Instructor's Manual

Digital Control System Analysis and Design

Third Edition

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CHAPTER 1

1-1. (a) $C(s) = G_1(s)\,E(s) + G_2(s)\,E(s) = [G_1(s) + G_2(s)]\,E(s)$

$$\therefore \frac{C(s)}{E(s)} = \underline{G_1(s) + G_2(s)}$$

(b) $C(s) = G_2(s)\,M(s) = G_1(s)\,G_2(s) \quad \therefore \dfrac{C(s)}{E(s)} = \underline{G_1(s)\,G_2(s)}$

1-2. (a) $C(s) = G_p(s)M(s) = G_c(s)\,G_p(s)\,E(s) = G_c(s)\,G_p(s)\,[R(s) - H(s)\,C(s)]$

$[1 + G_c(s)\,G_p(s)\,H(s)]\,C(s) = G_c(s)\,G_p(s)\,R(s)$

$$\therefore \underline{\frac{C(s)}{R(s)} = \frac{G_c(s)\,G_p(s)}{1 + G_c(s)\,G_p(s)\,H(s)}}$$

(b) $C(s) = G_3(s)\,M(s) = G_3(s)[G_1(s)\,E(s) + G_2(s)\,R(s)]$

$= G_3(s)\,G_1(s)\,[R(s) - H(s)\,C(s)] + G_2(s)\,G_3(s)\,R(s)$

$[1 + G_1(s)\,G_3(s)\,H(s)]\,C(s) = [G_1(s) + G_2(s)]\,G_3(s)\,R(s)$

$$\therefore \underline{\frac{C(s)}{R(s)} = \frac{[G_1(s) + G_2(s)]\,G_3(s)}{1 + G_1(s)\,G_3(s)\,H(s)}}$$

(c) $C(s) = G_2(s)\,M(s) = G_1(s)\,G_2(s)\,E(s) = G_1(s)\,G_2(s)[R(s) - H_2(s)M(s)$

$- H_1(s)\,C(s)]$ and $M(s) = C(s)/G_2(s)$

$$\therefore \left[1 + \frac{G_1 G_2 H_2}{G_2} + G_1 G_2 H_1\right] C(s) = G_1 G_2\,R(s)$$

$$\therefore \frac{C(s)}{R(s)} = \underline{\frac{G_1(s)\,G_2(s)}{1 + G_1(s)\,H_2(s) + G_1(s)\,G_2(s)\,H_1(s)}}$$

1-3. (a) $\dfrac{C(s)}{R(s)} = \underline{\dfrac{G_c(s)\,G_p(s)}{1 + G_c(s)\,G_p(s)\,H(s)}}$ (b) $\dfrac{C(s)}{R(s)} = \underline{\dfrac{G_1(s)\,G_3(s) + G_2(s)\,G_3(s)}{1 + G_1(s)\,G_3(s)\,H(s)}}$

(c) $\dfrac{C(s)}{R(s)} = \dfrac{G_1(s)\,G_2(s)}{1 + G_1(s)\,H_2(s) + G_1(s)\,G_2(s)\,H_1(s)}$

1-4. (a) $G_p(s) = \dfrac{C(s)}{M(s)} = \dfrac{5}{0.2s + 1} = \dfrac{25}{s + 5} \Rightarrow (s + 5)C(s) = 25\,M(s)$

$$\therefore \underline{\frac{dc(t)}{dt} + 5c(t) = 25\,m(t)}$$

1-4.(b) $m(t) = 10e(t) = 10[r(t) - c(t)]$

$\therefore \dot{c}(t) + 5c(t) = 250[r(t) - c(t)]$

$\therefore \underline{\dot{c}(t) + 255c(t) = 250\,r(t)}$

(c) $sC(s) + 255C(s) = 250R(s)$

$\therefore \underline{\dfrac{C(s)}{R(s)} = \dfrac{250}{s+255}}$

(d) $\dfrac{C(s)}{R(s)} = \dfrac{(10)\frac{25}{s+5}}{1+(10)\frac{25}{s+5}(1)} = \underline{\dfrac{250}{s+255}}$

(e) open-loop: $\tau = \underline{0.2\,s}$

closed-loop: $\tau = 1/255 = \underline{0.00392\,s}$

1-5. (a) $G(s) = \dfrac{C(s)}{M(s)} = \dfrac{3s+8}{s^2+2s+2}$

$\therefore (s^2+2s+2)C(s) = (3s+8)M(s)$

$\underline{\ddot{c}(t) + 2\dot{c}(t) + 2c(t) = 3\dot{m}(t) + 8m(t)}$

(b) $m(t) = 2e(t) = 2[r(t) - c(t)]$

$\therefore \ddot{c}(t) + 2\dot{c}(t) + 2c(t) = 6[\dot{r}(t) - \dot{c}(t)] + 16[r(t) - c(t)]$

$\therefore \underline{\ddot{c}(t) + 8\dot{c}(t) + 18c(t) = 6\dot{r}(t) + 16r(t)}$

(c) $(s^2+8s+18)C(s) = (6s+16)R(s)$

$\therefore \underline{\dfrac{C(s)}{R(s)} = \dfrac{6s+16}{s^2+8s+18}}$

(d) $\dfrac{C(s)}{R(s)} = \dfrac{(2)\frac{3s+8}{s^2+2s+2}}{1+(2)\frac{3s+8}{s^2+2s+2}(1)} = \underline{\dfrac{6s+16}{s^2+8s+18}}$

(e) open-loop: $s^2+2s+2 \Rightarrow s = \underline{-1 \pm j}$

$\therefore$ term $= A_1 e^{-t}\cos(t+\theta_1)$; $\tau = 1/1 = \underline{1\,s}$

closed-loop: $s^2+8s+18 \Rightarrow s = \underline{-4 \pm j\sqrt{2}}$

$\therefore$ term $= A_2 e^{-4t}\cos(\sqrt{2}\,t+\theta_2)$; $\tau = 1/4 = \underline{0.25\,s}$

1-6. (a) $G(s) = \dfrac{C(s)}{M(s)} = \dfrac{5}{s^2+2s+2} \Rightarrow (s^2+2s+2)C(s) = 5M(s)$

$\therefore \underline{\ddot{c}(t) + 2\dot{c}(t) + 2c(t) = 5m(t)}$

1-6.(b) $E(s) = R(s) - (3s+1)C(s)$

$\therefore\ e(t) = r(t) - 3\dot{c}(t) - c(t)$

$\therefore\ \ddot{c}(t) + 2\dot{c}(t) + 2c(t) = 5[2e(t)] = 10r(t) - 30\dot{c}(t) - 10c(t)$

$\therefore\ \underline{\ddot{c}(t) + 32\dot{c}(t) + 12c(t) = 10r(t)}$

(c) $(s^2 + 32s + 12)C(s) = 10R(s)$

$$\therefore\ \frac{C(s)}{R(s)} = \underline{\frac{10}{s^2+32s+12}}$$

(d)
$$\frac{C(s)}{R(s)} = \frac{(2)\frac{5}{s^2+2s+2}}{1 + (2)\frac{5}{s^2+2s+2}(3s+1)} = \underline{\frac{10}{s^2+32s+12}}$$

(e) open-loop: $\tau = \underline{1\ s}$, from Problem 1-5(e)

closed-loop: poles $s = -16 \pm \sqrt{244} = -31.62, -0.38$

$\therefore\ \tau_1 = 1/31.62 = \underline{0.0316\ s}$; $\tau_2 = 1/0.38 = \underline{2.63\ s}$

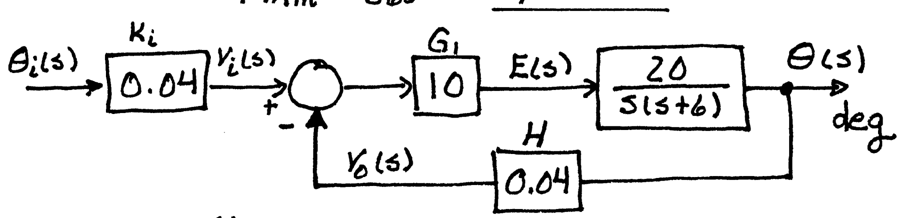
1-7. (a)
$$\Theta(s) = \frac{20}{s^2(s+6)} = \frac{3.33}{s^2} + \frac{k}{s} + \frac{5/9}{s+6}$$

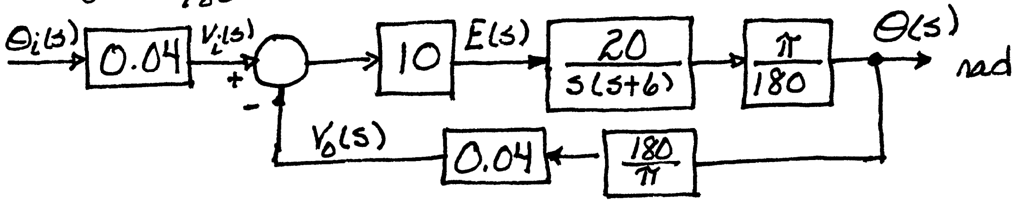
$$k = \frac{d}{ds}\left[\frac{20}{s+6}\right]_{s=0} = \left.\frac{-20}{(s+6)^2}\right|_{s=0} = -\frac{5}{9}$$

$\therefore\ \theta_{ss}(t) = \underline{\left(3.33t - \frac{5}{9}\right) \text{ in degrees}}$

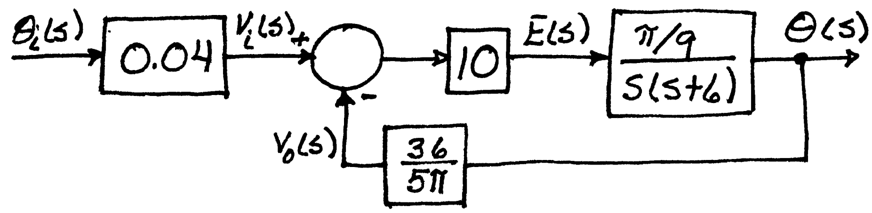
$\dot{\theta}_{ss}(t) = 3.33$ deg/s

$$\therefore\ 3.33\,\frac{d}{s} \times \frac{60\ s}{1\ min} \times \frac{1\ rev}{360^\circ} = \underline{\frac{5}{9}\ rpm}$$

(b)


(c) 1 degree $= \frac{\pi}{180}$ rad


1-7.(c)



(d) $$\frac{\theta(s)}{\theta_i(s)} = \frac{(0.04)(10)\frac{20}{s(s+6)}}{1+(0.04)(10)\frac{20}{s(s+6)}} = \frac{8}{s^2+6s+8}$$

$$\theta(s) = \frac{8}{(s+2)(s+4)} \times \frac{10}{s} = \frac{10}{s} + \frac{-20}{s+2} + \frac{10}{s+4}$$

$$\therefore \theta(t) = \underline{10 - 20e^{-2t} + 10e^{-4t}}, \quad t \geq 0$$

(e) $\epsilon^{-t/\tau} \Rightarrow \frac{1}{\tau_1} = 2,\ \underline{\tau_1 = 0.5},\ \underline{\tau_2 = 1/4 = 0.25},\ \therefore t_{ss} \approx 4(0.5) = \underline{2s}$

1-8. (1-15) $$\frac{K_T/JR_a}{s\left(s+\frac{BR_a+K_TK_b}{JR_a}\right)} = \frac{20}{s(s+6)}, \ \therefore \frac{K_T}{JR_a} = 20; \ \frac{BR_a+K_TK_b}{JR_a} = 6$$

(1-16),(1-17), $x_1 = \theta$; $x_2 = \dot{\theta} = \dot{x}_1$

(1-17) $$\dot{x}_2(t) = \ddot{\theta}(t) = -6x_2(t) + 20e(t)$$
$$= -6x_2(t) + 20\left(10[0.04\theta_i(t) - 0.04\theta(t)]\right)$$
$$= -6x_2(t) + 8\theta_i(t) - 8x_1(t)$$

$$\therefore \underline{\dot{x}}(t) = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 8 \end{bmatrix} \theta_i(t)$$

$$\theta(t) = [1 \quad 0]\, \underline{x}(t)$$

1-9. (a) From Problem 1-7(d), $\frac{\theta(s)}{\theta_i(s)} = \frac{8}{s^2+6s+8}$

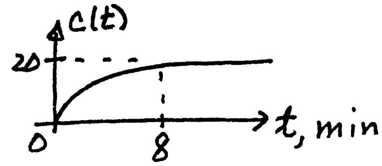
(b) 1 deg $= (\pi/180)$ rad

$$\therefore \frac{\theta(s)}{\theta_i(s)} = \frac{8}{s^2+6s+8} \times \frac{\pi}{180} = \underline{\frac{2\pi/45}{s^2+6s+8}}$$

(c) $$\frac{\theta(s)}{\theta_i(s)} = \frac{K_iG_1G_2}{1+G_1G_2H} = \frac{\left(\frac{1}{25}\right)(10)\left(\frac{\pi/9}{s(s+6)}\right)}{1+\frac{(10)(\pi/9)}{s(s+6)}\left(\frac{36}{5\pi}\right)} = \underline{\frac{2\pi/45}{s^2+6s+8}}$$

1-10.(a) $G(s) = \frac{2}{s+0.5} = \frac{K}{\tau s+1} = \frac{K/\tau}{s+1/\tau}$; $\therefore \tau = \underline{2\text{ min}}$

(b) $C(s) = \frac{2}{s+0.5} \times \frac{5}{s} = \frac{20}{s} + \frac{-20}{s+0.5}$



$\therefore c(t) = \underline{20(1-\epsilon^{-0.5t})}, \; t \geq 0$

$c_{ss}(t) = \underline{20\ ^\circ C}$

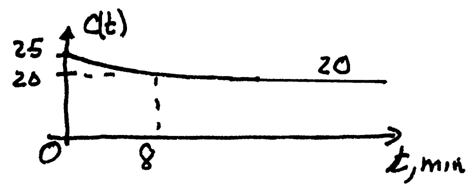
(c) $\frac{C(s)}{E(s)} = \frac{2}{s+0.5}$

$\therefore \dot{c}(t) + 0.5c(t) = 2e(t)$

$sC(s) - c(0) + 0.5C(s) = 2E(s)$

$\therefore C(s) = \frac{2E(s)}{s+0.5} + \frac{c(0)}{s+0.5} \Rightarrow c(t) = \overbrace{20(1-\epsilon^{-0.5t})}^{\text{from (b)}} + 25e^{-0.5t}$

$\therefore c(t) = \underline{20+5\epsilon^{-0.5t}}, \; t \geq 0$



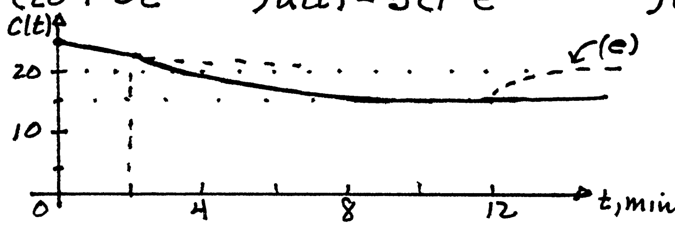
(d) Disturbance response:

$C_d(s) = \frac{-2.5}{s+0.5} \cdot \frac{1}{s} = \frac{-5}{s} + \frac{5}{s+0.5}$

$\therefore c_d(t) = \underline{-5(1-\epsilon^{-0.5(t-2)})u(t-2)}$, since door opened at $t_0 = 2$.

$\therefore$ from (c) and (d),

$c(t) = (20+5\epsilon^{-0.5t})u(t) - 5(1-\epsilon^{-0.5(t-2)})u(t-2)$



(e) $c(12) = 20 + 5e^{-6} - 5 + 5e^{-5} \approx \underline{15}$

From (c), with $c(0) = 15$

$c_1(t) = 20(1-\epsilon^{-0.5t}) + 15\epsilon^{-0.5t} = 20 - 5e^{-0.5t}$

$\therefore \underline{c(t) = [20 - 5\epsilon^{-0.5(t-12)}]u(t-12), \quad t \geq 12}$

1-11. (a) $\tau = 2$ min $\rightarrow$ 120 s

$$\therefore \frac{C(s)}{E(s)} = \frac{K}{\tau s+1} = \frac{4}{2s+1} \Rightarrow \frac{4}{120s+1} = \underline{\frac{0.0333}{s+0.00833}}$$

(b) (i) From Problem 1-4(b), $c(t) = 20(1-\epsilon^{-0.5t}),\ t \geq 0$

$$\therefore c(1) = \underline{20(1-\epsilon^{-0.5})}$$

(ii) $$C(s) = \frac{0.0333}{s+0.00833} \times \frac{5}{s} = \frac{20}{s} + \frac{-20}{s+0.00833}$$

$$\therefore c(t) = \underline{20(1-\epsilon^{-0.00833t})},\ t \geq 0$$

$$c(1) = 20(1-\epsilon^{-0.00833(60)}) = \underline{20(1-\epsilon^{-0.5})}$$

1-12. (a) $$\Theta(s) = \frac{1}{Js^2}T(s) = \frac{K}{Js^2}E(s) = \frac{K}{Js^2}[\Theta_c(s) - \Theta(s)]$$

$$\therefore \left[1+\frac{K}{Js^2}\right]\Theta(s) = \frac{K}{Js^2}\Theta_c(s)$$

$$\therefore \frac{\Theta(s)}{\Theta_c(s)} = \frac{K/Js^2}{1+\frac{K}{Js^2}} = \underline{\frac{K/J}{s^2+K/J}}$$

(b) (1-4): $x_1 = \theta$; (1-5): $x_2 = \dot{\theta} = \dot{x}_1$

$$\therefore \dot{x}_2(t) = \ddot{\theta}(t) = \frac{1}{J}\tau(t) = \frac{K}{J}e(t) = \frac{K}{J}[\theta_c(t) - \theta(t)]$$

$$= \frac{K}{J}[\theta_c(t) - x_1(t)]$$

$$\therefore \begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ K/J & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ K/J\end{bmatrix}\theta_c(t)$$

$$y(t) = [1 \quad 0]\underline{x}(t)$$

1-13. (a) From Problem 1-12, $\frac{\Theta(s)}{\Theta_c(s)} = \frac{36}{s^2+36}$

$$\therefore \Theta(s) = \frac{36}{s^2+36} \times \frac{20}{s} = \frac{720}{s(s^2+36)} = \frac{20}{s} + \frac{as+b}{s^2+36}$$

$$= \frac{20s^2+720+as^2+bs}{s(s^2+36)};\ \therefore b = \underline{0},\ a = \underline{-20}$$

$$\therefore \Theta(s) = \frac{20}{s} + \frac{-20s}{s^2+36} \Rightarrow \theta(t) = \underline{20[1-\cos 6t]},\ t \geq 0$$

1-13. (b) From (a), $\ddot{\theta}(t) + 36\theta(t) = 36\,\theta_c(t)$

$\therefore\ (s^2+36)\Theta(s) - s\,\theta(0) - \dot{\theta}(0) = 36\,\Theta_c(s)$

$\therefore\ \Theta(s) = \dfrac{36}{s^2+36}\Theta_c(s) + \dfrac{10s}{s^2+36} + \dfrac{30}{s^2+36}$

$= \dfrac{20}{s} - \dfrac{20s}{s^2+36} + \dfrac{10s}{s^2+36} + \dfrac{30}{s^2+36}$

$= \dfrac{20}{s} - \dfrac{10s}{s^2+36} + \dfrac{30}{s^2+36}$

$\therefore\ \theta(t) = \underline{20 - 10\cos 6t + 5\sin 6t}\,,\ t \geq 0$

(c) $\theta(0) = 20 - 10 = \underline{10^\circ}$ ✓

$\dot{\theta}(t) = 60\sin 6t + 30\cos 6t \Rightarrow \dot{\theta}(0) = \underline{30^\circ/s}$ ✓

From (b), $\ddot{\theta} + 36\theta = 720$

$\therefore\ 360\cos 6t - 180\sin 6t + 720 - 360\cos 6t$
$+ 180\sin 6t = 720$

$\therefore\ \underline{720 = 720}$ ✓

1-14. From Problem 1-12,

$\Theta(s) = \dfrac{K/J}{s^2+K/J} \times \dfrac{5}{s} = \dfrac{a^2 5}{(s^2+a^2)s}\,,\qquad a^2 = K/J$

$\therefore\ \Theta(s) = \dfrac{5}{s} - \dfrac{5s}{s^2+a^2}$

$\therefore\ \theta(t) = \underline{5 - 5\cos at}\,,\ t \geq 0$

$10\ \text{cy/min} \times \dfrac{1\ \text{min}}{60\ \text{s}} = \dfrac{1}{6}\ \text{cy/s} \Rightarrow 2\pi\left(\tfrac{1}{6}\right)\text{rad/s} = a$

$\therefore\ a = \pi/3 = \sqrt{K/J}$, $\therefore$ K and J cannot be determined without additional data.

1-15 (a) $\Theta(s) = \frac{1}{s}\dot{\Theta}(s) = \frac{1}{Js^2}T(s) = \frac{K}{Ts^2}[\Theta_c(s) - K_v\dot{\Theta}(s) - \Theta(s)]$

$\dot{\Theta}(s) = s\,\Theta(s)$

$\therefore\ \Theta(s) = \frac{K}{Js^2}[\Theta_c(s) - (K_v s + 1)\Theta(s)$

1-15.(a) $\therefore \dfrac{\Theta(s)}{\Theta_c(s)} = \dfrac{K}{Js^2+KK_v s+K} = \dfrac{K/J}{s^2+\frac{KK_v}{J}s+\frac{K}{J}}$

(b) $x_1(t) = \theta(t)$; $x_2(t) = \dot{x}_1(t) = \dot{\theta}(t)$

$\therefore \dot{x}_2(t) = \ddot{\theta}(t) = \frac{1}{J}\tau(t) = \frac{K}{J}[\theta_c(t) - K_v x_2(t) - x_1(t)]$

$\therefore \dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ -K/J & -KK_v/J \end{bmatrix}\underline{x}(t) + \begin{bmatrix} 0 \\ K/J \end{bmatrix}\theta_c(t)$

$\theta(t) = [1 \quad 0]\,\underline{x}(t)$

(C) $|sI-A| = \begin{vmatrix} s & -1 \\ K/J & s+KK_v/J \end{vmatrix} = \underline{s^2+\frac{KK_v}{J}s+\frac{K}{J}}$

1-16. $\Theta_a(s) = \frac{1}{100}\Theta_m(s)$ $\quad \frac{1}{100s}\dot{\Theta}_m(s) = \frac{2}{s(0.5s+1)}E_a(s) = \frac{2K}{s(0.5s+1)}M(s)$

$\therefore \underline{\dfrac{\Theta_a(s)}{M(s)} = \dfrac{2K}{s(0.5s+1)}}$; $\underline{\dfrac{\Theta_a(s)}{E_a(s)} = \dfrac{2}{s(0.5s+1)}}$

1-17.(a) $\dot{\Theta}_a(s) = \frac{2}{0.5s+1} \times \frac{24}{s} = \frac{48}{s(0.5s+2)} = \frac{96}{s(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+2}$

$= \underline{\frac{48}{s} - \frac{48}{s+2}} \Rightarrow \underline{\dot{\theta}_a(t) = 48 - 48\epsilon^{-2t},\ t \geq 0}$

$\therefore \dot{\theta}_{ass}(t) = 48°/s \times \frac{60s}{1min} \times \frac{1 rev}{360°} = 8$ rpm

$\dot{\theta}_{mss}(t) = 100\,\dot{\theta}_a(t) = \underline{800 \text{ rpm}}$

(b) From (a), $\dot{\theta}_a(t) = 48°/s$

(c) From (a), $\dot{\theta}_a(t) = 48(1-\epsilon^{-2t})$, $t \geq 0$

$\therefore \tau = \underline{0.5s}$

$\epsilon^{-2t_1} = 0.01 \Rightarrow 2t_1 = 4.60 \Rightarrow t_1 = \underline{2.30s}$

(d) $K = 2.4 \Rightarrow e_a(t) = \underline{24V}$, rated voltage

CHAPTER 2

2-1. $e(t) = t; \quad E(z) = 0 + Tz^{-1} + 2Tz^{-2} + \cdots = \dfrac{Tz}{(z-1)^2}$

2-2. (a) $E(z) = 1 + \epsilon^{-T}z^{-1} + \epsilon^{-2T}z^{-2} + \cdots$

$= 1 + (\epsilon^{-T}z^{-1})^1 + (\epsilon^{-T}z^{-1})^2 + \cdots = \dfrac{1}{1-\epsilon^{-T}z^{-1}} = \underline{\dfrac{z}{z-\epsilon^{-T}}}$

(b) $E(z) = 1 + (0.9512\, z^{-1})^1 + (0.9512z^{-1})^2 + \cdots$

$= \dfrac{z}{z - 0.9512}$

(c) $\epsilon^{-bT}\Big|_{T=0.2} = \epsilon^{-0.2b} = 0.5$

$\therefore -0.2b = \ln(0.5) = -0.6931 \Rightarrow b = \underline{-3.466}$

2-3. (a) $e(t) = \epsilon^{-at} \Rightarrow E(z) = 1 + \epsilon^{-aT}z^{-1} + \epsilon^{-2aT}z^{-2} + \cdots = \underline{\dfrac{z}{z-\epsilon^{-aT}}}$

(b) $e(t) = \epsilon^{-(t-T)}u(t-T)$

$E(z) = z^{-1} + \epsilon^{-T}z^{-2} + \epsilon^{-2T}z^{-3} + \cdots = z^{-1}\left[\dfrac{z}{z-\epsilon^{-T}}\right] = \underline{\dfrac{1}{z-\epsilon^{-T}}}$

(c) $e(t) = \epsilon^{-(t-5T)}u(t-5T)$

$E(z) = z^{-5} + \epsilon^{-T}z^{-6} + \epsilon^{-2T}z^{-7} + \cdots = z^{-5}\left[\dfrac{z}{z-\epsilon^{-T}}\right] = \underline{\dfrac{1}{z^4(z-\epsilon^{-T})}}$

2-4. $E_1(s) = \dfrac{2}{s(s+2)} = \dfrac{1}{s} + \dfrac{-1}{s+2}$

$\therefore e_1(t) = (1-\epsilon^{-2t})u(t) \Rightarrow e_1(kT) = (1-\epsilon^{-2kT})u(kT)$

$\therefore E_1(z) = (1 + z^{-1} + z^{-2} + \cdots) - (1 - \epsilon^{-2T}z^{-1} + \epsilon^{-4T}z^{-2} + \cdots)$

$= \dfrac{1}{1-z^{-1}} - \dfrac{1}{1-\epsilon^{-2T}z^{-1}} = \dfrac{z}{z-1} - \dfrac{z}{z-\epsilon^{-2T}} = \underline{\dfrac{(1-\epsilon^{-2})z}{(z-1)(z-\epsilon^{-2})}}, \ T=1$

$E(z) = E_1(z) - z^{-5}E_1(z) = \dfrac{(1-\epsilon^{-2})(z^5-1)}{z^4(z-1)(z-\epsilon^{-2})} = \underline{\dfrac{0.8647(z^5-1)}{z^4(z-1)(z-0.1353)}}$

2-5. (a) $e(k) = \sum\limits_{\text{residues}} \dfrac{0.1z^{k-1}}{z(z-0.9)} = \sum\limits_{\text{residues}} \dfrac{0.1z^{k-2}}{z-0.9}$

2-5. (a) $k=0$: $\text{fcn} = \dfrac{0.1}{z^2(z-0.9)}$, $\therefore$ $\text{residue}\Big|_{z=0.9} = \dfrac{0.1}{(0.9)^2} = \underline{0.1235}$

$$\text{residue}\Big|_{z=0} = \frac{d}{dz}\left[\frac{0.1}{z-0.9}\right]_{z=0} = \frac{-0.1(1)}{(z-0.9)^2}\Bigg|_{z=0} = \frac{-0.1}{(0.9)^2} = \underline{-0.1235}$$

$\therefore$ $e(0) = \underline{0}$

$$k=1:\ e(1) = \frac{0.1}{z-0.9}\Bigg|_{z=0} + \frac{0.1}{z}\Bigg|_{z=0.9} = \underline{0}$$

$$k=10:\ e(10) = \underline{0.1(0.9)^8}$$

(b) $e(0) = \lim\limits_{z\to\infty} E(z) = \lim\limits_{z\to\infty} \dfrac{0.1}{z(z-0.9)} = \underline{0}$

(c) $\dfrac{E(z)}{z} = \dfrac{0.1}{z^2(z-0.9)} = \dfrac{k_1}{z^2} + \dfrac{k_2}{z} + \dfrac{k_3}{z-0.9}$

$k_1 = \dfrac{-0.1}{0.9} = \underline{-\dfrac{1}{9}}$; $k_3 = \dfrac{0.1}{(0.9)^2} = \underline{\dfrac{1}{8.1}}$

$k_2 = \dfrac{d}{dz}\left[\dfrac{0.1}{z-0.9}\right]_{z=0} = \underline{\dfrac{-1}{8.1}}$, from (a)

$$\therefore e(k) = \frac{-1}{8.1}\,\delta(k) - \frac{1}{9}\,\delta(k-1) + \frac{1}{8.1}(0.9)^k$$

$x(0) = -\dfrac{1}{8.1} + 0 + \dfrac{1}{8.1} = \underline{0}$ ✓ ; $x(1) = -0 - \dfrac{1}{9} + \dfrac{0.9}{8.1} = \underline{0}$ ✓

$x(10) = -0 - 0 + \dfrac{0.1}{(0.9)^2}(0.9)^{10} = \underline{0.1(0.9)^8}$ ✓

(d) $E(z) = \dfrac{1.98z}{z^5 + \cdots} = 1.98z^{-4} + (\cdot)z^{-5} + (\cdot)z^{-6} + \cdots$

$\therefore$ $e(0) = e(1) = e(2) = e(3) = \underline{0}$; $e(4) = \underline{1.98}$

(e) $E(z) = \dfrac{2z}{z-0.8} = \dfrac{2z}{z-\epsilon^{-aT}}$ $\therefore$ $\epsilon^{-aT} = 0.8 \Rightarrow aT = 0.2231$

$\therefore$ $a = \dfrac{0.2231}{0.1} = 2.231$, $\therefore$ $\underline{e(t) = 2\epsilon^{-2.231t}u(t)}$

(f) $E(z) = \dfrac{2z}{z-(-0.8)}$; $\therefore$ $\epsilon^{-aT}\epsilon^{j\pi} = -0.8 \Rightarrow aT = 2.231$

$\therefore$ $\underline{e(t) = 2e^{-2.231t}\cos 10\pi t}$ where $\dfrac{\omega_s}{2} = 10\pi$

(g) (e) $e(k) = (0.8)^k$; (f) $e(k) = (-0.8)^k$

$\therefore$ sign alternates on $e(k)$.

2-6. (a) $\mathcal{Z}[e(t-2T)u(t-2T)] = \dfrac{(z^3-2z)z^{-2}}{z^4-0.9z^2+0.8}$

(b) $e(0)=0,\ e(1)=1$

$\therefore\ \mathcal{Z}[e(t+T)u(t)] = z[E(z) - e(0) - e(1)z^{-1}]$

$= z\left[\dfrac{z^3-2z}{z^4-0.9z^2+0.8} - \dfrac{1}{z}\right] = \dfrac{-1.1z^2+0.8}{z^4-0.9z^2+0.8}$

(c) $\mathcal{Z}[e(t-T)u(t-2T)] = e(T)z^{-2} + e(2T)z^{-3} + \cdots$

$= z^{-1}[E(z) - e(0)] = z^{-1}E(z)$, since $e(0)=0$

$= \dfrac{z^2-z}{z^4-0.9z^2+0.8}$

2-7. (a) $e(\infty) = \lim_{z\to1}(z-1)E(z) = \left.\dfrac{z(z-1)}{(z+1)^2}\right|_{z=1} = \underline{0}$

(b) $e(k) = \mathcal{Z}^{-1}\left[\dfrac{z}{(z-1)^2}\right] = k(-1)^k$, $\therefore\ e(\infty)$ unbounded

(c) (a) $e(\infty) = \lim_{z\to1}(z-1)\dfrac{z}{(z-1)^2}$, $\therefore$ unbounded

(b) $e(k) = k$, $\therefore$ unbounded

(d) (a) $e(\infty) = \lim_{z\to1}(z-1)\dfrac{z}{(z-0.9)^2} = \underline{0}$

(b) $e(k) = k(0.9)^k$; $\therefore\ e(\infty) \to \underline{0}$

(e) (a) $e(\infty) = \lim_{z\to1}(z-1)\dfrac{z}{(z-1.1)^2} = \underline{0}$

(b) $e(k) = k(1.1)^k$; $\therefore\ e(\infty)$ is unbounded.

2-8. (a) i)

$$
\begin{array}{r l}
 & 0.5z^{-1} + 0.8z^{-2} + 0.98z^{-3} + \cdots \\
z^2 - 1.6z + 0.6 \,\big) & 0.5z \\
 & \underline{0.5z - 0.8 + 0.3z^{-1}} \\
 & 0.8 - 0.3z^{-1} \\
 & \underline{0.8 - 1.28z^{-1} + \cdots} \\
 & 0.98z^{-1} + \cdots
\end{array}
$$

ii) $\dfrac{E(z)}{z} = \dfrac{0.5}{(z-1)(z-0.6)} = \dfrac{1.25}{z-1} + \dfrac{-1.25}{z-0.6}$; $\therefore\ E(z) = \dfrac{1.25z}{z-1} - \dfrac{1.25z}{z-0.6}$

$\therefore\ e(k) = \underline{1.25(1 - 0.6^k)u(k)}$

(iii) $z^{k-1}E(z) = \dfrac{0.5z^k}{(z-1)(z-0.6)}$

$$e(k) = \frac{0.5(1)^k}{1-0.6} + \frac{0.5(0.6)^k}{0.6-1} \underline{= 1.25(1-0.6^k)u(k)}$$

(iv) $E_1(z) = \frac{0.5z}{z-0.6} \Rightarrow e_1(k) = 0.5(0.6)^k$

$E_2(z) = \frac{1}{z-1} \Rightarrow e_2(0) = 0\,;\ e_2(k) = 1,\ k \geq 1$

$e(0) = e_1(0)e_2(0) = (0.5)(0) = \underline{0}$

$e(1) = e_1(0)e_2(1) + e_1(1)e_2(0) = (0.5)(1) + (0.3)(0) = \underline{0.5}$

$e(2) = e_1(0)e_2(2) + e_1(1)e_2(1) + e_1(2)e_2(0)$

$\quad = 0.5\times 1 + 0.3\times 1 + 0.18\times 0 = \underline{0.8}$

$e(3) = 0.5\times 1 + 0.3\times 1 + 0.18\times 1 + 0.108\times 0 = \underline{0.98}$

(b) $e(0) = 0$

$e(k) = 1.25 - 2.083(0.6)^k,\ k \geq 1$

$E(z) = \underline{0.5z^{-2} + 0.8z^{-3} + 0.98z^{-4} + 1.088z^{-5} + \cdots}$

(c) $e(0) = 0\,;\ e(k) = 2.5 - 3.33(0.6)^k,\ k \geq 1$

$E(z) = \underline{0.5z^{-1} + 1.30z^{-2} + 1.78z^{-3} + 2.068z^{-4} + 2.2408z^{-5} + \cdots}$

(d) $e(k) = 0.75 + 0.25(0.6)^k$

$\underline{E(z) = 1 + 0.9z^{-1} + 0.84z^{-2} + 0.804z^{-3} + \cdots}$

(e)
```
num=[0 0 0.5];
den=[1 -1.6 0.6];
[r,p,k]=residue(num,den)
```

2-9. (a) poles: $z = \frac{2\cos a \pm \sqrt{4\cos^2 a - 4}}{2} = \cos(a) \pm j\sin(a)$

$\therefore$ pole $= \cos a$, provided $\sin a = 0 \Rightarrow a = 0, \pm\pi, \pm 2\pi, \ldots, \pm n\pi$

Then $\cos a = (-1)^n \quad \therefore$ poles $= \underline{\cos a}$

(b) $E(z) = \frac{z(z-\cos a)}{(z-\cos a)(z-\cos a)} = \underline{\frac{z}{z-\cos a}}\ ,\ a = \pm n\pi,\ n = 0, 1, \ldots$

(c) $E(z) = \frac{z}{z-\cos a} = \underline{\frac{z}{z-1}}\ ,\ \therefore \underline{\cos a = 1}\ ,\ a = 0, \pm 2\pi, \pm 4\pi, \ldots$

2-10. $x(k) - 3x(k-1) + 2x(k-2) = e(k)\,,\quad e(k) = \begin{cases} 1, & k = 0, 1 \\ 0, & k \geq 2 \end{cases}$

(a) $x(0) = e(0) = 1$

$x(1) = e(1) + 3x(0) = 4$

2-10. (a) $x(2) = e(2) + 3x(1) - 2x(0) = 10$

$x(3) = 0 + 3(10) - 2(4) = 22$

$x(4) = 0 + 3(22) - 2(10) = 46$

(b) $[1 - 3z^{-1} + 2z^{-2}]X(z) = E(z) = 1 + z^{-1} = \frac{z+1}{z}$

$X(z) = \frac{z^2}{(z-1)(z-2)} \times \frac{z+1}{z} = \frac{z(z+1)}{(z-1)(z-2)} = z\left[\frac{-2}{z-1} + \frac{3}{z-2}\right]$

$\therefore x(k) = \underline{-2 + 3(2)^k}$

(c) No, since the final value does not exist.

2-11. (a) $E(z) = \mathcal{z}[u(k-1)] = z^{-1}\left[\frac{z}{z-1}\right] = \frac{1}{z-1}$

$[z^2 - \frac{3}{4}z + \frac{1}{8}]Y(z) = E(z)$

$\frac{Y(z)}{z} = \frac{1}{z(z-\frac{1}{2})(z-\frac{1}{4})} \cdot \frac{1}{z-1} = \frac{-8}{z} + \frac{8/3}{z-1} + \frac{-16}{z-1/2} + \frac{64/3}{z-1/4}$

$\therefore y(k) = \underline{-8\delta(0) + 8/3 - 16(\frac{1}{2})^k + \frac{64}{3}(\frac{1}{4})^k}$

$\therefore y(0) = \underline{0}$; $y(1) = \underline{0}$; $y(2) = \underline{0}$; $y(3) = \underline{1}$; $y(4) = \underline{\frac{7}{4}}$

(b) $y(k+2) = e(k) + \frac{3}{4}y(k+1) - \frac{1}{8}y(k)$

$y(2) = 0 + \frac{3}{4}(0) - \frac{1}{8}(0) = \underline{0}$

$y(3) = 1 + \frac{3}{4}(0) - \frac{1}{8}(0) = \underline{1}$

$y(4) = 1 + \frac{3}{4}(1) - \frac{1}{8}(0) = \underline{7/4}$

(c) (a) $y(k+2) - \frac{3}{4}y(k+1) + \frac{1}{8}y(k) = 0$

$\therefore z^2[Y(z) - y(0) - y(1)z^{-1}] - \frac{3}{4}z[Y(z) - y(0)] + \frac{1}{8}Y(z) = 0$

$\therefore [z^2 - \frac{3}{4}z + \frac{1}{8}]Y(z) = z^2 - 2z - \frac{3}{4}z$

$\therefore Y(z) = z\left[\frac{z - 1/4}{(z-1/2)(z-1/4)}\right] = z\left[\frac{-9}{z-1/2} + \frac{10}{z-1/4}\right] \Rightarrow y(k) = \underline{-9(\frac{1}{2})^k + 10(\frac{1}{4})^k}$

$y(0) = \underline{1}$, $y(1) = \underline{-2}$, $y(2) = \underline{-13/8}$, $y(3) = \underline{-31/32}$ $y(4) = \underline{-67/128}$

(b) $y(k+2) = \frac{3}{4}y(k+1) - \frac{1}{8}y(k)$

$y(2) = \frac{3}{4}(-2) - \frac{1}{8}(1) = \underline{-13/8}$ $y(4) = \frac{3}{4}(-\frac{31}{32}) - \frac{1}{8}(-\frac{13}{8}) = -\frac{67}{128}$

$y(3) = \frac{3}{4}(-\frac{13}{8}) - \frac{1}{8}(-2) = \underline{-31/32}$

2-12.(a) $[1 - z^{-1} + z^{-2}]X(z) = E(z) = \frac{z}{z-1}$

$X(z) = \frac{z^3}{(z-1)(z^2-z+1)}$, poles: $z = \frac{1}{2} \pm j\frac{\sqrt{3}}{2} = 1\angle{\pm 60^\circ}$

$\frac{X(z)}{z} = \frac{1}{z-1} + \frac{k_1}{z-p_1} + \frac{k_1^*}{z-p_1^*}$ with $p = 1\angle{60^\circ}$

$k_1 = \left.\frac{z^2}{(z-1)(z-1\angle{-60^\circ})}\right|_{z=1\angle 60^\circ} = \frac{1\angle 120^\circ}{(.5+j.866-1)(.5+j.866-.5+j.866)}$

$= \frac{1\angle 120^\circ}{1\angle 120^\circ[j2(0.866)]} = 0.5774\angle{-90^\circ}$

$\therefore aT = \ln(|p_1|) = \underline{0}$; $bT = \arg P_1 = \underline{\frac{\pi}{3}}$

$A = 2|k_1| = \underline{1.155}$; $\theta = \arg k_1 = \underline{-90}^\circ$

$\therefore x(k) = 1 + 1.155\cos(\frac{\pi}{3}k - 90^\circ) = 1 + 1.155\sin(\frac{\pi}{3}k)$

$\therefore x(0) = \underline{1}$, $x(1) = \underline{2}$, $x(2) = \underline{2}$

(b)
$$\begin{array}{r|l} & 1 + 2z^{-1} + 2z^{-2} + \cdots \\ \hline z^3 - 2z^2 + 2z - 1 & z^3 \\ & \underline{z^3 - 2z^2 + 2z - 1} \\ & 2z^2 - 2z + 1 \\ & \underline{2z^2 - 4z + 4 - 2z^{-1}} \\ & 2z + \cdots \end{array}$$

$\therefore x(0) = \underline{1}$

$x(1) = \underline{2}$

$x(2) = \underline{2}$

(c) $x(k) = 1 + x(k-1) - x(k-2)$

$x(0) = 1 + 0 - 0 = \underline{1}$

$x(1) = 1 + 1 - 0 = \underline{2}$

$x(2) = 1 + 2 - 1 = \underline{2}$

(d) No, 3 poles for $X(z)$ on the unit circle.

