current motors

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ABSTRACT

In this study, electromechanical equations of separately excited DC motors are examined and the exact electrical equivalent circuit related to mechanical equations is introduced. The motor has mechanical variables and components in both armature circuit and output circuit related to load. The electrical equivalent circuit having the same equation structure with mechanical system is given. By using the similarity of equations, all of mechanical variables and components are expressed in terms of electrical circuit elements and electrical variables. Thus, all variables and components in the model are electrical although the DC motor system contains mechanical components and variables. The transient-state and steady-state analysis of DC motor with load and without load are obtained by using the exact electrical equivalent circuit in the numerical example.

1. Introduction

DC motors are still the commonly used machines as variable speed drives. The methods used for modeling DC machines are far simpler and more straightforward than those used for modeling AC machines. Especially, since developments in the design of controlled rectifiers and DC–DC converters, the control of DC motors is realized more easily. For power electronic DC drive applications, the most commonly used DC machines are separately excited DC machines and permanent magnet DC machines. The main advantage of separately excited DC machines is that the armature and field windings are fed from different sources. This property allows to obtain the desired speed-torque characteristics. It is very common application to control the speed of DC motor by changing its terminal voltage. The desired variable voltage is obtained by a controlled rectifier or DC–DC converter.

The armature current and field current, consequently the torque and flux, in these motors are controlled separately from each other. It allows motors to have high performance. In the system analysis, it is a desired property to use a linear model. Therefore, in the control of motor, the field current is taken as a constant value and the torque is directly proportional to the armature current. The motor drive system is controlled by only one variable, armature current.

The general structures, characteristics, analysis, dynamic equations, control of DC motors are given in many textbooks [1–4] and some papers [5–9]. Large-signal analysis of DC motor drive system is obtained by using state-space averaging technique in [10]. Ahmed [11] gives modeling and simulation of ac–dc buck–boost converter fed DC motor with uniform PWM technique. Some models and simulation techniques related to DC series motor are presented in [12,13]. The paper [14] is related to the current control of brushless DC motor based on a common dc signal for space operated vehicles. A simple model for a thyristor-driven DC motor considering continuous and discontinuous current modes is established in [15]. The paper [16] investigates steady-state and dynamic performance analysis of PV supplied DC motors fed from intermediate power converter. Gargouri [17] describes a motor which uses an electronic commutator substituting the mechanical commutator and brushes in the DC machine. The simulation and analysis of DC motor driver system for the hybrid electric vehicles is given in [18]. Some analysis and modeling methods related to permanent magnet DC motors are presented in [19,20]. Mazouz and Midoun [21] propose a technique for the identification of the maximum power point (MPP) based on fuzzy logic. They made a complete modeling of the entire system which consists of a photovoltaic array supplying, through a DC converter, a DC machine coupled to a centrifugal pump. The paper [22] describes finite element modeling and analysis of brushless DC motor for direct-drive application with low speed and high torque capability. An intelligent control method for the maximum power point tracking of a photovoltaic system under variable temperature and insolation conditions is proposed in [23]. This method uses a fuzzy logic controller applied to a DC–DC converter device.

In this study, the exact electrical equivalent circuit of a separately excited DC motor is introduced. Although the DC motor system contains mechanical components and variables, all variables and components in the model are electrical. The paper is organized as follows. Section 2 gives the electromechanical equations related

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to DC motor with load, the electrical equivalent circuit of armature and the state-space model of dynamic system. The exact electrical equivalent circuit related to the whole electromechanical system is introduced in Section 3. The transient-state and steady-state analysis of DC motor with load and without load by using the developed electrical equivalent circuit are realized in Section 4. The paper concludes in Section 5.

