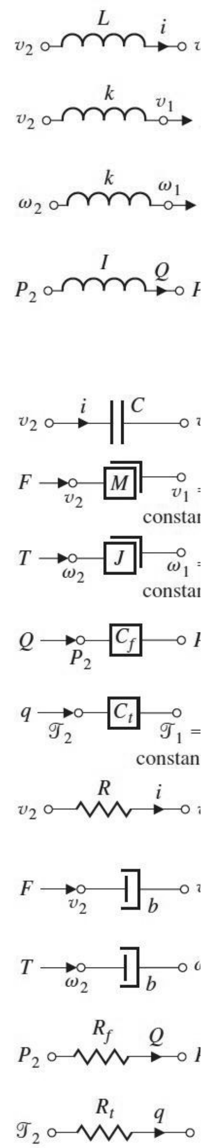
Table 2.2 Summary of Governing Differential Equations for Ideal Elements

Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\dfrac{di}{dt}$	$E = \dfrac{1}{2}Li^2$	
	Translational spring	$v_{21} = \dfrac{1}{k}\dfrac{dF}{dt}$	$E = \dfrac{1}{2}\dfrac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \dfrac{1}{k}\dfrac{dT}{dt}$	$E = \dfrac{1}{2}\dfrac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\dfrac{dQ}{dt}$	$E = \dfrac{1}{2}IQ^2$	

Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Capacitive storage	Electrical capacitance	$i = C\dfrac{dv_{21}}{dt}$	$E = \dfrac{1}{2}C{v_{21}}^2$	
	Translational mass	$F = M\dfrac{dv_2}{dt}$	$E = \dfrac{1}{2}M{v_2}^2$	
	Rotational mass	$T = J\dfrac{d\omega_2}{dt}$	$E = \dfrac{1}{2}J{\omega_2}^2$	
	Fluid capacitance	$Q = C_f\dfrac{dP_{21}}{dt}$	$E = \dfrac{1}{2}C_f{P_{21}}^2$	
	Thermal capacitance	$q = C_t\dfrac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	
Energy dissipators	Electrical resistance	$i = \dfrac{1}{R}v_{21}$	$\mathcal{P} = \dfrac{1}{R}{v_{21}}^2$	
	Translational damper	$F = bv_{21}$	$\mathcal{P} = b{v_{21}}^2$	
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b{\omega_{21}}^2$	
	Fluid resistance	$Q = \dfrac{1}{R_f}P_{21}$	$\mathcal{P} = \dfrac{1}{R_f}{P_{21}}^2$	
	Thermal resistance	$q = \dfrac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \dfrac{1}{R_t}\mathcal{T}_{21}$	



Percent Peak Overshoot Versus Damping Ratio for Second-Order System							
Damping ratio	0.9	0.8	0.7	0.6	0.5	0.4	0.3
Percent overshoot	0.2	1.5	4.6	9.5	16.3	25.4	37.2

$$T_s = 4\tau = \frac{4}{\zeta\omega_n} \qquad T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \qquad P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

Table 2.3 Important Laplace Transform Pairs

$f(t)$	$F(s)$
Step function, $u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}}$
$f^{(k)}(t) = \frac{d^k f(t)}{dt^k}$	$s^k F(s) - s^{k-1} f(0^-) - s^{k-2} f'(0^-) - \ldots - f^{(k-1)}(0^-)$

$f(t)$	$F(s)$
$\int_{-\infty}^{t} f(t)\, dt$	$\frac{F(s)}{s} + \frac{1}{s}\int_{-\infty}^{0} f(t)\, dt$
Impulse function $\delta(t)$	1
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{1}{\omega}[(\alpha - a)^2 + \omega^2]^{1/2} e^{-at} \sin(\omega t + \phi)$, $\phi = \tan^{-1}\frac{\omega}{\alpha - a}$	$\frac{s+\alpha}{(s+a)^2+\omega^2}$
$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t, \zeta < 1$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
$\frac{1}{a^2+\omega^2} + \frac{1}{\omega\sqrt{a^2+\omega^2}} e^{-at} \sin(\omega t - \phi)$, $\phi = \tan^{-1}\frac{\omega}{-a}$	$\frac{1}{s[(s+a)^2+\omega^2]}$
$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \phi\right)$, $\phi = \cos^{-1}\zeta, \zeta < 1$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$
$\frac{\alpha}{a^2+\omega^2} + \frac{1}{\omega}\left[\frac{(\alpha - a)^2 + \omega^2}{a^2+\omega^2}\right]^{1/2} e^{-at} \sin(\omega t + \phi)$. $\phi = \tan^{-1}\frac{\omega}{\alpha - a} - \tan^{-1}\frac{\omega}{-a}$	$\frac{s+\alpha}{s[(s+a)^2+\omega^2]}$