2-13.(a) $z^2[X(z) - x(0) - x(1)z^{-1}] + 3z[X(z) - x(0)] + 2X(z) = E(z) = 1$

$\therefore X(z) = \frac{1 + z^2 - z + 3z}{z^2 - 3z + 2} = \frac{z^2 + 2z + 1}{z^2 + 3z + 2} = \underline{\frac{z+1}{z+2}}$

$\therefore X(z) = z\left[\frac{z+1}{z(z+2)}\right] = z\left[\frac{1/2}{z} + \frac{1/2}{z+2}\right]$

$\therefore x(k) = \frac{1}{2}\delta(k) + \frac{1}{2}(-2)^k$

2-13.(b) $x(0) = \underline{1}$, $x(1) = \underline{-1}$, $x(2) = \underline{2}$, $x(3) = \underline{-4}$

(c)

$$
\begin{array}{r|l}
 & 1 - z^{-1} + 2z^{-2} - 4z^{-3} + \cdots \\
z+2 & z+1 \\
 & \underline{z+2} \\
 & -1 \\
 & \underline{-1 - 2z^{-1}} \\
 & 2z^{-1} \\
 & \underline{2z^{-1} + 4z^{-2}} \\
 & -4z^{-2} \\
 & \cdots
\end{array}
$$

(d) $x(k+2) = e(k) - 3x(k+1) - 2x(k)$

$x(2) = 1 - 3(-1) - 2(1) = \underline{2}$

$x(3) = 0 - 3(2) - 2(-1) = \underline{-4}$

2-14.(a)

```
x0 = 0;
x1 = 0;
x2 = 0;
for k = 0:5;
   x3 = 2.2*x2 - 1.57*x1 + 0.36*x0 + 1
   x0 = x1;
   x1 = x2;
   x2 = x3;
end
```

(b) $x(k+3) = e(k) + 2.2\,x(k+2) - 1.57x(k+1) + 0.36\,x(k)$

$x(3) = 1 + 0 - 0 + 0 = \underline{1}$

$x(4) = 1 + 2.2(1) - 0 + 0 = \underline{3.2}$

$x(5) = 1 + 2.2(3.2) - 1.57(1) = \underline{6.47}$

(c) $[z^3 - 2.2z^2 + 1.57z - 0.36]\,X(z) = E(z) = \dfrac{z}{z-1}$

$$X(z) = \frac{z}{(z-1)(z^3 - 2.2z^2 + 1.57z - 0.36)}$$

$$
\begin{array}{r|l}
 & z^{-3} + 3.2z^{-4} + 6.47z^{-5} + \cdots \\
z^4 - 3.2z^3 + 3.77z^2 - 1.93z + 0.36 & z \\
 & \underline{z - 3.2 + 3.77z^{-1} \cdots} \\
 & 3.2 - 3.77z^{-1} \\
 & \underline{3.2 - 10.24z^{-1} + \cdots} \\
 & 6.47z^{-1} + \cdots \\
 & \cdots
\end{array}
$$

$\therefore$ $x(3) = \underline{1}$

$x(4) = \underline{3.2}$

$x(5) = \underline{6.47}$

2-15.(a) $y(k+1) = y(k) + Tx(k)$

(b) $zY(z) = Y(z) + TX(z) \Rightarrow \dfrac{Y(z)}{X(z)} = \dfrac{T}{z-1}$

2-15. (c) $y(k+1) = y(k) + T x(k+1)$

(d) $zY(z) = Y(z) + TzX(z) \Rightarrow \dfrac{Y(z)}{X(z)} = \dfrac{Tz}{z-1}$

(e) $y(1) = y(0) + Tx(0)$

$y(2) = y(1) + Tx(1) = y(0) + T(x(0) + x(1))$

$y(3) = y(2) + Tx(2) = y(0) + T[x(0) + x(1) + x(2)]$

$\therefore \quad y(k) = y(0) + T\sum_{n=0}^{k-1} x(n)$

(f) $y(1) = y(0) + Tx(1)$

$y(2) = y(1) + Tx(2) = y(0) + T[x(1) + x(2)]$

$\therefore \quad y(k) = y(0) + T\sum_{n=1}^{k} x(n)$

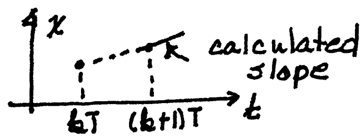
2-16. (a) $y(k+1) = y(k) + T\dfrac{x(k) + x(k+1)}{2}$

(b) $zY(z) = Y(z) + \frac{T}{2}[X(z) + zX(z)] \Rightarrow \underline{Y(z) = \frac{T}{2}\,\frac{z+1}{z-1}X(z)}$

2-17. (a) $TzW(z) = zX(z) - X(z)$

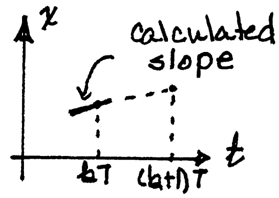
$w(k+1) = \frac{1}{T}[x(k+1) - x(k)]$

(b)



(c) $TW(z) = zX(z) - X(z)$

$w(k) = \frac{1}{T}[x(k+1) - x(k)]$



2.18. (a) $y(k) = \beta_2 e(k) + \beta_1 e(k-1) + \beta_0 e(k-2) - \alpha_1 y(k-1) - \alpha_0 y(k-2)$

(b) $[1 + \alpha_1 z^{-1} + \alpha_0 z^{-2}]\,Y(z) = [\beta_2 + \beta_1 z^{-1} + \beta_0 z^{-2}]\,E(z)$

$$\underline{\frac{Y(z)}{E(z)} = \frac{\beta_2 z^2 + \beta_1 z + \beta_0}{z^2 + \alpha_1 z + \alpha_0}}$$

(c) $f(k) = e(k) - a_1 f(k-1) - a_0 f(k-2)$

$y(k) = b_2 f(k) + b_1 f(k-1) + b_0 f(k-2)$

(d) $F(z) = E(z) - (a_1 z^{-1} + a_0 z^{-2})F(z) \Rightarrow F(z) = \dfrac{E(z)}{1 + a_1 z^{-1} + a_0 z^{-2}}$

2.18.(d) $Y(z) = (b_2 + b_1 z^{-1} + b_0 z^{-2}) F(z) = \underline{\dfrac{b_2 z^2 + b_1 z + b_0}{z^2 + a_1 z + a_0} E(z)}$

(e) $\underline{\alpha_i = a_i}$ and $\underline{\beta_i = b_i}$, $i = 1, 2$

(f)

```
ykminus2 = 0;
ykminus1 = 0;
ekminus2 = 0;
ekminus1 = 0;
ek = 1;
for k = 0:5
   yk=b2*ek+b1*ekminus1+b0*ekminus2-a1*ykminus1-a0*ykminus2;
   [k, ek, yk]
   ekminus2 = ekminus1;
   ekminus1 = ek;
   ykminus2 = ykminus1;
   ykminus1 = yk;
end
```

(g)

```
fkminus2 = 0;
fkminus1 = 0;
ek = 1;
for k = 0:5
   fk=ek-a1*fkminus1-a0*fkminus2;
   yk = b2*fk+b1*fkminus1+b0*fkminus2;
   [k, ek, yk]
   fkminus2 = fkminus1;
   fkminus1 = fk;
end
```

2-19.(a) $f_1(k) = g_1 f_1(k-1) - g_2 f_2(k-1) + g_3 e(k)$

$f_2(k) = g_1 f_2(k-1) + g_1 f_1(k-1) + g_4 e(k)$

$y(k) = b_2 e(k) + f_2(k-1)$

(b) (1) $F_1(z) = g_1 z^{-1} F_1(z) - g_2 z^{-1} F_2(z) + g_3 E(z)$

(2) $F_2(z) = g_1 z^{-1} F_2(z) + g_2 z^{-1} F_1(z) + g_4 E(z)$

(3) $Y(z) = b_0 E(z) + z^{-1} F_2(z)$

$\therefore$ (1) $(z - g_1) F_1(z) + g_2 F_2(z) = g_3 z E(z)$

(2) $-g_2 F_1(z) + (z - g_1) F_2(z) = g_4 z E(z)$

$$\therefore F_2(s) = \frac{\begin{vmatrix} z-g_1 & g_3 z E(z) \\ -g_2 & g_4 z E(z) \end{vmatrix}}{\begin{vmatrix} z-g_1 & g_2 \\ -g_2 & z-g_1 \end{vmatrix}} = \frac{(g_4 z^2 - g_1 g_4 z + g_2 g_3 z)}{(z-g_1)^2 + g_2^2} E(z)$$

2.19. (b) $\therefore \dfrac{Y(z)}{E(z)} = b_2 + \dfrac{g_4 z + g_2 g_3 - g_1 g_4}{(z-g_1)^2 + g_2^2}$

Also, $D(z) = b_2 + \dfrac{Re(A) + j\,Im(A)}{z - Re(P) - j\,Im(P)} + \dfrac{Re(A) - j\,Im(A)}{z - Re(P) + j\,Im(P)}$

$= b_2 + \dfrac{\frac{1}{2}(g_4 - j g_3)}{z - g_1 - j g_2} + \dfrac{\frac{1}{2}(g_4 + j g_3)}{z - g_1 + j g_2}$

$= b_2 + \dfrac{g_4 z - g_1 g_4 + g_2 g_3}{(z-g_1)^2 + g_2^2}$ ✓

(c) $D(z) = b_0 + \dfrac{g_2 g_3 z^{-2} + g_4(1 - g_1 z^{-1})}{1 - g_1 z^{-1} - g_1 z^{-1} + g_1^2 z^{-2} + g_2^2 z^{-2}}$

$= b_0 + \dfrac{g_4 z + g_2 g_3 - g_1 g_4}{z^2 - 2 g_1 z + g_1^2 + g_2^2}$

(d)

```
f1kminus1 = 0;
f2kminus1 = 0;
ek = 1;
for k = 0:5
   yk = b0*ek+f2kminus1;
   [k, ek, yk]
   f1k = g1*f1kminus1 - g2*f2kminus1 + g3*ek;
   f2k = g1*f2kminus1 + g2*f1kminus1 + g3*ek;
   f1kminus1 = f1k;
   f2kminus1 = f2k;
end
```

2-20 (a) $\beta_2 = 2$, $\beta_1 = -2.4$, $\beta_0 = 0.72$, $\alpha_1 = -1.4$, $\alpha_0 = 0.98$

(b) $b_2 = 2$, $b_1 = -2.4$, $b_0 = 0.72$, $a_1 = -1.4$, $a_0 = 0.98$

(c) poles: $z = \dfrac{1.4 \pm (1.4^2 - 4(0.98))^{1/2}}{2} = 0.7 \pm j0.7 = 0.99\angle \pm 45^\circ$

$D(z) = 2 + \dfrac{A}{z - 0.7 - j0.7} + \dfrac{A^*}{z - 0.7 + j0.7}$

$\therefore A = \left.\dfrac{2z^2 - 2.4z + 0.72}{z - 0.7 + j0.7}\right|_{z = 0.99\angle 45^\circ} = \dfrac{j1.96 - (1.68 + j1.68) + 0.72}{j1.4}$

$= \underline{0.2 + j\,0.6857}$

$\therefore$ $g_1 = 0.7$ $\quad g_3 = 1.371$

$g_2 = 0.7$ $\quad g_4 = 0.4$

2-20.(d)

```
num=[2 -2.4 .72];
den=[1 -1.4 0.98];
[r,p,k]=residue(num,den)
```

(e) $\Delta = 1 - (0.7z^{-1} + 0.7z^{-1} + 0.4z^{-2}) + 0.49z^{-2}$

$= 1 - 1.4z^{-1} + 0.98z^{-2}$

$D(z) = 2 + \frac{1}{\Delta}[1.371(0.7)z^{-2} + 0.4z^{-1}(1 + 0.7z^{-1})]$

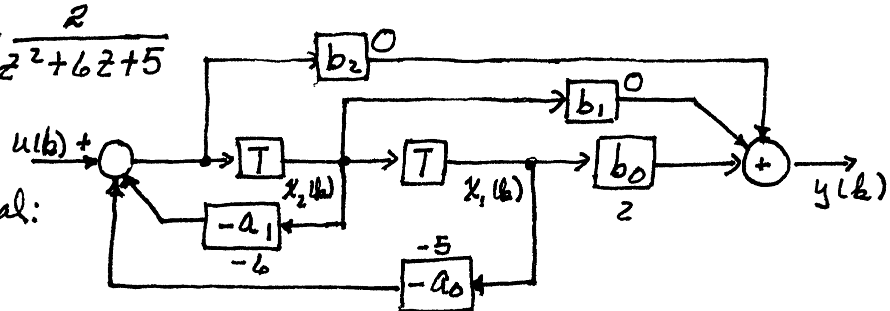
$= 2 + \dfrac{0.4z - 1.24}{z^2 - 1.4z + 0.98} = \underline{\dfrac{2z^2 - 2.4z + 0.72}{z^2 - 1.4z + 0.98}}$

2-21.

```
s1 = 0;
e = 0;
for k=0:5
   s2 = e - s1;
   m = 0.5*s2 - s1;
   [k,m]
   s1 = s2;
   e = e + 1;
end
```

2-22. (a) $\dfrac{Y(z)}{U(z)} = \dfrac{2}{z^2 + 6z + 5}$

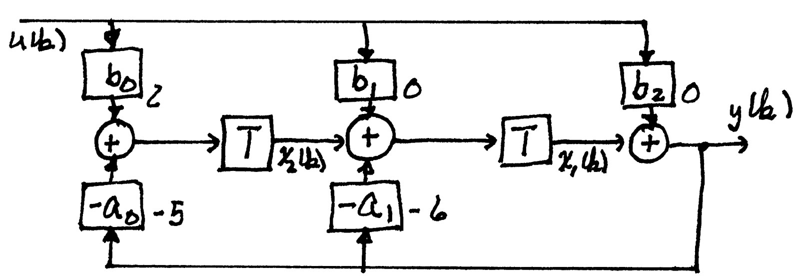
(1) control canonical:



$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$

$y(k) = [2 \quad 0]\,\underline{x}(k)$

(2) observer canonical:



$\underline{x}(k+1) = \begin{bmatrix} -6 & 1 \\ -5 & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(k)$

$y(k) = [1 \quad 0]\,\underline{x}(k)$

2-22. (b) $\dfrac{Y(z)}{U(z)} = \dfrac{z+2}{z^2+6z+5}$ (1) control canonical: $\underline{x}(k+1)$ = same as (a)

$y(k) = [2 \quad 1]\underline{x}(k)$

(2) observer canonical:

$$\underline{x}(k+1) = \begin{bmatrix} -6 & 1 \\ -5 & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0]\,\underline{x}(k)$$

(c) $\dfrac{Y(z)}{U(z)} = \dfrac{3z^2+z+2}{z^2+6z+5}$ (1) control canonical: $\underline{x}(k+1)$ = same as (a)

$y(k) = [-13 \quad -17]\underline{x}(k) + 3u(k)$

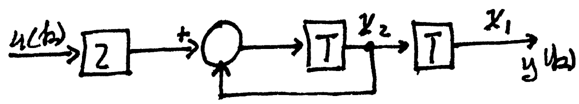
(2) observer canonical:

$$\underline{x}(k+1) = \begin{bmatrix} -6 & 1 \\ -5 & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0]\,\underline{x}(k) + 3u(k)$$

2-23. (a) $G(z) = G_1(z)\,G_2(z) = \dfrac{2}{z^2 - z} = \dfrac{2z^{-2}}{1 - z^{-1}}$

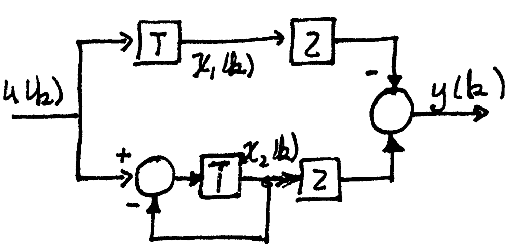
(1)



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0]\,\underline{x}(k)$$

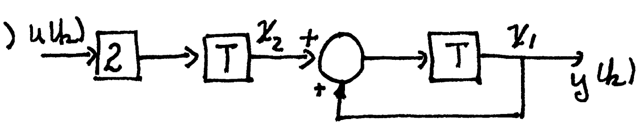
(2) $G(z) = \dfrac{2}{z(z-1)} = \dfrac{-2}{z} + \dfrac{2}{z-1} = G_1(z) + G_2(z)$



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = [-2 \quad 2]\,\underline{x}(k)$$

(3)



$$\underline{x}(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0]\,\underline{x}(k)$$

2-23(b) (1) $zI-A=\begin{bmatrix} z & -1 \\ 0 & z-1 \end{bmatrix}$; $|zI-A| = z^2-z=\Delta$

$$G(z)=C(zI-A)^{-1}B=\frac{1}{\Delta}[1 \quad 0]\begin{bmatrix} z-1 & 1 \\ 0 & z \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix}=\frac{1}{\Delta}[z-1 \quad 1]\begin{bmatrix} 0 \\ 2 \end{bmatrix}=\underline{\frac{2}{z(z-1)}}$$

(2) $zI-A=\begin{bmatrix} z & 0 \\ 0 & z-1 \end{bmatrix}$; $|zI-A|=\Delta=z^2-z$

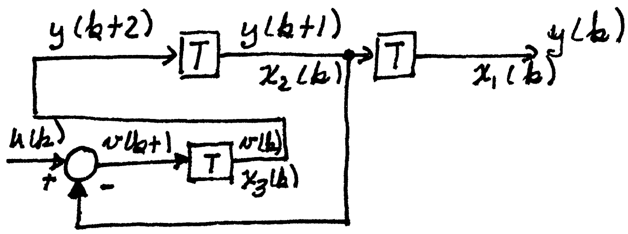
$$G(z)=C(zI-A)^{-1}B=\frac{1}{\Delta}[-1 \quad 1]\begin{bmatrix} z-1 & 0 \\ 0 & z \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix}=\frac{1}{\Delta}[-1 \quad 1]\begin{bmatrix} 2z-2 \\ 2z \end{bmatrix}=\underline{\frac{2}{z(z-1)}}$$

(3) $zI-A=\begin{bmatrix} z-1 & -1 \\ 0 & z \end{bmatrix}$; $|zI-A|=z^2-z=\Delta$

$$G(z)=C(zI-A)^{-1}B=\frac{1}{\Delta}[1 \quad 0]\begin{bmatrix} z & 1 \\ 0 & z-1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix}=\frac{1}{\Delta}[z \quad 1]\begin{bmatrix} 0 \\ 2 \end{bmatrix}=\underline{\frac{2}{z(z-1)}}$$

(c)
```
A = [0 1;0 1]; B = [0;2]; C = [1 0]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

2-24. (a)



$$\underline{x}(k+1)=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}\underline{x}(k)+\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u(k)$$

$$\underline{y}_o(k)=\begin{bmatrix} x_2(k) \\ x_3(k) \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\underline{x}(k) \quad ; \ \underline{y}_o(k)=\text{output}$$

(b) $\underline{x}(k+1)=$ same as (a)

$$\underline{y}_o(k)=\begin{bmatrix} x_1(k) \\ x_3(k) \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\underline{x}(k)$$

(c) $\underline{x}(k+1)=$ same as (a)

$$y_o(k)=x_3(k)=[0 \quad 0 \quad 1]\underline{x}(k)$$

(d)

$$zI-A=\begin{bmatrix} z & -1 & 0 \\ 0 & z & -1 \\ 0 & 1 & z \end{bmatrix} \ ; \ |zI-A|=z^3-(-z)=z^3+z=\Delta$$

2-24.(d)
$$\operatorname{cof}(zI-A)=\begin{bmatrix} z^2+1 & z^2 & 0 \\ z & z^2 & z \\ 1 & z & z^2 \end{bmatrix};\quad (zI-A)^{-1}=\frac{1}{\Delta}\begin{bmatrix} z^2+1 & z & 1 \\ z^2 & z^2 & z \\ 0 & z & z^2 \end{bmatrix}$$

$$\therefore \frac{Y_o(z)}{U(z)} = C(zI-A)^{-1}B = \frac{1}{\Delta}[0 \quad 0 \quad 1]\begin{bmatrix} \quad \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

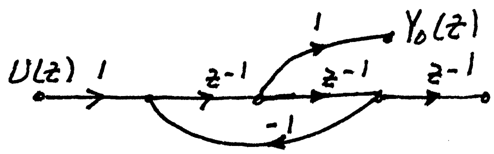
$$= \frac{1}{\Delta}[0 \quad z \quad z^2]\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{z^2}{z^3-z} = \underline{\frac{z}{z^2+1}}$$

(e) $z^2 Y(z) - V(z) = 0 \quad \Rightarrow Y(z) = \frac{1}{z^2}V(z)$

$z\,V(z) + z\,Y(z) = z\,V(z) + \frac{1}{z}V(z) = U(z)$

$$\therefore \frac{Y(z)}{U(z)} = \frac{Y_o(z)}{U(z)} = \frac{1}{z+\frac{1}{z}} = \underline{\frac{z}{z^2+1}}$$

(f) From (a):

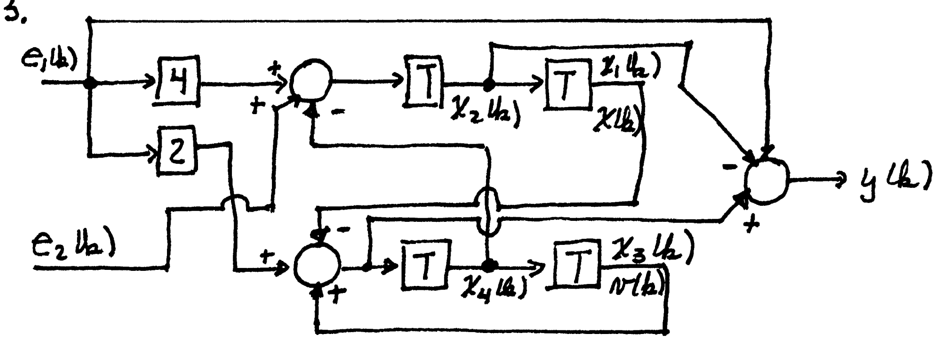


$$\therefore \frac{Y_o(z)}{U(z)} = \frac{z^{-1}}{1+z^{-2}} = \frac{z}{z^2+1}$$

(g)

```
A = [0 1 0;0 0 1;0 -1 0]; B = [0; 0; 1]; C = [0 0 1]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

2-25.



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0 & 0 \\ 4 & 1 \\ 0 & 0 \\ 2 & 0 \end{bmatrix}\underline{e}(k)$$

2-25. $y(k) = x_4(k+1) - x_2(k) + e_1(k) = -x_1(k) + x_3(k) - x_2(k) + e_1(k)$

$\therefore y(k) = [-1 \quad -1 \quad 1 \quad 0]\, \underline{x}(k) + [1 \quad 0]\, \underline{e}(k)$

2-26 (a) $zI - A = \begin{bmatrix} z & -1 \\ 0 & z-3 \end{bmatrix}$; $\Delta = |zI - A| = z(z-3) = \Delta$

$$\frac{Y(z)}{U(z)} = C(zI-A)^{-1}B = \frac{1}{\Delta}[-2 \quad 1]\begin{bmatrix} z-3 & 1 \\ 0 & z \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\Delta}[-2z+6 \quad z-2]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \underline{\frac{-z+4}{z(z-3)}}$$

(b) $P = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$; $P^{-1} = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$

$$A_w = P^{-1}AP = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$$

$$= \underline{\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}}$$

$$B_w = P^{-1}B = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \underline{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

$$C_w = CP = [-2 \quad 1]\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = [-1 \quad 3]$$

$$\therefore \underline{w}(k+1) = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} w(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = [-1 \quad 3]\, w(k)$$

(c) $zI - A_w = \begin{bmatrix} z-2 & -2 \\ -1 & z-1 \end{bmatrix}$; $\Delta = |zI - A_w| = z^2 - 3z + 2 - 2 = z(z-3)$

$$\frac{Y(z)}{U(z)} = C_w(zI - A_w)^{-1}B_w = \frac{1}{\Delta}[-1 \quad 3]\begin{bmatrix} z-1 & 2 \\ 1 & z-2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\Delta}[-1 \quad 3]\begin{bmatrix} z-1 \\ 1 \end{bmatrix} = \underline{\frac{-z+4}{z(z-3)}}$$

(d) $|zI - A| = \begin{vmatrix} z & 1 \\ 0 & z-3 \end{vmatrix} = \underline{z^2 - 3z}$; $|zI - A_w| = \begin{vmatrix} z-2 & -2 \\ -1 & z-1 \end{vmatrix} = z(z-3)$

$\therefore z_1 = 0,\ z_2 = 3$

$|A| = \begin{vmatrix} 0 & 1 \\ 0 & 3 \end{vmatrix} = \underline{0} = z_1 z_2$; $|A_w| = \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = \underline{0}$

$\mathrm{tr}\, A = \underline{3} = z_1 + z_2$; $\mathrm{tr}\, A_w = \underline{3}$

2-26 (e)

```
A = [0 1;0 3]; B = [1;1]; C = [-2 1]; D = 0;
[num, den] = ss2tf(A,B,C,D)

pause

A = [2 2;1 1]; B = [1;0]; C = [-1 3]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

2-27. (a) Let z_1, z_2 be the characteristic value of A.

$$zI-A=\begin{bmatrix} z & -1 \\ 0 & z-3 \end{bmatrix}, \quad \therefore |zI-A| = z(z-3); \quad \therefore \underline{z_1=0}, \ \underline{z_2=3}$$

(b) $$(z_1 I - A)\underline{m}_1 = \begin{bmatrix} 0 & -1 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} m_{11} \\ m_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} -m_{21}=0 \\ -3m_{21}=0 \end{array}$$

$$\therefore m_{21}=0, \text{ let } m_{11}=1, \ \therefore \underline{m}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$(z_2 I - A)\underline{m}_2 = \begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} m_{12} \\ m_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 3m_{12} - m_{22} = 0$$

$$\therefore \text{ let } m_{12}=1, \ m_{22}=3, \ \therefore \underline{m}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\therefore M = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}, \quad |M| = 3, \quad M^{-1} = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix}$$

$$M^{-1}AM = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}$$

(c) $$B_w = M^{-1}B = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$$

$$C_w = CM = \begin{bmatrix} -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \end{bmatrix}$$

$$\therefore \underline{w}(k+1) = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}\underline{w}(k) + \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}u(k)$$

$$y(k) = \begin{bmatrix} -2 & 1 \end{bmatrix}\underline{w}(k)$$

(d) See Problem 2-26(c) for the first transfer function.

$$zI - A_w = \begin{bmatrix} z & 0 \\ 0 & z-3 \end{bmatrix}; \quad |zI-A_w| = z(z-3) = \Delta$$

$$\frac{Y(z)}{U(z)} = C_w(zI-A_w)^{-1}B_w = \frac{1}{\Delta}\begin{bmatrix} -2 & 1 \end{bmatrix}\begin{bmatrix} z-3 & 0 \\ 0 & z \end{bmatrix}\begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$$

$$= \frac{1}{\Delta}\begin{bmatrix} -2z+6 & z \end{bmatrix}\begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix} = \frac{-\frac{4}{3}z + 4 + \frac{1}{3}z}{\Delta} = \underline{\frac{-z+4}{z(z-3)}}$$

2-27. (e)

```
A = [0 1;0 3]; B = [1;1]; C = [-2 1]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

```
A = [0 0;0 -3]; B = [.6667; .3333]; C = [-2 1]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

2-28. (a) $\dfrac{Y(z)}{U(z)} = \dfrac{-z+4}{z(z-3)}$; from Problem 2-26(c)

(b) $Y(z) = \dfrac{(-z+4)z}{z(z-3)(z-1)}$

$$\frac{Y(z)}{z} = \frac{-z+4}{z(z-1)(z-3)} = \frac{4/3}{z} + \frac{-3/2}{z-1} + \frac{1/6}{z-3}$$

$$\therefore y(k) = \begin{cases} 4/3 - 3/2 + 1/6 = \underline{0}, & k=0 \\ \underline{-3/2 + 1/6(3)^k}, & k \geq 1 \end{cases}$$

$\therefore y(0) = \underline{0}$ $\quad y(2) = -\frac{3}{2} + \frac{3}{2} = \underline{0}$

$y(1) = -\frac{3}{2} + \frac{1}{2} = \underline{-1}$

(c) From Problem 2.26(c),

$$\Phi(z) = z(zI-A)^{-1} = z\begin{bmatrix} \frac{z-3}{z(z-3)} & \frac{1}{z(z-3)} \\ 0 & \frac{z}{z(z-3)} \end{bmatrix} = z\begin{bmatrix} \frac{1}{z} & \frac{-1/3}{z} + \frac{1/3}{z-3} \\ 0 & \frac{1}{z-3} \end{bmatrix}$$

$$\therefore \Phi(k) = \begin{bmatrix} \delta(k) & -\frac{1}{3}\delta(k) + \frac{1}{3}(3)^k \\ 0 & (3)^k \end{bmatrix}$$

(d) $$y(k) = \sum_{j=0}^{k-1} C\,\Phi(k-1-j)Bu(j) = \sum_{j=0}^{k-1} [-2 \quad 1]\,\Phi(k-1-j)\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \sum_{j=0}^{k-1} [-2 \quad 1]\begin{bmatrix} \frac{2}{3}\delta(k-1-j) + \frac{1}{3}(3)^{k-j-1} \\ (3)^{k-j-1} \end{bmatrix} = \sum_{j=0}^{k-1}\left[-\frac{4}{3}\delta(k-j-1) + \frac{1}{3}(3)^{k-j-1}\right]$$

$$= \underline{\sum_{j=0}^{k-1}\left[-\frac{4}{3}\delta(k-1-j) + \frac{1}{3}(3)^{k-1-j}\right]}$$

$y(0) = \underline{0}$; $\quad y(1) = -\frac{4}{3}\delta(0) + \frac{1}{3}(3)^0 = -\frac{4}{3} + \frac{1}{3} = \underline{-1}$

$y(2) = -\frac{4}{3}\delta(1) + \frac{1}{3}(3)^1 - \frac{4}{3}\delta(0) + \frac{1}{3}(3)^0 = 1 - \frac{4}{3} + \frac{1}{3} = \underline{0}$

(e) $\underline{x}(1) = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $y(1) = [-2 \quad 1]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \underline{-1}$

$\underline{x}(2) = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$; $y(2) = [-2 \quad 1]\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \underline{0}$

2-29. (a) From Problem 2-30(a),

2-30.(a) $\frac{Y(z)}{U(z)} = C(zI-A)^{-1}B = [1 \quad 2]\begin{bmatrix} \frac{1}{z-1} & 0 \\ 0 & \frac{1}{z-0.5}\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}$

$= [1 \quad 2]\begin{bmatrix}\frac{2}{z-1} \\ \frac{1}{z-0.5}\end{bmatrix} = \frac{2}{z-1} + \frac{2}{z-0.5} = \underline{\frac{4z-3}{(z-1)(z-0.5)}}$

(b) $P = \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$, $P^{-1} = \begin{bmatrix}1/2 & 1/2\\-1/2 & 1/2\end{bmatrix}$

$\therefore A_W = P^{-1}AP = \begin{bmatrix}1/2 & 1/2\\-1/2 & 1/2\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1/2\end{bmatrix}\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix} = \begin{bmatrix}1/2 & 1/4\\-1/2 & 1/4\end{bmatrix}\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix} = \begin{bmatrix}3/4 & -1/4\\-1/4 & 3/4\end{bmatrix}$

$B_W = P^{-1}B = \begin{bmatrix}1/2 & 1/2\\-1/2 & 1/2\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}3/2\\-1/2\end{bmatrix}$

$C_W = CP = [1 \quad 2]\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix} = [3 \quad 1]$

$\therefore \underline{w}(k+1) = \begin{bmatrix}3/4 & -1/4\\-1/4 & 3/4\end{bmatrix}\underline{w}(k) + \begin{bmatrix}3/2\\-1/2\end{bmatrix}u(k)$

$y(k) = [3 \quad 1]\underline{x}(k)$

(c) $zI - A_W = \begin{bmatrix}z-3/4 & 1/4\\1/4 & z-3/4\end{bmatrix}$, $|zI-A_W| = z^2 - 1.5z + \frac{9}{16} - \frac{1}{16} = z^2 - 1.5z + 0.5 = \Delta$

$\frac{Y(z)}{U(z)} = C_W(zI-A_W)^{-1}B_W = [3 \quad 1]\frac{1}{\Delta}\begin{bmatrix}z-3/4 & -1/4\\-1/4 & z-3/4\end{bmatrix}\begin{bmatrix}3/2\\-1/2\end{bmatrix}$

$= \frac{1}{\Delta}[3z-2.5 \quad z-1.5]\begin{bmatrix}3/2\\-1/2\end{bmatrix} = \frac{4z-3}{(z-1)(z-0.5)}$

(d) $|zI-A| = \begin{vmatrix}z-1 & 0\\0 & z-0.5\end{vmatrix} = \underline{z^2 - 1.5z + 0.5}$; $|zI-A_W| = \underline{z^2 - 1.5z + 0.5}$

$\therefore z_1 = 1$, $z_2 = 0.5$

$|A| = \begin{vmatrix}1 & 0\\0 & 0.5\end{vmatrix} = \underline{0.5} = z_1 z_2$; $|A_W| = \begin{vmatrix}3/4 & -1/4\\-1/4 & 3/4\end{vmatrix} = \frac{9}{16} - \frac{1}{16} = \underline{0.5}$

$\text{tr}\, A = \underline{1.5} = z_1 + z_2$; $\text{tr}\, A_W = \underline{1.5}$

2-30.(a) $zI - A = \begin{bmatrix}z-1 & 0\\0 & z-0.5\end{bmatrix}$; $|zI-A| = \Delta = (z-1)(z-0.5)$

$(zI - A^{-1}) = \frac{1}{\Delta}\begin{bmatrix}z-0.5 & 0\\0 & z-1\end{bmatrix} = \begin{bmatrix}\frac{1}{z-1} & 0\\0 & \frac{1}{z-0.5}\end{bmatrix}$

2-30 (a) $\therefore \Phi(k) = \mathcal{Z}^{-1}\begin{bmatrix} \frac{z}{z-1} & 0 \\ 0 & \frac{z}{z-0.5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0.5^k \end{bmatrix}$

$\therefore \underline{x}(k) = \Phi(k)\,\underline{x}(0) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5^k \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2(0.5)^k \end{bmatrix}$

(b) $y(k) = Cx(k) = [1 \quad 2]\begin{bmatrix} 1 \\ 2(0.5)^k \end{bmatrix} = \underline{1+4(0.5)^k}$

(c) $\Phi(0) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5^0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

(d) $x(k)\Big|_{k=0} = \begin{bmatrix} 1 \\ 2(0.5)^k \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(e) From (b), $y(0) = \underline{5}$ $\quad y(2) = \underline{2}$

$y(1) = \underline{3}$ $\quad y(3) = \underline{1.5}$

$y(0) = C\underline{x}(0) = [1 \quad 2]\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \underline{5}$

$x(1) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad y(1) = [1 \quad 2]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \underline{3}$

$x(2) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \quad y(2) = [1 \quad 2]\begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = \underline{2}$

$\underline{x}(3) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}\begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.25 \end{bmatrix}, \quad y(3) = [1 \quad 2]\begin{bmatrix} 1 \\ 0.25 \end{bmatrix} = \underline{1.5}$

(f)
```
A = [1 0;0 .5]; B = [2; 1]; C = [1 2];
x=[1; 2];
u = 0;
for k = 0:3
   x1 = A*x + B*u;
   y = C*x;
   [k,y]
   x = x1;
end
```

2-31. (a) $zI - A = \begin{bmatrix} z-1.1 & -1 \\ 0.3 & z \end{bmatrix}; \quad |zI-A| = \Delta = z^2 - 1.1z + 0.3 = (z-0.5)(z-0.6)$

$(zI-A)^{-1} = \frac{1}{\Delta}\begin{bmatrix} z & 1 \\ -0.3 & z-1.1 \end{bmatrix}$

2-31 (a) $\Phi(k) = \mathcal{Z}^{-1}[z(zI-A)^{-1}] = \mathcal{Z}^{-1}\left(z\begin{bmatrix} \frac{z}{(z-0.5)(z-0.6)} & \frac{1}{(z-0.5)(z-0.6)} \\ \frac{-0.3}{(z-0.5)(z-0.6)} & \frac{z-1.1}{(z-0.5)(z-0.6)} \end{bmatrix}\right)$

$$= \mathcal{Z}^{-1}\left(z\begin{bmatrix} \frac{-5}{z-.5} + \frac{6}{z-.6} & \frac{-10}{z-.5} + \frac{10}{z-.6} \\ \frac{3}{z-.5} + \frac{-3}{z-.6} & \frac{6}{z-.5} + \frac{-5}{z-.6} \end{bmatrix}\right)$$

$$= \begin{bmatrix} -5(0.5)^k + 6(0.6)^k & -10(0.5)^k + 10(0.6)^k \\ 3(0.5)^k - 3(0.6)^k & 6(0.5)^k - 5(0.6)^k \end{bmatrix}$$

$$\therefore \underline{x}(k) = \Phi(k)\underline{x}(0) = \begin{bmatrix} \quad \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -15(0.5)^k + 14(0.6)^k \\ 9(0.5)^k - 7(0.6)^k \end{bmatrix}$$

(b) $y(k) = C\underline{x}(k) = [1 \quad -1]\,\underline{x}(k) = -24(0.5)^k + 21(0.6)^k$

(c) $\Phi(0) = \begin{bmatrix} -5+6 & -10+10 \\ 3-3 & 6-5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

(d) $\underline{x}(k)\Big|_{k=0} = \begin{bmatrix} -15+14 \\ 9-7 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(e) From (b), $y(0) = \underline{-3}$ $\qquad y(2) = \underline{1.56}$

$y(1) = \underline{0.6}$ $\qquad y(3) = \underline{1.536}$

$y(0) = C\underline{x}(0) = [1 \quad -1]\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \underline{-3}$

$\underline{x}(1) = \begin{bmatrix} 1.1 & 1 \\ -0.3 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.3 \end{bmatrix}$; $y(1) = [1 \quad -1]\begin{bmatrix} 0.9 \\ 0.3 \end{bmatrix} = \underline{0.6}$

$\underline{x}(2) = \begin{bmatrix} 1.1 & 1 \\ -0.3 & 0 \end{bmatrix}\begin{bmatrix} 0.9 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 1.29 \\ -0.27 \end{bmatrix}$; $y(2) = [1 \quad -1]\begin{bmatrix} 1.29 \\ -0.27 \end{bmatrix} = \underline{1.56}$

$\underline{x}(3) = \begin{bmatrix} 1.1 & 1 \\ -0.3 & 0 \end{bmatrix}\begin{bmatrix} 1.29 \\ -0.27 \end{bmatrix} = \begin{bmatrix} 1.149 \\ -0.387 \end{bmatrix}$; $y(3) = [1 \quad -1]\begin{bmatrix} 1.149 \\ -0.387 \end{bmatrix} = \underline{1.536}$

(f)

```
A = [1.1 1;-0.3 0]; B = [1; 1]; C = [1 -1];
x=[-1; 2];
u = 0;
for k = 0:3
   x1 = A*x + B*u;
   y = C*x;
   [k,y]
   x = x1;
end
```

2-32.

$$\underline{x}(k+1) = \begin{bmatrix} -a_{n-1} & 1 & 0 & \cdots & 0 \\ -a_{n-2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -a_0 & 0 & 0 & \cdots & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} a_{n-1} \\ b_{n-2} \\ \vdots \\ b_0 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0 \quad 0 \quad \cdots \quad 0]\,\underline{x}(k)$$

2-33. $\underline{x}(k+1) = A\,\underline{x}(k)$; $\underline{x}(k) = \Phi(k)\,\underline{x}(0)$

$\therefore \Phi(k+1)\,\underline{x}(0) = A\,\Phi(k)\,\underline{x}(0)$

Since this is true for any $\underline{x}(0)$, $\therefore$ $\underline{\Phi(k+1) = A\,\Phi(k)}$

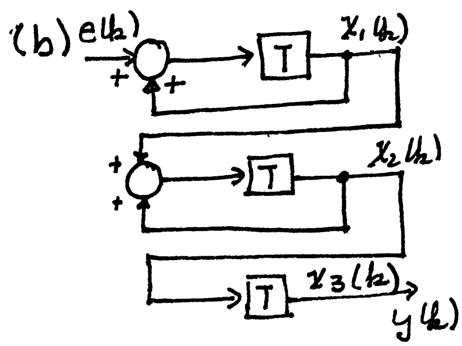
2-34 (a) $zI - A = \begin{bmatrix} z-1 & 0 & 0 \\ -1 & z-1 & 0 \\ 0 & -1 & z \end{bmatrix}$; $\Delta = z^3 - 2z^2 + z = z(z-1)^2$

$$\operatorname{cof}(zI-A) = \begin{bmatrix} z(z-1) & z & 1 \\ 0 & z(z-1) & z-1 \\ 0 & 0 & (z-1)^2 \end{bmatrix}, \quad (zI-A)^{-1} = \begin{bmatrix} \frac{1}{z-1} & 0 & 0 \\ \frac{1}{(z-1)^2} & \frac{1}{z-1} & 0 \\ \frac{1}{z(z-1)^2} & \frac{1}{z(z-1)} & \frac{1}{z} \end{bmatrix}$$

$$G(z) = C(zI-A)^{-1}B = [0 \quad 0 \quad 1](zI-A)^{-1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \left[\frac{1}{z(z-1)^2} \quad \frac{1}{z(z-1)} \quad \frac{1}{z}\right]\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{z(z-1)^2} = \underline{\frac{1}{z^3 - 2z^2 + z}}$$

(b)



(c) $\Delta = 1 - z^{-1} - z^{-1} + z^{-2} = 1 - 2z^{-1} + z^{-2}$

$$\therefore G(z) = \frac{z^{-3}}{\Delta} = \frac{1}{z^3 - 2z^2 + z}$$

2-34. (d)

```
A = [1 0 0;1 1 0;0 1 0]; B = [1;0;0]; C = [0 0 1]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

2-35. $C_W(zI-A_W)^{-1}B_W + D_W = CP[zI - P^{-1}AP]^{-1}P^{-1}B + D$

$$= CP[zP^{-1}IP - P^{-1}AP]^{-1}P^{-1}B + D$$

$$= CP[P^{-1}(zI-A)P]^{-1}P^{-1}B + D$$

$$= CPP^{-1}(zI-A)^{-1}PP^{-1}B + D \quad ; \text{ since } (ABC)^{-1} = C^{-1}B^{-1}A^{-1}$$

$$= C(zI-A)^{-1}B + D$$

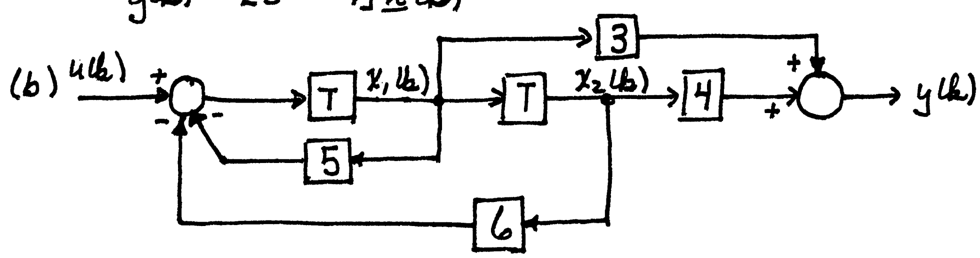
2-36. (a)

```
n = [0 3 4];
d = [1 5 6];
[A,B,C,D] = tf2ss(n,d)
```

$$\underline{x}(k+1) = \begin{bmatrix} -5 & -6 \\ 1 & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = [3 \quad 4]\,\underline{x}(k)$$

(b)



(c) The control canonical form with the states renumbered.

CHAPTER 3

3-1. (a) $E^*(s) = \sum_{n=0}^{\infty} e(nT)\epsilon^{-nTs}$ (b) $E(z) = \sum_{n=0}^{\infty} e(nT) z^{-n}$

(c) $E^*(s) = E(z)\Big|_{z=\epsilon^{sT}}$

3-2. (a) 1. No frequencies in $e(t)$ greater than $\omega_s/2$.

2. An ideal low-pass filter follows the sampler.

(b) None

(c) The signal can be recovered to a sufficient degree of accuracy.