2. Electromechanical model of DC motor

The continuous-time electromechanical equations related to a separately excited DC motor circuit in Fig. 1 are given in Eqs. (1a) and (1b). Eq. (1a) is the electrical circuit equation of armature. Eq. (1b) is the mechanical equation of DC motor with load.

$$E_c + R_a i_a(t) + L_a \frac{di_a}{dt} = U_a \tag{1a}$$

$$B_1 \omega_m + J\frac{d\omega_m}{dt} = T_e - T_L = T_a \tag{1b}$$

where L_a is the armature inductance; R_a the armature resistance; i_a the armature current, U_a the terminal voltage of DC motor, E_c the back emf; T_L the load torque, T_e the electromechanical (air gap) torque, T_a the acceleration torque, J the torque of inertia, B_1 the viscous friction coefficient and ω_m is the speed of motor.

The electrical variable of DC motor is armature current, the mechanical variable is speed. In Eqs. (1a) and (1b), the back emf (E_c) is proportional to speed, the produced torque (T_e) is proportional to armature current as shown in Eqs.(2a) and (2b). Where, K_b is equal for both torque constant and back emf constant in a separately excited DC motor. At low speeds, back emf is small, at high speeds, back emf is bigger.

$$E_c = K_b \omega_m \tag{2a}$$

$$T_e = K_b i_a \tag{2b}$$

By substituting the back emf, E_c, in Eq. (2a) into Eq. (1a) and the produced torque, T_e, in Eq. (2b) into Eq. (1b), the electrical circuit equation of armature and the mechanical equation of DC motor are rearranged as;

$$K_b \omega_m + R_a i_a(t) + L_a \frac{di_a}{dt} = U_a \tag{3a}$$

$$B_1 \omega_m + J\frac{d\omega_m}{dt} = K_b i_a(t) - T_L \tag{3b}$$

The electrical equivalent circuit of armature, related to Eq. (3a), is given in Fig. 2. The back emf is modeled by a dependent voltage source whose value is controlled by speed, a mechanical variable.

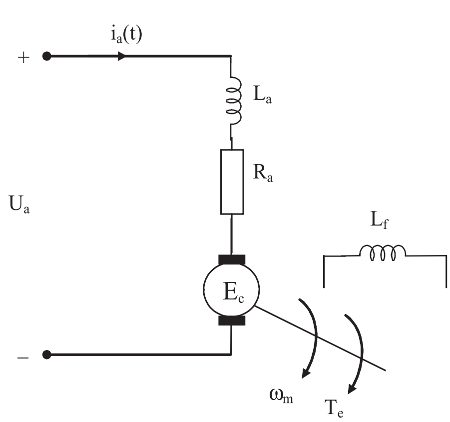

Fig. 1. Armature equivalent circuit of DC motor.

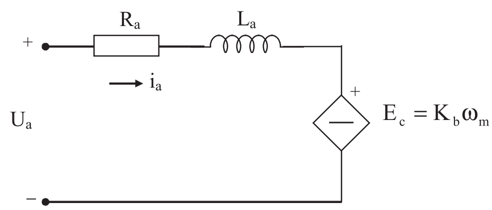

Fig. 2. Electrical equivalent circuit of armature.

Eqs. (3a) and (3b) constitute the dynamic model of DC motor with load. By taking account these equations together, the state-space model of electromechanical system is obtained as follows.

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} \\ \frac{K_b}{J} & -\frac{B_1}{J} \end{bmatrix}\begin{bmatrix} i_a(t) \\ \omega_m(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}\begin{bmatrix} U_a \\ T_L \end{bmatrix} \tag{4}$$

The system model contains both a mechanical variable, ω_m, and a electrical variable, i_a. Since $\frac{di_a}{dt} = 0$ and $\frac{d\omega_m}{dt} = 0$ in steady-state, the steady-state equations of system are obtained as;

$$\begin{bmatrix} R_a & K_b \\ K_b & -B_1 \end{bmatrix}\begin{bmatrix} I_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} U_a \\ T_L \end{bmatrix} \tag{5}$$

3. Model of electrical equivalent circuit

There are two dependent variables in the system model of DC motor, Eqs. (3) or (4). The back emf related to electrical circuit of armature, in Eqs. (2a) or (3a), is dependent on speed, a mechanical variable. The produced torque related to mechanical equation of DC motor, in Eqs. (2b) or (3b), is dependent on current, an electrical variable.