3-3. $E(s) \xrightarrow[T]{\;/\;} E^*(s) \longrightarrow \boxed{G_{ho}(s)} \longrightarrow \bar{E}(s)$

$E_1(s) \longrightarrow \bar{E}_1(s) \qquad E_2(s) \longrightarrow \bar{E}_2(s)$

Let $E = E_1 + E_2$

$\therefore\ E^* = (E_1 + E_2)^* = E_1^* + E_2^*$

$\therefore\ \bar{E} = G_{ho}(E_1^* + E_2^*) = G_{ho}E_1^* + G_{ho}E_2^*$

3-4. $E^*(s) = \sum$ residues of $E(\lambda)\dfrac{1}{1-\epsilon^{-T(s-\lambda)}}$

(a) $$E^*(s) = \frac{20}{(s+5)(1-\epsilon^{-T(s-\lambda)})}\Bigg|_{\lambda=-2} + \frac{20}{(s+2)(1-\epsilon^{-T(s-\lambda)})}\Bigg|_{\lambda=-5}$$

$$= \underline{\frac{20/3}{1-\epsilon^{-T(s+2)}} + \frac{-20/3}{1-\epsilon^{-T(s+5)}}}$$

(b) $$E^*(s) = \frac{5}{(s+1)(1-\epsilon^{-T(s-\lambda)})}\Bigg|_{\lambda=0} + \frac{5}{s(1-\epsilon^{-T(s-\lambda)})}\Bigg|_{\lambda=-1}$$

$$= \underline{\frac{5}{1-\epsilon^{-Ts}} - \frac{5}{1-\epsilon^{-T(s+1)}}}$$

3-4.(c) $E^*(s) = \left.\dfrac{\lambda+2}{(\lambda+1)(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=0} + \left.\dfrac{\lambda+2}{\lambda(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=-1} = \underline{\dfrac{2}{1-\epsilon^{-Ts}} - \dfrac{1}{1-\epsilon^{-T(s+1)}}}$

(d) $(\text{residue})_{\lambda=0} = \dfrac{d}{d\lambda}\left[\dfrac{\lambda+2}{(\lambda+1)(1-\epsilon^{-T(s-\lambda)})}\right]_{\lambda=0}$

$$= \left.\frac{(\lambda+1)(1-\epsilon^{-T(s-\lambda)}) - (\lambda+2)(\lambda+1)(-T\epsilon^{-T(s-\lambda)}) - (\lambda+2)(1-\epsilon^{-T(s-\lambda)})}{(\lambda+1)^2(1-\epsilon^{-T(s-\lambda)})^2}\right|_{\lambda=0}$$

$$= \frac{-(1-\epsilon^{-Ts}) + 2T\epsilon^{-Ts}}{(1-\epsilon^{-Ts})^2}$$

$(\text{residue})_{\lambda=-1} = \left.\dfrac{\lambda+2}{\lambda^2(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=-1} = \dfrac{1}{1-\epsilon^{-T(s+1)}}$

$\therefore E^*(s) = \underline{\dfrac{-(1-\epsilon^{-Ts}) + 2T\epsilon^{-Ts}}{(1-\epsilon^{-Ts})^2} + \dfrac{1}{1-\epsilon^{-T(s+1)}}}$

(e) $E^*(s) = \sum_{\text{residues}} \dfrac{\lambda^2+5\lambda+6}{\lambda(\lambda+4)(\lambda+5)(1-\epsilon^{-T(s-\lambda)})} = \underline{\dfrac{3/10}{1-\epsilon^{-Ts}} + \dfrac{-1/2}{1-\epsilon^{-T(s+4)}} + \dfrac{6/5}{1-\epsilon^{-T(s+5)}}}$

(f) $s = -1 \pm j2$

$E^*(s) = \left.\dfrac{2}{(\lambda+1+j)(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=-1+j2} + \left.\dfrac{2}{(\lambda+1-j2)(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=-1-j2}$

$= \underline{\dfrac{-j/2}{1-\epsilon^{-T(s+1-j2)}} + \dfrac{j/2}{1-\epsilon^{-T(s+1+j2)}}}$

3-5.(a) $E^*(s) = 1 + \epsilon^{aT}\epsilon^{-Ts} + \epsilon^{2aT}\epsilon^{-2Ts} + \cdots = 1 + \epsilon^{(a-s)T} + [\epsilon^{(a-s)T}]^2 + \cdots$

$= \underline{\dfrac{1}{1-\epsilon^{(a-s)T}}}$

(b) $e(t) = \epsilon^{a(t-2T)}u(t-2T)$

$E^*(s) = \epsilon^{-2Ts} + \epsilon^{aT}\epsilon^{-3Ts} + \epsilon^{2aT}\epsilon^{-4Ts} + \cdots$

$= \epsilon^{-2Ts}(1 + \epsilon^{aT}\epsilon^{-Ts} + \epsilon^{2aT}\epsilon^{-2Ts} + \cdots) = \underline{\dfrac{\epsilon^{-2Ts}}{1-\epsilon^{(a-s)T}}}$

(c) From (b), $E^*(s) = \underline{\dfrac{\epsilon^{-2Ts}}{1-\epsilon^{(a-s)T}}}$

(d) $E^*(s) = \epsilon^{aT/2}\epsilon^{-Ts} + \epsilon^{3aT/2}\epsilon^{-2Ts} + \epsilon^{5aT/2}\epsilon^{-3Ts} + \cdots$

3-5.(d) $E^*(s) = \epsilon^{aT/2}\epsilon^{-Ts}(1 + \epsilon^{aT}\epsilon^{-Ts} + \epsilon^{2aT}\epsilon^{-2Ts} + \cdots)$

$$= \underline{\frac{\epsilon^{aT/2}\epsilon^{-Ts}}{1-\epsilon^{(a-s)T}}}$$

3-6. (a) $E^*(s) = 1 + \epsilon^{-3T}\epsilon^{-Ts} + \epsilon^{-6T}\epsilon^{-2Ts} + \cdots$

(b) $E^*(s) = \dfrac{1}{1-\epsilon^{-T(s+3)}}$

(c) $E(s) = \dfrac{1}{s+3}$. $\therefore$ from (3-11),

$$E^*(s) = \frac{1}{2} + \frac{1}{T}\left[\frac{1}{s+3} + \frac{1}{s+3+j\omega_s} + \frac{1}{s+3+j2\omega_s} + \cdots + \frac{1}{s+3-j\omega_s} + \frac{1}{s+3-j2\omega_s} + \cdots\right]$$

3-7. $E^*(s) = e(0) + e(T)\epsilon^{-Ts} + e(2T)\epsilon^{-2Ts} + \cdots = [e(t)]^*$

Let: $e_1(t) = e(t-kT)u(t-kT)$

$\therefore E_1^*(s) = e(0)\epsilon^{-kTs} + e(T)\epsilon^{-(k+1)Ts} + \cdots$

$$= \epsilon^{-kTs}[e(0) + e(T)\epsilon^{-Ts} + \cdots] = \epsilon^{-kTs}E^*(s)$$

$\therefore$ from (3-14),

$$E_1^*(s) = \epsilon^{-kTs}\Big[\sum_{\substack{\text{poles} \\ \text{of } E(\lambda)}} \text{residues of } \frac{E(\lambda)}{1-\epsilon^{-T(s-\lambda)}}$$

3-8. $E(s) = \dfrac{1-\epsilon^{-Ts}}{s(s+1)}$; define $E_1(s) = \dfrac{1}{s(s+1)}$

$$\therefore E_1^*(s) = \left.\frac{1}{(\lambda+1)(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=0} + \left.\frac{1}{\lambda(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=-1}$$

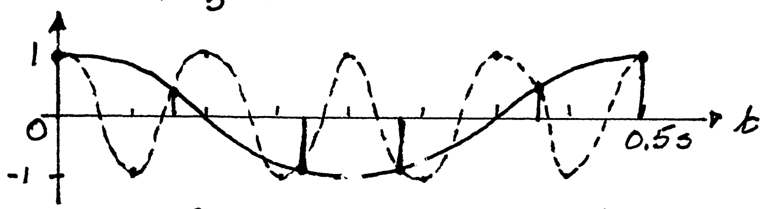
$$= \frac{1}{1-\epsilon^{-Ts}} - \frac{1}{1-\epsilon^{-T(s+1)}}$$

$E(s) = E_1(s) - E_1(s)\epsilon^{-Ts}$; $\therefore E^*(s) = E_1^*(s) - E_1^*(s)\epsilon^{-Ts}$

$$= \left[\frac{1}{1-\epsilon^{-Ts}} - \frac{1}{1-\epsilon^{-T(s+1)}}\right](1-\epsilon^{-Ts}) = \frac{\epsilon^{-Ts}(1-\epsilon^{-T})}{1-\epsilon^{-T(s+1)}}$$

3-9. (a) (i) $\omega_1 = \underline{4\pi}$, $\omega_s = \frac{2\pi}{T} = \underline{20\pi}$ (ii) $\omega_2 = 16\pi = \frac{4\omega_s}{5} = \underline{4\omega_1}$

$\omega_1 = \frac{\omega_s}{5}$, $T_1 = 0.5$

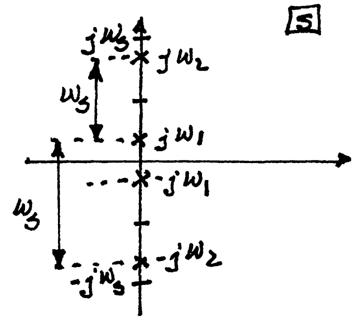


$\cos 72^\circ = 0.309$ $\qquad \cos(4\times 72^\circ) = 0.309$

$\cos 144^\circ = -0.809$ $\qquad \cos(4\times 144^\circ) = -0.809$

$\cos 216^\circ = -0.809$ $\qquad \cos(4\times 216^\circ) = -0.809$

$\cos 280^\circ = 0.309$ $\qquad \cos(4\times 280^\circ) = 0.309$



$\underline{\omega_1 = \omega_s - \omega_2}$

$\underline{-\omega_1 = -\omega_s + \omega_2}$

(b) $\omega_3 = \omega_s + \omega_1 = 20\pi + 4\pi = 24\pi \Rightarrow e_3(t) = \cos(24\pi t)$

3-10. $E(s) = \dfrac{s+2}{s(s+1)}$, $\therefore$ poles: $\underline{s = 0, -1}$, zero: $\underline{s = -2}$

From Problem 3-4(c): $E^*(s) = \dfrac{2}{1-\epsilon^{-Ts}} - \dfrac{1}{1-\epsilon^{-T(s+1)}}$

$$= \frac{1 + \epsilon^{-Ts} - 2\epsilon^{-Ts}\epsilon^{-T}}{(1-\epsilon^{-Ts})(1-\epsilon^{-T(s+1)})}$$

$\underline{\text{zeros}}$: $1 + \epsilon^{-Ts}(1 - 2\epsilon^{-T}) = 0 \Rightarrow \epsilon^{Ts} = 2\epsilon^{-T} - 1$

$\therefore \epsilon^{\sigma T} = |2\epsilon^{-T} - 1|$

$$\epsilon^{j\omega T} = 1\,\underline{/\omega T} = \begin{cases} 1\ , & 2\epsilon^{-T} - 1 > 0 \Rightarrow \omega T = \pm n\pi\ , n = 0, 2, 4, \ldots \\ -1\ , & 2\epsilon^{-T} - 1 < 0 \Rightarrow \omega T = \pm n\pi\ , n = 1, 3, 5, \cdots \end{cases}$$

$\therefore \underline{\omega} = \pm n\frac{\pi}{T} = \underline{\pm n\frac{\omega_s}{2}}$

$\therefore \underline{\text{zeros}}$: $\sigma \pm jn\frac{\omega_s}{2}$ $\begin{cases} n = 0, 2, 4 & \text{for } (1) \\ \quad \text{or} & \\ n = 1, 3, 5 & \text{for } (-1) \end{cases}$

$\underline{\text{poles}}$: $s = \pm jn\omega_s\ , -1 \pm jn\omega_s\ ;\ n = 0, 1, 2, \ldots$

3-11. Consider $E_1(s) = \dfrac{1}{s^2(s+1)}$; Then $E^*(s) = E_1^*(s)\left[2(1-\epsilon^{-Ts})^2\right]$

$$(\text{residue})\Big|_{\lambda=0} = \frac{d}{d\lambda}\left[\frac{1}{(\lambda+1)(1-\epsilon^{-T(s-\lambda)})}\right]_{\lambda=0}$$

$$= \frac{-(1-\epsilon^{-T(s-\lambda)}) - (-T\epsilon^{-T(s-\lambda)})(\lambda+1)}{(\lambda+1)^2(1-\epsilon^{-T(s-\lambda)})^2}\Bigg|_{\lambda=0} = \frac{-1+\epsilon^{-Ts}+T\epsilon^{-Ts}}{(1-\epsilon^{-Ts})^2}$$

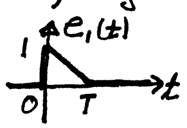
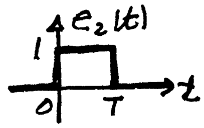
$$(\text{residue})_{\lambda=-1} = \left[\frac{1}{\lambda^2(1-\epsilon^{-T(s-\lambda)})}\right]_{\lambda=-1} = \frac{1}{1-\epsilon^{-T(s+1)}}$$

$$\therefore E_1^*(s) = \frac{1}{1-\epsilon^{-T(s+1)}} + \frac{-1+\epsilon^{-Ts}+T\epsilon^{-Ts}}{(1-\epsilon^{-Ts})^2}$$

$$\therefore E^*(s) = \frac{1-\epsilon^{-Ts}-1+\epsilon^{-Ts}+T\epsilon^{-Ts}}{(1-\epsilon^{-T(s+1)})(1-\epsilon^{-Ts})^2}\left[2(1-\epsilon^{-Ts})^2\right] = \underline{\frac{2T\epsilon^{-Ts}}{1-\epsilon^{-T(s+1)}}}$$

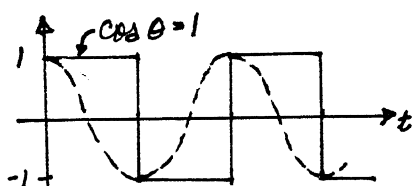
3-12. (a) $e(kT) = \begin{cases} 1 & , k=0 \\ 0 & , k \geq 1 \end{cases}$

(b) No, any $e(t)$ with $e(0)=1$ and $e(kT)=0,\ k\geq 1$

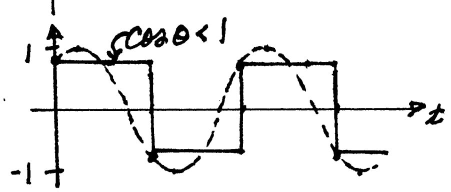
(c)  

(d) $e_1(t) = (1-\frac{1}{T}t)[u(t)-u(t-T)]$; $e_2(t) = u(t)-u(t-T)$

3-13. $e(t) = \cos(\frac{\omega_s}{2}t+\theta)$



$\theta = 0°$



$\theta > 0°$

(b) Output is a square wave with amplitude equal to $\cos\theta$.

$\therefore \bar{e}(t) = A_0 + A_1 \cos\frac{\omega_s}{2}t + B_1 \sin\frac{\omega_s}{2}t + \cdots$

$A_1 = \underline{0}$, since $\bar{e}(t)$ is odd.

3-13.(b) $B_1 = \frac{2}{T}\int_0^T \bar{e}(t)\sin(\frac{\omega_s}{2}t)\,dt = \frac{4}{T}\int_0^{T/2} \cos\theta \sin(\omega_s/2\, t)\,dt$

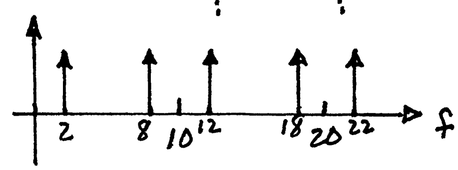
$= \frac{4\cos\theta}{T}\left[-\frac{\cos(\frac{\omega_s}{2}t)}{\omega_s/2}\right]_0^{T/2} = \underline{\frac{4}{\pi}\cos\theta}$, since $\omega_s = \frac{2\pi}{T}$

$\therefore$ 1st harmonic $= \underline{(\frac{4}{\pi}\cos\theta)\sin(\frac{\omega_s}{2}t)}$

3-14. (a) $f_s = 10$ Hz, $f_1 = 2$ Hz. (b) $f_s = 10$ Hz., $f_1 = 8$ Hz

	a	b
f_1	2	8
$f_s - f_1$	8	2
$f_s + f_1$	12	18
$2f_s - f_1$	18	12
$2f_s + f_1$	22	28
	28	22
⋮	⋮	⋮

(c)



$f = 12, 18, 22, 28, 32, \ldots$
$52, 58, 62, \ldots$

3-15. (a) 4, 7 rad/s

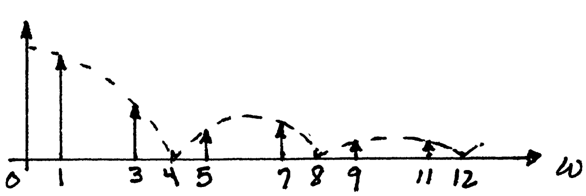
(b)

4	7
$22 \pm 4 = \underline{18, 26}$	$22 \pm 7 = \underline{15, 29}$
$44 \pm 4 = \underline{40, 48}$	$44 - 7 = \underline{37}$

(c) Same as (b)

(d) Same as (b)

3-16. (a) Frequencies in $\bar{e}(t)$: $\underline{7}$; $4 \pm 7 = \underline{-3}, \underline{10}$; $8 \pm 7 = \underline{1}, \underline{15}$; $12 \pm 7 = \underline{5}, \underline{19}$

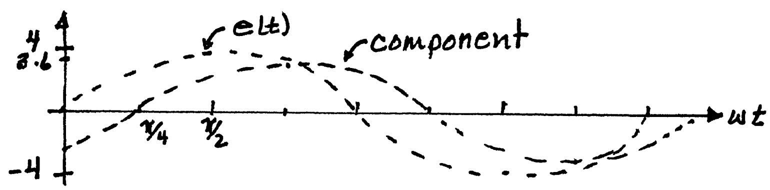


$\underline{\omega = 1}$

(b) $\frac{1}{T}G_{ho}(j1) = \frac{\sin(\frac{\pi(1)}{4})}{\pi(1)/4}\, e^{-j\pi/4} = 0.900\,\angle{-45^\circ}$

$\therefore$ component $= (4)(0.9)\sin(t - 45^\circ) = \underline{3.6 \sin(t - 45^\circ)}$

3.16.(c)



(d) $\frac{1}{T}G_{ho}(j7) = \frac{\sin(\pi 7/4)}{\pi 7/4} e^{-j\pi 7/4} = (-0.129)e^{-j315^\circ} = 0.129\angle{-135^\circ}$

$\therefore$ component $= 4(0.129)\sin(7t - 135^\circ) = \underline{0.516 \sin(7t-135^\circ)}$

ratio $= \frac{3.6}{0.516} = \underline{6.977}$ (7)

3-17. (a) phase of z.o.h. $= \pi\frac{\omega}{\omega_s} \Rightarrow \frac{180^\circ(10)}{\omega_s} = 10^\circ$, $\therefore \omega_s = \underline{180}$ rad/s

$T = 2\pi/\omega_s = 2\pi/180 = \underline{0.0349}$ s

(b) $\frac{180^\circ}{\omega_s} = 5^\circ \Rightarrow \omega_s = \underline{360}$ rad/s ; $T = \frac{2\pi}{360} = \underline{0.0175}$ s

(c) $\frac{180^\circ(10)}{\omega_s} = 20^\circ$, $\therefore \omega_s = \underline{90 \text{ rad/s}}$; $T = \underline{0.0698}$ s

3-18. (a) $\frac{1}{T}G_{ho}(j2) = \frac{\sin\left(\pi \frac{2\pi(2)}{2\pi(12)}\right)}{2\pi/12} \epsilon^{-j\pi/6} = \frac{0.50\angle{-30^\circ}}{0.5236} = \underline{0.9549\angle{-30^\circ}}$

(b) $\frac{1}{T}G_{ho}(j10) = \frac{\sin(\pi\, 10/12)}{10\pi/12} e^{-j10\pi/12} = \frac{0.50\angle{-150^\circ}}{2.618} = \underline{0.1910\angle{-150^\circ}}$

(c) (a) $\frac{1}{T}|G_{h1}(j\omega)| = \left[1 + \left(\frac{2\pi\omega}{\omega_s}\right)^2\right]^{1/2}\left[\frac{1}{T}G_{ho}(j\omega)\right]^2$

$\frac{1}{T}|G_{h1}(j2)| = \left[1 + \left(\frac{2\pi}{6}\right)^2\right]^{1/2}(0.9549)^2 = \underline{1.3203}$

$\underline{\angle G_{h1}(j2)} = \tan^{-1}\left(\frac{2\pi}{6}\right) - \left(\frac{2\pi}{6}\right) = 46.32^\circ - 60^\circ = \underline{-13.7^\circ}$

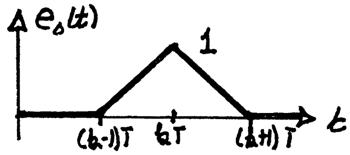
(b) $\frac{1}{T}|G_{h1}(j10)| = \left[1 + \left(\frac{20\pi}{12}\right)^2\right]^{1/2}[0.1910]^2 = \underline{0.194}$

$\underline{\angle G_{h1}(j10)} = \tan^{-1}\left(\frac{20\pi}{12}\right) - \left(\frac{20\pi}{12}\right) = 79.2^\circ - 300^\circ = \underline{-220.18^\circ}$

(d) The components at $\omega = 10$ are approximately equal.
The component at $\omega = 10$ for the 1st order hold is approximately 30% too large, while that for the zero-order-hold is approximately 10% too small. $\therefore$ the zero-order hold is better in this case.

3-19. Assume: input $= e_i(t) = \delta(t-kT)$, $\therefore E_i(s) = \epsilon^{-kTs}$

Then



$$\therefore e_o(t) = \frac{1}{T}\{t-(k-1)T\}u[t-(k-1)T] - \frac{2}{T}\{t-kT\}u[t-kT] + \frac{1}{T}\{t-(k+1)T\}u[t-(k+1)T]$$

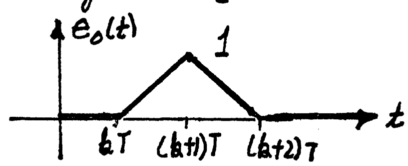
$$\therefore E_o(s) = \frac{1}{Ts^2}\epsilon^{-(k-1)Ts} - \frac{2}{Ts^2}\epsilon^{-kTs} + \frac{1}{Ts^2}\epsilon^{-(k+1)Ts}$$

$$\therefore G_{ha}(s) = \frac{E_o(s)}{E_i(s)} = \underline{\frac{\epsilon^{Ts}}{Ts^2}(1-\epsilon^{-Ts})^2}$$

Not physically realizable, since an advance in time is required.

3-20. Assume: input $= e_i(t) = \delta(t-kT)$, $\therefore E_i(s) = \epsilon^{-kTs}$

Then



$$e_o(t) = \frac{1}{T}\{t-kT\}u(t-kT) - \frac{2}{T}\{t-(k+1)T\}u[t-(k+1)T] + \frac{1}{T}\{t-(k+2)T\}u[t-(k+2)T]$$

$$E_o(s) = \frac{1}{Ts^2}\epsilon^{-kTs} - \frac{2}{Ts^2}\epsilon^{-(k+1)Ts} + \frac{1}{Ts^2}\epsilon^{-(k+2)Ts}$$

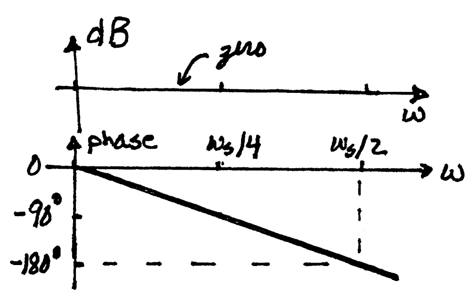
$$\therefore G_{hb}(s) = \frac{E_o(s)}{E_i(s)} = \underline{\frac{1}{Ts^2}(1-\epsilon^{-Ts})^2}$$

This data hold is physically realizable.

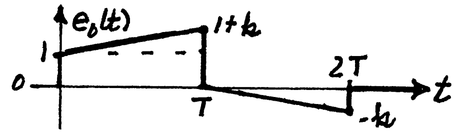
3-21. From Problems 3-19 and 3-20,

$$\frac{G_{ha}(s)}{G_{hb}(s)} = \epsilon^{-j\omega T} = \underline{1\angle -\omega T}$$

$$-\omega T\Big|_{\omega=\omega_s/2} = -\frac{\pi}{T}T = \underline{-\pi}$$



3-22. $E_i(s) = 1$;



$$e_o(t) = u(t) + \frac{k}{T}\, t\, u(t) - (1+k)\, u(t-T) - \frac{2k}{T}(t-T)\, u(t-T)$$
$$+ k\, u(t-2T) + \frac{k}{T}(t-2T)\, u(t-2T)$$

$$E_o(s) = \frac{1}{s}\left[1 - (1+k)\epsilon^{-Ts} + k\epsilon^{-2Ts}\right] + \frac{k}{Ts^2}\left(1 - 2\epsilon^{-Ts} + \epsilon^{-2Ts}\right)$$

$$\therefore G_{hk}(s) = \frac{E_o(s)}{E_i(s)} = (1 - k\epsilon^{-Ts})\left(\frac{1-\epsilon^{-Ts}}{s}\right) + \frac{k}{T}\left(\frac{1-\epsilon^{-Ts}}{s}\right)^2$$

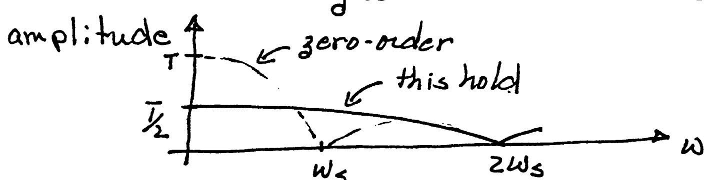
3-23. $\bar{e}(t) = e(0)[u(t) - u(t-\frac{T}{2})] + e(T)[u(t-T) - u(t-\frac{3T}{2})] + \cdots$

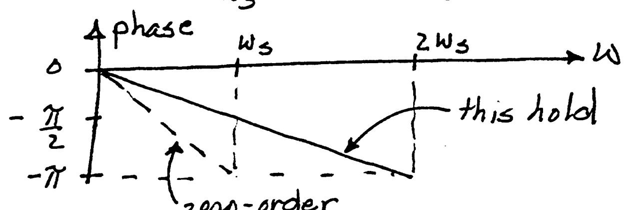
$$\therefore \bar{E}(s) = e(0)\left[\frac{1}{s} - \frac{\epsilon^{-\frac{T}{2}s}}{s}\right] + e(T)\left[\frac{\epsilon^{-Ts}}{s} - \frac{\epsilon^{-\frac{3Ts}{2}}}{s}\right] + \cdots$$

$$= \left[\frac{1-\epsilon^{-\frac{T}{2}s}}{s}\right]\left[e(0) + e(T)\epsilon^{-Ts} + \cdots\right]$$

$$\therefore G_h(s) = \frac{1-\epsilon^{-\frac{T}{2}s}}{s}$$

$$G_h(j\omega) = \frac{1-\epsilon^{-j\frac{\omega T}{2}}}{j\omega}\epsilon^{j\frac{\omega T}{4}}\epsilon^{-j\frac{\omega T}{4}} = \frac{T}{2}\left[\frac{\sin\left(\frac{\pi\omega}{2\omega_s}\right)}{\frac{\pi\omega}{2\omega_s}}\right]\epsilon^{-j\frac{\pi\omega}{2\omega_s}}$$





Amplitude characteristic of zero-order much better.
Phase characteristic of zero-order has more lag.

3-24. $V_o = V_{fs}[A_1 2^{-1} + A_2 2^{-2} + A_3 2^{-3} + A_4 2^{-4}]$; $A_i = 0$ or 1

(a) $V_o = 10[2^{-1} + 2^{-2} + 2^{-3} + 2^{-4}] = 10(0.9375) = \underline{9.375}\text{ V}$

(b) $\Delta V = 10(2^{-4}) = \underline{0.625\text{V}}$

(c) $V_o = 10[A_1 2^{-1} + \cdots + A_3 2^{-3}]$

$\therefore V_o = \underline{0,\ 1.25,\ 2.50,\ 3.75,\ 5.00,\ 6.25,\ 7.50,\ 8.75}$

3-24 (d) $V_o = 10[A_1 2^{-1} + \cdots + A_n 2^{-n}]$

$\therefore 10(2^{-n}) = 0.005\ V$

$2^{-n} = 0.0005 = \frac{1}{2,000}$; $\therefore 2^{10} = 1024$ and $2^{11} = 2048$

$\therefore$ <u>11</u> bits required

3-25. (a) Maximum error is a change in the least significant bit.

$\therefore (\text{error})_{max} = 10\left(\frac{1}{256}\right) = 0.03906\ V \approx 0.04\ V$

(b) In Fig. 3-22, $V_x < V_R$.

In (3-41), $V_R = 10\left[\frac{1}{2^6}\right] = 0.15625$

The next lower value is, in binary, 0000 0011.

$\therefore V_R = 10\left[\frac{1}{2^7} + \frac{1}{2^8}\right] = 10\left[\frac{3}{256}\right] = 0.1171875$

$\therefore 0.1171875 < V_x < 0.15625$

3-26. $n = 12$, $f_c = 5$ MHz, $\therefore T = \frac{1}{5\times 10^6} = \underline{0.2\ \mu sec}$

(a) $2^{12} = 4,096$; time $= (4096)(0.2\ \mu sec) = \underline{819.2\ \mu sec}$

(b) time $= (2^{12} + 2^{12})(0.2\ \mu sec) = \underline{1.6384}$ msec

(c) time $= \frac{n}{f_c} = \frac{12}{5\times 10^6} = \underline{2.4\ \mu sec}$

3-27. (1) $\underline{1}000 \Rightarrow 5(\frac{1}{2}) = 2.5V$ $\quad \therefore \underline{1}000 \Rightarrow \underline{2.5V}$

(2) $1\underline{1}00 \Rightarrow 5(\frac{3}{4}) = 3.75V$ $\quad \therefore \underline{1}\underline{0}00 \Rightarrow \underline{2.5V}$

(3) $10\underline{1}0 \Rightarrow (5)(\frac{5}{8}) = 3.125V$ $\quad \therefore \underline{1}\underline{0}\underline{1}0 \Rightarrow \underline{3.125V}$

(4) $101\underline{1} \Rightarrow (5)(\frac{11}{16}) = 3.438V$ $\quad \therefore 1011 \Rightarrow \underline{3.438V}$

3-28. For the 8-bit converter, $V_R = V_R \sum_{i=1}^{8} A_i 2^{-i}$

for $N = (1000\ 0000)_2 \Rightarrow V_R = 10\left[\frac{(1000\ 0000)_2}{2^8}\right] = 10\left[\frac{128}{256}\right] = 5.00\ V$

for $N = (1000\ 0000)_2 \Rightarrow V_R = 10\left[\frac{129}{256}\right] = 5.03906\ V$

(a) $V_R > V_x$ $\therefore N = (1000\ 0001)_2 \Rightarrow V_o = 5.03906\ V$

3-28. (b) $V_R > V_x$, $v_0 = 5.03906\ V$ } either
$V_R < V_x$, $v_0 = 5.00\ V$ }

(c) $V_R < V_x$, $v_0 = 5.00\ V$

(d) $V_R < V_x$, $v_0 = 5.00\ V$

(e) Parallel converter has roundoff - 5.01 is closer to 5.00 than to 5.03906 , $\therefore\ v_0 = 5.00\ V$

3-29. (a) $N_{A/D} = (1000\ 0001)_2 \rightarrow (1000\ 0001\ 0000)_2$ to D/A

$$v_0 = 10\left[\frac{(1000\ 0001\ 0000)_2}{2^{12}}\right] = 10\left[\frac{(1000\ 0001)_2}{2^8}\right] = 5.03906\ V$$

$\therefore$ all answers are same as in 3 28.

3-30. (a) $N_{A/D} = (1000\ 0001)_2 \Rightarrow (0000\ 1000\ 0001)_2$ to D/A

$$\therefore\ v_0 = 10\left[\frac{(0000\ 1000\ 0001)_2}{2^{12}}\right] = 10\left[\frac{129}{2^{12}}\right] = \frac{5.03906}{16} = 0.3149\ V$$

These values are those of Problem 3-28 divided by 16.

(b) 0.3149 V } either
0.3125 V }

(c) 0.3125 V

(d) 0.3125 V

(e) 0.3125 V

CHAPTER 4

4-1. A pole at $\underline{s}$ transforms into a pole at $z = \epsilon^{sT}$.

(a) $s = -a + jb$, $a > 0$

$$|z| = |\epsilon^{(-a+jb)T}| = |\epsilon^{-aT}\epsilon^{jbT}| = \underline{|\epsilon^{-aT}| < 1}$$

(b) $s = j\omega \Rightarrow z = \epsilon^{j\omega T} = \underline{1\angle\omega T}$

(c) $s = a + jb$, $a > 0$

$$|z| = |\epsilon^{(a+jb)T}| = |\epsilon^{aT}\epsilon^{jbT}| = \underline{|\epsilon^{aT}| > 1}$$

4-2. (a) $s = 1$, $\therefore\ z = \epsilon^{T} = \epsilon^{0.05} = \underline{1.051}$

$s = -1$, $\therefore\ z = \epsilon^{-T} = \epsilon^{-0.05} = \underline{0.9512}$

(b) A pole at $\underline{s}$ transforms into a pole at $\underline{z} = \epsilon^{sT} = \underline{\epsilon^{0.05s}}$

(c) $$E(s) = \frac{s+2}{(s-1)(s+1)} = \frac{3/2}{s-1} + \frac{-1/2}{s+1}$$

$$\therefore\ E(z) = \frac{1.5z}{z-1.051} + \frac{-0.5z}{z-0.9512} = \frac{z^2 - 0.9013z}{(z-1.051)(z-0.9512)}$$

(d) $s = \underline{-2}$; $z = \underline{0}, \underline{0.9013}$

(e) No - both poles and zeros of $E(s)$ determine the zeros of $E(z)$.

4-3. (a) $$E(z) = \frac{20(\epsilon^{-0.2} - \epsilon^{-0.5})z}{3(z-\epsilon^{-0.2})(z-\epsilon^{-0.5})} = \frac{1.415z}{(z-0.8187)(z-0.6065)}$$

$\omega_s = 2\pi/T = \underline{20\pi}$

$E(s)$: $s = -2, -5$

$E^*(s)$: $s = -2 \pm jk20\pi$, $-5 \pm jk20\pi$

$E(z)$: $z = 0.8187, 0.6065$

(b) $$E(z) = \frac{5z(1-\epsilon^{-0.1})}{(z-1)(z-\epsilon^{-0.1})} = \frac{0.4758z}{(z-1)(z-0.9048)}$$

4-3.(b) poles: $E(s)$: $s = 0, -1$

$E^*(s)$: $s = \pm jk20\pi, -1 \pm jk20\pi$

$E(z)$: $z = 1, 0.9048$

(c) $E(s) = \dfrac{s+2}{s(s+1)} = \dfrac{2}{s} + \dfrac{-1}{s+1}$

$\therefore E(z) = \dfrac{2z}{z-1} + \dfrac{-z}{z-0.9048} = \dfrac{z(z-0.8096)}{(z-1)(z-0.9048)}$

poles: Same as (b).

(d) $E(s) = \dfrac{s+2}{s^2(s+1)} = \dfrac{2}{s^2} + \dfrac{-1}{s} + \dfrac{1}{s+1}$

$\therefore E(z) = \dfrac{0.2z}{(z-1)^2} + \dfrac{-z}{z-1} + \dfrac{z}{z-0.9048} = \dfrac{0.1048z^2 - 0.0858z}{(z-1)^2(z-0.9048)}$

poles: Same as (b)

(e) $E(s) = \dfrac{s^2+5s+6}{s(s+4)(s+5)} = \dfrac{0.3}{s} + \dfrac{-0.5}{s+4} + \dfrac{1.2}{s+5}$

$\therefore E(z) = \dfrac{0.3z}{z-1} - \dfrac{0.5z}{z-0.6703} + \dfrac{1.2z}{z-0.6065}$

$= \dfrac{z^3 - 1.5841z^2 + 0.6231z}{(z-1)(z-0.6703)(z-0.6065)}$

poles: $E(s)$: $s = 0, -4, -5$

$E^*(s)$: $s = \pm jk20\pi, -4 \pm jk20\pi, -5 \pm jk20\pi$

$E(z)$: $z = 1, 0.6703, 0.6065$

(f) $E(s) = \dfrac{2}{(s+1)^2 + 2^2}$, $\therefore$

$\therefore E(z) = \dfrac{2}{2}\left[\dfrac{z\epsilon^{-0.1}\sin 0.2}{z^2 - 2z\epsilon^{-0.1}\cos 0.2 + \epsilon^{-0.2}}\right] = \dfrac{0.1798z}{z^2 - 1.7736z + 0.8187}$

poles: $E(s)$: $s = -1 \pm j2 = 2.236\angle{\pm 116.6^\circ}$

$E^*(s)$: $s = -1 \pm j2 \pm jk20\pi = -1 \pm j(2 + k20\pi)$

$E(z)$: $z = 0.8868 \pm j0.1798 = 0.9048\angle{\pm 11.46^\circ}$

(g)
```
num=[0 1 5 6];
den=[1 9 20 0];
[r,p,k]=residue(num,den)
```

4-4. (a) $E(z) = \dfrac{(z^2-1)}{z^4}\,\mathcal{z}\left[\dfrac{1}{s(s+1)}\right] = \dfrac{z^2-1}{z^4}\,\dfrac{z(1-\epsilon^{-0.5})}{(z-1)(z-\epsilon^{-0.5})}$

$= \dfrac{0.3935(z+1)}{z^3(z-0.6065)}$

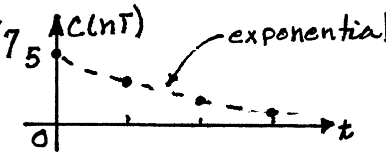
4-4. (b) $E(z) = (1-z^{-1})\,\mathcal{Z}\left[\dfrac{0.5s+1}{0.5s(s+0.25)}\right] = \dfrac{z-1}{z}\,\mathcal{Z}\left[\dfrac{8}{s} + \dfrac{-7}{s+0.25}\right]$

$= \dfrac{z-1}{z}\left[\dfrac{8z}{z-1} - \dfrac{7z}{z-0.9344}\right] = \underline{\dfrac{z-0.5152}{z-0.9394}}$

4-5. (a) $G(z) = \dfrac{z-1}{z}\,\mathcal{Z}\left[\dfrac{5s}{s(s+0.1)}\right] = \dfrac{z-1}{z} \times \dfrac{5z}{z-\epsilon^{-0.2}} = \dfrac{5(z-1)}{z-0.8187}$

$Y(z) = \left(\dfrac{z}{z-1}\right)\dfrac{5(z-1)}{z-0.8187} = \dfrac{5z}{z-0.8187}$

$\therefore\ c(nT) = \underline{5(0.8187)^n}$



(b) $M(s) = \dfrac{1}{s}$

$\therefore\ Y(s) = \dfrac{5s}{s(s+0.1)} = \dfrac{5}{s+0.1}$, $\quad\therefore\ y(t) = \underline{5\epsilon^{-0.1t}}$

$\therefore\ y(nT) = y(2n) = 5\epsilon^{-0.1(2n)} = 5(\epsilon^{-0.2})^n = \underline{5(0.8187)^n}$

(c) dc gain $= G_p(s)\big|_{s=0} = \dfrac{5(0)}{0+0.1} = \underline{0}$

dc gain $= G(z)\big|_{z=1} = \dfrac{5(1-1)}{1-0.8187} = \underline{0}$

(d) Yes - $c_{ss}(kT) = (1)(\text{dc gain}) = 0$ from (a), $\therefore$ dc gain $= 0$

$c_{ss}(t) = (1)(\text{dc gain}) = 0$ from (b), $\therefore$ dc gain $= 0$

4-6. (a) $G(z) = \dfrac{z-1}{z}\,\mathcal{Z}\left[\dfrac{5}{s(s+1)(s+2)}\right] = \dfrac{z-1}{z}\,\mathcal{Z}\left[\dfrac{2.5}{s} + \dfrac{-5}{s+1} + \dfrac{2.5}{s+2}\right]$

$= \dfrac{z-1}{z}\left[\dfrac{2.5z}{z-1} - \dfrac{5z}{z-0.9048} + \dfrac{2.5z}{z-0.8187}\right]$

$= \dfrac{z-1}{z}\left[\dfrac{0.0227z^2+0.0205z}{(z-1)(z-0.9048)(z-0.8187)}\right] = \dfrac{0.0227z+0.0205}{(z-0.9048)(z-0.8187)}$

$\therefore\ \dfrac{Y(z)}{z} = \dfrac{0.0227z+0.0205}{(z-1)(z-0.9048)(z-0.8187)} = \dfrac{2.5}{z-1} + \dfrac{-5}{z-0.9048} + \dfrac{2.5}{z-0.8187}$

$\therefore\ c(kT) = \underline{2.5 - 5(0.9048)^n + 2.5(0.8187)^n}$

(b) $M(s) = \dfrac{1}{s}$, $\quad\therefore\ Y(s) = \dfrac{5}{s(s+1)(s+2)} = \dfrac{2.5}{s} - \dfrac{5}{s+1} + \dfrac{2.5}{s+2}$

$\therefore\ y(t) = 2.5 - 5\epsilon^{-t} + 2.5\epsilon^{-2t}$

$\therefore\ y(nT) = 2.5 - 5(0.9048)^n + 2.5(0.8187)^n$

4-6. (c) dc gain $= G_p(s)\big|_{s=0} = \underline{2.5}$

$$\text{dc gain} = G(z)\big|_{z=1} = \frac{0.0227 + 0.0225}{(1-0.9048)(1-0.8187)} = \underline{2.5}$$

(d) $\underline{Yes}$ – $c_{ss}(kT) = (1)(\text{dc gain}) = \underline{2.5}$, from (a); $\therefore$ dc gain $= \underline{2.5}$

$c_{ss}(t) = (1)(\text{dc gain}) = \underline{2.5}$, from (b); $\therefore$ dc gain $= \underline{2.5}$

(b) (a) $$G(z) = \frac{z-1}{z}\mathcal{Z}\left[\frac{5}{s[(s+1)^2+1^2]}\right] = \frac{z-1}{z}\mathcal{Z}\left[\frac{2.5}{s} + \frac{-2.5(s+1)}{(s+1)^2+1} + \frac{-2.5}{(s+1)^2+1}\right]$$

$$= \frac{z-1}{z}\left[\frac{2.5z}{z-1} + \frac{-2.5z^2+2.251z}{z^2-1.801z+0.8187} + \frac{-0.2258}{z^2-1.801z+0.8187}\right]$$

$$= \frac{0.02337z + 0.02187}{z^2 - 1.801z + 0.8187} \Rightarrow \text{poles: } z = 0.9048\angle \pm 5.73^\circ$$
$$= 0.900 \pm j0.09033$$

$$\therefore \frac{Y(z)}{z} = \frac{0.02337z + 0.02187}{(z-1)(z-0.900-j0.0903)(z-0.900+j0.0903)}$$

$$= \frac{2.50}{z-1} + \frac{k_1}{z-0.900-j0.0903} + \frac{k_1^*}{z-0.900+j0.0903}$$

$$k_1 = \frac{0.02337(0.900+j0.0903)+0.02187}{(0.900+j0.0903-1)(j(2)(0.0903))}$$

$$= \frac{0.04295\angle 2.82^\circ}{(0.1347\angle 132.9^\circ)(0.1806\angle 90^\circ)} = 1.766\angle -225.9^\circ$$

From (2-29), (2-30): $aT = \ln(0.9048) = -1 \Rightarrow \underline{a = -1}$

$bT = 5.73^\circ = 0.1\text{ rad} \Rightarrow \underline{b = 1}$

$A = 2|k_1| = \underline{3.531}$, $\underline{\theta = 134.1^\circ}$

$\therefore y(nT) = \underline{2.5 + 3.531(0.9048)^n \cos(0.1n + 134.1^\circ)}$

(b) $M(s) = \frac{1}{s}$

$$\therefore Y(s) = \frac{5}{s(s^2+2s+2)} = \frac{2.5}{s} + \frac{-2.5(s+1)}{(s+1)^2+1} + \frac{-2.5}{(s+1)^2+1}$$

$$\therefore y(t) = 2.5 - 2.5(\epsilon^{-t}\cos t + \epsilon^{-t}\sin t)$$
$$= 2.5 + 3.535\epsilon^{-t}\cos(t + 135^\circ)$$

$$\therefore y(nT) = 2.5 + 3.535\epsilon^{-0.1n}\cos(0.1n + 135^\circ)$$
$$= \underline{2.5 + 3.535(0.9048)^n \cos(0.1n + 135^\circ)}$$

4-6.(b) (c) $G_p(s)\big|_{s=0} = 5/2 = \underline{2.5}$

$$G(z)\big|_{z=1} = \frac{0.02337+0.02187}{1-1.800+0.8187} \approx \underline{2.5}$$

(d) Same as (d) of part (a).

4-7.(a) dc gain $= G(z)\big|_{z=1} = 0$, $\therefore$ $G(z)$ has a zero at $z=1$.

(b) dc gain $= G(z)\big|_{z=1} = G_p(s)\big|_{s=0}$, $\therefore G_p(s)$ has a zero at $s=0$.

$\therefore$ let $G_p(s) = s\,G_{p_1}(s)$

$$G(z) = \mathcal{Z}\left\{\left(\frac{1-\epsilon^{-Ts}}{s}\right)(s\,G_{p_1}(s))\right\} = \frac{z-1}{z}\,\mathcal{Z}\left[G_{p_1}(s)\right]$$

$\therefore$ $G(z)$ has a zero at $z=1$.

(c) Note: $\mathcal{Z}[1-\epsilon^{-Ts}] = \frac{z-1}{z}$. $\therefore$ this term cancels the pole at $z=1$.

(d) dc gain $= \lim_{z\to 1} G(z)$ $\therefore$ $G(z)$ has a pole at $z=1$.

(e) $G_p(s)$ has a pole at $s=0$. $\therefore$ let $G_p(s) = \frac{1}{s}G_{p_1}(s)$

$$\therefore G(z) = \frac{z-1}{z}\,\mathcal{Z}\left[\frac{1}{s^2}G_{p_1}(s)\right] = \frac{z-1}{z}\;\frac{N(z)}{(z-1)^2 D_1(z)} = \frac{N(z)}{z(z-1)D_1(z)}$$

4-8. $$C(z) = \mathcal{Z}\left[\frac{1}{s(s+0.5)}\right]\frac{z-1}{z}\,\mathcal{Z}\left[\frac{3}{(s+1)(s+2)}\right]$$

$$\mathcal{Z}\left[\frac{1}{s(s+0.5)}\right] = \frac{1}{0.5}\;\frac{z(1-\epsilon^{-0.5})}{(z-1)(z-\epsilon^{-0.5})} = \frac{0.7869z}{(z-1)(z-0.6065)}$$

$$\frac{z-1}{z}\,\mathcal{Z}\left[\frac{3}{s(s+1)(s+2)}\right] = \frac{z-1}{z}\,\mathcal{Z}\left[\frac{1.5}{s}+\frac{-3}{s+1}+\frac{1.5}{s+2}\right]$$

$$= \frac{z-1}{z}\left[\frac{1.5z}{z-1}+\frac{-3z}{z-0.3679}+\frac{1.5z}{z-0.1353}\right] = \frac{0.5990z+0.2184}{(z-0.3679)(z-0.1353)}$$

$$\therefore \frac{C(z)}{z} = \frac{0.4714z+0.1719}{(z-0.6065)(z-0.3679)(z-0.1353)(z-1)}$$

$$= \frac{-10.35}{z-0.6065}+\frac{9.845}{z-0.3679}+\frac{-2.487}{z-0.1353}+\frac{2.991}{z-1}$$

$$\therefore c(nT) = \underline{2.991 - 10.35(0.6065)^n + 9.845(0.3679)^n - 2.487(0.1353)^n}$$

4-9 (a) $C_u(z) = \mathcal{Z}\left[\frac{1-\epsilon^{-Ts}}{s}\right]\mathcal{Z}\left[\frac{2}{s^2}\right] = \mathcal{Z}\left[\frac{1}{s^2}\right] = \frac{2Tz}{(z-1)^2}$

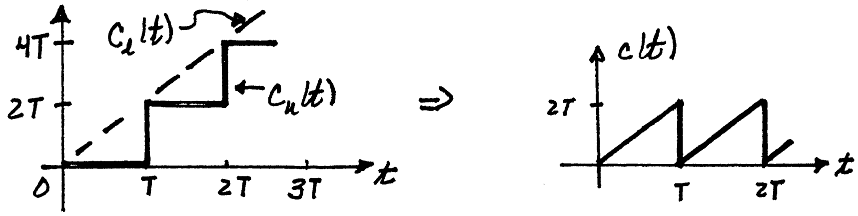
$C_\ell(z) = \frac{z-1}{z}\mathcal{Z}\left[\frac{2}{s^2}\right]\left[\frac{z}{z-1}\right] = \mathcal{Z}\left[\frac{1}{s^2}\right] = \frac{2Tz}{(z-1)^2}$

$\therefore C(z) = C_\ell(z) - C_u(z) = 0$

(b) No effect. The output of the sampler and data hold at the sampling instants is equal to the input.

(c) $C_u(t)$ = output of data hold.

$C_\ell(t)$ = integral of step function $= 2t$



(d) $c_u(t) = c_\ell(t)$, $\quad \therefore c(t) = 0$

4-10. (a) (i) $C(z) = G_3(z)D(z)E(z) + \overline{G_1G_2E}(z)$

(ii) $C(z) = \overline{G_1G_3G_4E}(z) + \overline{G_2G_4}(z)\overline{G_1E}(z)$

(iii) $C(z) = \overline{G_1G_2G_3E}(z) + G_3'(z)D(z)\overline{G_1E}(z)$; $G_3'(z) = \left(\frac{1-\epsilon^{-Ts}}{s}\right)G_3(s)$

(b) (i) $G_3(s)$

(ii) $G_2(s)$

(iii) $G_3'(s)$

4-11. (a) $M(z) = (2 - z^{-1})E(z)$

$\therefore m(kT) = 2e(kT) - e[(k-1)T] = 2u(kT) - u[(k-1)T]$

$\therefore m(0) = 2, \quad m(k) = 1, \quad k \geq 1$

(b) $c(kT) = 2c_1(kT) - c_1[(k-1)T]$

(c) $c_1(kT) = (1-\epsilon^{-kT})u(kT)$

$c(kT) = 2(1-\epsilon^{-kT})u(kT) - u[(k-1)T] + \epsilon^{-(k-1)T}u[(k-1)T]$

$= \begin{cases} 0, & k=0 \\ 1 - (2-\epsilon^{-T})\epsilon^{-kT}, & k \geq 1 \end{cases}$

4-11.(d) Example 4.3, $C_1(kT) = \dfrac{z(1-\epsilon^{-T})}{(z-1)(z-\epsilon^{-T})}$

$$\therefore C(z) = 2\,C_1(z) - z^{-1}C_1(z) = \frac{2z(1-\epsilon^{-T})}{(z-1)(z-\epsilon^{-T})} - \frac{1-\epsilon^{-T}}{(z-1)(z-\epsilon^{-T})}$$

$$= \frac{(2z-1)(1-\epsilon^{-T})}{(z-1)(z-\epsilon^{-T})}$$

4-12. (a) dc gain $= D(1) = \dfrac{4.5(1-0.9)}{1-0.85} = \underline{3}$

(b) 1. Use hardware configuration of Figure P4-12.

2. Apply a unit step input voltage and let the output settle to a constant value.

3. Use an oscilloscope to measure the output voltage, which should be equal to 3 V.

4-13.(a) $M(z) = -0.9z^{-1}M(z) + 0.2\,E(z)$

$$\therefore D(z) = \frac{M(z)}{E(z)} = \frac{0.2z}{z-0.9}$$

$$G(z) = \frac{z-1}{z}\,\mathcal{Z}\left[\frac{1}{s(s+0.2)}\right] = \frac{z-1}{z}\left[\frac{1}{0.2}\,\frac{z(1-\epsilon^{-0.2})}{(z-1)(z-\epsilon^{-0.2})}\right]$$

$$= \frac{0.9063}{z-0.8187}$$

$$\therefore \frac{C(z)}{E(z)} = D(z)\,G(z) = \frac{0.1813z}{(z-0.9)(z-0.8187)}$$

(b) $D(1)\,G(1) = \dfrac{0.1813}{(1-0.9)(1-0.8187)} = \underline{10}$

(c) $D(1) = \dfrac{0.2}{1-0.9} = \underline{2}\ ;\quad G_p(0) = \dfrac{1}{0.2} = \underline{5}$

$D(1)\,G_p(0) = (2)(5) = \underline{10}$

(d) $C_{ss}(kT) = D(1)\,G(1) \times (1) = \underline{10}$

(e) $C(z) = z\left[\dfrac{0.1813z}{(z-1)(z-0.9)(z-0.8187)}\right] = \dfrac{10}{z-1} + \dfrac{-20.07}{z-0.9} + \dfrac{10.07}{z-0.8187}\Big]$

$\therefore c(kT) = 10 - 20.07(0.9)^k + 10.07(0.8187)^k, \quad \therefore C_{ss}(kT) = \underline{10}$

4-13. (f) If the sum of the partial-fraction coefficients is not zero, then

$$C(z) = z\,C_1(z) = \frac{a z^3 + \cdots}{z^3 + \cdots} = a + (\;)z^{-1} + (\;)z^{-2} + \cdots$$

Hence $c(0)$ is not zero, and the plant responds instantly to a step input. This does not occur.

4-14. (a) $zM(z) - 0.995M(z) = 0.5(z-0.98)E(z)$

$$\therefore D(z) = \frac{M(z)}{E(z)} = \underline{\frac{0.5(z-0.98)}{z-0.995}}$$

$$G(z) = \frac{z-1}{z}\mathcal{Z}\left[\frac{5}{s(s+1)(s+2)}\right] = \underline{\frac{0.0227z + 0.0205}{(z-0.9048)(z-0.8187)}}, \text{ from Prob 4-6}$$

$$\therefore \frac{C(z)}{E(z)} = D(z)G(z) = \underline{\frac{(z-0.98)(0.01135z + 0.01025)}{(z-0.995)(z-0.9048)(z-0.8187)}}$$

(b) $$D(1)G(1) = \frac{(1-0.98)(0.01135+0.01025)}{(1-0.995)(1-0.9048)(1-0.8187)} = \underline{5}$$

(c) $$D(1) = \frac{0.5(1-0.98)}{1-0.995} = \underline{2}\;; \quad G_p(0) = \frac{5}{2} = \underline{2.5}$$

$$\therefore D(1)G_p(0) = (2)(2.5) = \underline{5}$$

(d) $c_{ss}(kT) = D(1)G(1) \times (1) = \underline{5}$

(e) $$\frac{C(z)}{z} = \frac{(z-0.98)(0.01135z + 0.01025)}{(z-1)(z-0.995)(z-0.9048)(z-0.8187)} = C_1(z)$$

$$= \frac{5}{z-1} + \frac{-4.064}{z-0.0995} + \frac{-2.087}{z-0.9048} + \frac{1.145}{z-0.8187}$$

$$\therefore \underline{c(kT) = 5 - 4.064(0.995)^k - 2.087(0.9048)^k + 1.145(0.8187)^k}$$

(f) See Problem 4-13(f)

4-15. (a) $$m(t) \rightarrow \boxed{A/D} \rightarrow \boxed{D/A} \rightarrow \boxed{K} \rightarrow \boxed{\frac{200}{s(0.5s+1)}} \rightarrow \boxed{\frac{1}{100}} \rightarrow \theta_a(t) \qquad T = 0.1$$

(b) $\underline{\pm 10V}$. Then the input motor voltage is in the range of ± 24 V, with 24 V the rated motor voltage.

4-15.(c) $\dfrac{\Theta_a(s)}{E_a(s)} = \dfrac{2}{s(0.5s+1)} = \dfrac{4}{s(s+2)}$

(d) $\dfrac{\Theta_a(z)}{M(z)} = \dfrac{z-1}{z}\,\mathcal{Z}\left[\dfrac{9.6}{s^2(s+2)}\right] = 9.6\left(\dfrac{z-1}{z}\right)\left(\dfrac{1}{4}\right)\dfrac{z[(0.2-1+0.8187)z+(1-.8187-.1637)]}{(z-1)^2(z-0.8187)}$

$$= \frac{0.04488z+0.04206}{(z-1)(z-0.8187)}$$

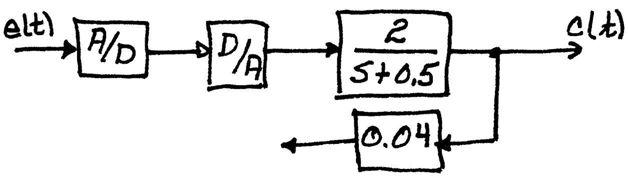
(e) $\Theta_a(s) = \dfrac{1}{s}\,\dfrac{9.6}{s(s+2)} = \dfrac{9.6}{s^2(s+2)} = \dfrac{4.8}{s^2} + \dfrac{-2.4}{s} + \dfrac{2.4}{s+2}$

$\therefore \Theta_a(t) = 4.8t - 2.4 + 2.4\epsilon^{-2t}$

$\therefore \Theta_{ass}(t) = 4.8t - 2.4$, $\dot{\Theta}_{ass}(t) = 4.8$ = shaft velocity

Constant voltage into a motor gives constant shaft velocity.

4-16. (a)



(b) $G(z) = \dfrac{z-1}{z}\,\mathcal{Z}\left[\dfrac{2}{s(s+0.5)}\right] = \dfrac{z-1}{z}\,\dfrac{2}{0.5}\left[\dfrac{z(1-\epsilon^{-0.5T})}{(z-1)(z-\epsilon^{-0.5T})}\right]$

$$= \frac{4(1-\epsilon^{-0.5T})}{z-\epsilon^{-0.5T}}$$

(c) $G_p(0) = \text{dc gain} = \dfrac{2}{0.5} = \underline{4}$, $\therefore c_{ss}(kT) = 10(4) = \underline{40^\circ C}$

(d) $G_d(0) = -\dfrac{2.5}{0.5} = -5$, $\therefore \Delta c_{ss}(kT) = (-5)(1) = \underline{-5^\circ C}$

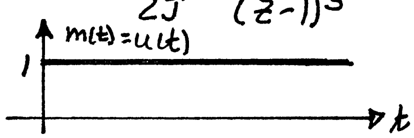
(e) $C(s) = E^*(s)\left(\dfrac{1-\epsilon^{-Ts}}{s}\right)\left(\dfrac{2}{s+0.5}\right) - \dfrac{2.5}{s+0.5}D(s)$

4-17. (a) $G(z) = K\left(\dfrac{z-1}{z}\right)\mathcal{Z}\left[\dfrac{1}{Js^3}\right] = \dfrac{z-1}{z}\left(\dfrac{K}{J}\right)\dfrac{T^2z(z+1)}{2(z-1)^3}$

$$= \frac{KT^2(z+1)}{2J\,(z-1)^2}$$

(b) $C(z) = \dfrac{KT^2}{2J}\,\dfrac{(z+1)z}{(z-1)^3}$, $\therefore C(kT) = \dfrac{K}{J}\,\dfrac{(kT)^2}{2} = \underline{\dfrac{K}{2J}(kT)^2}$

(c)



4-17. (d) $C(s) = \left(\frac{K}{Js^2}\right)\frac{1}{s} = \frac{K}{J}\cdot\frac{1}{s^3}$, $\therefore c(t) = \underline{\frac{K}{J}\left(\frac{t^2}{2}\right)}$

(e) $\underline{c(kT) = \frac{K}{2J}(kT)^2}$

4-18. (a) $KG(z) = \frac{z-1}{z} K\mathcal{Z}\left[\frac{20}{s^2(s+6)}\right]$

$$= \left(\frac{z-1}{z}\right)\frac{20K}{36}\left[\frac{z(0.3-1+0.7408)+(1-0.7408-0.2222)}{(z-1)^2(z-\epsilon^{-6T})}\right]$$

$$= \underline{K\left[\frac{0.02268z+0.02052}{(z-1)(z-0.7408)}\right]}$$

(b) $E(z) = 10$

$\therefore \frac{C(z)}{z} = \frac{10K(0.02268z+0.02052)}{z(z-1)(z-0.7408)} = \frac{0.277K}{z} + \frac{1.67K}{z-1} + \frac{-1.94K}{z-0.7408}$

$c(kT) = 0.277K\,\delta(k) + 1.67K - 1.94K(0.7408)^k$

$\therefore c_{ss}(kT) = \underline{1.67K}$

(c) In (b), $C(z) = \frac{az+\cdots}{z^2+\cdots} = az^{-1}+\cdots = C(0)+c(T)z^{-1}+\cdots$

$\therefore \underline{C(0)=0}$

In $c(kT)$ in (b), $c(0) = K[0.277 + 1.67 - 1.94] \underline{\approx 0}$

(d) A constant voltage is applied to the motor. Thus the motor speed increases to a constant value, and the shaft angle $\theta(t)$ is a ramp voltage.

(e) From (d), the antenna rotates at a constant rpm.

4-19. From tables:

(a) $E(s) = \frac{20}{(s+2)(s+5)} = \frac{20/3}{s+2} - \frac{20/3}{s+5}$

$\therefore E(z,m) = \frac{\frac{20}{3}\epsilon^{-2mT}}{z-\epsilon^{-2T}} - \frac{\frac{20}{3}\epsilon^{-5mT}}{z-\epsilon^{-5T}}$

(b) $E(s) = \frac{5}{s(s+1)} \Rightarrow E(z,m) = \frac{5}{z-1} - \frac{5\epsilon^{-mT}}{z-\epsilon^{-T}}$

(c) $E(s) = \frac{s+2}{s(s+1)} = \frac{2}{s} + \frac{-1}{s+1} \Rightarrow E(z,m) = \frac{2}{z-1} - \frac{\epsilon^{-mT}}{z-\epsilon^{-T}}$

4-19. (d) $E(s) = \dfrac{s+2}{s^2(s+1)} = \dfrac{2}{s^2} + \dfrac{-1}{s} + \dfrac{1}{s+1}$, from Problem 4-3(d)

$$E(z,m) = 2\left[\frac{mT}{z-1} + \frac{T}{(z-1)^2}\right] - \frac{1}{z-1} + \frac{\epsilon^{-mT}}{z-\epsilon^{-T}}$$

(e) $E(s) = \dfrac{s^2+5s+6}{s(s+4)(s+5)} = \dfrac{0.3}{s} + \dfrac{-0.5}{s+4} + \dfrac{1.2}{s+5}$, from Problem 4-3(e)

$$E(z,m) = \frac{0.3}{z-1} - \frac{0.5\epsilon^{-4mT}}{z-\epsilon^{-4T}} + \frac{1.2\epsilon^{-5mT}}{z-\epsilon^{-5T}}$$

(f) $E(s) = \dfrac{2}{(s+1)^2+2^2}$; $E(z,m) = \dfrac{2}{2}\left[\dfrac{\epsilon^{-mT}[z\sin(2mT) + \epsilon^{-T}\sin(1-mT)]}{z^2 - 2z\,\epsilon^{-T}\cos 2T + \epsilon^{-2T}}\right]$

4-20. Results of Problem 4-19 used.

(a) $E(z,m)\Big|_{m=0.7} = E(z,0.7) = \dfrac{\frac{20}{3}\epsilon^{-1.4T}}{z-\epsilon^{-2T}} - \dfrac{\frac{20}{3}\epsilon^{-3.5T}}{z-\epsilon^{-5T}}$

(b) $E(z,0.4) = \dfrac{5}{z-1} - \dfrac{5\epsilon^{-0.4T}}{z-\epsilon^{-T}}$

(c) $z^{-1}E(z,0.9) = \dfrac{2}{z(z-1)} - \dfrac{\epsilon^{-0.9T}}{z(z-\epsilon^{-T})}$

(d) $E(z,0.8) = 2\left[\dfrac{0.8T}{z-1} + \dfrac{T}{(z-1)^2}\right] - \dfrac{1}{z-1} + \dfrac{\epsilon^{-0.8T}}{z-\epsilon^{-0.8T}}$

(e) $E(z,0.7) = \dfrac{0.3}{z-1} - \dfrac{0.5\epsilon^{-2.8T}}{z-\epsilon^{-4T}} + \dfrac{1.2\epsilon^{-3.5T}}{z-\epsilon^{-5T}}$

(f) $E(z,0.25) = \dfrac{1}{2}\left[\dfrac{\epsilon^{-0.25T}[z\sin(0.5T) + \epsilon^{-T}\sin(1.5T)]}{z^2 - 2z\epsilon^{-T} + \epsilon^{-2T}}\right]$

4-21. $\dfrac{z-1}{z}\,\mathcal{Z}\left[\dfrac{G_p(s)}{s}\right] = \dfrac{2(z-1)}{z}\,\mathcal{Z}\left[\dfrac{\epsilon^{-3.33Ts}}{s(s+0.5)}\right] = \dfrac{2(z-1)}{z}z^{-3}\,\mathcal{Z}_m\left[\dfrac{1}{s(s+0.5)}\right]_{m=0.667}$

$$= \frac{2(z-1)}{z^4(0.5)}\left[\frac{1}{z-1} - \frac{\epsilon^{-(0.5)(0.667)(0.6)}}{z-\epsilon^{-(0.5)(0.6)}}\right] = \frac{4(z-1)}{z^4}\left[\frac{0.1814z+0.0778}{(z-1)(z-0.7408)}\right]$$

$$= \frac{0.7256z+0.3112}{z^4(z-0.7408)}$$

(a) From Problem 4-16, $G(z) = \dfrac{4(1-\epsilon^{-0.3})}{z-\epsilon^{-0.3}} = \dfrac{1.037}{z-0.7408}$

$\therefore C(z) = z\left[\dfrac{1.037}{(z-1)(z-0.7408)}\right] = \dfrac{4}{z-1} - \dfrac{4}{z-0.7408}$

$\therefore c(kT) = 4 - 4(0.7408)^k$

4-21.(b) $C(z) = \dfrac{(0.7256z - 0.3112)z}{z^4(z-1)(z-0.7408)} = z^{-4}C_1(z)$

$$\therefore \frac{C_1(z)}{z} = \frac{0.7256z - 0.3112}{(z-1)(z-0.7408)} = \frac{4}{z-1} + \frac{-3.2744}{z-0.7408}$$

$$\therefore c_1(kT) = 4 - 3.2744(0.7408)^k$$

$$c(kT) = [4 - 3.2744(0.7408)^{(k-4)}]u(k-4)$$

(c) $C(s) = \dfrac{2}{s(s+0.5)} = \dfrac{4}{s} - \dfrac{4}{s+0.5} \Rightarrow c(t) = 4(1 - \epsilon^{-0.5t})$

(d) $c(t-2) = 4[1 - \epsilon^{-0.5(t-2)}]u(t-2)$

(e) $c(kT) = 4[1 - \epsilon^{-(0.5)(0.6)k}] = 4[1 - (0.7408)^k]$

(f) $c(kT) = 4[1 - \epsilon^{1}\epsilon^{-0.5(kT)}]u(k-4)$

$$= [4 - 4\epsilon^{1}\epsilon^{-0.5(k-4)}\epsilon^{-1.2}]u(k-4)$$

$$= [4 - 3.2749(0.7408)^{(k-4)}]u(k-4)$$

4-22.(a) $D(z) = 1.2 + \dfrac{0.1z}{z-1} = \dfrac{1.3z-1}{z-1}$; From Prob. 4-21(a): $G(z) = \dfrac{1.037}{z-0.7408}$

$$\therefore \frac{C(z)}{z} = \frac{1.037(1.3z-1)}{(z-1)^2(z-0.7408)} = \frac{1.20}{(z-1)^2} + \frac{0.5705}{z-1} + \frac{-0.5705}{z-0.7408}$$

$$\therefore C(z) = \frac{2(0.6z)}{(z-1)^2} + \frac{0.5705z}{z-1} + \frac{-0.5705z}{z-0.7408}$$

$$\therefore c(kT) = \underline{2kT + 0.5705[1 - 0.7408^k]}$$

(b) The temperature increases without limit - Not physically realizable.

(c) $\dfrac{M(z)}{z} = \dfrac{1.3z-1}{(z-1)^2} = \dfrac{1.3(z-1)}{(z-1)^2} + \dfrac{0.3}{(z-1)^2} = \dfrac{0.3}{(z-1)^2} + \dfrac{1.3}{z-1}$

$$\therefore m(kT) = \underline{kT + 1.3}$$

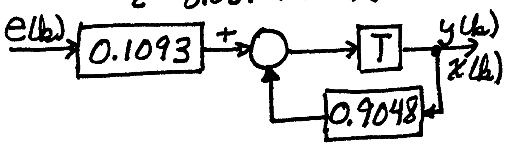
(d) The valve will reach full open. Then m(kT) is effectively constant and the temperature will settle to a constant value.

4-23.(a) $(s+0.05)Y(s) = 0.1M(s)$, $\therefore G_p(s) = \dfrac{0.1}{s+0.05}$

$$G(z) = \frac{Y(z)}{E(z)} = \frac{z-1}{z}\mathcal{Z}\left[\frac{0.1}{s(s+0.05)}\right]$$

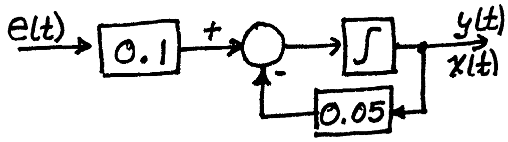
$$= \frac{z-1}{z}\left(\frac{0.1}{0.05}\right)\frac{z\left(1-\epsilon^{-(0.05)(2)}\right)}{(z-1)(z-0.9048)} = \underline{\frac{0.1903}{z-0.9048}}$$

(b)



$x(k+1) = 0.9048x(k) + 0.1093e(k)$
$y(k) = x(k)$

(c)



$\dot{x}(t) = -0.05x(t) + 0.1e(t)$
$y(t) = x(t)$

(d) $(sI-A)^{-1} = \dfrac{1}{s+0.05}$, $\therefore \Phi(t) = \epsilon^{-0.05t}$

$$\therefore A = \Phi(T) = \underline{0.9048}$$

$$B = B_c\int_0^2 \Phi(\tau)d\tau = B_c\int_0^2 \epsilon^{-0.05\tau}d\tau$$

$$= \frac{0.1}{-0.05}\epsilon^{-0.05\tau}\Big|_0^2 = 2[1-0.9048] = \underline{0.1903}$$

$$\therefore x(k+1) = 0.9048x(k) + 0.1093e(k)$$
$$y(k) = x(k)$$

(e) Same as (b).

(f) $$G(z) = \frac{0.1093z^{-1}}{1-0.9048z^{-1}} = \underline{\frac{0.1093}{z-0.9048}}$$

(g)

```
num=[0 0.1];
den=[1 0.05];
[Ac,Bc,C,D] = tf2ss(num,den)
[A,B] = c2d(Ac,Bc,2)
[n,d] = ss2tf(A,B,C,D)
pause
Ac = -.05; Bc = 0.1;
[A,B] = c2d(Ac,Bc,2)
```

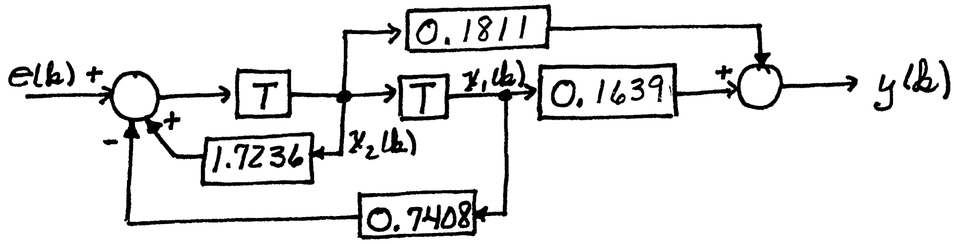
4-24.(a) $[s^2+0.15s+0.005]Y(s) = 0.1M(s) \Rightarrow G_p(s) = \dfrac{0.1}{s^2+0.15s+0.005}$

$$\therefore G(z) = \frac{Y(z)}{E(s)} = \frac{z-1}{z}\mathcal{Z}\left[\frac{0.1}{s(s+0.1)(s+0.05)}\right]$$

$$= \frac{z-1}{z}\mathcal{Z}\left[\frac{2}{s} + \frac{-2.5}{s+0.1} + \frac{0.5}{s+0.05}\right]$$

4-24. (a) $G(z) = \frac{z-1}{z}\left[\frac{2z}{z-1} + \frac{-2.5z}{z-0.8187} + \frac{0.5z}{z-0.9048}\right] = \frac{0.1811z + 0.1639}{z^2 - 1.7236z + 0.7408}$

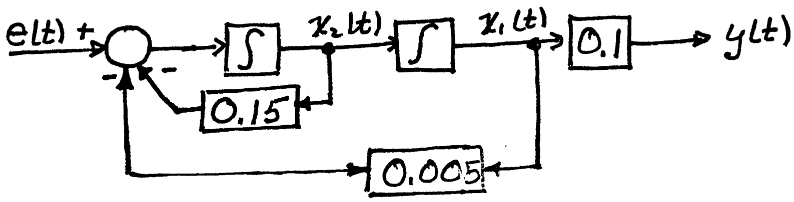
(b)



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.7408 & 1.7236 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e(k)$$

$$y(k) = [0.1639 \quad 0.1811]\underline{x}(k)$$

(c)



$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ -0.005 & -0.15 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e(t)$$

$$y(t) = [0.1 \quad 0]\underline{x}(t)$$

(d) $sI - A = \begin{bmatrix} s & -1 \\ 0.005 & s+0.15 \end{bmatrix}$, $\Delta = |sI-A| = s^2 + 0.15s + 0.005$
$= (s+0.1)(s+0.05)$

$$\therefore (sI-A)^{-1} = \begin{bmatrix} \frac{s+0.15}{(s+0.1)(s+0.05)} & \frac{1}{(s+0.1)(s+0.05)} \\ \frac{-0.005}{(s+0.1)(s+0.05)} & \frac{s}{(s+0.1)(s+0.05)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{s+0.1} + \frac{2}{s+0.05} & \frac{-20}{s+0.1} + \frac{20}{s+0.05} \\ \frac{0.1}{s+0.1} + \frac{-0.1}{s+0.05} & \frac{2}{s+0.1} + \frac{-1}{s+0.05} \end{bmatrix} = \Phi(s)$$

$$\therefore \Phi(t) = \begin{bmatrix} -\epsilon^{-0.1t} + 2\epsilon^{-0.05t} & -20\epsilon^{-0.1t} + 20\epsilon^{-0.05t} \\ 0.1\epsilon^{-0.1t} - 0.1\epsilon^{-0.05t} & 2\epsilon^{-0.1t} - \epsilon^{-0.05t} \end{bmatrix}$$

$$\therefore A = \Phi(2) = \begin{bmatrix} 0.9909 & 1.7220 \\ -0.00861 & 0.7326 \end{bmatrix}$$

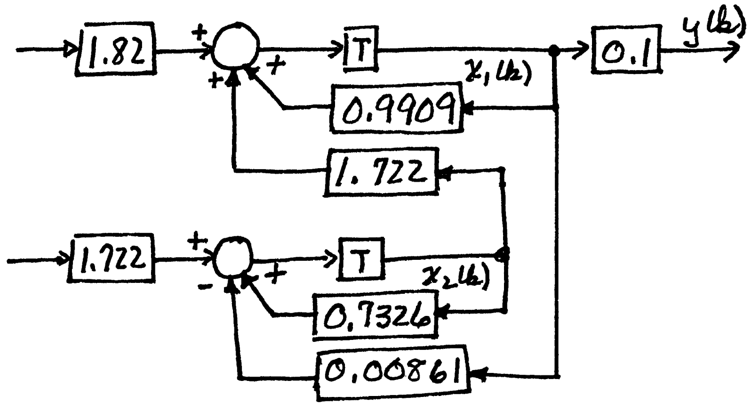
$$\int_0^2 \Phi(t)dt = \begin{bmatrix} 10\epsilon^{-0.1t} - 40\epsilon^{-0.05t} & 200\epsilon^{-0.1t} - 400\epsilon^{-0.05t} \\ -\epsilon^{-0.1t} + 2\epsilon^{-0.05t} & -20\epsilon^{-0.1t} + 20\epsilon^{-0.05t} \end{bmatrix}_0^2$$

4-24.(d) $\therefore \int_0^2 \Phi(t)\,d\tau = \begin{bmatrix} (\quad) & 1.82 \\ (\quad) & 1.722 \end{bmatrix} \Rightarrow B = \left[\int_0^2 \Phi_c(t)dt\right] B_c = \begin{bmatrix} 1.82 \\ 1.722 \end{bmatrix}$

$$\therefore \underline{x}(k+1) = \begin{bmatrix} 0.9909 & 1.7220 \\ -0.00861 & 0.736 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1.82 \\ 1.722 \end{bmatrix} e(k)$$

$$y(k) = \begin{bmatrix} 0.1 & 0 \end{bmatrix} \underline{x}(k)$$

(e)



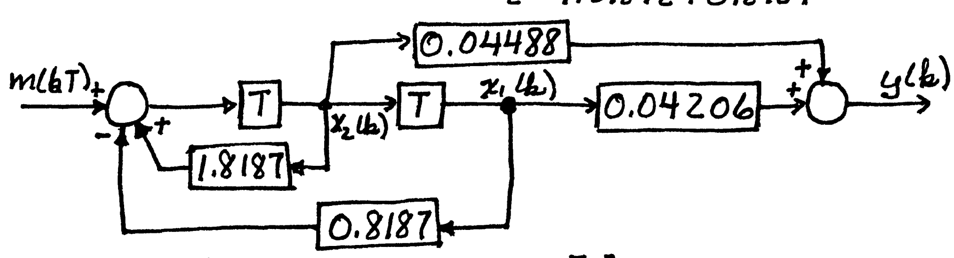
(f) $$Y(z) = \frac{0.182z^{-1}(1-0.7326z^{-1}) + 0.2965z^{-2}}{1-0.9909z^{-1}-0.7326z^{-1}+0.01483z^{-2}+0.7260z^{-2}} = \underline{\frac{0.182z+0.1630}{z^2-1.7235z+0.7408}}$$

(g)

```
num=[0 0 0.1];
den=[1 0.15 0.005];
[Ac,Bc,C,D] = tf2ss(num,den)
[A,B] = c2d(Ac,Bc,2)
[n,d] = ss2tf(A,B,C,D)
pause
Ac = [0 1;-.005 -.15]; Bc = [0;1];
[A,B] = c2d(Ac,Bc,2)
```

4-25.(a) From Problem P4-15: $G(z) = \dfrac{0.04488z+0.04206}{z^2-1.8187z+0.8187}$

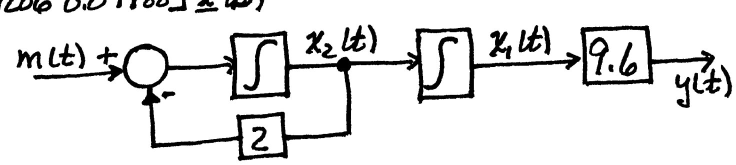
(b)



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.8187 & 1.8187 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} m(k)$$

$$y(k) = \begin{bmatrix} 0.04206 & 0.04488 \end{bmatrix} \underline{x}(k)$$

(c) $KG_p(s) = \dfrac{9.6}{s(s+2)}$



4-25. (c) $\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} m(t)$

$y(t) = [9.6 \quad 0]\, \underline{x}(t)$

(d) $sI - A_c = \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}$, $|sI - A_c| = \Delta = s^2 + 2s$

$$(sI - A_c)^{-1} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix} \Rightarrow \Phi_c(t) = \begin{bmatrix} 1 & 0.5(1-\epsilon^{-2t}) \\ 0 & \epsilon^{-2t} \end{bmatrix}$$

$$\therefore A = \Phi_c(T) = \begin{bmatrix} 1 & 0.09063 \\ 0 & 0.8187 \end{bmatrix}$$

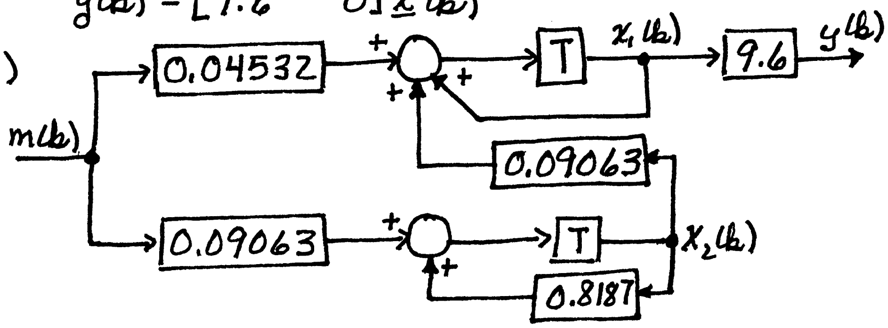
$$\int_0^{0.1} \Phi_c(t)\,dt = \begin{bmatrix} t & 0.5t + 0.25\epsilon^{-2t} \\ 0 & -0.5\epsilon^{-2t} \end{bmatrix}_0^{0.1} = \begin{bmatrix} 0.1 & 0.04532 \\ 0 & 0.09063 \end{bmatrix}$$

$$B = \left[\int_0^{0.1} \Phi_c(t)\,dt\right] B_c = \begin{bmatrix} 0.04532 \\ 0.09063 \end{bmatrix}$$

$$\therefore \underline{x}(k+1) = \begin{bmatrix} 1 & 0.09063 \\ 0 & 0.8187 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0.04532 \\ 0.09063 \end{bmatrix} m(k)$$

$$y(k) = [9.6 \quad 0]\, \underline{x}(k)$$

(e)



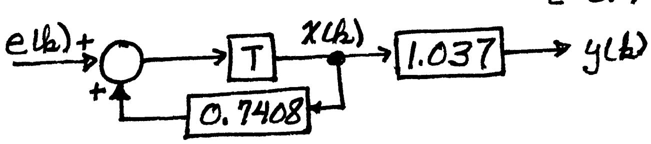
$$\text{(f)}\quad \frac{Y(z)}{M(z)} = \frac{0.04351z^{-1}(1-0.8187z^{-1}) + 0.07885z^{-2}}{1 - z^{-1} - 0.8187z^{-1} + 0.8187z^{-2}} = \frac{0.04351z + 0.04323}{z^2 - 1.8187z + 0.8187}$$

(g)

```
num=[0 0 9.61];
den=[1 2 0];
[Ac,Bc,C,D] = tf2ss(num,den)
[A,B] = c2d(Ac,Bc,0.1)
[n,d] = ss2tf(A,B,C,D)
pause
Ac = [0 1;0 -2]; Bc = [0;1];
[A,B] = c2d(Ac,Bc,0.1)
```

4-26. (a) From Problem 4-16 (a), $G(z) = \dfrac{1.037}{z - 0.7408} = \dfrac{C(z)}{E(z)}$

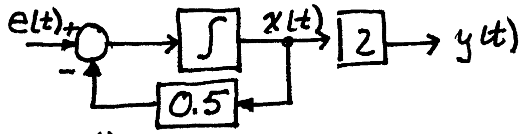
(b)



$x(k+1) = 0.7408\,x(k) + e(k)$

$y(k) = 1.037\,x(k)$

4-26. (c) $G_p(s) = \dfrac{2}{s+0.5}$



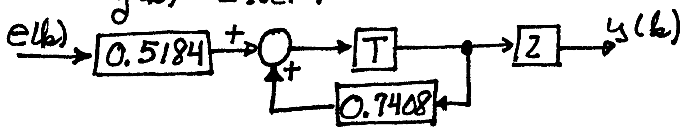
$$\dot{x}(t) = -0.5\,x(t) + e(t)$$
$$y(t) = 2\,x(t)$$

(d) $\Phi_c(t) = \mathcal{L}^{-1}[(sI-A_c)^{-1}] = \mathcal{L}^{-1}\left[\dfrac{1}{s+0.5}\right] = \epsilon^{-0.5t}$

$$A = \Phi_c(T) = \epsilon^{-0.5(0.6)} = 0.7408$$
$$B = B_c\int_0^T \Phi_c(t)\,dt = (1)\int_0^{0.3} e^{-0.5t}\,dt = -\frac{1}{0.5}e^{-0.5t}\Big|_0^{0.3}$$
$$= 2(1-0.7408) = 0.5184$$

$$\therefore\; x(k+1) = 0.7408\,x(k) + 0.5184\,e(k)$$
$$y(k) = 2\,x(k)$$

(e)



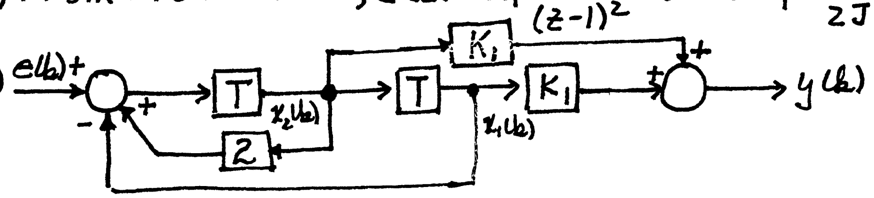
(f) $\dfrac{Y(z)}{E(z)} = \dfrac{(0.5184)z^{-1}(2)}{1-0.7408z^{-1}} = \dfrac{1.037}{z-0.7408}$

(g)

```
num=[0 2];
den=[1 0.5];
[Ac,Bc,C,D] = tf2ss(num,den)
[A,B] = c2d(Ac,Bc,0.6)
[n,d] = ss2tf(A,B,C,D)
pause
Ac = -.5; Bc = 1;
[A,B] = c2d(Ac,Bc,0.6)
```

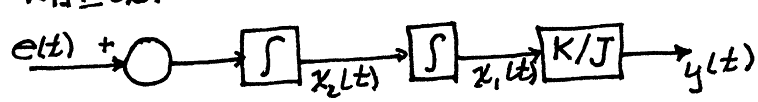
4-27. (a) From Problem 4-17, $G(z) = K_1\dfrac{z+1}{(z-1)^2}$ with $K_1 = \dfrac{KT^2}{2J} = \dfrac{K}{2J}$, $T=1$

(b)



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e(k)$$
$$y(k) = [K_1 \quad K_1]\,\underline{x}(k)$$

(c) $G_p(s) = \dfrac{K}{Js^2}$



$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\underline{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e(t)$$
$$y(t) = [K/J \quad 0]\,\underline{x}(t)$$

4-27.(d) $sI - A_c = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$, $\Delta = |sI - A_c| = s^2$, $\therefore (sI - A) = \begin{bmatrix} 1/s & 1/s^2 \\ 0 & 1/s \end{bmatrix}$

$$\therefore \Phi_c(t) = \mathcal{L}^{-1}[(sI - A_c)^{-1}] = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \Rightarrow A = \Phi_c(T) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

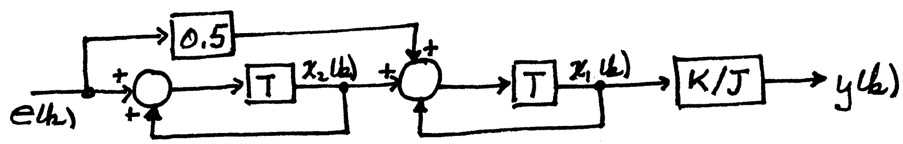
$$\int_0^1 \Phi_c(t)dt = \begin{bmatrix} t & t^2/2 \\ 0 & t \end{bmatrix}_0^1 = \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore B = \left[\int_0^1 \Phi_c(t)dt \right] B_c = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

$$\therefore \underline{x}(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} e(k)$$

$$y(k) = [K/J \quad 0]\, \underline{x}(k)$$

(e)



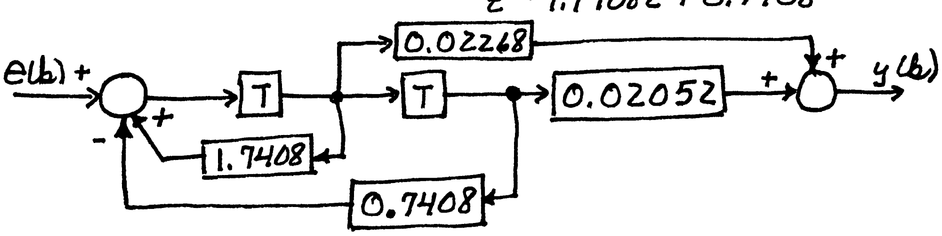
(f) $$G(z) = \frac{0.5 \frac{K}{J} z^{-1}(1 - z^{-1}) + \frac{K}{J} z^{-2}}{1 - z^{-1} - z^{-1} + z^{-2}} = \frac{\frac{K}{2J}(z+1)}{(z-1)^2}$$

(g)

```
num=[0 0 1];
den=[1 0 0];
[Ac,Bc,C,D] = tf2ss(num,den)
[A,B] = c2d(Ac,Bc,1)
[n,d] = ss2tf(A,B,C,D)
pause
Ac = [0 1;0 0]; Bc = [0;1];
[A,B] = c2d(Ac,Bc,1)
```

4-28.(a) From Problem 4-18, $G(z) = \dfrac{0.02268z + 0.02052}{z^2 - 1.7408z + 0.7408}$

(b)



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.7408 & 1.7408 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e(k)$$

$$y(k) = [0.02052 \quad 0.02268]\, \underline{x}(k)$$

(c) $G_p(s) = \dfrac{20}{s(s+6)}$

4-28.(c) $\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -6 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e(t)$

$y(t) = [20 \quad 0]\underline{x}(k)$

(d) $sI - A_c = \begin{bmatrix} s & -1 \\ 0 & s+6 \end{bmatrix}$, $\Delta = |sI - A_c| = s^2 + 6s$

$\therefore (sI - A_c)^{-1} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+6)} \\ 0 & \frac{1}{s+6} \end{bmatrix}$, $\therefore \Phi_c(t) = \begin{bmatrix} 1 & \frac{1}{6}(1-\epsilon^{-6t}) \\ 0 & \epsilon^{-6t} \end{bmatrix}$

$\therefore A = \Phi_c(T) = \begin{bmatrix} 1 & 0.04320 \\ 0 & 0.7408 \end{bmatrix}$

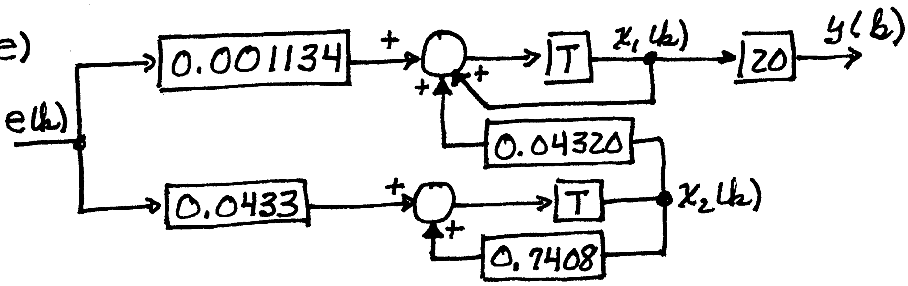
$$\int_0^{0.05} \Phi_c(t)dt = \begin{bmatrix} t & t/6 + (1/36)\epsilon^{-6t} \\ 0 & -(1/6)\epsilon^{-6t} \end{bmatrix}_0^{0.05} = \begin{bmatrix} 0.05 & 0.001134 \\ 0 & 0.04320 \end{bmatrix}$$

$$B = \left[\int_0^{0.05} \Phi_c(t)dt\right] B_c = \begin{bmatrix} 0.001134 \\ 0.04320 \end{bmatrix}$$

$$\therefore \underline{x}(k+1) = \begin{bmatrix} 1 & 0.04320 \\ 0 & 0.7408 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0.001134 \\ 0.04320 \end{bmatrix} e(k)$$

$y(k) = [20 \qquad 0]\underline{x}(k)$

(e)



(f) $$\frac{Y(z)}{E(z)} = \frac{0.02268z^{-1}(1-0.7408z^{-1}) + 0.03741\, z^{-2}}{1 - z^{-1} - 0.7408z^{-1} + 0.7408z^{-2}}$$

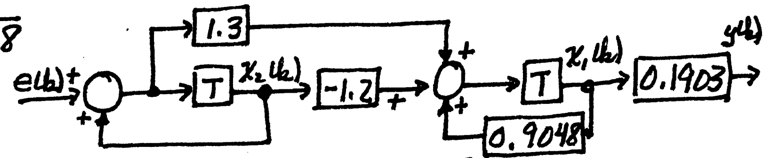
$$= \frac{0.02268z + 0.02061}{z^2 - 1.7408z + 0.7408}$$

(g)

```
num=[0 0 20];
den=[1 6 0];
[Ac,Bc,C,D] = tf2ss(num,den)
[A,B] = c2d(Ac,Bc,.05)
[n,d] = ss2tf(A,B,C,D)
pause
Ac = [0 1;0 -6]; Bc = [0;1];
[A,B] = c2d(Ac,Bc,.05)
```

4-29. $D(z) = 1.2 + \dfrac{0.1z}{z-1} = \dfrac{1.3z - 1.2}{z-1}$

(a) $G(z) = \dfrac{0.1903}{z - 0.9048}$



$$x_1(k+1) = 0.9408x_1(k) - 1.2x_2(k) + 1.3[e(k) + x_2(k)]$$

$$= 0.9408x_1(k) + 0.1x_2(k) + 1.3e(k)$$

$$x_2(k+1) = x_2(k) + e(k)$$

$$\therefore \underline{x}(k+1) = \begin{bmatrix} 0.9408 & 0.1 \\ 0 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1.3 \\ 1 \end{bmatrix} e(k)$$

$$y(k) = [0.1903 \quad 0] \underline{x}(k)$$

(b) Same filter simulation diagram as in (a).