Again, let's consider the mechanical equation of DC motor, in Eq. (6). By taking the structure of this equation into consideration, an electrical equivalent circuit [24] having the same equation structure can be obtained.

$$J\frac{d\omega_m}{dt} + B_1 \omega_m = T_e - T_L \tag{6}$$

The equation of parallel RC circuit in Fig. 3 is

$$i_C(t) + i_R(t) = J_m - J_l$$

$$C\frac{dU_c}{dt} + GU_C(t) = J_m - J_l \tag{7}$$

The source J_m in Fig. 3 is a controlled current source. By using the structural similarity between Eq. (6) and Eq. (7), mechanical variables and components are expressed in terms of electrical variables and components. In consequence of this similarity, the produced torque is analogous to the controlled current source, the load torque is analogous to the independent current source, the speed is analogous to the capacitor voltage, the viscous friction coefficient is analogous to the conductance, the torque of inertia is analogous to the capacitance.

$$T_e \text{ (Nm)} \rightarrow J_m \text{ (A)} \tag{8a}$$

$$T_L \text{ (Nm)} \rightarrow J_l \text{ (A)} \tag{8b}$$

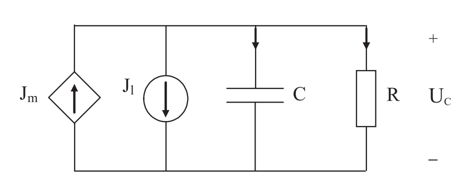

Fig. 3. Parallel RC circuit.

$$\omega_m \text{ (rad/s)} \rightarrow U_C \text{ (V)} \tag{8c}$$

$$B_1 \text{ (Nm/rad/s)} \rightarrow G\ (\Omega^{-1}) \tag{8d}$$

$$J\ (\text{kg}-\text{m}^2) \rightarrow C\ (F) \tag{8e}$$

Considering the expressions in Eq. (8), the circuit given in Fig. 3 represents the electrical equivalent circuit related to the mechanical equations of DC motor. Thus, all of mechanical variables and components are expressed in terms of electrical variables and components.

The dependent source in Fig. 3, equivalent to produced torque, can be taken as a current source controlled by the armature current. The expression of this source is as follows.

$$J_m = T_e = K_b i_a \tag{9}$$

After expressing the produced torque by the controlled current source, Eq. (7) is rearranged;

$$C\frac{dU_c}{dt} + GU_C(t) = K_b i_a(t) - J_l \tag{10}$$

The back emf of DC motor in Eq. (2a) is dependent on speed, a mechanical variable. Since the speed is equivalent to capacitor voltage according to Eq. (8c), the back emf will be also dependent on voltage, an electrical variable. Therefore, the back emf in Fig. 2 is expressed as a voltage controlled voltage source, as in Fig. 4.

$$E_c = K_b U_C \tag{11}$$

After expressing the back emf by the controlled voltage source, Eq. (3a) is rearranged;

$$U_a = R_a i_a(t) + L_a\frac{di_a}{dt} + K_b U_C(t) \tag{12}$$

Taking the circuits in Figs. 2 and 3 into consideration together, the exact electrical equivalent circuit of DC motor is obtained as in Fig. 4. The circuit model does not contain any mechanical variable and component. The controlled sources depend on electrical variables. Thus, the transient analysis and the dynamic characteristics related to DC motor can be obtained in terms of electrical variables in this model. Moreover, any computer circuit simulator (PSPICE, etc.) can be used for analysis of DC motor because all components and variables are electrical.