From Problem 4-27(d):

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} m(k)$$

$$m(k) = [-1.2 + 1.3]x_3(k) + 1.3e(k) = 0.1x_3(k) + 1.3e(k)$$

$$\therefore \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0.05 \\ 0 & 1 & 0.1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0.65 \\ 1.3 \\ 1 \end{bmatrix} e(k)$$

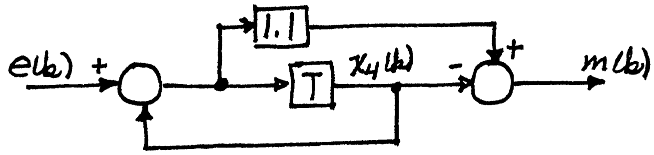
$$y[k] = [K/J \quad 0 \quad 0] \underline{x}(k)$$

(c)

```
nd = [1.3 -1.2]; dd = [1 -1];
ng = [0 0.1903]; dg = [1 -0.9048];
n = conv(nd,ng)
d = conv(dd,dg)
pause
A = [.9408 .1;0 1]; B = [1.3; 1];
C = [0.1903 0]; D = 0;
[num,dem] = ss2tf(A,B,C,D)


nd = [1.3 -1.2]; dd = [1 -1];
ng = [0 0.5 0.5]; dg = [1 -2 1];
n = conv(nd,ng)
d = conv(dd,dg)
pause
A = [1 1 .05;0 1 .1;0 0 1]; B = [.65;1.3;1];
C = [1 0 0]; D = 0;
[num,dem] = ss2tf(A,B,C,D)
```

4-30. $D(z) = 1 + \frac{0.1z}{z-1} = \frac{1.1z-1}{z-1}$



$m(k) = -x_4(k) + 1.1[e(k) + x_4(k)] = 0.1\, x_4(k) + 1.1\, e(k)$

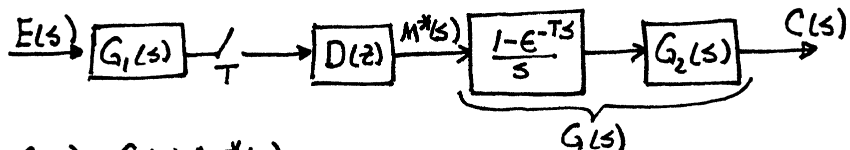
$\therefore\ \underline{x}(k+1) = A\underline{x}(k) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}[0.1x_4(k) + 1.1e(k)]$

and $x_4(k+1) = x_4(k) + e(k)$

$$\therefore\ \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 0.9 & 1 & 2 & 0 \\ 0 & 0.9 & 0 & 0.1 \\ -1 & 0 & 0.5 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \overbrace{\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix}}^{\underline{x}_e(k)} + \begin{bmatrix} 0 \\ 1.1 \\ 1.1 \\ 1 \end{bmatrix} e(k)$$

$y(k) = [1 \quad 1.5 \quad 2.3 \quad 0]\, \underline{x}_e(k)$

4-31. (a)



$C(s) = G(s)M^*(s)$

$C(z) = G(z)M(z) = G(z)D(z)\,\overline{G_1E}(z)$

(b) Cannot factor $E(z)$ from $\overline{G_1E}(z)$.

(c) Assume that $e(t)$ changes so slowly that the system can be accurately approximated by placing a sampler/data-hold in front of $G_1(s)$. Then

$C(z) = G(z)D(z)\,\mathfrak{z}\left[\frac{1-\epsilon^{-Ts}}{s} \cdot G_1(s)\right]E(z).$

4-32. (a) $(1-0.5)\dot{x}(t) = -2x(t) + 3u(t)$

$\therefore\ \dot{x}(t) = -4x(t) + 6u(t)$

$y(t) = -4x(t) + 6u(t) + 4u(t) = -4x(t) + 10u(t)$

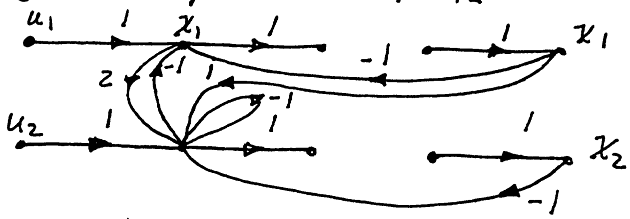
4-32.(b) $\underline{\dot{x}} = \begin{bmatrix} 0 & 2 \\ -1 & -1 \end{bmatrix} \underline{\dot{x}} + \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \underline{u} = A_1 \underline{\dot{x}} + A_2 \underline{x} + B_1 \underline{u}$

$[I - A_1]^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix}; \quad [I - A_1]^{-1} A_2 = \begin{bmatrix} 0 & -1/2 \\ 1/2 & -1/4 \end{bmatrix}$

$[I - A_1]^{-1} B_1 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix}; \quad C_1 [I - A_1]^{-1} A_2 = [\frac{1}{2} \ \ -\frac{1}{4}]; \quad C_1 [I - A_1]^{-1} B_1 = [-\frac{1}{4} \ \ \frac{1}{4}]$

$\therefore \ \underline{\dot{x}}(t) = \begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix} \underline{x}(t) + \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix} \underline{u}(t)$

$y(t) = [\frac{1}{2} \ \ -\frac{1}{4}] \underline{x}(t) + [-\frac{1}{4} \ \ \frac{1}{4}] \underline{u}(t)$



$\Delta = 1 - (-2 - 1) = 4$

$\dot{x}_1 = \frac{-1(1+1)+2}{4} x_1 + \frac{-2}{4} x_2 + \frac{1(1+1)}{4} u_1 + \frac{2}{4} u_2$

$\dot{x}_2 = \frac{1+1}{4} x_1 + \frac{-1}{4} x_2 + \frac{-1}{4} u_1 + \frac{1}{4} u_2$

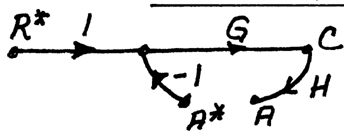
$y = \frac{1+1}{4} x_1 + \frac{-1}{4} x_2 + \frac{-1}{4} u_1 + \frac{1}{4} u_2$

CHAPTER 5

5-1.(a) $C(s) = G(s)[R^*(s) - H^*(s)\,C^*(s)]$

$$\therefore\ C(z) = \frac{G(z)}{1 + G(z)H(z)} R(z)$$

(b)



$A = GH[R^* - A^*]$

$$\therefore A^* = \frac{\overline{GH}^* R^*}{1 + \overline{GH}^*}$$

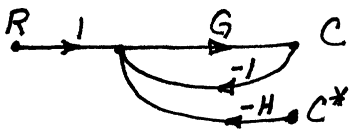
$$\therefore C = G[R^* - A^*] = G\left[\frac{R^*}{1 + \overline{GH}^*}\right]$$

$$\therefore C(z) = \frac{G(z)}{1 + \overline{GH}(z)} R(z)$$

(c) $E = R - GHE^* \Rightarrow E^* = R^* - \overline{GH}^* E^* \Rightarrow E(z) = \dfrac{R(z)}{1 + \overline{GH}(z)}$

$$C = GE^* \Rightarrow C(z) = \frac{G(z)}{1 + \overline{GH}(z)} R(z)$$

(d)

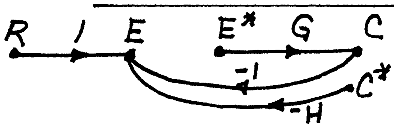


$$C = \frac{GR}{1+G} - \frac{GH}{1+G} C^*$$

$$C(z) = \left[\frac{GR}{1+G}\right](z) - \left[\frac{GH}{1+G}\right](z)\, C(z)$$

$$\therefore C(z) = \frac{\left[\frac{GR}{1+G}\right](z)}{1 + \left[\frac{GH}{1+G}\right](z)}$$

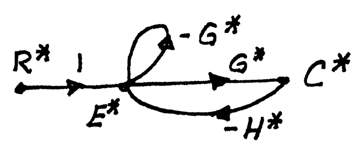
(e)



$E = R - GE^* - HC^*$

$C = GE^*$

$$\left.\begin{aligned} E^* &= R^* - G^*E^* - H^*C^* \\ C^* &= G^*E^* \end{aligned}\right\}$$



$$\therefore\ C(z) = \frac{G(z)}{1 + G(z) + G(z)H(z)} R(z)$$

5-2.(a) $C = G_2 A^*,\ \therefore A = G_1[R^* - \overline{G_2 H}^* A^*] \Rightarrow A(z) = \dfrac{G_1(z) R(z)}{1 + G_1(z)\overline{G_2 H}(z)}$

$$\therefore C(z) = \frac{G_1(z)\, G_2(z)}{1 + G_1(z)\overline{G_2 H}(z)} R(z)$$

5-2.(b) $C = G_1 G_2 [R^* - H^* C^*]$

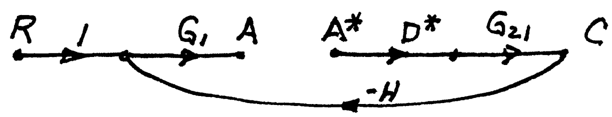
$$\therefore C(z) = \frac{\overline{G_1 G_2}(z)}{1 + \overline{G_1 G_2}(z) H(z)} R(z)$$

(c) $C = \dfrac{G_1 G_2 R}{1 + G_1 G_2 H} - \dfrac{G_2 H}{1 + G_1 G_2 H} C^*$

$$\therefore C(z) = \frac{\left[\dfrac{G_1 G_2 R}{1 + G_1 G_2 H}\right](z)}{1 + \left[\dfrac{G_2 H}{1 + G_1 G_2 H}\right](z)}$$

(d) $C = \dfrac{GR}{1 + GH_2} - \dfrac{GH_1}{1 + GH_2} D^* C^*$

$$\therefore C(z) = \frac{\left[\dfrac{GR}{1 + GH_2}\right](z)}{1 + D(z)\left[\dfrac{GH_1}{1 + GH_2}\right](z)}$$

(e) 

$G_{21} = \left[\dfrac{1 - \epsilon^{-Ts}}{s}\right] G_2$

$$A = G_1 R - G_1 G_2 H D^* A^*$$

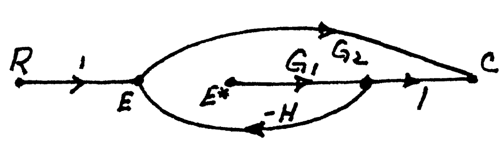
$\therefore A^* = \dfrac{\overline{G_1 R}^*}{1 + \overline{G_1 G_2 H}^* D^*}$ and $C = G_{21} D^* A^*$

$$\therefore C(z) = \frac{G_{21}(z) D(z) \overline{G_1 R}(z)}{1 + \overline{G_1 G_{21} H}(z) D(z)}$$

5-3. (a) From Problem 5-1(b): $C(z) = \dfrac{G(z)}{1 + \overline{GH}(z)} R(z)$

(b) From Problem 5-1(c): $C(z) = \dfrac{G(z)}{1 + \overline{GH}(z)} R(z)$

(c) In each system, $E^* = R^* - \overline{GH}^* E^*$. Hence both $C(s)$ and $C(z)$ are the same for each system.

5-4.(a) 

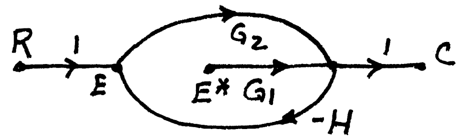
$$E = R - G_1 H E^*$$

$$\therefore E^* = \frac{R^*}{1 + \overline{GH}^*}$$

5-4. (a) $C = G_2 R - G_1 G_2 H E^* + G_1 E^*$

$$\therefore\ C(z) = \overline{G_2 R}(z) + \frac{G_1(z) - \overline{G_1 G_2 H}(z)}{1 + \overline{G_1 H}(z)} R(z)$$

(b)

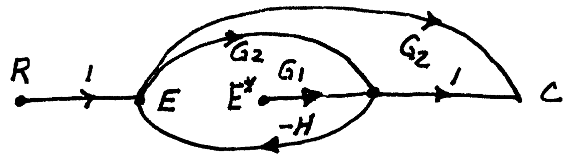


$$E = \frac{R}{1+G_2H} - \frac{G_1 H}{1+G_2 H} E^* \Rightarrow E^* = \frac{\left[\frac{R}{1+G_2H}\right]^*}{1 + \left[\frac{G_1 H}{1+G_2H}\right]^*}$$

$$C = \frac{G_2 R}{1+G_2H} + \frac{G_1}{1+G_2H} E^*$$

$$\therefore\ C(z) = \left[\frac{G_2 R}{1+G_2H}\right](z) + \frac{\left[\frac{G_1}{1+G_2H}\right](z)}{1 + \left[\frac{G_1 H}{1+G_2H}\right](z)} \left[\frac{R}{1+G_2H}\right](z)$$

(c)



$$E = R - H[G_1 E^* + G_2 E] \Rightarrow E = \frac{R}{1+G_2H} - \frac{G_1 H}{1+G_2H} E^*$$

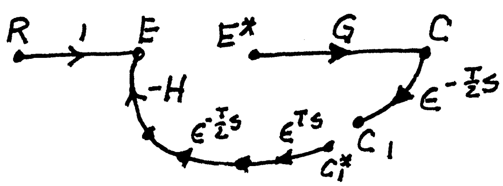
$$E^* = \frac{\left[\frac{R}{1+G_2H}\right]^*}{1 + \left[\frac{G_1 H}{1+G_2H}\right]^*}$$

$$C = G_2 E + G_1 E^* + G_2 E = 2 G_2 E + G_1 E^*$$

$$= \frac{2G_2 R}{1+G_2H} + \frac{G_1 \left[\frac{R}{1+G_2H}\right]^*}{1 + \left[\frac{G_1 H}{1+G_2H}\right]^*}$$

$$\therefore\ C(z) = \left[\frac{2G_2 R}{1+G_2H}\right](z) + \frac{G_1(z)\left[\frac{R}{1+G_2H}\right](z)}{1 + \left[\frac{G_1 H}{1+G_2H}\right](z)}$$

5-5. (a)



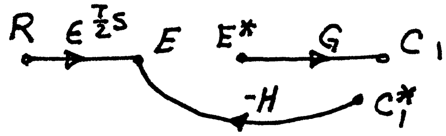
$$E(z) = R(z) - z\, C_1(z) H(z,m)\Big|_{m=\frac{1}{2}}$$

$$C_1(z) = G(z,m)\Big|_{m=\frac{1}{2}} E(z)$$

$$\therefore\ E(z) = R(z) - z H(z,\tfrac{1}{2}) G(z,\tfrac{1}{2}) E(z)$$

$$C(z) = G(z)E(z) = \frac{G(z)}{1 + zG(z,\tfrac{1}{2})H(z,\tfrac{1}{2})}$$

(b) Equivalent:



$$E = R\epsilon^{\frac{T}{2}s} - HC_1$$

$$C_1 = GE^* = G(R\epsilon^{\frac{T}{2}s})^* - GHC_1^*$$

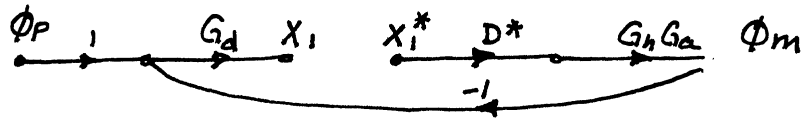
$$\therefore C_1(z) = \frac{G(z)\,\mathcal{Z}[R\epsilon^{\frac{T}{2}s}]}{1 + \overline{GH}(z)} = \frac{G(z)}{1 + \overline{GH}(z)}\, zR(z,\tfrac{1}{2})$$

$$C_1(z) = c(\tfrac{T}{2}) + c(\tfrac{3T}{2})z^{-1} + c(\tfrac{5T}{2})z^{-2} + \cdots$$

$$\therefore C(z) = C_1(z)z^{-\frac{1}{2}} = \frac{G(z)}{1 + \overline{GH}(z)}\, z^{\frac{1}{2}} R(z,\tfrac{1}{2})$$

$$= c(\tfrac{T}{2})z^{-\frac{1}{2}} + c(\tfrac{3T}{2})z^{-\frac{3}{2}} + \cdots, \qquad z^{\frac{1}{2}}\Big|_{z=\epsilon^{Ts}} = \epsilon^{\frac{T}{2}s}$$

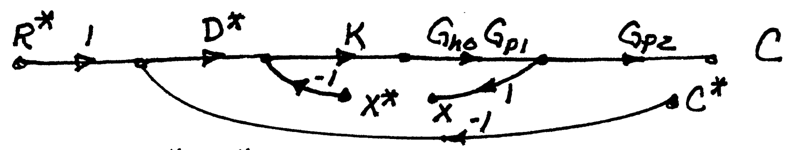
5-6.



$$X_1 = G_d\phi_P - G_dG_hG_aD^*X_1^* \Rightarrow X_1^* = \frac{\overline{G_d\phi_P}^*}{1 + D^*\overline{G_dG_hG_a}^*}$$

$$\therefore \phi_m(z) = \overline{G_hG_a}(z)D(z)X_1(z) = \frac{\overline{G_hG_a}(z)\,\overline{G_d\phi_P}(z)\,D(z)}{1 + \overline{G_dG_hG_a}(z)D(z)}$$

5-7.



Let: $G_{ho}G_{p1} = G_1$

$G_{ho}G_{p1}G_{p2} = G_2$

$$X = -KG_1X^* + D^*KG_1(R^* - C^*)$$

$$X(z) = \frac{D(z)KG_1(z)}{1 + KG_1(z)}[R(z) - C(z)]$$

$$C = -KG_2X^* + KD^*G_2(R^* - C^*)$$

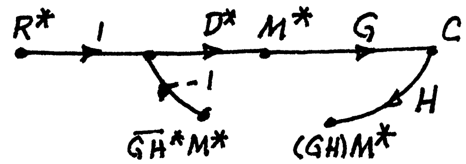
$$C(z) = -KG_2(z)\left[\frac{KD(z)G_1(z)}{1 + KG_1(z)}\right][R(z) - C(z)] + KD(z)G_2(z)[R(z) - C(z)]$$

$$\therefore C(z)[1 + KG_1(z)] = -KG_2(z)D(z)KG_1(z)[R(z) - C(z)]$$

$$+ D(z)KG_2(z)[1 + KG_1(z)][R(z) - C(z)]$$

5-7. $\therefore\ C(z) = \dfrac{K\,D(z)\,G_2(z)\,R(z)}{1 + K G_1(z) + K\,D(z)\,G_2(z)}$

5-8. (a)



$G(s) = \left[\dfrac{1-\epsilon^{-Ts}}{s}\right] G_p(s)$

$M^* = D^*(R^* - \overline{GH}^* M^*) \Rightarrow M(z) = \dfrac{D(z)\,R(z)}{1 + D(z)\,\overline{GH}(z)}$

$C = G M^* \Rightarrow \underline{C(z) = \dfrac{D(z)\,G(z)}{1 + D(z)\,\overline{GH}(z)}\,R(z)}$

$C(s) = G(s)\,M^*(s) = \dfrac{G(s)}{1 + D^*(s)\,\overline{GH}^*(s)}\,D^*(s)\,R^*(s)$

(b) The signals out of the D/A's are identical.

5-9. (a) $G(s)$, $H(s)$
(b) $G(s)$
(c) $G(s)$
(d) $H(s)$
(e) $G(s)$, $H(s)$

5-10. (a) $G_1(s)$, $G_2(s)$
(b) $G_1(s)$, $H(s)$
(c) $H_2(s)$
(d) $H_1(s)$
(e) $G_2(s) = \left(\dfrac{1-\epsilon^{-Ts}}{s}\right) G_{p2}(s)$

5-11. (a) $C(s) = G(s)\,R(s) - G(s)\,H(s)\,\epsilon^{-0.1Ts}\,D^*(s)\,C^*(s)$

$\therefore\ C(z) = \overline{GR}(z) - \overline{GH}(z,m)\big|_{m=0.9}\,D(z)\,C(z)$

$\therefore\ C(z) = \dfrac{\overline{GR}(z)}{1 + D(z)\,\overline{GH}(z, 0.9)}$

(b) $C(z)$ same as in (a).

5-12.(a) $C = GD^*E^*$

$$E = R - H_k G D^* E^* \Rightarrow E^* = \frac{R^*}{1 + D^* G^* H_k}$$

$$\therefore C(z) = \frac{D(z)\,G(z)}{1 + D(z)G(z)H_k} R(z)$$

(b) $C = -G_d R_d + G D^* E^*$

$$E = -H_k G D^* E^* + H_k G_d R_d \Rightarrow E^* = \frac{\overline{H_k G_d R_d}^*}{1 + D^* G^* H_k}$$

$$C(z) = -\overline{G_d R_d}(z) + \frac{G(z)D(z)H_k \overline{G_d R_d}(z)}{1 + D(z)G(z)H_k} = \frac{-\overline{G_d R_d}(z)}{1 + D(z)G(z)H_k}$$

(c) $$C(z) = \frac{D(z)G(z)R(z) - \overline{G_d R_d}(z)}{1 + D(z)G(z)H_k}$$

5-13.(a) $\pm 135° \times 0.07\ \text{V/degree} = \pm 9.45\text{ V}$

$\therefore$ choose A/D with ± 10V input.

(b) $$G(z) = \frac{z-1}{z} \mathcal{Z}\left[\frac{2}{s^2(0.5s+1)}\right]$$

$$M = D^*[\theta_c - H_k K G M^*]$$

$$\therefore M^* = \frac{D^* \theta_c^*}{1 + K D^* G^* H_k}$$

$$\theta_a(z) = K G(z) M(z) = \frac{K D(z) G(z)}{1 + K D(z) G(z) H_k} \theta_c(z)$$

(c) From Problem 4-15, $$KG(z) = \frac{0.04488z + 0.04206}{z^2 - 1.8187z + 0.8187}$$

$$\therefore \frac{\theta_a(z)}{\theta_c(z)} = \frac{0.04488z + 0.04206}{z^2 - 1.8187z + 0.8187 + 0.00314z + 0.00294}$$

$$= \frac{0.04488z + 0.04206}{z^2 - 1.8156z + 0.8216}$$

(d)

```
num=[0 0 9.6];
den=[1 2 0];
[Ac,Bc,C,D] = tf2ss(num,den)
[A,B] = c2d(Ac,Bc,.1)
[n,d] = ss2tf(A,B,C,D)
tnum = n
tden = d + 0.07*n
```

5-14. (a) $\theta_{max}(t) = 360°$, maximum sensor output $= (0.02)(360°) = 7.2V$

$\therefore$ input for $360° = 7.2V$

input for $0° = 0V$

$\therefore$ need A/D range 0-7.2V, $\therefore$ choose <u>0-10V</u> range.

5-14. (b) From (a), sensor output $= (0.02)(70°) = \underline{1.4V}$

$\therefore$ input $= 1.4V$

(c) (a) maximum sensor output $= (0.02)(\pm 180°) = \pm 3.6V$

$\therefore$ choose $\pm 5V$ range

(b) From (b), $\underline{1.4V}$

(d) $G(z) = \mathcal{Z}\left[\frac{1-\epsilon^{-Ts}}{s} G_p(s)\right]$

$$M^*(s) = D^*[R^* - KG^*H_k M^*] \Rightarrow M^* = \frac{D^*R^*}{1 + KD^*G^*H_k}$$

$$C = KG^*M^* \Rightarrow C(z) = \frac{KD(z)G(z)}{1 + KD(z)G(z)H_k} R(z)$$

(e) $$G(z) = \frac{z-1}{z}\mathcal{Z}\left[\frac{1}{Js^3}\right] = \frac{z-1}{z}\,\frac{(1)^2 z(z+1)}{0.1(2)(z-1)^3} = \frac{5(z+1)}{(z-1)^2}$$

$$\therefore \frac{KD(z)G(z)}{1 + KD(z)G(z)H_k} = \frac{2[5(z+1)]}{z^2 - 2z + 1 + 10(z+1)0.02}$$

$$\therefore \text{system transfer function} = \frac{10(z+1)}{z^2 - 1.8z + 1.2}$$

(f)

```
num=[0 0 20];
den=[1 0 0];
[Ac,Bc,C,D] = tf2ss(num,den)
[A,B] = c2d(Ac,Bc,1)
[n,d] = ss2tf(A,B,C,D)
tnum = n
tden = d + 0.02*n
```

5-15. (a) Sensor output $= 0.4(30°) = 12$

$\therefore 30° \longrightarrow r(k) = 12$

$-30° \longrightarrow r(k) = -12$

5-15 (b) $M^* = D^*(R^* - H_k K G^* M^*)$; $\quad G(s) = \frac{1-\epsilon^{-Ts}}{s} G_p(s)$

$$\therefore M^* = \frac{D^* R^*}{1 + K D^* G^* H_k}$$

$$C(s) = K G(s) M^*(s) \Rightarrow C(z) = \frac{K D(z) G(z)}{1 + K D(z) G(z) H_k} R(z)$$

(c) From Problem 4-18,

$$G(z) = \frac{0.02268z + 0.02052}{z^2 - 1.7408z + 0.7408}$$

$$\therefore \frac{C(z)}{R(z)} = \frac{0.4536z + 0.4104}{z^2 - 1.7408z + 0.7408 + 0.1814z + 0.1642}$$

$$= \frac{0.4536z + 0.4104}{z^2 - 1.5594z + 0.9050}$$

(d)

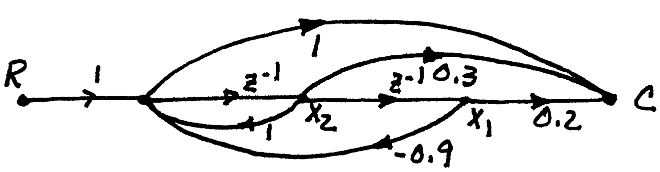
```
num=[0 0 400];
den=[1 6 0];
[Ac,Bc,C,D] = tf2ss(num,den)
[A,B] = c2d(Ac,Bc,.05)
[n,d] = ss2tf(A,B,C,D)
tnum = n
tden = d + 0.4*n
```

5-16. (a) $\frac{C(z)}{R(z)} = \frac{1 + 0.3z^{-1} + 0.2z^{-2}}{1 - z^{-1} + 0.9z^{-2}}$

$$(1 - z^{-1} + 0.9z^{-2}) C(z) = (1 + 0.3z^{-1} + 0.2z^{-2}) R(z)$$

$$\therefore c(k) = c(k-1) - 0.9c(k-2) + r(k) + 0.3r(k-1) + 0.2r(k-2)$$

(b)



Let: $y(k) = c(k)$
$u(k) = r(k)$

$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.9 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = [0.2 - 0.9 \quad 0.3 + 1]\, \underline{x}(k) + u(k) = [-0.7 \quad 1.3]\, \underline{x}(k) + u(k)$$

(c) $zI - A = \begin{bmatrix} z & -1 \\ 0.9 & z-1 \end{bmatrix}$, $\quad |zI - A| = \Delta = z^2 - z + 0.9$

$$\therefore (zI - A)^{-1} = \frac{1}{\Delta} \begin{bmatrix} z-1 & 1 \\ -0.9 & z \end{bmatrix}$$

5-16. (c) $\dfrac{Y(z)}{U(z)} = C(zI-A)^{-1}B + D = \dfrac{1}{\Delta}\begin{bmatrix}-0.7 & 1.3\end{bmatrix}\begin{bmatrix} z-1 & 1 \\ -0.9 & z \end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} + 1$

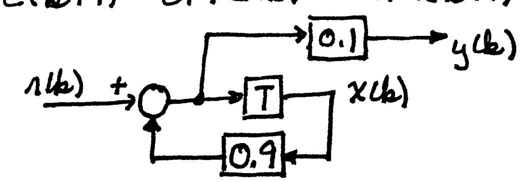
$= \dfrac{1}{\Delta}\begin{bmatrix}-0.7 & 1.3\end{bmatrix}\begin{bmatrix}1\\z\end{bmatrix} + 1 = \dfrac{1.3z-0.7}{z^2-z+0.9} + 1 = \dfrac{z^2+0.3z+0.2}{z^2-z+0.9}$

(d)

```
A = [0 1;-.9 1]; B = [0;1]; C = [-0.7 1.3]; D = 1;
[num, den] = ss2tf(A,B,C,D)
```

5-17. (a) (a) $c(k+1) = 0.9c(k) + 0.1r(k+1)$

(b)



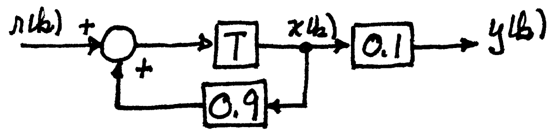
$x(k+1) = 0.9x(k) + r(k)$

$y(k) = 0.09x(k) + 0.1r(k)$

(c) $\dfrac{Y(z)}{R(z)} = C(zI-A)^{-1}B + D = \dfrac{0.09}{z-0.9} + 0.1 = \dfrac{0.1z}{z-0.9}$

(b) (a) $c(k+1) = 0.9c(k) + 0.1r(k)$

(b)



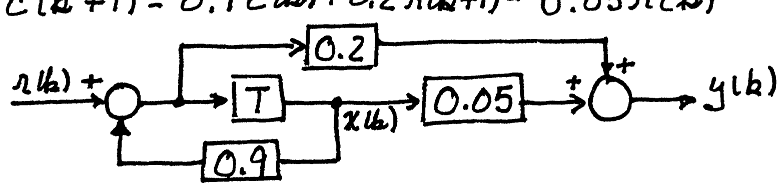
$x(k+1) = 0.9x(k) + r(k)$

$y(k) = 0.1x(k)$

(c) $\dfrac{Y(z)}{R(z)} = C(zI-A)^{-1}B = (0.1)\dfrac{1}{z-0.9}(1) = \dfrac{0.1}{z-0.9}$

(c) (a) $c(k+1) = 0.9c(k) + 0.2r(k+1) - 0.05r(k)$

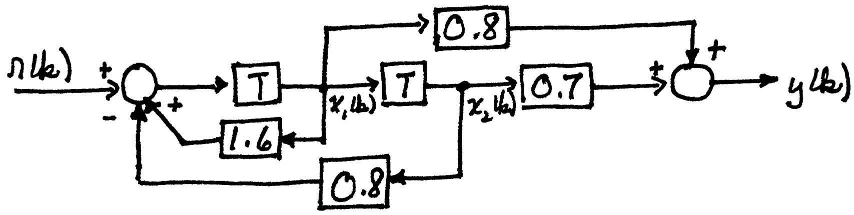
(b)



(c) $\dfrac{Y(z)}{R(z)} = C(zI-A)^{-1}B + D = \dfrac{0.13}{z-0.9} + 0.2 = \dfrac{0.2z-0.05}{z-0.9}$

(d) (a) $c(k+2) = 1.6c(k+1) - 0.8c(k) + 0.8r(k+1) + 0.7r(k)$

5-17.(d) (b)



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.8 & 1.6 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(k)$$

$$y(k) = [0.7 \quad 0.8]\, \underline{x}(k)$$

(c) $$(zI - A) = \begin{bmatrix} z & -1 \\ 0.8 & z-1.6 \end{bmatrix}; \quad |zI - A| = \Delta = z^2 - 1.6z + 0.8$$

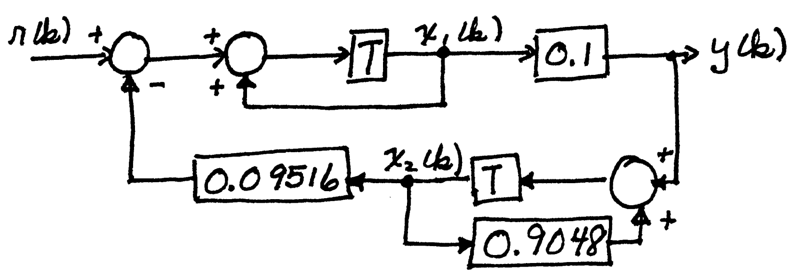
$$\therefore \frac{Y(z)}{R(z)} = [0.7 \quad 0.8] \frac{1}{\Delta} \begin{bmatrix} z-1.6 & 1 \\ -0.8 & z \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\Delta} [0.7 \quad 0.8] \begin{bmatrix} 1 \\ z \end{bmatrix}$$

$$= \frac{0.8z + 0.7}{z^2 - 1.6z + 0.8}$$

5-18 (a) $$G(z) = \frac{z-1}{z} \mathcal{Z}\left[\frac{1}{s^2}\right] = \frac{z-1}{z} \frac{0.1z}{(z-1)^2} = \frac{0.1}{z-1}$$

$$H(z) = \frac{z-1}{z} \mathcal{Z}\left[\frac{1}{s^2+s}\right] = \frac{z-1}{z} \frac{z(1-\epsilon^{-0.1})}{(z-1)(z-\epsilon^{-0.1})} = \frac{0.09516}{z - 0.9048}$$

(b)



(c) $$\underline{x}(k+1) = \begin{bmatrix} 1 & -0.09516 \\ 0.1 & 0.9048 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(k)$$

(d) $$1 + G(z)H(z) = 0 = z^2 - 1.9048z + 0.9048 + 0.009516$$

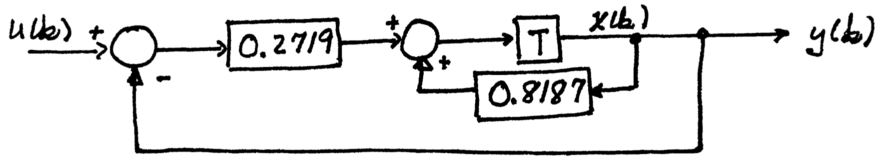
$$= \underline{z^2 - 1.9048z + 0.9143}$$

(e) $$|zI - A| = \begin{vmatrix} z-1 & 0.09516 \\ -0.1 & z-0.9048 \end{vmatrix} = z^2 - 1.9048z + 0.9048 + 0.009516$$

$$= \underline{z^2 - 1.9048z + 0.9143}$$

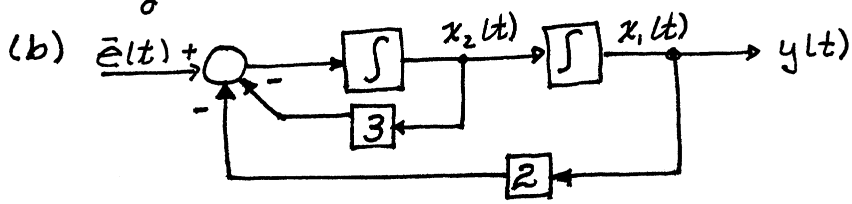
5-19. (a) $\dfrac{Y(s)}{M(s)} = G_p(s) = \dfrac{3}{s+2}$

$$G(z) = \frac{z-1}{z}\mathcal{Z}\left[\frac{3}{s(s+2)}\right] = \left(\frac{z-1}{z}\right)\frac{3}{2}\frac{z(1-\epsilon^{-2T})}{(z-1)(z-\epsilon^{-2T})} = \frac{0.2719}{z-0.8187}$$



$$\therefore x(k+1) = [0.8187 - 0.2719]x(k) + 0.2719u(k)$$
$$= 0.5468x(k) + 0.2719u(k)$$
$$y(k) = x(k)$$

(b)



$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\underline{x}(t) + \begin{bmatrix} 0 \\ 6 \end{bmatrix}\bar{e}(t)$$

$$y(t) = [1 \quad 0]\underline{x}(t)$$

$$(sI-A)^{-1} = \begin{bmatrix} s & 1 \\ 2 & s+3 \end{bmatrix}^{-1} = \begin{bmatrix} \dfrac{s+3}{(s+1)(s+2)} & \dfrac{1}{(s+1)(s+2)} \\ \dfrac{-2}{(s+1)(s+2)} & \dfrac{s}{(s+1)(s+2)} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{2}{s+1} + \dfrac{-1}{s+2} & \dfrac{1}{s+1} + \dfrac{-1}{s+2} \\ \dfrac{-2}{s+1} + \dfrac{2}{s+2} & \dfrac{-1}{s+1} + \dfrac{2}{s+2} \end{bmatrix}$$

$$\therefore \Phi_c(t) = \begin{bmatrix} 2\epsilon^{-t} - \epsilon^{-2t} & \epsilon^{-t} - \epsilon^{-2t} \\ -2\epsilon^{-t} + 2\epsilon^{-2t} & -\epsilon^{-t} + 2\epsilon^{-2t} \end{bmatrix}$$

$$A = \Phi_c(0.1) = \begin{bmatrix} 0.9909 & 0.0861 \\ -0.1722 & 0.7326 \end{bmatrix}$$

$$\int_0^T \Phi_c(t)\,dt = \begin{bmatrix} -2\epsilon^{-t} + 0.5\epsilon^{-2t} & -\epsilon^{-t} + 0.5\epsilon^{-2t} \\ 2\epsilon^{-t} - \epsilon^{-2t} & \epsilon^{-t} - \epsilon^{-2t} \end{bmatrix}_0^{0.1}$$

$$B = \left[\int_0^T \Phi_c(t)\,dt\right]B_c = \begin{bmatrix} (\quad) & 0.00455 \\ (\quad) & 0.0861 \end{bmatrix}\begin{bmatrix} 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 0.0273 \\ 0.5166 \end{bmatrix}$$

5-19. (b) $BC = \begin{bmatrix} 0.0273 \\ 0.5116 \end{bmatrix} [1 \quad 0] = \begin{bmatrix} 0.0273 & 0 \\ 0.5116 & 0 \end{bmatrix}$

$$\underline{x}(k+1) = A\underline{x}(k) + Be(k) = A\underline{x}(k) + B[u(k) - y(k)]$$
$$= [A - BC]\underline{x}(k) + Bu(k)$$
$$\therefore \underline{x}(k+1) = \begin{bmatrix} 0.9636 & 0.0861 \\ -0.6888 & 0.7326 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0.0273 \\ 0.5166 \end{bmatrix} u(k)$$
$$y(k) = [1 \quad 0]\underline{x}(k)$$

5-20. $\underline{x}(k+1) = A\underline{x}(k) + Bm(k)$

$$y(k) = C\underline{x}(k) + Dm(k)$$
$$m(k) = r(k) - y(k) = r(k) - C\underline{x}(k) - Dm(k)$$
$$\therefore \quad m(k) = [1/(1+D)][r(k) - C\underline{r}(k)]$$
$$\therefore \underline{x}(k+1) = A\underline{x}(k) + B(1+D)^{-1}[r(k) - C\underline{x}(k)]$$
$$\therefore (1) \quad \underline{x}(k+1) = [A - (1+D)^{-1}BC]\underline{x}(k) + (1+D)^{-1}Br(k)$$
$$y(k) = C\underline{x}(k) + D(1+D)^{-1}[r(k) - C(k)]$$
$$\therefore \quad y(k) = [C - D(1+D)^{-1}C]\underline{x}(k) + D(1+D)^{-1}r(k)$$
$$\therefore (2) \quad y(k) = (1+D)^{-1}C\underline{x}(k) + D(1+D)^{-1}r(k)$$

5-21. (a) $m(k) = r(k) - y(k) = r(k) - 0.2x(k) - 0.5m(k)$

$$\therefore \quad m(k) = 0.667r(k) - 0.1333x(k)$$
$$\therefore \quad x(k+1) = 0.7x(k) + 0.2r(k) - 0.04x(k)$$
$$y(k) = 0.2x(k) + 0.3333r(k) - 0.0667x(k)$$
$$\therefore \quad x(k+1) = 0.66x(k) + 0.2r(k)$$
$$y(k) = 0.1333x(k) + 0.3333r(k)$$

(b) $m(k) = r(k) - y(k) = r(k) - C\underline{x}(k)$

$$\therefore \underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.9 & 1.3 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix} [r(k) - 1.2x_1(k) + 0.7x_2(k)]$$

5-21.(b) $\therefore\ \underline{x}(k+1) = \begin{bmatrix} 0.12 & 1.07 \\ -0.96 & 1.335 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix} r(k)$

$$y(k) = [1.2 \quad -0.7]\, \underline{x}(k)$$

(c) $\quad m(k) = r(k) - C\underline{x}(k) = r(k) - x_1(k)$

$$\therefore\ \underline{x}(k+1) = \begin{bmatrix} -0.5 & 0 & 0 \\ -1 & 0.9 & 1 \\ -3 & 0 & 0.9 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} r(k)$$

$$y(k) = [1 \quad 0 \quad 0]\, \underline{x}(k)$$

5-22. $\quad D(z) = 1.1 + \dfrac{0.02z}{z-1} = \dfrac{1.12z - 1.1}{z-1}$

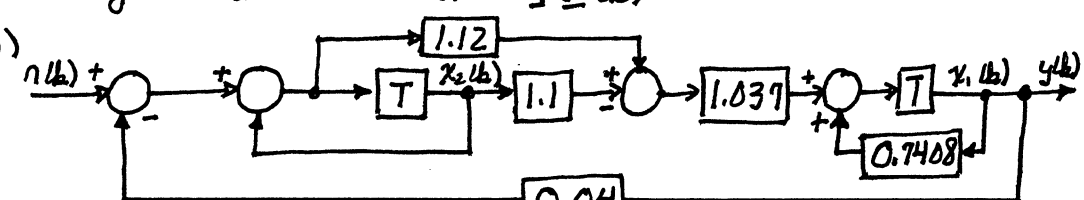
(a) From Problem 4-21, $G(z) = \dfrac{1.037}{z - 0.7408}$

$$\therefore\ \frac{D(z)G(z)}{1 + D(z)G(z)H} = \frac{1.1614z - 1.140}{z^2 - 1.6943z + 0.7001}$$

$$\therefore\ \underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.6952 & 1.6945 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(k)$$

$$y(k) = [1.1614 \quad -1.140]\, \underline{x}(k)$$

(b)



$$\underline{x}(k+1) = \begin{bmatrix} 0.6943 & 0.02074 \\ -0.04 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1.1614 \\ 1 \end{bmatrix} r(k)$$

$$y(k) = [1 \quad 0]\, \underline{x}(k)$$

(c) $\quad zI - A = \begin{bmatrix} z - 0.6943 & -0.02074 \\ 0.04 & z-1 \end{bmatrix}, \ \Delta = |zI - A| = z^2 - 1.6943z + 0.6953$

$$\frac{D(z)G(z)}{1 + D(z)G(z)H} = C(zI-A)^{-1}B = \frac{1}{\Delta}[1 \quad 0] \begin{bmatrix} z-1 & 0.02074 \\ -0.04 & z - 0.6943 \end{bmatrix} \begin{bmatrix} 1.1614 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\Delta}[z-1 \quad 0.02074] \begin{bmatrix} 1.1614 \\ 1 \end{bmatrix} = \frac{1.6164z - 1.140}{z^2 - 1.6943z + 0.6953}$$

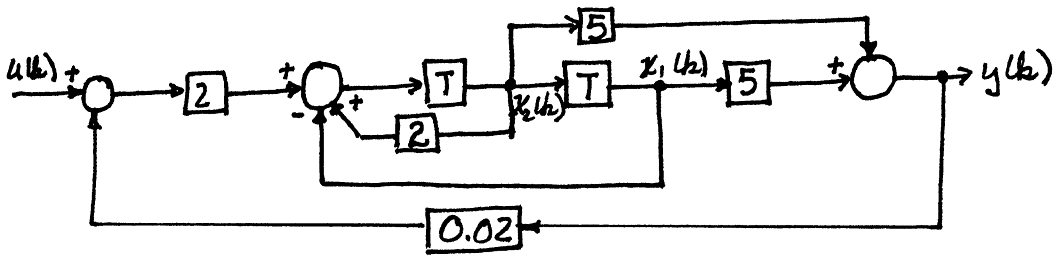
5.22.(a) The states of (a) are related to those of (b) by a similarity transformation: $\underline{x}_{(a)}(t) = P\underline{x}_{(b)}(t)$

5-23. (a) From Problem 5-14, $\dfrac{Y(z)}{U(z)} = \dfrac{10(z+1)}{z^2-1.8z+1.2}$

$$\therefore \underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -1.2 & 1.8 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(k)$$

$$y(k) = [10 \quad 10]\underline{x}(k)$$

(b) From Problem 5-14: $G(z) = \dfrac{5(z+1)}{z^2-2z+1}$



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -1.2 & 1.8 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix}u(k)$$

$$y(k) = [5 \quad 5]\underline{x}(k)$$

(c) $zI - A = \begin{bmatrix} z & -1 \\ 1.2 & z-1.8 \end{bmatrix}$, $\Delta = |zI-A| = z^2-1.8z+1.2$

$$\frac{Y(z)}{U(z)} = C(zI-A)^{-1}B = [5 \quad 5]\frac{1}{\Delta}\begin{bmatrix} z-1.8 & 1 \\ -1.2 & z \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

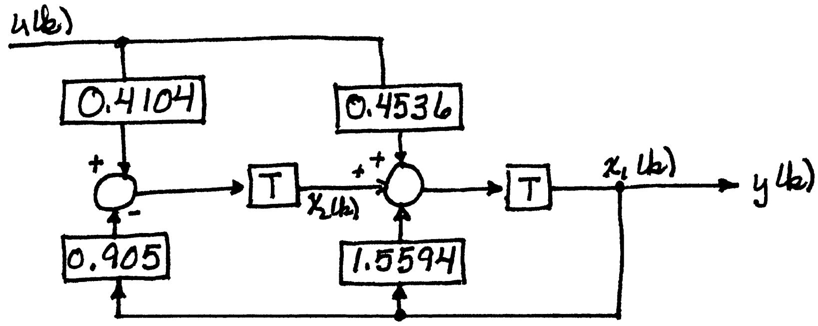
$$= [5 \quad 5]\frac{1}{\Delta}\begin{bmatrix} 2 \\ 2z \end{bmatrix} = \frac{10z+10}{z^2-1.8z+1.2}$$

(d)

```
A = [0 1;-1.2 1.8]; B = [0;2]; C = [5 5]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

5-24. (a) From Problem 5-15: $\dfrac{Y(z)}{U(z)} = \dfrac{0.4536z+0.4104}{z^2-1.5594z+0.9050}$

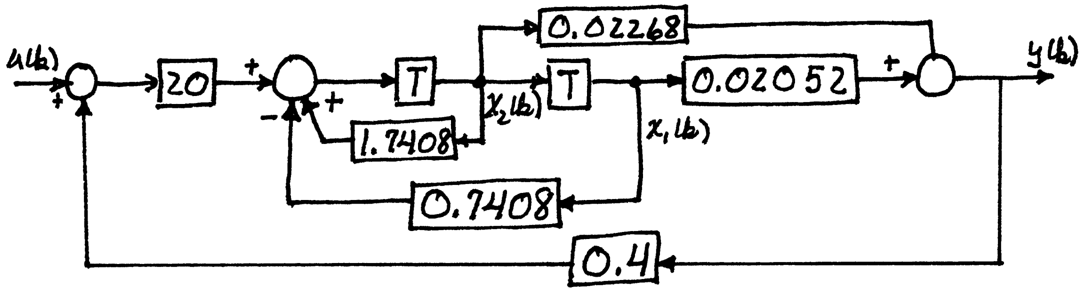
5-24.(a)



$$\underline{x}(k+1) = \begin{bmatrix} 1.5594 & 1 \\ -0.905 & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0.4536 \\ 0.4140 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0]\, \underline{x}(k)$$

(b) From Problem 5-15: $G(z) = \dfrac{0.02268z + 0.02052}{z^2 - 1.7408z + 0.7408}$



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.905 & 1.5594 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 20 \end{bmatrix} u(k)$$

$$y(k) = [0.02052 \quad 0.02268]\, \underline{x}(k)$$

(c) $$zI - A = \begin{bmatrix} z & -1 \\ 0.905 & z-1.5594 \end{bmatrix}, \quad \Delta = |zI - A| = z^2 - 1.5594z + 0.905$$

$$\frac{Y(z)}{U(z)} = C(zI - A)^{-1}B = [0.02052 \quad 0.02268] \frac{1}{\Delta} \begin{bmatrix} z-1.5594 & 1 \\ -0.905 & z \end{bmatrix} \begin{bmatrix} 0 \\ 20 \end{bmatrix}$$

$$= \frac{0.4536z + 0.4104}{z^2 - 1.5594z + 0.905}$$

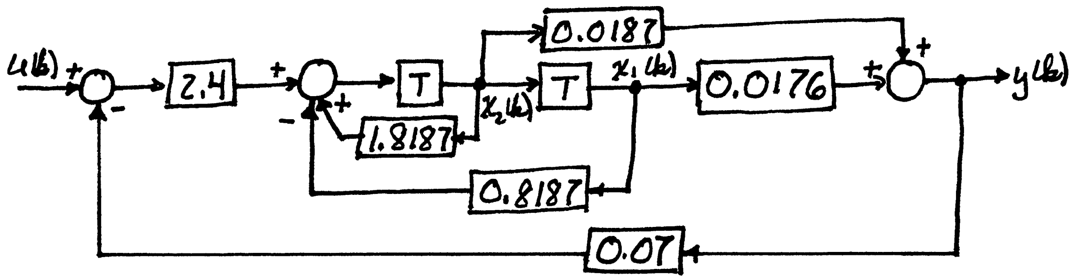
(d)
```
A = [0 1;-.905 1.5594]; B = [0;20];
C = [.02052 .02268]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

5-25.(a) From Problem 5-13: $\dfrac{Y(z)}{U(z)} = \dfrac{0.04488z+0.04206}{z^2-1.8156z+0.8216}$

$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.8216 & 1.8156 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = [0.04488 \quad 0.04206]\,\underline{x}(k)$$

(b) From Problem 5-13, $G(z) = \dfrac{0.0187z+0.0176}{z^2-1.8187z+0.8187}$



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.8217 & 1.8156 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0 \\ 2.4 \end{bmatrix} u(k)$$

$$y(k) = [0.0176 \quad 0.0187]\,\underline{x}(k)$$

(c) $z I - A = \begin{bmatrix} z & -1 \\ 0.8217 & z-1.8156 \end{bmatrix}$, $\Delta = |zI - A| = z^2 - 1.8156z + 0.8217$

$$\frac{Y(z)}{U(z)} = C(zI-A)^{-1}B = \frac{1}{\Delta}C\begin{bmatrix} z-1.8156 & 1 \\ 0.8217 & z \end{bmatrix}\begin{bmatrix} 0 \\ 2.4 \end{bmatrix}$$

$$= \frac{1}{\Delta}[0.0176 \quad 0.0187]\begin{bmatrix} 2.4 \\ 2.4z \end{bmatrix} = \frac{0.04488z + 0.04224}{z^2 - 1.8156z + 0.8217}$$

5-26.(a) $\underline{v}(k+1) = A_1\,\underline{v}(k) + B_1 m(k) + B_2 e(k)$

$$m(k) = C_1\,\underline{v}(k) + D_1 e(k)$$

$$y(k) = C\,\underline{v}(k) + D_2 e(k)$$

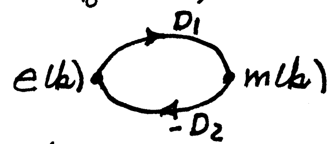
Solving,

$$\underline{v}(k+1) = \left[A_1 + \frac{D_2}{1+D_1D_2}(B_1 - D_2B_2)(C_1 - D_1C) - B_2C\right]\underline{v}(k)$$

$$+ \left[B_2 + \frac{D_1}{1+D_1D_2}B_1 - \frac{D_1D_2}{1+D_1D_2}B_2\right]u(k)$$

$$y(k) = \left[C + \frac{D_2}{1+D_1D_2}(C_1 - D_1C)\right]\underline{v}(k) + \frac{D_1D_2}{1+D_1D_2}u(k)]$$

5-26. (b) $m(k)$ is a direct function of $e(k)$, and $e(k)$ is a direct function of $m(k)$.



(c) The state equations do not exist.

(d) Delays cannot be ignored if this results in an algebraic loop with gain equal to -1.

CHAPTER 6

6-1. (a) $G(z) = \frac{z-1}{z} \mathfrak{z}\left[\frac{0.5}{s(s+0.5)}\right] = \frac{z-1}{z} \frac{(1-\epsilon^{-1})}{(z-1)(z-\epsilon^{-1})} = \frac{0.6321}{z-0.3679}$

$$\frac{G(z)}{1+G(z)} = \frac{0.6321}{z-0.2642}$$

$$\frac{C(z)}{z} = \frac{0.6321}{(z-1)(z+0.2642)} = \frac{0.5}{z-1} + \frac{-0.5}{z+0.2642}$$

$$\therefore \; c(kT) = \underline{0.5[1-(-0.2642)^k]}$$

(b) $\frac{G_p(s)}{1+G_p(s)} = \frac{0.5}{s+1}$

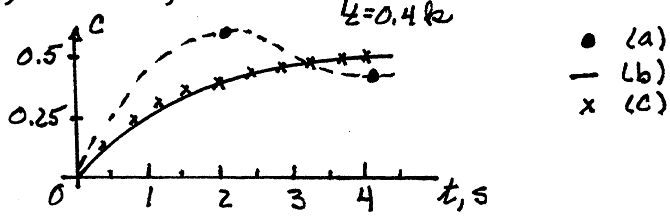
$$C(s) = \frac{0.5}{s(s+1)} = \frac{0.5}{s} - \frac{0.5}{s+1} \Rightarrow \underline{c(t) = 0.5(1-\epsilon^{-t})}$$

(c) $G(z) = \frac{0.1813}{z-0.8187}$, $\frac{G(z)}{1+G(z)} = \frac{0.1813}{z-0.6374}$

$$\therefore \; \frac{C(z)}{z} = \frac{0.1813}{(z-1)(z-0.6374)} = \frac{0.5}{z-1} + \frac{0.5}{z-0.6374}$$

$$\therefore \; c(kT) = \underline{0.5[1-(0.6374)^k]}$$

In (b), $t = 0.4k$, and $c(t)\big|_{t=0.4k} = 0.5[1-\epsilon^{-0.4k}] = 0.5[1-(0.6703)^k]$



(d) (a) $T(1) = 0.5$ (c) $T(1) = 0.5$

(b) $T(0) = 0.5$

dc gain is a function of only $G_p(0)$ and is independent of T.

6-2. (a) $\tau = \frac{-T}{\ln r} = \frac{-2}{\ln(0.2642)} = \underline{1.503 \text{ s}}$

(b) $\epsilon^{-t/\tau} = 0.02 \quad \therefore \; t = -(1.503)\ln(0.02) = \underline{5.88\text{s}} \cong 4\tau$

(c) (a) $\tau = \frac{-0.4}{\ln(0.6374)} = \underline{0.888\text{s}}$; (b) $t = -\tau \ln(0.02) = \underline{3.47\text{s}} \approx 4\tau$

(d) (a) $\tau = \underline{1\text{s}}$; (b) $t = -\tau\ln(0.02) = \underline{3.91\text{s}} \approx 4\tau$

6-3. (a) From Problem 6-1(a): $G(z) = \dfrac{0.6321}{z-0.3678}$

From Example 6.1: $C(s) = G(s)\dfrac{R(z)}{1+G(z)}\Big|_{z=\epsilon^{Ts}}$

$$\frac{1}{1+G} = \frac{z-0.3678}{z+0.2642} = 1 - 0.632z^{-1} + \cdots$$

$$C_1(s) = \frac{0.5}{s(s+0.5)} = \frac{1}{s} - \frac{1}{s+0.5}$$

$$c_1(t) = (1-\epsilon^{-0.5t})$$

$$\therefore\ c(t) = (1-\epsilon^{-0.5t})u(t) - 0.632(1-\epsilon^{-0.5(t-2)})u(t-2) + \cdots$$

$$\therefore\ c(t)\big|_{t=1} = 1-\epsilon^{-0.5} = \underline{0.3935}$$

(b) $c(t)\big|_{t=3} = 1-\epsilon^{-1.5} - 0.632(1-\epsilon^{-0.5}) = \underline{0.5281}$

(c) The output c(t) cannot change instantaneously for step inputs.

(d) (a) $c_1(2) = 1-\epsilon^{-1}$; (b) $c(2) = 1-\epsilon^{-1} - (0) = 1-\epsilon^{-1}$

6-4. (a) From Problem 4-21: $G(z) = \dfrac{1.037}{z-0.7408}$

$$T(z) = \frac{G(z)}{1+G(z)H} = \frac{1.037}{z-0.7408+1.037(0.04)} = \frac{1.037}{z-0.700}$$

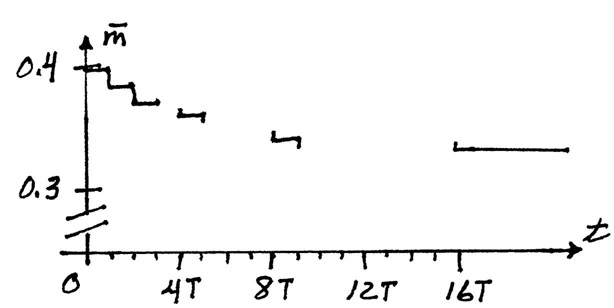
$$C(z) = R(z)T(z) = \frac{0.4z}{z-1}\,\frac{1.037}{z-0.7} = z\left[\frac{1.383}{z-1} + \frac{-1.383}{z-0.7}\right]$$

$$\therefore\ \underline{c(kT) = 1.383[1-(0.7)^k]}$$

$\underline{r(t) = 0.4\,V}$, since the sensor output = 0.4 with c(t) = 10°C.

(b) $m(kT) = 0.4[1-(0.1)c(kT)]$

k	c(kT)	m(kT)
0	0	0.4
1	0.416	0.384
2	0.704	0.372
4	1.05	0.360
8	1.30	0.348
16	1.38	0.345
∞	1.383	0.345



(c) $c_{ss}(kT) = \lim_{z\to 1}(z-1)C(z) = \dfrac{0.415}{z-0.7}\Big|_{z=1} = \underline{1.383}$

6-4. (d) $T(z) = \dfrac{G(z)}{1+G(z)H}$; $G(z) = \dfrac{1.037(\frac{K}{2})}{z-0.7408}$

$$T(z) = \frac{0.519K}{z-0.7408+0.519K(0.04)} = \frac{0.519K}{z-0.7408+0.0207K}$$

$$\text{dc gain} = T(1) = \frac{0.519K}{0.259+0.0207K}$$

$$\lim_{K\to\infty} T(1) = \frac{0.519K}{0.0207K} = \underline{25}$$

$\therefore C_{ss}(kT) = (0.4)(25) = 10$, $\therefore$ no steady-state error.

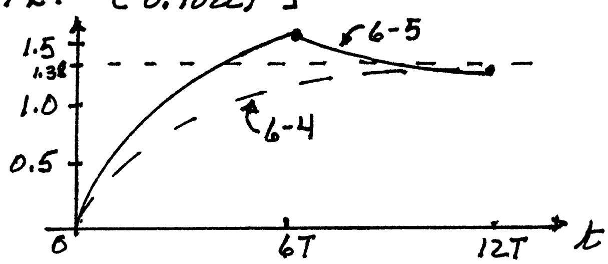
6-5. (a) $G(z) = \left(\dfrac{z-1}{z}\right)(2)\mathcal{Z}\left[\dfrac{1}{s(s+0.5)}\right] = \dfrac{z-1}{z}(2)\dfrac{2z(1-\epsilon^{-3})}{(z-1)(z-\epsilon^{-3})} = \dfrac{3.801}{z-0.04979}$

$$\therefore T(z) = \frac{G(z)}{1+G(z)H} = \frac{3.801}{z-0.04979+3.801(0.04)} = \frac{3.801}{z+0.1022}$$

$$C(z) = \frac{0.4z}{z-1}\,\frac{3.801}{z+0.1022} = \frac{1.379z}{z-1} + \frac{-1.379z}{z+0.1022}$$

$$\therefore c(kT) = 1.379[1-(-0.1022)^k]$$

k	$c(kT)$
0	0
1	1.52
2	1.365
⋮	⋮
∞	1.379



(b) At $t=0$, $m(t)=1$, and doesn't change until $t=6$.

$$\therefore C(s) = \frac{0.4}{s} \times \frac{2}{s+0.5} = \frac{0.8}{s(s+0.5)} = \frac{1.6}{s} - \frac{1.6}{s+0.5}$$

$$\therefore c(t) = 1.6(1-\epsilon^{-0.5t}),\quad 0 \le t \le 6$$

6-6. (a) From Problem 6-4, $T(z) = \dfrac{1.037}{z-0.700}$

$$\therefore \tau = \frac{-T}{\ln r} = \frac{-0.6}{\ln(0.7)} = \underline{1.68\,s}$$

(b) $\epsilon^{-t/\tau} = 0.02 \Rightarrow t = -\tau\ln(0.02) = -(1.68)\ln(0.02) = \underline{6.57\,s} \approx 4\tau$

(c) From Problem 6-5, (a) $T(z) = \dfrac{1.90}{z+0.1022}$

$$\therefore \tau = \frac{-T}{\ln r} = \frac{-6}{\ln(0.1022)} = \underline{2.63\,s}$$

6-6. (c) (b) $t = -\tau \ln(0.02) = \underline{10.3\,s} \approx 4\tau$

(d) $T(s) = \dfrac{G_p(s)}{1+G_p(s)H} = \dfrac{2}{s+0.5+2(0.04)} = \dfrac{2}{s+0.58}$

$\therefore \tau = \dfrac{1}{0.58} = \underline{1.72\,s} \Rightarrow \epsilon^{-t/\tau} = 0.02, \therefore t = -(1.72)\ln(0.02) = \underline{6.73\,s} \approx 4\tau$

6-7. (a) $r(t) = 1.4V$, since the sensor output $= 1.4$ with $c(t) = 20^0$

$$G(z) = \left(\frac{z-1}{z}\right)\mathcal{Z}\left[\frac{4}{s^2(s+2)}\right] = \frac{4}{2}\,\frac{(z-1)}{z}\left[\frac{z(0.2-1+\epsilon^{-0.2}) + (1-\epsilon^{-0.2}-0.2\epsilon^{-0.2})}{2(z-1)^2(z-\epsilon^{-0.2})}\right]$$

$$= \frac{0.01873z+0.01752}{(z-1)(z-0.8187)}$$

$$T(z) = \frac{KG(z)}{1+KG(z)H} = \frac{10(0.01873z+0.01752)}{z^2-1.8187z+0.8187+0.01311z+0.01226}$$

$$= \frac{0.1873z+0.1752}{z^2-1.8056z+0.8310}$$

$$C(z) = \left(\frac{1.4z}{z-1}\right)T(z) = \frac{1.4z(0.1873z+0.1752)}{(z-1)(z^2-1.8056z+0.8310)}$$

(b) $c_{ss}(kT) = \lim_{z\to 1}(z-1)C(z) = \dfrac{1.4(0.1873+0.1752)}{1-1.8056+0.8310} = \underline{20}$

(c) poles: $z = 0.9028 \pm j0.1262 = 0.9116\underline{/\pm 7.96^0}$

$\therefore \tau = -\dfrac{T}{\ln r} = \dfrac{-0.1}{\ln(0.9116)} = \underline{1.08s}$; $t \approx 4\tau = \underline{4.32s}$

6-8 (a) The open-loop system has the characteristic of $G_p(s)$.

$\therefore$ poles at: $s_1 = 0$, $\underline{\tau_1 = \infty}$

$s = -2$, $\underline{\tau_2 = 0.5s}$

(b) From Problem 6-7, char. eq.: $z^2 - 1.8056z + 0.8310 = 0$

$\underline{\text{zeros}}$: $z = 0.9116\underline{/\pm 7.96^0} = 0.9116\underline{/\pm 0.139\,\text{rad}}$

$\tau = \dfrac{-T}{\ln r} = \dfrac{-0.1}{\ln(0.9116)} = \underline{1.08\,s}$

$\omega_n = \dfrac{1}{T}\sqrt{\ln^2 r + \theta^2} = \dfrac{1}{0.1}\left[\ln^2(0.9116) + (0.139)^2\right]^{1/2} = \underline{1.670}$

$\zeta = \dfrac{-\ln r}{\sqrt{\ln^2 r + \theta^2}} = \underline{0.554}$

6-8. (c) (a) same as (a)

(b) $T(s) = \dfrac{KG_p(s)}{1+KG_p(s)H} = \dfrac{(10)\frac{4}{s^2+2s}}{1+0.07\frac{40}{s^2+2s}}$

$$= \frac{40}{s^2+2s+2.8} = \frac{40}{s^2+2\zeta\omega_n s+\omega_n^2}$$

poles: $s = \dfrac{-2\pm\sqrt{4-11.2}}{2} = -1 \pm j1.3416 = 1.673\,\underline{/\pm 126.7^\circ}$

$\therefore \tau = \underline{1s}$ $\qquad \zeta = \dfrac{2}{2(1.67)} = \underline{0.5988}$

$\omega_n = (2.8)^{1/2} = \underline{1.67}$

(d) (b) $\zeta = 0.554$, $-\zeta\pi/\sqrt{1-\zeta^2} = -(0.554)\pi/0.8325 = -2.091$

$\therefore$ % overshoot $= (\epsilon^{-2.091})100 = \underline{12.4\%}$

(c) $-\zeta\pi/\sqrt{1-\zeta^2} = -2.349$, % overshoot $= (\epsilon^{-2.349})100 = \underline{9.5\%}$

6-9. (a) $G(z) = \dfrac{z-1}{z}\mathfrak{z}\left[\dfrac{10}{s^3}\right] = 10\left(\dfrac{z-1}{z}\right)\dfrac{(0.1)^2 z(z+1)}{2(z-1)^3} = \dfrac{0.05(z+1)}{(z-1)^2}$

$(s+a)$ has the time constant $\tau = 1/a$.

$\therefore$ $\underline{\tau = \infty}$ at both poles.

The characteristics of the plant are not changed by sampling.

(b) $T(z) = \dfrac{KG}{1+KG(z)H_k} = \dfrac{5(z+1)}{z^2-2z+1+0.1(z+1)} = \dfrac{5(z+1)}{z^2-1.9z+1.1}$

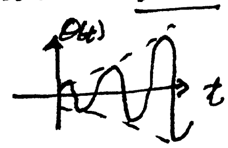
poles: $z = 0.95 \pm j0.4444 = 1.049\,\underline{/\pm 25.07}$

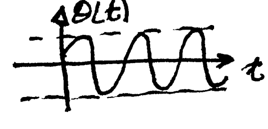
$\therefore$ unstable

(c) (a) $G_p(s) = \dfrac{10}{s^2}$ $\quad \therefore \tau = \infty$ at both poles

(b) $T(s) = \dfrac{KG_p(s)}{1+KG_p(s)H_k} = \dfrac{1000}{s^2+20}$, poles: $s = \pm j4.472$

$\therefore$ marginally stable $-$ $\underline{\tau = \infty}$ for both poles.

(d) sampled-data: 

analog: 

6-10. (a) From Example 6.2: $G(z) = \dfrac{(T-1+\epsilon^{-T})z + (1-\epsilon^{-T} - T\epsilon^{-T})}{(z-1)(z-\epsilon^{-T})}$

$T = 0.5$, $G(z) = \dfrac{0.1065z + 0.09020}{(z-1)(z-0.6065)}$

$\therefore$ Char. eq.: $z^2 - 1.6065z + 0.6065 + 0.1065z + 0.09020$
$= z^2 - 1.500z + 0.6967 = 0$

zeros: $z = 0.75 \pm j0.3663 = 0.8347\angle{\pm 26.03°} = 0.8347\angle{\pm 0.454 \text{ rad}}$

$\tau = -\dfrac{T}{\ln r} = \underline{2.767 s}$

$\omega_n = \frac{1}{T}[\ln^2 r + \theta^2] = \underline{0.9773}$

$\zeta = \dfrac{-\ln r}{[\ln^2 r + \theta^2]^{1/2}} = \underline{0.3698}$

(b) $T = 0.1$, $G(z) = \dfrac{0.004837z + 0.004679}{(z-1)(z-0.9048)}$

$\therefore$ Char. eq.: $z^2 - 1.9048z + 0.9048 + 0.004837z + 0.004679$
$= z^2 - 1.900z + 0.9095$

zeros: $z = 0.950 \pm j0.0837 = 0.9537\angle{\pm 5.033°}$
$= 0.9537\angle{\pm 0.0878 \text{ rad}}$

$\tau = \dfrac{-T}{\ln r} = \underline{2.109 s}$

$\omega_n = \frac{1}{T}[\ln^2 r + \theta^2] = \underline{0.9978}$

$\zeta = \dfrac{-\ln r}{[\ln^2 r + \theta^2]^{1/2}} = \underline{0.4751}$

(c) $T(s) = \dfrac{G_p(s)}{1+G_p(s)} = \dfrac{1}{s^2+s+1} = \dfrac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$= \dfrac{1}{(s+0.5)^2 + 0.75}$

$\therefore \tau = \frac{1}{0.5} = \underline{2 s}$; $\omega_n^2 = 1 \Rightarrow \omega_n = \underline{1}$

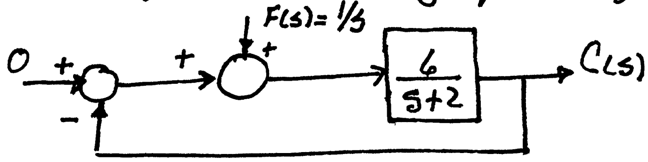
$2\zeta\omega_n = 1 \Rightarrow \zeta = \dfrac{1}{(2)(1)} = 0.5$

(d)

T	τ	ω_n	ζ	$f_s = 1/T$
1	4.365	0.9191	0.25	1
0.5	2.77	0.977	0.369	2
0.1	2.11	0.998	0.475	10
analog	2	1	0.5	$\to\infty$

6-10.(d) As the sampling frequency decreases: τ increases
ω_n increases
ζ decreases

6-11.(a) Can ignore sampling if all signals are constant.



$$C(s) = \frac{G_p(s)}{1+G_p(s)} F(s) = \frac{6}{s+8}\cdot\frac{1}{s} = \frac{6/8}{s} + \frac{-6/8}{s+8}$$

$$\therefore C_{ss}(t) = \frac{6}{8} = \underline{0.75}$$

(b) $$G(z) = \frac{z-1}{z}\mathcal{Z}\left[\frac{6}{s(s+2)}\right] = \frac{z-1}{z}\left[\frac{6z(1-\epsilon^{-0.2})}{(2)(z-1)(z-\epsilon^{-0.2})}\right] = \frac{0.5438}{z-0.8187}$$

$$\mathcal{Z}\left[\frac{6}{s(s+2)}\right] = \frac{0.5438z}{(z-1)(z-0.8187)} = \overline{FG_p}(z)$$

$$C(z) = \frac{\overline{FG_p}(z)}{1+D(z)G(z)} = \frac{\dfrac{0.5438z}{(z-1)(z-0.8187)}}{1+\left(\dfrac{1.1z-1}{z-1}\right)\cdot\left(\dfrac{0.5438}{z-0.8187}\right)}$$

$$= \frac{0.5438z}{z^2-1.221z+0.2749}$$

poles: $z = 0.924, 0.298$ $\therefore$ no poles at $z=1$.

$$\therefore C_{ss}(kT) = \lim_{z\to 1}(z-1)C(z) = \underline{0}$$

6-12.(a) (i) $\tau = \dfrac{-T}{\ln r} = \dfrac{-0.1}{\ln(0.999)} = \underline{99.95s}$, exponential

(ii) $\tau = \dfrac{-0.1}{\ln(0.99)} = \underline{9.95s}$, exponential

(iii) $\tau = \dfrac{-0.1}{\ln(0.9)} = \underline{0.95s}$, exponential

(iv) $\tau = \dfrac{-0.1}{\ln(0.9)} = \underline{0.95s}$; $\omega = \dfrac{\omega_s}{2} = \dfrac{\pi}{0.1} = 31.42$ rad/s
damped sinusoid

6-12. (a) (v) zeros: $z = 0.965, 0.885$

exponential, $\tau_1 = \frac{0.1}{\ln(0.965)} = \underline{2.81s}$; $\tau_2 = \frac{0.1}{\ln(0.885)} = \underline{0.819\,s}$

(vi) zeros: $z = 1, -1$ $z = 1$ - constant output

$z = -1$ - sinusoidal output, $\omega = \omega_s/2 = 31.42$ rad/s

(vii) zeros: $z = 1.01, 0.99$ $\therefore$ <u>unstable</u>

(viii) zeros: $z = 0.837\underline{/\pm 44.2^\circ}$, $0.837\underline{/\pm 0.771\text{ rad}}$

$\therefore$ damped sinusoid

$$\tau = \frac{-T}{\ln r} = \underline{0.56s} \qquad \zeta = \frac{-\ln r}{[\ln^2 r + \theta^2]^{1/2}} = \underline{0.156}$$

$$\omega_n = \frac{1}{T}[\ln^2 r + \theta^2]^{1/2} = \underline{11.38}\text{ rad/s}$$

(b) (i) $\underline{\tau = 999.5s}$ (ii) $\underline{\tau = 99.5s}$

(iii) $\underline{\tau = 9.5s}$ (iv) $\underline{\tau = 9.5s}$, $\underline{\omega = 3.142}$ rad/s

(v) $\underline{\tau_1 = 28.1s}$, $\underline{\tau_2 = 8.19s}$ (vi) $\underline{\omega = 3.142\text{ rad/s}}$

(vii) unstable (viii) $\underline{\tau = 5.6s}$, $\underline{\omega_n = 1.138}$ rad/s, $\underline{\zeta = 0.156}$

(c) τ, ω_n vary with T, ζ independent of T.

6-13. (a) From Problem 6-1, $T(z) = \frac{0.6321}{z + 0.2642}$

$$\therefore \tau = \frac{-T}{\ln r} = \frac{-1}{\ln(0.2642)} = \underline{0.751s}$$

(b)
$$G(z) = \frac{z-1}{z}\mathcal{z}\left[\frac{0.5\epsilon^{-0.2Ts}}{s(s+0.5)}\right] = \frac{z-1}{z}\mathcal{z}_m\left[\frac{0.5}{s(s+0.5)}\right]_{m=0.8}$$

$$= \frac{z-1}{z}\left[\frac{1}{z-1} - \frac{\epsilon^{-0.8}}{z-\epsilon^{-1}}\right] = \frac{0.5507z + 0.0814}{z(z-0.3679)}$$

$\therefore$ char. eq.: $1 + G(z) = 0 = z^2 - 0.3679z + 0.5507z + 0.0814$

$= z^2 + 0.1828z + 0.0814 = 0$

zeros: $z = 0.2853\underline{/\pm 108.7^\circ}$

$$\therefore \tau = \frac{-T}{\ln r} = \frac{-1}{\ln(0.2853)} = \underline{0.7973}$$

(c) From Problem 6-1, $G(z) = \frac{0.6321}{z(z-0.3679)}$

char. eq.: $z^2 - 0.3679z + 0.6321 = 0$

6-13.(c) zeros: $z = 0.7950 \angle \pm 26.6^\circ$

$$\tau = \frac{-T}{\ln \Omega} = \underline{4.36\,s}$$

(d) Delay slows the system response.

6-14.(a) (i) type 0 (ii) type 0 (iii) type 1

(iv) type 2 (v) type 0

(b) (i), (ii), (v) one pole at $z=1$, with the system stable.

(iii), (iv) system stable only

(c) (i), (ii), (v) 2 poles at $z=1$, with the system stable

(iii) 1 pole at $z=1$, with the system stable.

(iv) system stable only.