Eqs. (10) and (12) constitute the electrical dynamic model of DC motor. By taking account of these equations together, the state-space model of electrical system is given in Eq. (13).

$$\frac{d}{dt}\begin{bmatrix} i_a \\ U_C \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} \\ \frac{K_b}{C} & -\frac{G}{C} \end{bmatrix}\begin{bmatrix} i_a(t) \\ U_C(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{C} \end{bmatrix}\begin{bmatrix} U_a \\ J_l \end{bmatrix} \tag{13}$$

Eq. (13) represents the exact electrical equivalent of electromechanical system model in Eq. (4).

3.1. Steady-state model

In steady-state, in Fig. 4, the voltage across the inductor and the current through the capacitor are zero. All circuit variables are time-invariant. The steady-state circuit model of system is given in Fig. 5.

Since $\frac{di_a}{dt} = 0$ and $\frac{dU_C}{dt} = 0$ in steady-state, the steady-state equations of system whose dynamic model is given in Eq. (13) are obtained as follows.

$$\begin{bmatrix} R_a & K_b \\ K_b & -G \end{bmatrix}\begin{bmatrix} I_a \\ U_C \end{bmatrix} = \begin{bmatrix} U_a \\ J_l \end{bmatrix} \tag{14}$$

Eq. (14) represents the exact electrical equivalent of steady-state electromechanical system model in Eq. (5).

3.2. State without load

In the exact electrical equivalent circuits of Figs. 4 and 5, the independent current source J_l express a load torque. For both transient-state and steady-state analysis of DC motor without load, the independent current source in Figs. 4 and 5 is taken as an open circuit and $J_l = 0$ in Eqs. (13) and (14). This state is shown in the example.

In order to prove the proposed model valid under different operating conditions, for both without load state and different load torque states, the transient and steady-state analysis are realized by the proposed exact electrical equivalent circuit.

4. Numerical example

The parameters of a separately excited DC motor in Fig. 1 are $R_a = 0.5\ \Omega$, $L_a = 3$ mH, $K_b = 0.8$ V/rad/s, $J = 0.0167$ kg m^2, $B_1 = 0.01$ Nm/rad/s. Terminal voltage of motor is $U_a = 220$ V. (a) Obtain the transient-state analysis related to the speed (ω_m) of DC motor without load ($T_L = 0$), with load 1 ($T_L = 50$ Nm) and load 2 ($T_L = 100$ Nm), (b) When the load torque is increased gradually ($T_L = 0 \rightarrow 50$ Nm $\rightarrow 100$ Nm), obtain the transient-state analysis related to the produced torque (T_e) and the speed (ω_m) of DC motor, (c) Find the speed of DC motor without load in steady-state, (d) Find the speed of DC motor with load 1 ($T_L = 50$ Nm) and load 2 ($T_L = 100$ Nm) in steady-state.

According to Eq. (8), the parameters of exact electrical equivalent circuit are determined as;

$$C = J = 0.0167F$$

$$R = 1/B_1 = 1/0.01 = 100\Omega$$

$$J_1 = T_L(A)$$

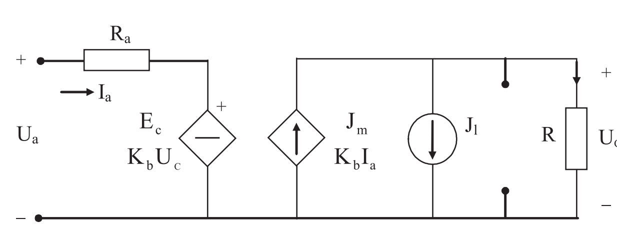

Fig. 5. Steady-state equivalent circuit of DC motor.