6-15. (a) $(1-z^{-1})M(z) = (1-0.9z^{-1})E(z)$

$$D(z) = \frac{M(z)}{E(z)} = \frac{z-0.9}{z-1} \quad \therefore \text{ system is type } \underline{1}$$

(b) Type 1 system, ∴ no steady-state error, ∴ $c_{ss}(kT) = \underline{1}$

(c) $$G(z) = \frac{z-1}{z}\mathcal{Z}\left[\frac{1}{s+1}\right] = \frac{z-1}{z}\,\frac{z(1-\epsilon^{-1})}{(z-1)(z-\epsilon^{-1})} = \frac{0.6321}{z-0.3679}$$

$$\frac{D(z)G(z)}{1+D(z)G(z)} = \frac{0.6321(z-0.9)}{z^2-1.368z+0.6379+0.6321z-0.5689}$$

$$= \frac{0.6321(z-0.9)}{z^2-0.7358z-0.201}$$

∴ poles: $z = 0.9479, -0.2121$

$$\tau_1 = \frac{-T}{\ln \Omega} = \frac{-1}{\ln(0.9479)} = \underline{18.7\,s}, \quad \tau_2 = \underline{0.645\,s}$$

$$\therefore 4\tau_1 = \underline{74.8}\text{ s}$$

(d) $$\frac{C(z)}{z} = \frac{0.6321(z-0.9)}{(z-1)(z-0.9479)(z+0.2121)} = \frac{1}{z-1} + \frac{-0.500}{z-0.9479} + \frac{-0.500}{z+0.2121}$$

$$\therefore c(kT) = \underline{1-(0.9479)^k - 0.5(-0.2121)^k}$$

(b) $c_{ss}(kT) = \underline{1}$ (c) $c(75) = 1 - .5(0.9479)^{75} - 5(-0.2121)^{75}$

$= 1 - 0.0090 - (\;)\times 10^{-51} \approx \underline{1}$

6-16. (a) From Problem 6-15, type $\underline{1}$

(b) $K_v = \lim_{z \to 1} \frac{1}{T}(z-1)D(z)G(z) = \frac{0.6321(1-0.9)}{(1-0.3679)} = \underline{0.1}$

$\therefore e_{ss} = \frac{1}{K_v} = \underline{10}$

$\therefore e_{ss}(kT) = r_{ss}(kT) - c_{ss}(kT)$

$\therefore c_{ss}(kT) = r_{ss}(kT) - e_{ss}(kT) = \underline{kT - 10}$

(c) From Problem 6-15, time to reach steady-state $\approx \underline{74.8\text{s}}$

(d) $\mathcal{z}[t] = \frac{Tz}{(z-1)^2} = \frac{z}{(z-1)^2}$

From Problem 6-15:

$$\frac{C(z)}{z} = \frac{0.6321(z-0.9)}{(z-1)^2(z-0.9499)(z+0.2121)}$$

$$= \frac{1}{(z-1)^2} + \frac{-10.1}{z-1} + \frac{9.60}{z-0.9479} + \frac{0.413}{z+0.2121}$$

$\therefore c_{ss}(kT) = kT - 10.1 \approx kT - 10$ (numerical errors)

$e_{ss}(kT) = kT - c_{ss}(kT) \approx \underline{10}$

6-17. (a) $G(z) = \frac{z-1}{z}\mathcal{z}\left[\frac{1}{s+1}\right] = \frac{z-1}{z}\frac{z}{z-\epsilon^{-1}} = \frac{z-1}{z-0.3679}$

$\therefore D(z)G(z) = \frac{z-0.9}{z-0.3679}$ $\therefore$ system type $= 0$

(b) $D(1)G(1) = \frac{1-0.9}{1-0.3679} = 0.158$

$T(1) = \frac{0.158}{1+0.158} = \underline{0.1364} = c_{ss}(kT)$

(c) $1 + G(z) = z - 0.3679 + z - 0.9 = 2(z-0.634) = 0$

$\therefore \tau = \frac{-T}{\ln r} = \frac{-1}{\ln(0.634)} = \underline{2.19\text{s}}$, $\therefore 4\tau = \underline{8.72\text{s}}$

(d) $\frac{D(z)G(z)}{1+D(z)G(z)} = \frac{z-0.9}{z-0.3679+z-0.9} = \frac{z-0.9}{2z-1.2679} = \frac{0.5z-0.45}{z-0.634}$

$\therefore \frac{C(z)}{z} = \frac{0.5z-0.45}{(z-1)(z-0.634)} = \frac{0.1366}{z-1} + \frac{0.3634}{z-0.634}$

$\therefore c(kT) = 0.1366 + 0.3634(0.634)^k$

$\therefore c_{ss}(kT) = \underline{0.1364}$

6-18. (a) $\dfrac{G_p(s)}{1+G_p(s)} = \dfrac{1}{s+1+1} = \dfrac{1}{s+2} = \dfrac{C(s)}{R(s)}$

$(s+2)C(s) = R(s)$

$\dot{c}(t) + 2c(t) = r(t), \quad r(t) = 1$

$\dot{c}(t) = 1 - 2c(t), \qquad t \geq 0$

(b) $H = 0.25s, \; c(0) = 0$

$c(kH) = c[(k-1)H] + H\{1 - 2 * c[(k-1)H]\}$

$= c[(k-1)H](1-2H) + H = 0.5c[(k-1)H] + 0.25$

kH	c(kH)	c(t)[exact, (d)]
0	0	0
0.25	0.25	0.197
0.50	0.375	0.316
0.75	0.4375	0.388
1.00	0.4688	0.432
1.25	0.4844	0.459
1.50	0.4922	0.475

(c) $\dfrac{C(s)}{\bar{M}(s)} = \dfrac{1}{s+1} \Rightarrow \dot{c} + c = \bar{m}\,; \; \dot{c}(t) = \bar{m}(t) - c(t)$

$\bar{m} \quad = 1 - c$

$c(kH) = c[(k-1)H] + H\{\bar{m}[(k-1)H] - c[(k-1)H]$

$= [1-H]c[(k-1)H] + H\bar{m}[(k-1)H] = 0.75c[(k-1)H] + 0.25\bar{m}[(k-1)H]$

kH	c(kH)	$\bar{m}$(kH)	c(kH) (exact (d))
0	0	1	0
0.25	0.25	1	
0.50	0.4375	0.5625	0.3935
0.75	0.4688	0.5625	
1.00	0.4922	0.5078	0.477
1.25	0.4961	0.5078	
1.50	0.4990		0.4952

(d) (b) $C(s) = \dfrac{1}{s(s+2)} = \dfrac{1/2}{s} - \dfrac{1/2}{s+2} \Rightarrow c(t) = \frac{1}{2}(1 - \epsilon^{-2t})$

(c) $G(z) = \dfrac{z-1}{z}\mathcal{Z}\left[\dfrac{1}{s(s+1)}\right] = \dfrac{0.3935}{z - 0.6065}$

$\dfrac{G(z)}{1+G(z)} = \dfrac{0.3935}{z - 0.2130}$

6-18. (d) $\therefore C(z) = \dfrac{0.3935z}{(z-1)(z-0.2130)} = \dfrac{0.50z}{z-1} + \dfrac{-0.50z}{z-0.2130}$

$\therefore c(kT) = 0.5[1 - (0.2130)^k]$

6-19. (a) $zX(z) - zx(0) = X(z) - HX(z)$

$\therefore [z - (1-H)]X(z) = zx(0)$

$X(z) = \dfrac{zx(0)}{z-(1-H)} \Rightarrow x(kH) = x(0)(1-H)^k$

(b) $x(1) = (1)(0.9)^{10} = \underline{0.3487}$

(c) $x(1) = (1)(0.99)^{100} = \underline{0.3660}$

(d) (b) % error $= \left(\dfrac{0.3678 - 0.3487}{0.3678}\right)100 = 5.19\%$

(c) % error $= \left(\dfrac{0.3678 - 0.3660}{0.3678}\right)100 = 0.489\%$

6-20. (a) $\dot{x}(t) = -x(t) + r(t)$

$x(kH) = x[(k-1)H] + H\dot{x}[(k-1)H] = x[(k-1)H] + H[-x[(k-1)H] + r[(k-1)H]]$

$\therefore x(kH) = (1-H)x[(k-1)H] + r[(k-1)H]$

(b) $X(z) = (1-H)z^{-1}X(z) + Hz^{-1}R(z)$

$\therefore X(z) = \dfrac{H}{z-(1-H)}R(z) = \dfrac{H}{z-(1-H)} \cdot \dfrac{z}{z-1}$

$\therefore \dfrac{X(z)}{z} = \dfrac{H}{(z-1)[z-(1-H)]} = \dfrac{1}{z-1} + \dfrac{-1}{z-(1-H)}$

$\therefore x(kH) = \underline{1-(1-H)^k}$

$\therefore x(1) = 1-(0.9)^{10} = \underline{0.6513}$

(c) $x(1) = 1-(0.99)^{100} = \underline{0.6340}$

(d) $sX(s) + X(s) = \dfrac{1}{s}$

$X(s) = \dfrac{1}{s(s+1)} = \dfrac{1}{s} + \dfrac{-1}{s+1} \Rightarrow x(t) = 1-e^{-t}$

$\therefore x(1) = \underline{0.6321}$

(e) (b) % error $= \left|\dfrac{0.6321-0.6513}{0.6321}\right|(100) = \underline{3.037}\%$

(c) % error $= \left|\dfrac{0.6321-0.6340}{0.6321}\right|(100) = \underline{0.301}\%$

6-21. $\dot{x}(t) = -x(t) \Rightarrow A=-1,\ B=0$

(a) (6-32) $\dot{x}(k-1) = -x(k-1)$

(6-33) $x(k) = x(k-1) - Hx(k-1) = (1-H)x(k-1)$

(6-34) $\dot{x}(k) = -x(k) = -(1-H)x(k-1)$

(6-35) $x(k) = x(k-1) + \frac{H}{2}[-x(k-1) - (1-H)x(k-1)]$

$= [1 - \frac{H}{2} - \frac{H}{2} - \frac{H^2}{2}]x(k-1) = [1 - H + \frac{H^2}{2}]x(k-1)$

$\therefore\ \underline{x(k+1) = (1 - H + \frac{H^2}{2})x(k) = a\,x(k)}$

(b) $zX(z) - zx(0) = aX(z) \Rightarrow X(z) = \underline{\frac{z\,x(0)}{z-a}}$

$\therefore\ x(kH) = a^k x(0) = (1 - H + \frac{H^2}{2})^k x(0)$

$\therefore\ x(1) = (0.905)^{10}(1) = 0.3685$

(c) $1 - 0.33333 + \frac{(0.33333)^2}{2} = 0.7222$

$x(1) = (0.7222)^3(1) = \underline{0.3767}$

(d) $sX(s) - x(0) + X(s) = 0$

$X(s) = \frac{x(0)}{s+1} \Rightarrow x(t) = x(0)\epsilon^{-t}$

$\therefore\ x(1) = \underline{0.3679}$

(e) (b) % error $= \left|\frac{0.3679 - 0.3685}{0.3679}\right|(100) = \underline{0.163\,\%}$

(c) % error $= \left|\frac{0.3679 - 0.3767}{0.3679}\right|(100) = \underline{2.39\,\%}$

6-22. $\dot{x}(t) = -x(t) + r(t),\ \therefore A=-1,\ B=1$

(a) (6-32) $\dot{x}(k-1) = -x(k-1) + r(k-1)$

(6-33) $x(k) = x(k-1) + H[-x(k-1) + r(k-1)]$

$= (1-H)x(k-1) + Hr(k-1)$

(6-34) $\dot{x}(k) = (-1)x(k) + r(k)$

$= -(1-H)x(k-1) - Hr(k-1) + r(k)$

(6-35) $x(k) = x(k-1) + \frac{H}{2}[-x(k-1) + r(k-1) - (1-H)x(k-1) - Hr(k-1) + r(k)]$

$\therefore\ \underline{x(k) = [1 - \frac{H}{2} - \frac{H}{2} + \frac{H^2}{2}]x(k-1) + (\frac{H}{2} - \frac{H^2}{2})r(k-1) + \frac{H}{2}r(k)}$

$= a\,x(k-1) + c\,r(k-1) + b\,r(k)$

6-22. (b) $(1-az^{-1})X(z) = (b+cz^{-1})R(z)$

$\therefore X(z) = \dfrac{bz+c}{z-a} R(z)$

$a = 1-0.1+\dfrac{(0.1)^2}{2} = \underline{0.905}$; $b = \dfrac{0.1}{2} = \underline{0.05}$, $c = \dfrac{0.1}{2} - \dfrac{(0.1)^2}{2} = \underline{0.045}$

$\therefore \dfrac{X(z)}{z} = \dfrac{0.05z+0.045}{z-0.905} \cdot \dfrac{1}{z-1} = \dfrac{1}{z-1} + \dfrac{-0.95}{z-0.905}$

$\therefore x(k) = 1 - 0.95(0.905)^k$

$x(1) = 1 - 0.95(0.905)^{10} = \underline{0.650}$

(c) $a = \underline{0.7222}$, $b = \underline{0.16667}$, $c = \underline{0.1111}$

$\therefore \dfrac{X(z)}{z} = \dfrac{0.16667z+0.1111}{(z-1)(z-0.7222)} = \dfrac{1}{z-1} + \dfrac{-0.8332}{z-0.7222}$

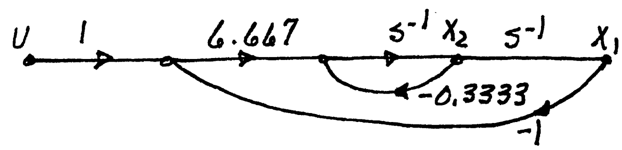
$\therefore x(k) = 1 - 0.8332(0.7222)^k$

$x(1) = 1 - (0.8332)(0.7222)^3 = \underline{0.6862}$

(d) See Problem 6-20(d), $x(1) = \underline{0.6321}$

(e) (b) % error $= \left|\dfrac{0.6321-0.650}{0.6321}\right|(100) = \underline{2.83\%}$

(c) % error $= \left|\dfrac{0.6321-0.6862}{0.6321}\right|(100) = \underline{8.56\%}$

6-23. 

$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ -6.667 & -0.3333 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 6.67 \end{bmatrix} u(t)$

$\underline{x}(k) = \underline{x}(k-1) + H\dot{\underline{x}}(k-1)$

$\dot{\underline{x}}(k) = A\underline{x}(k-1) + Bu(k-1)$

6-23. Computer program - MATLAB

```
A = [0 1; -6.667 -0.3333]; B = [0; 6.667];
u = 1;
H = 0.25;
x = [0; 0];
for k = 0:4;
   t = k*H;
   [t, x(1)]
   xd = A*x + B*u;
   x = x + H*xd;
end
```

results:

Time	output
0	0
0.25	0
0.5	0.4167
0.75	1.2153
1.00	2.1905

H has been chosen too large. The system is stable, but the simulation is unstable.

CHAPTER 7

7-1. Char. eq.: $(z-p_1)^r(z-p_{r+1})\cdots(z-p_n)=0$

In (7-1):

$$C(z)=\frac{k_1 z}{z-p_1}+\cdots+\frac{k_r f_r(z)}{(z-p_1)^r}+\frac{k_{r+1}z}{z-p_{r+1}}+\cdots+\frac{k_n z}{z-p_n}$$

where, from the table in the appendix,

$$\mathfrak{Z}^{-1}\left[\frac{k_i f_i(z)}{(z-p_i)^i}\right]=k_i\,(k)^{i-1}p_i^{\,k}$$

$\therefore\ c(k)=k_1 p_1^k + k_2\,k\,(p_2^k)+\cdots+k_r(p)^{r-1}(p_i^{\,r})+\cdots$

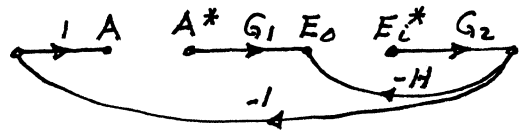
now

$$\lim_{k\to\infty} k^{i-1}p_i^{\,k}=\lim_{k\to\infty}\frac{k^{i-1}}{p_i^{-k}}=\lim_{k\to\infty}\frac{(i-1)k^{i-2}}{-k(p_i)^{k-1}}$$

$$=\ \cdots\ =\lim_{k\to\infty}\frac{(i-1)(i-2)\cdots k^0}{-k(\qquad)}$$

The numerator is constant and the denominator is unbounded for $|p_i|<1$. Thus all terms in $c(k)$ of the form $(k)^{i-1}(p_i)^k\to 0$ for $|p_i|<1$

7-2.



$A=-G_2E_i^* \ \Rightarrow\ A^*=-G_2^*E_i^*$

$E_0=G_1A^*-G_2HE_i^* \ \Rightarrow\ E_0^*=-G_1^*G_2^*E_i^*-\overline{G_2H}^*E_i^*$

Let $E_0^*=E_i^*$

$\therefore\ (1+G_1^*G_2^*+\overline{G_2H}^*)E_i^*=0$

$\therefore\ \underline{1+G_1(z)\,G_2(z)+\overline{G_2H}(z)=0}$

7-3.(a) $T(z)=\dfrac{N(z)}{D(z)}=\dfrac{N(z)}{(z-z_1)(z-z_2)\cdots(z-z_n)}$

7-3. (a) $\dfrac{C(z)}{z} = \dfrac{N(z)}{(z-1)(z-z_1)\cdots(z-z_n)} = \dfrac{k_0}{z-1} + \dfrac{k_1}{z-z_1} + \cdots + \dfrac{k_n}{z-z_n}$

$\therefore\ c(kT) = k_0 + k_1(z_1)^k + \cdots + k_n(z_n)^k\ , \quad |z_i|<1,\ i=1,2,\ldots,n$

$\therefore\ \lim\limits_{k\to\infty} c(kT) = \underline{k_0}$

(b) $T(z) = \dfrac{N(z)}{D(z)}, \quad \therefore\ N(z) = (z-1)N_1(z) \qquad [T(1) = 0]$

(c) $R(z) = 1$

$\dfrac{C(z)}{z} = \dfrac{k_1}{z-z_1} + \dfrac{k_2}{z-z_2} + \cdots + \dfrac{k_n}{z-z_n}$

$\therefore\ c(kT) = k_1(z_1)^k + \cdots + k_n(z_n)^k$

$\therefore\ \lim\limits_{k\to\infty} c(kT) = 0, \quad |z_i|<1$

7-4. $T=0.5$, char. eq.: $(z-0.9)(z-0.8)(z^2-1.9z+1)=0$

complex roots: $z = 0.95 \pm j0.3122 = \underline{1\angle\pm 18.19^\circ} = \underline{1\angle\pm 0.3175}$ rad

(a) $k_1(0.9)^k,\ k_2(0.8)^k,\ A\cos(0.3175k+\theta)$

(b) marginally stable

(c) root $= 1\angle\pm\omega T$, $\therefore\ \omega T = 0.3175 \Rightarrow \omega = \dfrac{0.3175}{0.5} = \underline{0.635}$ rad/s

7-5. (a) (i) $Q(1) = 1.3 - 1.1 > 0$ ✓

$(-1)^2 Q(-1) = 1.3 + 1.1 > 0$ ✓

$|a_0| < a_2 \Rightarrow 0.3 < 1$ ✓

$\therefore$ <u>stable</u>

(ii) $Q(1) = 1.25 - 1 > 0$ ✓

$(-1)^2 Q(-1) = 1 + 1 + 0.25 > 0$ ✓

$|a_0| < a_2 \Rightarrow 0.25 < 1$ ✓

$\therefore$ <u>stable</u>

(iii) $Q(1) = 0.7 - 0.1 > 0$ ✓

$(-1)^2 Q(-1) = 1 + 0.1 - 0.3 > 0$ ✓

$|a_0| < a_2 \Rightarrow 0.3 < 1$ ✓

$\therefore$ <u>stable</u>

(iv) $Q(1) = 1 - 0.25 > 0$ ✓

$(-1)^2 Q(-1) = 1 - 0.25 > 0$ ✓

$|a_0| < a_2 \Rightarrow 0.25 < 1$ ✓

$\therefore$ <u>stable</u>

7-5. (v) $Q(1) = 1 - 1.6 + 1 > 0\checkmark$

$(-1)^2 Q(-1) = 1 + 1.6 + 1 > 0 \checkmark$

$|a_0| < a_2 \Rightarrow 1 < 1^{\times}$

$\therefore$ <u>not stable</u>

(vi) $Q(1) = 1 - 1.9 + 0.89 > 0^{\times}$

$\therefore$ <u>not stable</u>

(vii) $Q(1) = 1 - 2.2 + 1.55 - 0.35 = 0 \therefore$ <u>not stable</u>

(viii) $Q(1) = 1 - 1.9 + 1.4 - 0.45 > 0 \checkmark$

$(-1)^3 Q(-1) = -1(-1 - 1.9 - 1.4 - 0.45) > 0 \checkmark$

$|a_0| < a_3 \Rightarrow 0.45 < 1$

z^0	z^1	z^2	z^3
-0.45	1.4	-1.9	1
1	-1.9	1.4	-0.45
-0.798	1.27	-0.545	

$$b_0 = \begin{vmatrix} -0.45 & 1 \\ 1 & -0.45 \end{vmatrix} = -0.798$$

$$b_1 = \begin{vmatrix} -0.45 & -1.9 \\ 1 & 1.4 \end{vmatrix} = 1.27$$

$$b_2 = \begin{vmatrix} -0.45 & 1.4 \\ 1 & -1.9 \end{vmatrix} = -0.545$$

$|b_0| > |b_2| \Rightarrow 0.798 > 0.545$

$\therefore$ <u>stable</u>

(b) (i) zeros: $0.5, 0.6 \Rightarrow k_1(0.5)^n,\ k_2(0.6)^n$

(ii) zeros: $0.5, 0.5 \Rightarrow k_1(0.5)^n,\ k_2 n(0.5)^n$

(iii) zeros: $-0.5, 0.6 \Rightarrow k_1(-0.5)^n,\ k_2(0.6)^n$

(iv) zeros: $-0.5, 0.5 \Rightarrow k_1(-0.5)^n,\ k_2(0.5)^n$

(v) zeros: $1\,\underline{/\pm 0.643}$ rad $\Rightarrow A\cos(0.643n + \theta)$

(vi) zeros: $0.9, 1.1 \Rightarrow k_1(0.9)^n,\ k_2(1.1)^n$

(vii) zeros: $1, 0.7, 0.5 \Rightarrow k_1, k_2(0.7)^n, k_3(0.5)^n$

(viii) zeros: $0.9, 0.707\underline{/\pm \pi/4} \Rightarrow k_1(0.9)^n;\ A(0.707)^n \cos(\frac{\pi}{4}n + \theta)$

(c) (v) $A\cos(0.643n + \theta)$, <u>marginally stable</u>

(vi) $k_2(1.1)^n$, <u>unstable</u>

(vii) $k_1(1)^n = k_1$, <u>marginally stable</u>

7-6. (a) $G(z) = \frac{z-1}{z}\mathcal{Z}\left[\frac{1}{s(s+1)}\right] = \frac{z-1}{z}\frac{(1-\epsilon^{-T})z}{(z-1)(z-\epsilon^{-T})} \Rightarrow \frac{0.6321}{z - 0.3678}$

7-6. (a) $\therefore\ 1 + KG(z) = 0 = z - 0.3679 + 0.6321K$

(b) $z = 0.3679 - 0.6321K > -1 \Rightarrow K < \dfrac{1.3679}{0.6321} = \underline{2.164}$

$z = 0.3679 - 0.6321K < 1 \Rightarrow K \underline{> -1}$

$\therefore\ \underline{-1 < K < 2.164}$

(c) $K = -1,\ z = 1 \Rightarrow k(1)^n = \underline{k}$

(d) $K = 2.164,\ z = -1 \Rightarrow \underline{k(-1)^n}$

(e)
```
K = -1;
for k=1:2
   q = [1 (-0.3679 + 0.6321*K)];
   roots(q)
   K = 2.164;
end
```

7-7. (a) From Problem 7-6: $G(z) = \dfrac{0.6321}{z - 0.3679}$

$\therefore$ char. eq.: $\underline{z - 0.3679 + 0.6321K = 0}$

(b) $z = \dfrac{1 + \frac{T}{2}w}{1 - \frac{T}{2}w} = \dfrac{1 + 0.5w}{1 - 0.5w} = \dfrac{2 + w}{2 - w}$

$\therefore$ char. eq.: $2 + w + (2 - w)(-0.3679 + 0.6321K) = 0$

$\therefore\ (1.3679 - 0.6321K)w + (1.2642 + 1.2642K) = 0$

$\therefore$

$$\begin{array}{l|ll} w^1 & 1.3679 - 0.6321K & \Rightarrow K < 2.164 \\ w^0 & 1.2642 + 1.2642K & \Rightarrow K > -1 \end{array}$$

$\therefore\ \underline{-1 < K < 2.164}$

(c) $Q(1) > 0 \Rightarrow 1 - 0.3679 + 0.6321K > 0 \Rightarrow K > -1$

$(-1)Q(-1) > 1 \Rightarrow 1 + 0.3679 - 0.6321K > 0 \Rightarrow K > -2.164$

$|a_0| < 1 \Rightarrow |-0.3679 + 0.6321K| < 1 \Rightarrow 1 < K < 2.164$

$\therefore\ \underline{-1 < K < 2.164}$

(d) $\underline{K = 2.164}$: $w = \dfrac{1.2642 + 1.2642K}{1.3679 - 0.632K} = \dfrac{(\ )}{1.3679 - 1.3679}$ $\therefore$ unbounded

$z = 0.3679 - 0.6321(2.164) = -1 = 1\angle\pi = 1\angle\omega T$

$\therefore\ \omega = \pi = \underline{3.1416}$ rad/s

7-7. (e) $1 + K G_p(s) = 1 + \frac{K}{s+1} \Rightarrow s+1+K=0$

$\therefore\ s = -1-K \Rightarrow \underline{K > -1}$

(f) Sampling destabilizes this system.

(g)

```
K = -1;
w0 = 1.2642 + 1.2642*K
K = 2.164;
w1 = 1.3679 - 0.6321*K
```

7-8. (a) From Problem 7-6, $z - \epsilon^{-T} + (1-\epsilon^{-T})K = 0$, char. eq.

(b) From Problem 7-6, $T=1 \Rightarrow \underline{K < 2.164}$

$T = 0.1 \Rightarrow z - 0.9048 + 0.09516 = 0$

$\therefore\ z = 0.9048 - 0.09516K > -1 \;=\; \underline{K < 20.02}$

$T = 0.01 \Rightarrow z - 0.99005 + 0.00995K = 0$

$\therefore\ z = 0.99005 - 0.00995K > -1 \Rightarrow K < 200.0$

(c) From Problem 7-7, $K < \infty$

(d) Reducing T improves stability for this system.

(e)

```
T = 0.1;
K = 20.02;
for k=1:2
   q = [1 (-exp(-T) + (1 - exp(-T))*K)]
   roots(q)
   T = .01;
   K = 200.0;
end
```

7-9. (a) $1 + KG(z)H = 0 = 1 + \frac{(1.037)K}{z - 0.7408}(0.04) = 0$

$\therefore$ char. eq.: $\underline{Q(z) = z - 0.7408 + 0.04148K = 0}$

(b) $G(w) = \dfrac{1.037}{\frac{1+0.3w}{1-0.3w} - 0.7408} = \dfrac{-0.5958w + 1.986}{w + 0.4964}$

$1 + KG(w)H = 0 = 1 + \dfrac{0.04K(-0.5958w + 1.986)}{w + 0.4964}$

$\therefore$ char. eq.: $\underline{(1 - 0.02383K)w + 0.4964 + 0.07944K = 0}$

7-9. (b) $\therefore$
$$\begin{array}{c|l} w^1 & 1-0.02383K \quad \Rightarrow \quad K<41.96 \\ w^0 & 0.4964+0.07944K \quad \Rightarrow \quad K>-6.249 \end{array}$$

$\therefore$ *stable:* $\underline{-6.249 < K < 41.96}$

(c) From (a):

$\underline{z^1}$	$\underline{z^0}$
1	$0.04148K-0.7408$

$$Q(1) = 1-0.7408+0.04148K>0 \quad \Rightarrow \quad K>-6.249$$
$$(-1)Q(-1) = 1+0.7408-0.04148K>0 \quad \Rightarrow \quad K<41.96$$
$$|a_0|<a_1: \ |0.04148K-0.7408K|<1 \quad \Rightarrow \text{ same as above}$$
$$\therefore \quad \underline{-6.249 < K < 41.96}$$

(d) $$G(z) = \frac{z-1}{z}\mathfrak{z}\left[\frac{2}{s(s+0.5)}\right] = \frac{z-1}{z}\left(\frac{2}{0.5}\right)\left[\frac{z(1-\epsilon^{-0.03})}{(z-1)(z-\epsilon^{-0.03})}\right] = \frac{0.1182}{z-0.9704}$$

$$1+KG(z)H = 1+\frac{0.1182K}{z-0.9704}(0.04)=0$$
$$\therefore Q(z) = z-0.9704+0.004729K=0$$
$$Q(1)=1-0.9704+0.004729K>0 \quad \Rightarrow \quad K<-6.259$$
$$-Q(-1)=1+0.9704-0.004729K>0 \quad \Rightarrow \quad K>416.7$$
$$\therefore \quad \underline{-6.259<K<416.7}$$

(e) *As $T\to 0$, the range of K for stability becomes larger. Thus sampling tends to destabilize this system.*

(f)
```
T = 0.6;
K = -6.249;
q = [1 ((-exp(-T*.5)) + (1 - exp(-T*.5)*K*0.04*4))]
roots(q)
```

7-10. (a) $1+KG(z)H=0 = z^2-1.8187z+0.8187+0.001311Kz+0.001226K$

$\therefore$ char. eq.: $Q(z)=z^2-(1.8187-0.001311K)z+0.8187+0.001226K=0$

(b) $$z = \frac{1+\frac{T}{2}w}{1-\frac{T}{2}w} = \frac{1+0.05w}{1-0.05w} = \frac{20+w}{20-w}$$

$$\therefore Q(w) = (20+w)^2-(1.8187-0.001311K)(400-w^2)+(0.8187+0.001226K)(20-w)^2=0$$
$$\therefore (3.6374-0.000085K)w^2+(7.252-0.04904K)w+1.0148K=0$$

7-10. (b)

$$\begin{array}{c|ll}
w^2 & 3.6374-0.000085K & 1.0148K \\
w^1 & 7.252-0.04904K & \\
w^0 & 1.0148K &
\end{array}
\quad
\begin{array}{l}
\Rightarrow K<42{,}793 \\
\Rightarrow K<147.9 \\
\Rightarrow K>0
\end{array}$$

$\therefore$ for stability, $\underline{0<K<147.9}$

(c) $Q(1)>0 \Rightarrow 1-1.818T+0.001311K+0.8187+0.001226K>0$ $\therefore \underline{K>0}$

$(1)^2Q(-1)>0 \Rightarrow 1+1.8187-0.001311K+0.8187+0.001226K>0$

$\therefore K<42{,}793$

$|a_0|<a_2 \Rightarrow 0.8187+0.001226K<1$, $\therefore \underline{K<147.9}$

For Stability: $\underline{0<K<147.9}$

(d) $K=147.9$ From (a): char. eq. $z^2-1.6248z+1=0$

$\underline{\text{zeros}}$: $z=\underline{1\angle\pm35.67^\circ}=\underline{1\angle\pm0.6225 \text{ rad.}}$

From (b): char. eq. $3.6148w^2+150.09=0$

$\therefore w=\underline{\pm j6.437}$

(e) z-plane: $1\angle\omega T=1\angle 0.6225$ $\therefore \omega=\dfrac{0.6225}{0.1}=\underline{6.225 \text{ rad/s}}$

w-plane: $\omega_w=\underline{6.437}$

(f) (7-10) $\omega_w=\dfrac{2}{T}\tan\left(\dfrac{\omega T}{2}\right)=\dfrac{2}{0.1}\tan\left(\dfrac{0.6225}{2}\right)=\underline{6.433}$ ✓

(g)

```
K = 147.9;
q = [1 (- 1.8187 + 0.001311*K) (0.8187 + 0.001226*K)]
roots(q)
```

7-11. (a) $1+KG(z)H=0=z^2-1.7408z+0.7408+0.009072Kz+$

$\therefore$ char. eq.: $\underline{z^2+(0.009072K-1.7408)z+0.7408+0.008208K=0}$

(b) $z=\dfrac{1+\frac{T}{2}w}{1-\frac{T}{2}w}=\dfrac{1+0.025w}{1-0.025w}=\dfrac{40+w}{40-w}$ yields

$(40+w)^2+(0.009072K-1.7408)(1600-w^2)+(0.7408+0.008208K)(40-w)^2=0$

$\therefore \underline{(3.4816-0.000864K)w^2+(20.736-0.65664K)w+27.65K=0}$

$$\begin{array}{c|ll}
w^2 & 3.4816-0.000864K & 27.65K \\
w^1 & 20.736-0.65664K & \\
w^0 & 27.65K &
\end{array}
\quad
\begin{array}{l}
\Rightarrow K<4{,}029.6 \\
\Rightarrow K<31.58 \\
\Rightarrow K>0
\end{array}$$

$\therefore$ for stability: $\underline{0<K<31.58}$

7-11. (c) $Q(1)>0 \Rightarrow 1+0.009072K-1.7408+0.7408+0.008208K>0 \Rightarrow \underline{K>0}$

$(-1)^2Q(-1)>0 \Rightarrow 1-0.009072K+1.7408+0.7408+0.008208K>0 \Rightarrow \underline{K<4029.6}$

$|a_0|<a_2 \Rightarrow 0.7408+0.008208K<1 \qquad \Rightarrow \underline{K<31.58}$

$\therefore$ for stability: $\underline{0<K<31.58}$

(d) $K=31.58$, from (a): $z^2-1.4543z+1=0$

$\therefore z = 1\angle{\pm 43.36^\circ} = 1\angle{\pm 0.7567}\ \text{rad}$

from (b): $3.4543w^2+873.2=0 \Rightarrow w=\pm j15.90=\pm j\omega_w$

(e) z-plane: $1\angle\omega T = 1\angle 0.7567 \Rightarrow \omega = 0.7567/0.05 = \underline{15.13\ \text{rad/s}}$

w-plane: $\omega_w = \underline{15.90}$

(f) (7-10) $\omega_w = \frac{2}{T}\tan\left(\frac{\omega T}{2}\right) = 40\tan\left(\frac{0.7567}{2}\right) = \underline{15.90}$ ✓

(g)
```
K = 31.58;
q = [1 (.009072*K - 1.7408) (0.7408 + 0.008208*K)]
roots(q)
```

7-12. (a) $1+KG(z)H = z^2-2z+1+0.001Kz+0.001K=0$

$\therefore$ char. eq.: $\underline{z^2+(0.001-2)z+(0.001K+1)=0}$

(b) $z = \dfrac{\frac{2}{T}+w}{\frac{2}{T}-w} = \dfrac{20+w}{20-w}$

$\therefore (20+w)^2+(0.001K-2)(400-w^2)+(0.001K+1)(20-w)^2=0$

$\therefore \underline{4w^2-0.04Kw+0.08K=0}$

w^2	4	$0.08K$	
w^1	$-0.04K$		$\Rightarrow \underline{K<0}$
w^0	$0.08K$		$\Rightarrow \underline{K>0}$

$\therefore$ unstable for all K

(c) $Q(1)>0 \Rightarrow 1+0.001K-2+0.001K+1>0 \Rightarrow \underline{K>0}$

$(-1)^2Q(-1)>0 \Rightarrow 1+2-0.001K+0.001K+1=4>0$

$|a_0|<a_2 \Rightarrow 0.001K+1<1 \qquad \Rightarrow \underline{K<0}$

$\therefore$ unstable for all K

7-13. (a) $1+G(z) = z^2 - 1.6z + 0.6 + Kz + 0.8K = 0$

$\therefore$ char. eq.: $\underline{z^2 - (1.6-K)z + 0.6 + 0.8K = 0}$

$z = \dfrac{1+w}{1-w} \Rightarrow (1+w)^2 + (K-1.6)(1-w^2) + (0.6+0.8K)(1-w)^2 = 0$

$\therefore$ char. eq.: $\underline{(3.2-0.2K)w^2 + (0.8-1.6K)w + 1.8K = 0}$

$$\begin{array}{c|cc l}
w^2 & 3.2-0.2K & 1.8K & \Rightarrow \underline{K<16} \\
w^1 & 0.8-1.6K & & \Rightarrow \underline{K<0.5} \\
w^0 & 1.8K & & \Rightarrow \underline{K>0}
\end{array}$$

$\therefore$ for stability: $\underline{0<K<0.5}$

(b) $Q(1)>0 \Rightarrow 1-1.6+K+0.6+0.8K>0 \Rightarrow \underline{K>0}$

$(-1)^2 Q(-1)>0 \Rightarrow 1+1.6-K+0.6+0.8K>0 \Rightarrow \underline{K<16}$

$|a_0|<a_2 \Rightarrow 0.6+0.8K<1 \Rightarrow \underline{K<0.5}$

$\therefore$ for stability: $\underline{0<K<0.5}$

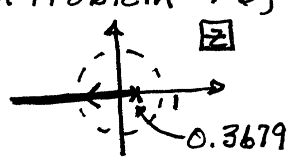
(c) $K=0.5$; char. eq.: $3.1w^2+0.9=0$

$\therefore$ zeros: $w = \underline{\pm j0.5388}$ $\therefore$ marginally stable

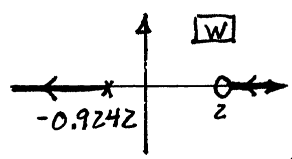
(d) $K=0.5$; char. eq.: $z^2-1.1z+1=0$

$\therefore$ zeros: $z = 0.55 \pm j0.8352 = \underline{1\angle\pm 56.6^\circ}$, $\therefore$ marginally stable

7-14. (a) From Problem 7-6, $G(z) = \dfrac{0.6321}{z-0.3679}$



(b) $z = \dfrac{2+w}{2-w}$ $\therefore$ $G(w) = \dfrac{0.6321(2-w)}{2+w-0.3679(2-w)} = \dfrac{-0.4621(w-2)}{w+0.9242}$



$\therefore$ effectively plotting for K negative

The locus passes through infinity.

(c) $z=-1$, $K = \left|\dfrac{1}{G(-1)}\right| = \left|\dfrac{-1-0.3679}{0.6321}\right| = 2.164$

(d) $w=\infty$, $K = \left|\dfrac{1}{G(\infty)}\right| = \left|\dfrac{1}{-0.4621}\right| = 2.164$

7-14. (e)

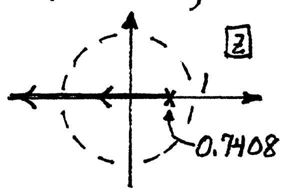
z-plane

```
k=0:0.2:5;
n = [0 0.6321];
d = [1 -0.3679];
r = rlocus(n,d,k);
[k',r]
pause
plot(real(r),imag(r),'x')
title('Root Locus')
```

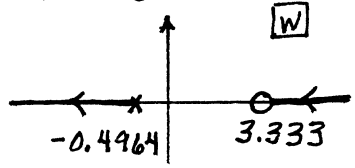
w-plane

```
k=0:0.2:5;
n = [-0.4621 0.9242];
d = [1 0.9242];
r = rlocus(n,d,k);
[k',r]
pause
plot(real(r),imag(r),'x')
title('Root Locus')
```

7-15. (a) From Problem 7-9, $G(z) = \dfrac{1.037}{z - 0.7408}$



(b) $z = \dfrac{1 + 0.3w}{1 - 0.3w} \Rightarrow G(w) = \dfrac{-0.5957(w - 3.333)}{w + 0.4964}$ $\therefore$ effectively plotting for K negative.



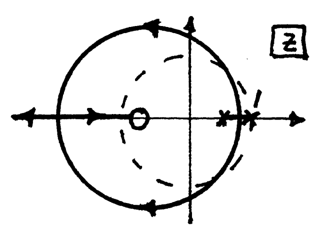
The locus passes through infinity.

(c) $K = \left|\dfrac{1}{G(-1)}\right| = \dfrac{1.7408}{1.037} = \underline{1.679}$

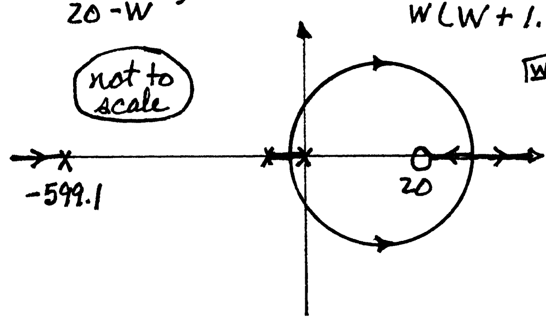
(d) $K = \left|\dfrac{1}{G(\infty)}\right| = \dfrac{1}{0.5957} = \underline{1.679}$

(e) See Problem 7-14 (e) for programs.

7-16. (a) $G(z) = \dfrac{0.01873(z + 0.9354)}{(z-1)(z-0.8187)}$



(b) $z = \dfrac{20 + w}{20 - w}$, $G(w) = \dfrac{-0.000333(w + 599.1)(w - 20)}{w(w + 1.994)}$ $\therefore$ effectively plotting for K negative



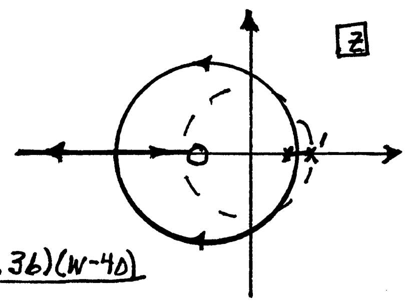
Locus passes through infinity.

7-16. (c) From Problem 7-10, zeros at $z = 1\angle \pm 35.7^\circ$ for $K = 147.9$

(d) From Problem 7-10, zeros at $w = \pm j6.437$ for $K = 147.9$

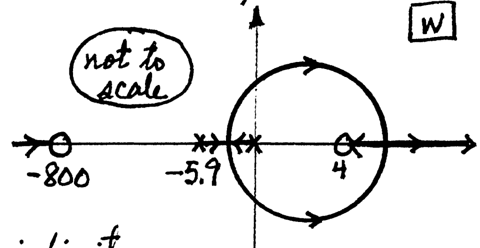
(e) See Problem 7-14(e) for programs.

7-17. (a) $G(z) = \dfrac{0.02268(z + 0.9048)}{(z-1)(z-0.7408)}$



(b) $z = \dfrac{40+w}{40-w}$; $G(w) = \dfrac{-0.0006202(w+800.36)(w-40)}{w(w+5.956)}$

$\therefore$ effectively plotting for K negative



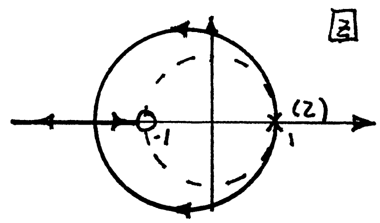
Locus passes through infinity.

(c) From Problem 7-11, zeros at $z = 1\angle \pm 43.4^\circ$ for $K = 31.58$

(d) From Problem 7-11, zero at $w = \pm j15.90$ for $K = 31.58$

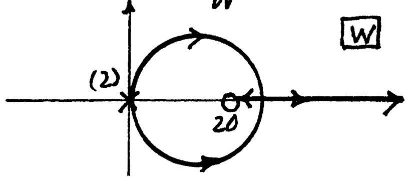
(e) See Problem 7-14(e) for programs.

7-18. (a) $G(z) = \dfrac{0.05(z+1)}{(z-1)^2}$



(b) $z = \dfrac{20+w}{20-w}$

$\therefore G(w) = \dfrac{-0.5(w-20)}{w^2}$ $\therefore$ effectively plotting for K negative.



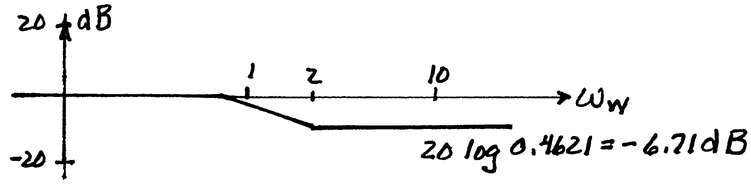
(c) Unstable for all K.

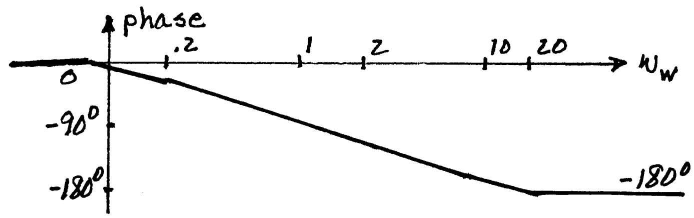
(d) Unstable for all K.

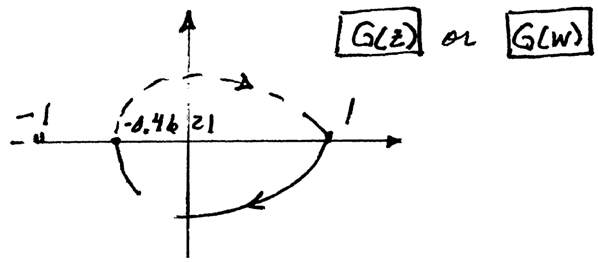
(e) See Problem 7-14(e) for programs.

7-19. (a) From Problem 7-14, stable for $K=1$.

(b) $G(w) = \dfrac{0.4621(2-w)}{w+0.9242} = \dfrac{1-w/2}{1+w/0.9242}$







(c) gain margin $= \dfrac{1}{0.4621} = \underline{2.16} \Rightarrow \underline{6.71\text{ dB}}$

$\phi_m = \underline{180^0}$

(d) $K = \dfrac{1}{0.4621} = \underline{2.16}$

(e) $\omega_w = \infty$, $z=-1 = 1\angle\omega T$, $\therefore \omega T = \pi \Rightarrow \omega = \frac{\pi}{T} = \frac{\omega_s}{2} = 3.14$ rad/s

(f) The problem should ask for Bode diagram.

```
num = [-0.4621 0.9242];
den = [1 0.9242];
bode(num,den)
```

7-20. (a) $z = \dfrac{1+aw}{1-aw} \Rightarrow w = \dfrac{1}{a}\dfrac{z-1}{z+1}$

$\therefore w = \dfrac{1}{a}\dfrac{z-1}{z+1}\Big|_{z=\epsilon^{j\omega T}} = j\dfrac{1}{a}\tan\left(\dfrac{\omega T}{2}\right)$

(b) $\omega_w = \dfrac{1}{a}\tan\left(\dfrac{\omega T}{2}\right)$

For $\frac{\omega T}{2} << 1$, $\omega_w \approx \dfrac{\omega T}{2a} = \dfrac{T}{2a}\omega$

7-20. (c) $|z|<1$ for stability — Let $z = z_r + j z_i$

then $|z|^2 = z_r^2 + z_i^2 < 1$ for stability

$$w = \frac{1}{a}\,\frac{z_r + j z_i - 1}{z_r + j z_i + 1} = \frac{1}{a}\,\frac{(z_r - 1) + j z_i}{(z_r + 1) + j z_i} \cdot \frac{(z_r+1) - j z_i}{(z_r+1) - j z_i}$$

$$\mathrm{Re}(w) = \frac{1}{a}\,\frac{z_r^2 - 1 + z_i^2}{(z_r+1)^2 + z_i^2} = \frac{1}{a}\,\frac{|z|^2 - 1}{b}, \quad b > 0$$

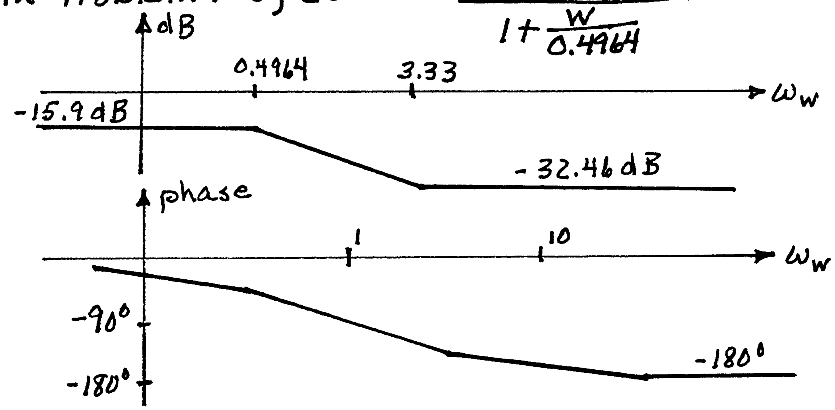
$\therefore$ $a > 0$, $\mathrm{Re}(w) < 0$, left-half plane for stability

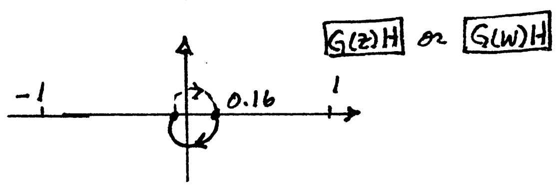
$a < 0$, $\mathrm{Re}(w) > 0$, right-half plane for stability

7-21. Output $= (5)(1.3)\cos(2t - 35°) = \underline{6.5 \cos(2t - 25°)}$

7-22. (a) From Problem 7-9, stable for $K = 1$.

(b) From Problem 7-15, $G(w)H = \dfrac{0.16\left(1 - \frac{w}{3.33}\right)}{1 + \frac{w}{0.4964}}$





(c) gain margin $= \dfrac{1}{(0.04)(0.5957)} = \underline{41.97} \Rightarrow \underline{32.46}\,$dB

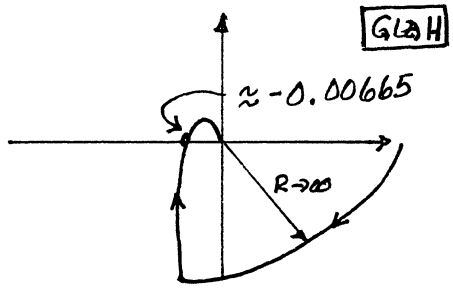
no phase margin

(d) $K = \underline{41.97}$ from (c)

(e) $\omega_w = \infty$, $z = -1 = 1\angle \omega T$, $\therefore \omega T = \pi$, $\omega = \frac{\pi}{T} = \frac{\omega_s}{2} = \underline{5.24 \text{ rad/s}}$

(f) See Problem 7-19(f).