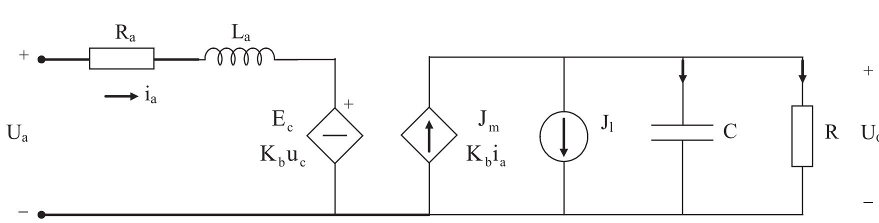

Fig. 4. Exact electrical equivalent circuit of DC motor.

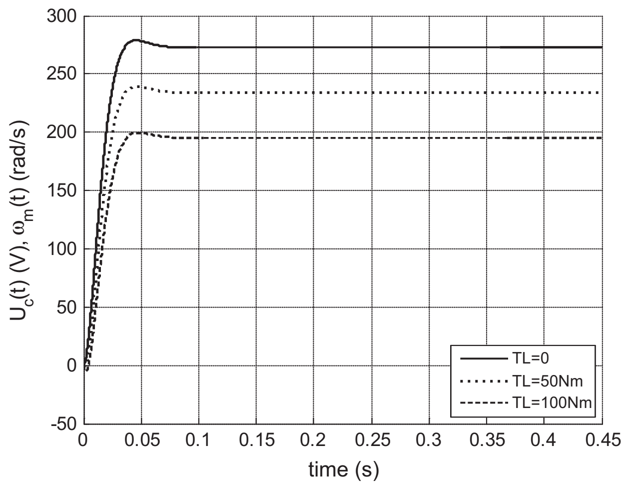

Fig. 6. Voltage of capacitor ($U_C(t)$) (accordingly, speed of DC motor ($\omega_m(t)$)).

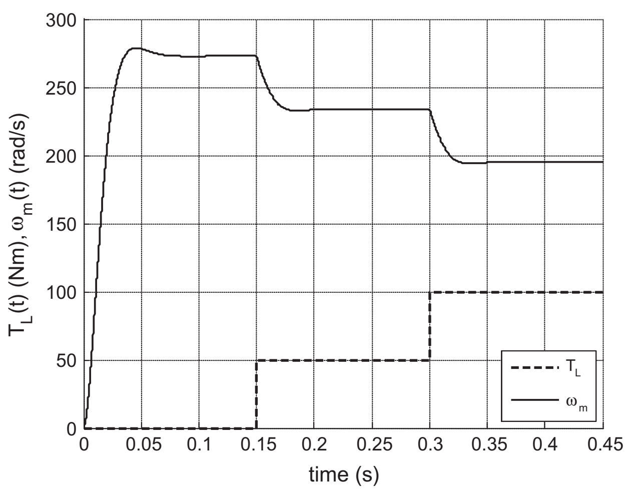

Fig. 7. Load torque (T_L) and speed of DC motor ($\omega_m(t)$) (accordingly, voltage of capacitor ($U_C(t)$)).

$$E_c = K_b U_c(V)$$

$$J_m = T_e = K_b i_a(A)$$

$$U_C = \omega_m(V)$$

4.1. Transient-state analysis related to the speed of DC motor without load, with load 1 and load 2

In the exact electrical equivalent circuit, given in Fig. 4, the independent current source is taken as $J_l = 0$ (without load, open circuit), $J_l = 50$ A (load 1) and $J_l = 100$ A (load 2), respectively. After substituting the element values into Eq. (13), the solution to dynamic analysis of capacitor voltage is given in Fig. 6. Besides, it is the speed characteristics of DC motor.

4.2. Transient-state analysis of DC motor for gradual load increase

In the state-space model of electrical system, given in Eq. (13), the independent current source is increased gradually as $J_l = 0$, $J_l = 50$ A and $J_l = 100$ A, respectively. Then, the characteristics of load torque and speed are given in Fig. 7, the produced torque is given in Fig. 8.

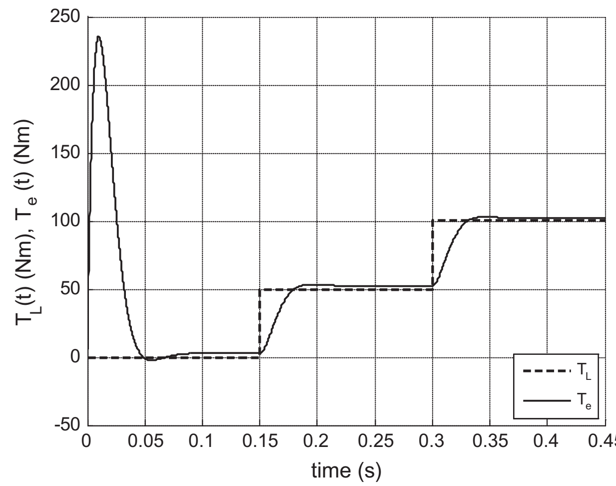

Fig. 8. Load torque (T_L) and produced torque (T_e) (accordingly, $J_m(t)$).

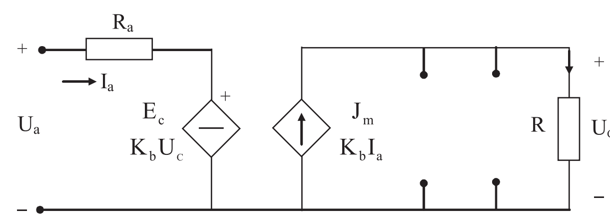

Fig. 9. Steady-state equivalent circuit of DC motor without load.

4.3. Steady-state analysis of DC motor without load

In steady-state without load, the independent current source in the exact equivalent circuit of Fig. 5 is taken as an open circuit, $J_l = 0$, as shown in Fig. 9. In this state, Eq. (14) is rearranged as

$$\begin{bmatrix} R_a & K_b \\ K_b & -G \end{bmatrix} \begin{bmatrix} I_a \\ U_C \end{bmatrix} = \begin{bmatrix} U_a \\ 0 \end{bmatrix} \tag{15}$$

After substituting the element values into Eq. (15), the results related to system are $I_a \cong 3.41$ A, $U_C = 272.8$ V. Consequently, the speed of DC motor without load in steady-state is $\omega_m = 272.8$ rad/s. For $T_L = J_l = 0$, this result harmonizes with the steady-state value of characteristic in Fig. 6.

4.4. Steady-state analysis of DC motor with load 1 and load 2

In steady-state model with load in Fig. 5, the independent current is taken as $J_l = 50$ A and $J_l = 100$ A, respectively. After substituting the element values into the system equations in Eq. (14), the results related to system are obtained as $I_a \cong 65.4$ A and $U_C \cong 234.1$ V for load 1, $I_a \cong 127.4$ A and $U_C \cong 195.3$ V for load 2. Consequently, the speeds of DC motor with load 1 and load 2 in steady-state are $\omega_m \cong 234.1$ rad/s and $\omega_m \cong 195.3$ rad/s, respectively. For $T_L(=J_l) = 50$ Nm and $T_L(=J_l) = 100$ Nm, the results harmonize with the steady-state values of characteristics in Fig. 6.

5. Conclusion

As in other electromechanical energy conversion machines, equations of DC motors contain both electrical variables and mechanical variables. In this study, all of variables and components in mechanical equations of separately excited DC motors are expressed in terms of electrical variables and components. Thus, the exact electrical equivalent circuit related to the whole electromechanical system of DC motor is obtained. In the numerical example, it is shown that various analysis and dynamic characteristics of DC motor can be obtained by this electrical equivalent circuit.

In order to prove the proposed model valid under different operating conditions, both without load state and different load torque states are investigated by the independent current source in the exact electrical equivalent circuit. In the study, while obtaining the exact electrical equivalent circuit, the excitation current and magnetic flux are assumed to be constant in order to obtain a linear model. But, the approach given can be generalized for variable flux applications.

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