7-23. (a)



(b) From (a), stable.

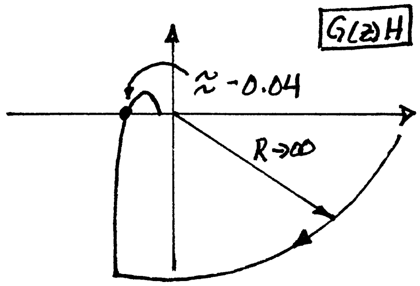
(c) Gain margin $\approx \frac{1}{(0.095)(0.07)} = \underline{150.4}$ (147.9, from Problem 7-10)

$\omega_w = 0.15$, dB ≈ 23, $\therefore \phi_m \approx 180^\circ - 95^\circ = \underline{85^\circ}$

(d) From (c), $K \approx 150.4$

(e) From Table P7-23, $\omega \approx \underline{6.5}$ rad/s with phase = -180°

7-24. (a)



(b) From (a), <u>stable</u>

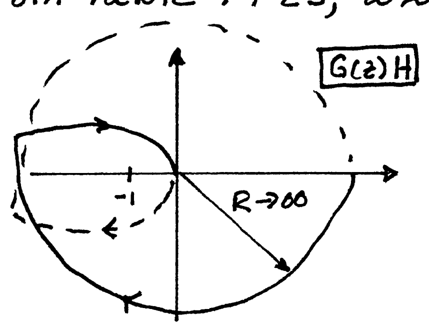
(c) Gain margin $\approx \frac{1}{0.04} = \underline{25}$ ($\underline{31.58}$ from Problem 7-11)

$\omega_w \approx 1.5$, dB ≈ 8, $\therefore \phi_m \approx 180^\circ - 105^\circ = \underline{75^\circ}$

(d) From (c), $K \approx \underline{25}$ (31.58)

(e) From Table P7-23, $\omega \approx 15$ rad/s

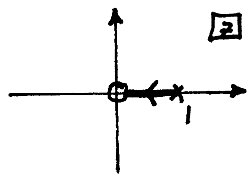
7-25. (a)



∴ always unstable.

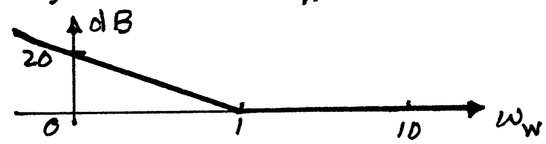
(b) Since K cannot affect the number of encirclements, the system is <u>unstable</u> for all K.

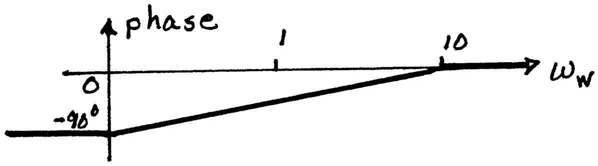
7-26. (a) $D(z)G(z) = \frac{z-0.9}{z-1} \times \frac{Kz}{z-0.9} = \underline{\frac{Kz}{z-1}}$



(b) $z = \frac{1+W}{1-W}$, $\therefore D(w)G(w) = \frac{K(1+W)}{1+W-1+W} = \frac{K}{2}\left(\frac{W+1}{W}\right)$

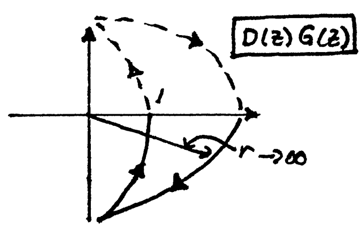
$K=2$, $D(w)G(w) = \frac{1+W}{W}$





$\therefore$ $\underline{\text{stable}}$ for all K

(c)



gain margin = $\underline{\infty}$
(stable for all $K > 0$)
$\phi_m = \underline{180°}$

7-27. (a) (i) $G^*(j\omega)$; $\quad 0 \le \omega \le \omega_s/2$

(ii) $G(z)$; $\quad z = \epsilon^{j\omega T}$, $0 \le \omega \le \omega_s/2$

(iii) $G(j\omega_w)$; $\quad 0 \le \omega_w < \infty$

(iv) $G^*(j\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} G(j\omega + jn\omega_s)$; $\quad 0 \le \omega \le \omega_s/2$

(b) $G^*(j\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} G(j\omega + jn\omega_s)$; $\quad 0 \le \omega \le \omega_s/2$

(c) Evaluate $G(z)$ for $z = \epsilon^{j\omega T}$. Also evaluate $\omega_w = \frac{2}{T}\tan\left(\frac{\omega T}{2}\right)$. Then plot $G(z)$ versus ω_w.

(d) With sufficient numerical accuracy, the two diagrams will be the same.

CHAPTER 8

All simulations performed by CTRL or CSP.
All designs verified by CTRL or CSP.

8-2. (a) $\omega_{w1} \approx 0.8$, $\phi_m \approx 30^\circ$

(b) $G(j\omega_w) = G(j0.6) = 1.461 \angle -136.9^\circ$

$\therefore K = \frac{1}{1.461} = \underline{0.684}$, $\phi_m = 180^\circ - 136.9^\circ = \underline{43.1^\circ}$

(c) $t_r \approx \underline{1.85s}$, $\underline{\approx 27\%}$ overshoot

(d) $-180^\circ + 45^\circ + 5^\circ = -130^\circ = \angle G(j\omega_{w1})$

$\therefore \omega_{w1} = 0.5 \Rightarrow \omega_{wo} = \underline{0.05}$

$|G(j\omega_{w1})| = 1.815 \Rightarrow \omega_{wp} = \frac{0.05}{1.815} = \underline{0.0275}$

$$\therefore D(j\omega_w) = \frac{1 + W/0.05}{1 + W/0.0275}$$

$$K_d = \frac{0.0275(2.05)}{0.05(2.0275)} = 0.5561$$

$$z_o = \frac{2-0.05}{2+0.05} = \underline{0.9512}\ ; \quad z_p = \frac{2-0.0275}{2+0.0275} = \underline{0.9729}$$

$$\therefore D(z) = \frac{0.5561(z-0.9512)}{z-0.9729} = \frac{0.5561z - 0.5290}{z-0.9729}$$

By computer: $\phi_m \approx 47^\circ$, G.M. ≈ 33.6 dB

(e) $t_r \approx \underline{2.1s}$, $\underline{\approx 23\%}$ overshoot

8-3. (a) $\omega_{w1} = 1$, $G(j\omega_{w1}) = 0.7614 \angle -159.1^\circ$

$\therefore \theta = 180^\circ + 45^\circ - (-159.1^\circ) = 24.1^\circ$

$$a_1 = \frac{1-(1)(0.7614)\cos\theta}{(1)(0.7614)\sin\theta} = \underline{0.9809}$$

$$b_1 = \frac{\cos\theta - (1)(0.7614)}{(1)\sin\theta} = \underline{0.3709}$$

8-3.(a) $D(w) = \dfrac{1+0.9809w}{1+0.3709w} = \dfrac{1+w/1.0194}{1+w/2.696}$

$K_d = \dfrac{2.696(3.0194)}{1.0194(4.696)} = \underline{1.700}$

$z_0 = \dfrac{2-1.0194}{3.0194} = \underline{0.3246}$; $z_p = \dfrac{2-2.696}{4.696} = \underline{-0.1482}$

$\therefore D(z) = \dfrac{1.700z - 0.5522}{z+0.1482}$

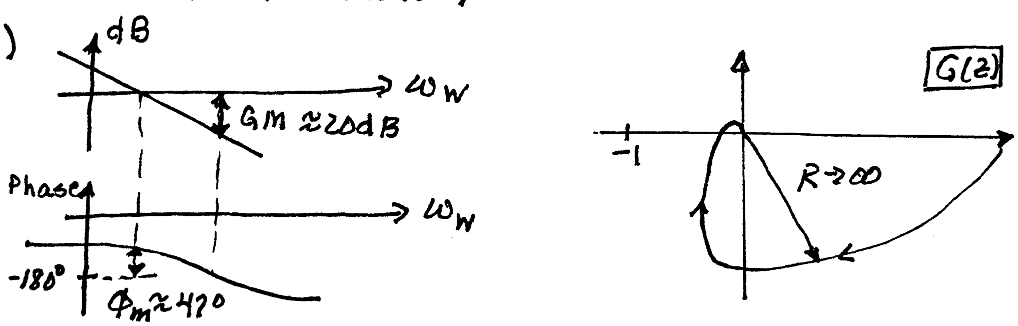
(b) $t_r \cong \underline{0.94\,s}$, $\underline{\approx 30\%}$ overshoot

(c)

	t_r	overshoot
gain	1.85	27%
phase-lag	2.1	23%
phase-lead	0.94	30%

8-4(a) $G(z) = \left(\dfrac{z-1}{z}\right) \dfrac{z[(0.2-1+\epsilon^{-0.2})z + (1-\epsilon^{-0.2}-0.2\epsilon^{-0.2})]}{(z-1)^2(z-\epsilon^{-0.2})}$

$= \dfrac{0.01873z + 0.01752}{(z-1)(z-0.8187)}$

(b) 

(c) $\lim_{\omega_W\to\infty} G(j\omega_W) = \lim_{z\to -1} = \dfrac{-0.01873+0.01752}{(-2)(-1.8187)} = \underline{-69.6\,dB}$

(d) $\omega_W \approx 0.80$, $\Phi_m \approx 180° - 133° \approx \underline{47°}$ $\therefore$ % overshoot $\approx 20\%$ Figure 8-2
$\omega_W \approx 3.0$, $GM \approx \underline{10 \approx 20dB}$

(e) Zeros are complex, since overshoot is present.

(f) Overshoot $\approx \underline{21\%}$, $t_r \approx \underline{2\,s}$

8-5.(a) $K_v = \lim_{z\to 1} \frac{1}{T}(z-1)G(z) = \frac{1}{0.2} \; \frac{0.01873+0.01752}{1-0.8187} = \underline{1}$

(a) $G(z) = \left[4\,G_{8.4}(z)\right] = \dfrac{0.07492z+0.07008}{(z-1)(z-0.8187)}$

(b) $20\log_{10}(4) = 12\,dB$ $\therefore$ the phase is the same as in Prob.8-4(b), with 12 added to the dB column. The magnitude on the Nyquist diagram is multiplied by 4, with the angles unchanged.

(c) From Problem 8-4(d),

$$\lim_{\omega_w \to \infty} G(j\omega_w) = (-0.3327\times10^{-3})(4) = \underline{-1.333\times10^{-3}}$$

$$\Rightarrow \underline{-57.6\,dB}$$

(d) In Table P8-4, multiply $|G(j\omega_w)|$ by 4

add 12 db to $|G(j\omega_w)|_{dB}$ column

$\therefore \omega_w \approx 2$, $\phi_m \approx 180° - 164° = \underline{16°}$

$\omega_w \approx 3$, G.M. $\approx 2.5 \approx -\underline{7\,dB}$

(b) $\phi_m \approx 16°$, from Figure 8-2, $\zeta \approx \underline{0.13}$, $M_r \approx \underline{3.25}$

and $M_p \approx \underline{1.65} \Rightarrow \underline{65\%}$ overshoot.

(c) Simulate by CTRL or CSP.

8-6.(a) $-180° + 45° + 5° = -130° = \underline{/G(j\omega_{w1})}$

$\therefore \omega_{w1} = 0.7 \Rightarrow \omega_{w0} = \underline{0.07}$

$4|G(j\omega_{w1})| = 4(1.172) = \underline{4.688}$

$\therefore \omega_{wp} = \dfrac{0.07}{4.688} = \underline{0.01493}$

$$\therefore D(j\omega_w) = \frac{4(1+w/0.07)}{1+w/0.01493}$$

$K_d = 4\left(\dfrac{0.01493(10.07)}{0.07(10.01493)}\right) = \underline{0.8587}$

$z_0 = \dfrac{10-0.07}{10+0.07} = \underline{0.9861}$; $z_p = \dfrac{10-0.01493}{10.01493} = \underline{0.99702}$

$$\therefore D(z) = \frac{0.8578z - 0.84676}{z-0.99702}$$

8-6. (b) $\phi_m \approx 45°$; from Figure 8-2, $\zeta \approx 0.42$, $\therefore$ % overshoot $\approx$ 25%

(c) From CTRL or CSP, 24% overshoot, $t_r \cong 1.67s$

8-7. (a) Choose $\omega_{w1} = 3$, $\therefore G(j3) = 0.1097\angle{-177.74°}$

$$\theta = 180° + 45° - (-177.74°) = 42.74°$$

$$a_1 = \frac{1 - 4(0.1097)\cos\theta}{(3)(0.1097)\sin\theta} = 3.034$$

$$b_1 = \frac{\cos\theta - (4)(0.1097)}{3\sin\theta} = 0.1468$$

$$\therefore D(w) = \frac{3.034w + 4}{0.1469w + 1}$$

By computer: $D(z) = \dfrac{14.01z - 10.74}{z - 0.1839}$

(b) From Problem 8-6(b): overshoot $\approx$ 25%

(c) By simulation: overshoot $\approx$ 26%, $t_r \approx 0.34s$

8-8. (a) From Table P8-4, $G(j0.8) = 0.9779\angle{-133.2°}$

$$G(j0.9) = 0.8280\angle{-137.1°}$$

$\therefore$ Interpolating, $G(j0.85) \approx 0.90\angle{-135°}$

Let $K = \dfrac{1}{0.90} = 1.11$ for $\phi_m = 45°$

Note that the gain is increased.

(b) overshoot $\approx$ 23%, $t_r \approx 1.4s$

8-9. $\theta < 0°$

1. $\angle G(j\omega_{w1}) < -180° + \phi_m$

$|D(j\omega_{w1})| \le a_0$, $|D||G| = 1$

$\therefore \dfrac{1}{|G(j\omega_{w1})|} \le a_0$

2. $\therefore |G(j\omega_{w1})| \ge \dfrac{1}{a_0}$

$b_1 > 0$, $\sin\theta < 0$

3. $\therefore a_0|G(j\omega_{w1})| > \cos\theta$

8-10. (a), (b), (c) : See Problem 8-2.

(d) Choose $\omega_1 = 0.5$ as in Problem 8-2.

$G(j0.5) = 1.815 \angle -130.1^\circ$

(8-56) $\theta = 180^\circ + 45^\circ - (-130.1^\circ) = -4.90^\circ$

(8-57) $K_P = \dfrac{\cos\theta}{1.815} = \underline{0.5490}$

(8-58) $K_I = \dfrac{-(0.5)\sin\theta}{1.815} = \underline{0.02353}$

(e) $t_r = \underline{2.15s}$, overshoot $= \underline{28\%}$

8-11. (a) Choose $\omega_1 = 1$, as in Problem 8-3.

$G(j1) = 0.7614 \angle -159.1^\circ$

(8-56) $\theta = 180^\circ + 45^\circ - (-159.1^\circ) = 24.1^\circ$

(8-57) $K_P = \dfrac{\cos\theta}{0.7614} = \underline{1.199}$

(8-58) $K_d = \dfrac{\sin\theta}{(1)(0.7614)} = \underline{0.5363}$

Actual phase margin, by computer, $\approx 33^\circ$

(b) $t_r = \underline{1.04s}$, overshoot $= \underline{45\%}$

8-12. (a) With the PI controller, the system is type $\underline{2}$.

$\therefore$ no steady-state error for a ramp input.

(b) As in Problem 8-6 (a), choose $\omega_1 = 0.7$.

$G(j0.7) = 1.172 \angle -128.95^\circ$

(8-56) $\theta = 180^\circ + 45^\circ - (-128.95^\circ) = -6.05$

(8-57) $K_P = \dfrac{\cos\theta}{1.172} = \underline{0.848}$

(8-58) $K_I = \dfrac{-(0.7)\sin\theta}{1.172} = \underline{0.06295}$

(c) $t_r = \underline{1.9s}$, overshoot $= 28\%$

8-13. (a) As in Problem 8-7, let $\omega_1 = 3$, $\therefore G(j3) = 0.1097\angle{-177.74^\circ}$

(8-56) $\theta = 180^\circ + 45^\circ - (-177.74^\circ) = 42.74^\circ$

Use $|G_e(j3)| = 4|G(j3)| = 0.4388$

(8-57) $K_P = \dfrac{\cos\theta}{0.4388} = \underline{1.674}$

(8-58) $K_D = \dfrac{\sin\theta}{(3)(0.4388)} = \underline{0.5155}$

(b) $t_r = \underline{0.28s}$, overshoot $\underline{= 42\%}$

By computer, actual phase margin $= 35^\circ$

8-14. (a) dc gain $= G_e(1) = \dfrac{0.04147}{1 - 0.7408} = 0.16$

$T(1) = \dfrac{D(1)G_e(1)}{1 + D(1)G_e(1)} = \dfrac{0.16}{1 + 0.16} = \underline{0.1379}$

(b) $K_p = \lim_{z \to 1} G_e(z) = 0.16$

$e_{ss}(kT) = \dfrac{1}{1 + K_p} = \dfrac{1}{1 + 0.16} = 0.862 \Rightarrow \underline{86.2\%}$

(c) $K_p = \lim_{z \to 1} D(z)G_e(z) = 0.16K$

$e_{ss}(kT) = 0.05 = \dfrac{1}{1 + K_p} \Rightarrow 1 + K_p = 20 \therefore \underline{K_p = 19}$

$\therefore K = \dfrac{19}{0.16} = \underline{118.8}$

(d) No - the system is unstable.

(e) $-180^\circ + 45^\circ + 5^\circ = 130^\circ$, $\therefore \omega_w = 4 \Rightarrow \omega_{wo} = \underline{0.4}$

$G(j4)H = 0.03077\angle{-133.1^\circ}$

$\therefore \omega_{wp} = \dfrac{0.4}{(118.8)(0.03077)} = \underline{0.1094}$

By computer, $D(z) = \underline{\dfrac{35.6z - 27.97}{z - 0.9364}}$

(f) 5 %

8-15. With the sensor output equal to unity, $c(t) = \dfrac{1}{0.04} = \underline{25\,^\circ C}$

Thus, unity on the input commands 25 °C on the output.

8-15. From Problem 8-14, $c_e(t) = 0.95$, $\therefore\ c(t) = \frac{0.95}{0.04} = \underline{23.75°C}$

$$\text{error} = \frac{25 - 23.75}{25} = 0.05 \Rightarrow \underline{5\%}$$

8-16. (a), (b), (c), (d) See Problem 8-14.

(e) $\omega_{w1} = 4.0$

(8-56) $\theta = 180° + 45° - (-133.1°) = -1.9°$

(8-57) $K_P = \frac{\cos\theta}{0.03077} = \underline{32.48}$

(8-58) $K_I = \frac{(-\sin\theta)(4)}{0.03077} = \underline{4.310}$

(f) $\underline{0\%}$

8-17. (a) With $R(s) = 0$ and $R_d(s) = \frac{1}{s}$, the dc gain in the disturbance path is:

$$\text{dc gain} = \frac{2.5}{0 + 0.5} = \underline{5}$$

The plant dc gain is:

$$\text{dc gain} = \frac{2}{0 + 0.5} = 4$$

Hence

$$c_{ss}(kT) = \frac{(5)(1)}{1 + (4)(0.04)} = \underline{4.31\ °C}$$

(b) The dc gain of the filter is 118.8. Hence

$$c_{ss}(kT) = \frac{5}{1 + (118.8)(4)(0.04)} = \underline{0.25\ °C}$$

(c) The dc gain of a PI filter is ∞ (unbounded).

Hence

$$c_{ss}(kT) = \frac{5}{1 + (\infty)(4)(0.04)} = \underline{0°C}$$

8-18. (a) $0.07|G(j\omega_1)| = 1$, $\therefore\ G(j\omega_1) = 14.3$

$\therefore\ \phi_m \approx 180° - 94° = \underline{86°}$

(b) Choose $\omega_{w1} = 1.0$, $0.07\,G(j\omega_{w1}) = 0.1253\,\underline{/-119.4°}$

By computer: $D(w) = \underline{\dfrac{6.1415w + 10}{1.078w + 1}}$

8-18. (b) and $D(z) = \dfrac{5.888z - 5.0016}{z - 0.9113}$

(c) Choose $\omega_{w1} = 6.0$, $\therefore\ 0.07G(j\omega_1) = 0.007681\angle{-177.74}$

$D(w) = \dfrac{30.164\,w + 10}{0.1614\,w + 1}$ by computer

$D(z) = \dfrac{145.0z - 140.29}{z - 0.5271}$ by computer

(d) Using CTRL,

(b) $t_r = \underline{1.12s}$, $\underline{23\%}$ overshoot, phase lag

(c) $t_r = \underline{0.20s}$, $\underline{15\%}$ overshoot, phase lead

8-19. (a) $\omega_{w1} = 1.0$, By computer: $K_P = 7.688$, $K_I = 2.145$

(b) $\omega_{w1} = 6.0$, By computer: $K_P = 95.6$, $K_D = 14.73$

(c) (a) $t_r = \underline{1.08s}$, $\underline{32\%}$ overshoot

$t_r = \underline{0.14s}$, $\underline{42\%}$ overshoot

8-20. (a) $\phi_m \approx 180^\circ - 130^\circ = 50^\circ$, $\omega_{w1} \cong \underline{4}$

(b) $\angle G(j\omega_{w1}) \approx -180^\circ + 45^\circ + 5^\circ = -130^\circ$, $\therefore\ \omega_{w1} = \underline{4}$

By computer: $D(w) = \dfrac{1.784w + 1}{6.259w + 1}$

$D(z) = \dfrac{0.2880z - 0.2800}{z - 0.9920}$

(c) Choose $\omega_{w1} = 20$.

By computer: $D(w) = \dfrac{0.1968w + 1}{0.0204w + 1}$

$D(z) = \dfrac{4.885z - 3.784}{z + 0.1015}$

(d) phase-lag: $t_r \approx \underline{0.26s}$, 27% overshoot

phase-lead: $t_r \approx \underline{0.05s}$, 30% overshoot

8-21. (a) From Problem 8-20(b): $\omega_{w1} = 4$, $K_p = 0.2863$, $K_I = 0.1140$

(b) From Problem 8-20(c), $\omega_{w1} = 20$ yields $\phi_m \approx 23°$

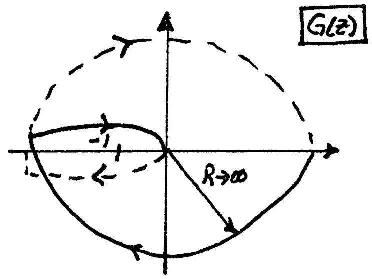
$\therefore$ let $\omega_{w1} = 9$, $K_p = 0.8752$, $K_D = 0.04227$

Then $\phi_m \approx 41°$ and $GM \approx 5.26 \Rightarrow 14.4\,dB$

(c) PI: $t_r \approx \underline{0.28s}$, $\underline{28\%}$ overshoot

PD: $t_r \approx \underline{0.114s}$, $\underline{28\%}$ overshoot

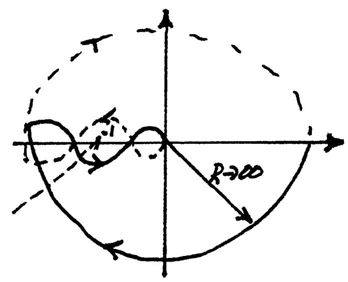
8-22. (a)



$Z = N + P = 2 + 0 = \underline{2}$

$\therefore$ unstable

(b)



$Z = N + P = 0 + 0 = \underline{0}$

$\therefore$ stable

Phase lead must be added, to bring the Nyquist diagram below the -1 point.

(c) $\omega_{w1} = 1$, $\quad 0.02G(j1) = 0.2002\angle{-182.86°}$

(8-32) $\quad \theta = 180° + 45° - (-182.86°) = 47.86°$

(8-33a) $\quad a_1 = \dfrac{1 - (1)(0.2002)\cos\theta}{(1)(0.2002)\sin\theta} = \underline{5.831}$

(8-33b) $\quad b_1 = \dfrac{\cos\theta - 0.2002}{(1)\sin\theta} = \underline{0.6348}$

By computer: $D(z) = \dfrac{8.587z - 8.441}{z - 0.8540}$

(d) $t_r \approx \underline{1.16s}$, $\underline{30\%}$ overshoot

8-23.(a) See Problem 8-22.

8-23.(b) From Problem 8-22 (c), $\omega_{w1} = 1$, $0.02G(1) = 0.2002\angle -182.86^\circ$

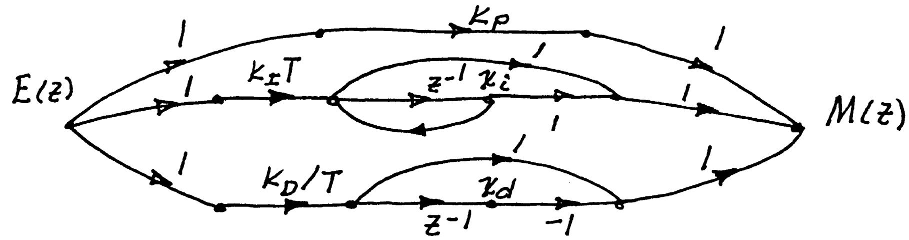
(8-56) $\theta = 180^\circ + 45^\circ - (-182.86^\circ) = 47.86^\circ$

(8-57) $K_P = \dfrac{\cos\theta}{0.2002} = \underline{3.351}$

(8-58) $K_D = \dfrac{\sin\theta}{0.2002} = \underline{3.704}$

(c) $t_r \approx \underline{1.04s}$, $\underline{36\%}$ overshoot

8-24. $D(z) = K_P + K_I T\left[\dfrac{1+z^{-1}}{1-z^{-1}}\right] + \dfrac{K_D}{T}[1 \quad z^{-1}]$



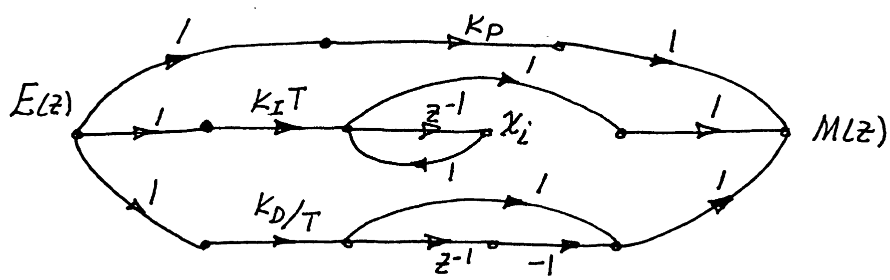
initial conditions on controller : $x_i(0)$, $x_d(0)$

$m(k) = K_p e(k) + K_I T e(k) + 2x_i(k) + \frac{K_D}{T} e(k) - x_d(k)$

$x_i(k+1) = x_i(k) + K_I T e(k)$

$x_d(k+1) = \frac{K_D}{T} e(k)$

8-25. $D(z) = K_P + K_I T\left[\dfrac{1}{1-z^{-1}}\right] + \dfrac{K_D}{T}[1-z^{-1}]$



Controller initial conditions : $x_i(0)$, $x_d(0)$

$m(k) = K_p e(k) + K_I T e(k) + x_i(k) + \frac{K_D}{T} e(k) - x_d(k)$

$x_i(k+1) = K_I T e(k) + x_i(k)$

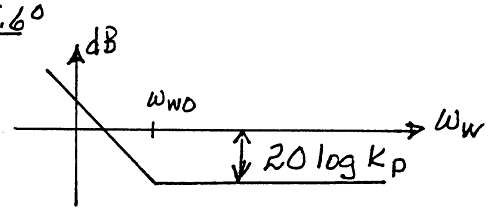
$x_d(k+1) = \frac{K_D}{T} e(k)$

8-26. From Table 8-1, for $\angle G(j\omega_{w1}) = -180^\circ + 60^\circ + 5^\circ = -115^\circ$,

$\omega_{w1} = 0.3$, $G(j0.3) = 3.16 \angle -115.6^\circ$

$\omega_{wo} = 0.1(0.3) = 0.03$

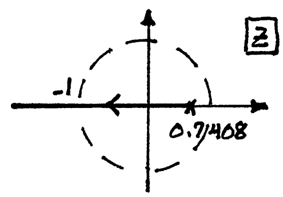
$K_P = \dfrac{1}{3.16} = 0.316$



$K_I = 0.03 K_P = 0.00949$

$D(z) = 0.316 + 0.000237 \dfrac{z+1}{z-1} \Rightarrow \phi_m = 59^\circ$, gain margin = 19dB

8-27. (a) $KG(z)H = \dfrac{0.04147}{z - 0.7408}$



$\therefore K = -\dfrac{1}{G(z)H|_{z=-1}} = 41.98$ $\quad\therefore$ stable for $\underline{K < 41.98}$

(b) $1 + KG(z)H = z - 0.7408 + 0.04147K = 0$

$\therefore z = 0.7408 - 0.04147 = \underline{0.6993}$

$\therefore \tau = \dfrac{-T}{\ln r} = \dfrac{-0.6}{\ln(0.6993)} = \underline{1.677\,s}$

(c) $K = 3 \Rightarrow K_u = 1, K_c = 3$

$\therefore K_d = \underline{\dfrac{1}{3}}$ $\qquad$ Let $z_p = 0.999$

$z_0 = 1 - \dfrac{1 - 0.999}{1/3} = 0.997$

$\therefore D(z) = \dfrac{0.3333(z - 0.997)}{z - 0.999} = \underline{\dfrac{0.3333z - 0.3323}{z - 0.999}}$

(d) $K_p = K\,D(z)G(z)H\Big|_{z=1} = (1)(1)\dfrac{(0.04147)}{1 - 0.7408} = 0.16$

$e_{ss} = \dfrac{1}{1 + 0.48} = 0.676 \Rightarrow \underline{67.6\%}$

(e) roots: $z = \underline{0.7271}$, $\underline{0.9989}$

```
ng = [0 .04147]; dg = [1 -.7408];
nd = [.3333 -.3323]; dd = [1 -.999];
q = conv(ng,nd) + conv(dg,dd)
r=roots(q)
```

8-28 (a) See Problem 8-27.

(b) From Problem 8-27, $z = 0.6993 \Rightarrow \tau = \underline{1.677s}$

(c) From Problem 8-14(c), $\underline{K = 118.8}$, unstable.

(d) $K_c = 118.8$, $K_u = 1$, $\therefore K_D = 1/118.8 = 0.008418$

Let $z_p = 0.99999$

$$\therefore z_0 = 1 - \frac{1 - 0.99999}{0.008418} = 0.998812$$

$$\therefore D(z) = \frac{0.008418(z - 0.998812)}{z - 0.99999} = \frac{0.008418z - 0.008408}{z - 0.99999}$$

(e) $z = 1, 0.7405$ — numerical problems

```
ng = [0 .04147]; dg = [1 -.7408];
nd = [.008418 -.008408]; dd = [1 -.99999];
q = conv(ng,nd) + conv(dg,dd)
r=roots(q)
```

8-29. (a), (b) See Problem 8-27(a), (b)

(c) $G(z)H = \frac{0.04147}{z - 0.7408}$, $z_d = -\epsilon^{-T/\tau} = -\epsilon^{-0.6/0.5} = \underline{0.3012}$

Let $z_0 = 0.7408$:

$$(8\text{-}76) \quad (1)\left(\frac{1 - z_p}{1 - 0.7408}\right)\left(\frac{0.3012 - 0.7408}{0.3012 - z_p}\right)\left(\frac{0.04147}{0.3012 - 0.7408}\right) = -1$$

$$\therefore 0.1560 - 0.1560 z_p = z_p - 0.3012 \Rightarrow z_p = 0.3990$$

$$K_d = \frac{1 - 0.3990}{1 - 0.7408} = 2.319$$

$$\therefore D(z) = \frac{2.319(z - 0.7408)}{z - 0.3990} = \frac{2.319z - 1.718}{z - 0.3990}$$

(d) zeros: $z = 0.7408, 0.3028$

```
ng = [0 .04147]; dg = [1 -.7408];
nd = [2.319 -1.718]; dd = [1 -0.3990];
q = conv(ng,nd) + conv(dg,dd)
r=roots(q)
```

8-30. (b) See Example 7.7, $K = \underline{0.196}$, $z = \underline{0.650}$

(c) $$\tau = \frac{-T}{\ln r} = \frac{-1}{\ln(0.650)} = \underline{2.32s}$$

8-30. (d) 1. $z_a = 0.650$, $K_u = 0.196$

2. $K_c = (1.8)(0.196) = 0.3528$

3. $K_d = \dfrac{K_u}{K_c} = \dfrac{0.196}{(0.196)(1.8)} = 0.5556$

4. $z_p = 0.999$

5. $z_o = 1 - \dfrac{1-0.999}{0.5556} = 0.9982$

$$\therefore D(z) = \frac{0.5556(z-0.9982)}{z-0.999} = \frac{0.5556z - 0.5546}{z-0.999}$$

(e) zeros: $z = 0.9982$, $0.5822 \pm j0.4196$

```
ng = [0 .368 .264]; dg = [1 -1.368 .368];
nd = [.5556 -.5546]; dd = [1 -0.999];
q = conv(ng,nd) + conv(dg,dd)
r=roots(q)
```

8-31. (a) From Example 8.7, $z = \epsilon^{-T/\tau} = \epsilon^{-0.1/2.03} = \underline{0.952}$, $K_u = 0.244$

2. $K_c = 0.5$

3. $K_d = K_u/K_c = 0.488$

4. $z_p = 0.999$

5. $z_o = 1 - \dfrac{1-0.999}{0.488} = 0.99795$

$$\therefore D(z) = \frac{0.488(z-0.99795)}{z-0.999} = \frac{0.488z - 0.487}{z-0.999}$$

(b) zeros: $z = 0.9979$, $0.9518 \pm j0.0471$

```
ng = [0 .004837 .004678]; dg = [1 -1.9048 .9048];
nd = [.488 -.487]; dd = [1 -0.999];
q = conv(ng,nd) + conv(dg,dd)
r=roots(q)
```

CHAPTER 9

Computer verifications using CTRL and CSP.

9-1.

$\zeta = 0.46$; $\tau = 0.5$, $\ln r = -\frac{T}{\tau} = -0.2$, $\therefore r = 0.8187$

$\theta^2 = \ln^2 r/\zeta^2 - \ln^2 r = 0.149$, $\therefore \theta = 0.386 = 22.1^\circ$

$\lambda_{1,2} = 0.8187 \angle \pm 22.1^\circ = 0.759 \pm j0.308$

$\alpha_c(z) = z^2 - 1.518z + 0.6703$

(9-9) $K_1 = 105(0.6703) - 1.518 + 1) = \underline{16.0}$

$K_2 = 14.67 - 5.34(0.6703) - 5.17(1.518) = \underline{3.24}$

9-2. (a) $T = 0.1$, $\tau = 0.8s$, $\zeta = 0.707$

$r = \epsilon^{-T/\tau} = \epsilon^{-0.1/0.8} = 0.8825$

$\theta = -\frac{\ln r}{\zeta}\sqrt{1-\zeta^2} = \frac{0.125}{0.707}(0.707) = 0.125 \Rightarrow 7.162^\circ$

$\therefore$ zeros: $z = 0.8825 \angle \pm 7.162^\circ = 0.8756 \pm j0.1100$

$\therefore \alpha_c(z) = (z - 0.8756 + j0.1100)(z - 0.8756 - j0.1100)$

$= z^2 - 1.7512z + 0.7788 = 0$

model: $\underline{x}(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$

$$\alpha_c(A) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix}\begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} - 1.7512\begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} + 0.7788\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0276 & 0.0147 \\ 0 & 0.0130 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix}\begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} = \begin{bmatrix} 0.0139 \\ 0.0862 \end{bmatrix}$$

$$[B \quad AB]^{-1} = \begin{bmatrix} 0.00484 & 0.0139 \\ 0.0952 & 0.0862 \end{bmatrix}^{-1} = \begin{bmatrix} -95.13 & 15.34 \\ 105.1 & -5.342 \end{bmatrix}$$

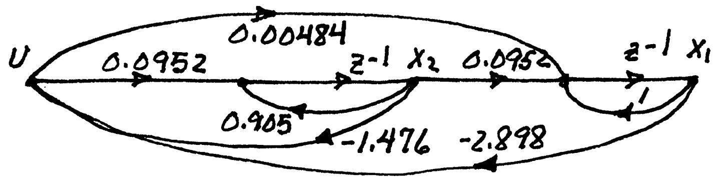
$$\therefore K = [0 \quad 1]\begin{bmatrix} -95.13 & 15.34 \\ 105.1 & -5.342 \end{bmatrix}\begin{bmatrix} 0.0276 & 0.0147 \\ 0 & 0.0130 \end{bmatrix} = \underline{[2.898 \quad 1.476]}$$

(b) $$BK = \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix}[2.898 \quad 1.476] = \begin{bmatrix} 0.0140 & 0.00714 \\ 0.2759 & 0.1405 \end{bmatrix}$$

9.2.(b) $zI-A+BK = \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0.9860 & 0.08806 \\ 0.2759 & 0.7645 \end{bmatrix} = \begin{bmatrix} z-0.9860 & -0.08806 \\ -0.2759 & z-0.7645 \end{bmatrix}$

$\therefore |zI-A+BK| = z^2 - 1.7505z + 0.7538 + 0.2429 = \underline{z^2 - 1.7505z + 0.7781}$

(c)



(d) $\underline{x}(k+1) = A_f\, \underline{x}(k) = (A-BK)\underline{x}(k)$

$a_{f11} = 1 - (2.898)(0.00484) = 0.9860$

$a_{f21} = -(2.898)(0.0952) = -0.2759$

$a_{f12} = 0.0952 - (1.476)(0.00484) = 0.08806$

$a_{f22} = 0.905 - (1.476)(0.0952) = 0.7645$

$\therefore A - BK = \begin{bmatrix} 0.9860 & 0.08806 \\ -0.2759 & 0.7645 \end{bmatrix}$

(e) 5 loops: $L_1 = 0.905z^{-1}$; $L_2 = z^{-1}$; $L_3 = (0.0952)(z^{-1})(-2.898)$
$= -0.1405z^{-1}$

$L_4 = (0.00484)z^{-1}(-2.898) = -0.01403z^{-1}$

$L_5 = (0.0952)z^{-1}(0.0952)z^{-1}(-2.898) = -0.02626z^{-2}$

$\Delta = 1 - L_1 - L_2 - L_3 - L_4 - L_5 + L_1L_2 + L_1L_4 + L_2L_3$

$= \underline{1 - 1.7505z^{-1} + 0.7781z^{-2}}$

9-3.(a) $\tau = 0.4s$, $z = \epsilon^{-T/\tau} = \epsilon^{-0.1/0.4} = 0.7788$

$\alpha_e(z) = (z-0.7788)^2 = \underline{z^2 - 1.5576z + 0.6065}$

$\alpha_e(A) = \begin{bmatrix} 1 & 0.1814 \\ 0 & 0.8190 \end{bmatrix} - \begin{bmatrix} 1.5576 & 0.1486 \\ 0 & 1.4096 \end{bmatrix} + \begin{bmatrix} 0.6065 & 0 \\ 0 & 0.6065 \end{bmatrix}$

$= \begin{bmatrix} 0.0489 & 0.0328 \\ 0 & 0.0159 \end{bmatrix}$

$CA = [1 \quad 0]A = [1 \quad 0.0952]$; $\begin{bmatrix} C \\ CA \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 0.0952 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -10.50 & 10.50 \end{bmatrix}$

$\therefore G = \alpha_e(A)\begin{bmatrix} C \\ CA \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.0489 & 0.0328 \\ 0 & 0.0159 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -10.50 & 10.50 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underline{\begin{bmatrix} 0.3444 \\ 0.1670 \end{bmatrix}}$

(b) $\alpha_e(z) = |zI - (A-GC)|$

$GC = \begin{bmatrix} 0.3444 \\ 0.1670 \end{bmatrix}[1 \quad 0] = \begin{bmatrix} 0.3444 & 0 \\ 0.1670 & 0 \end{bmatrix}$

9-3. (b) $zI - A + GC = \begin{bmatrix} z-1 & -0.0952 \\ 0 & z-0.905 \end{bmatrix} \begin{bmatrix} 0.3444 & 0 \\ 0.1670 & 0 \end{bmatrix} = \begin{bmatrix} z-0.6556 & -0.0952 \\ 0.1670 & z-0.905 \end{bmatrix}$

$\therefore |zI - A + GC| = \underline{z^2 - 1.561z + 0.6092} \approx \alpha_e(z)$

(c) $D_{ce}(z) = K[zI - A + BK + GC]^{-1}G$

$BK = \begin{bmatrix} 0.0140 & 0.00714 \\ 0.2759 & 0.1405 \end{bmatrix}$, from Problem 9-2(b)

$\therefore (zI - A + GC) + BK = \begin{bmatrix} z-0.6556 & -0.0952 \\ 0.1670 & z-0.905 \end{bmatrix} + \begin{bmatrix} 0.0140 & 0.0071 \\ 0.2759 & 0.1405 \end{bmatrix}$

$= \begin{bmatrix} z-0.6416 & -0.0881 \\ 0.4429 & z-0.7645 \end{bmatrix}$; $\therefore \Delta = \underline{z^2 - 1.406z + 0.5295}$

$[zI - A + BK + GC]^{-1} = \dfrac{1}{\Delta} \begin{bmatrix} z-0.7645 & 0.0881 \\ -0.4429 & z-0.7645 \end{bmatrix}$

$\therefore D_{ce}(z) = \dfrac{1}{\Delta} [2.898 \quad 1.476] \begin{bmatrix} z-0.7645 & 0.0881 \\ -0.4429 & z-0.6416 \end{bmatrix} \begin{bmatrix} 0.3444 \\ 0.1670 \end{bmatrix}$

$= [2.898z - 2.869 \quad 1.476z - 0.6917] \dfrac{1}{\Delta} \begin{bmatrix} 0.3444 \\ 0.1670 \end{bmatrix} = \underline{\dfrac{1.245z - 1.104}{z^2 - 1.406z + 0.5295}}$

(d) $1 + D_{ce}(z)\,G(z) = 0 = 1 + \dfrac{1.245z - 1.104}{z^2 - 1.406z + 0.5295} \times \dfrac{0.00484z + 0.00468}{z^2 - 1.905z + 0.905}$

$\Rightarrow z^4 - 3.311z^3 + 4.113z^2 - 2.280z + 0.4740 = 0$

$\alpha_c(z)\alpha_e(z) = (z^2 - 1.9512z + 0.7788)(z^2 - 1.5576z + 0.6065)$

$= z^4 - 3.309z^3 + 4.113z^2 - 2.275z + 0.4723 = 0$

9-4. (a) From Problem 9-3(a), $\alpha_e(z) = z - 0.7788$

From (9-57), $A_{aa} = 1 \quad A_{ab} = 0.0952 \quad B_a = 0.00484$

$A_{ba} = 0 \quad A_{bb} = 0.905 \quad B_b = 0.0952$

(9-64) $G = \alpha_e(A_{bb})(A_{ab})^{-1}(1) = \dfrac{0.905 - 0.7788}{0.0952} = \underline{1.326}$

(b) $\alpha_e(z) = |zI - A_{bb} + GA_{ab}| = z - 0.905 + (1.326)(0.0952)$

$= \underline{z - 0.7788}$

(c) $[zI - A_{bb} + GA_{ab} - (B_b - GB_a)K_b]^{-1}$

$= [z - 0.905 + 0.1262 + [0.0952 - 0.00642]1.476]^{-1} = (z - 0.6478)^{-1}$

9-4(c) $Gz + \{A_{ba} - GA_{aa} - K_1(B_b - GB_a)\}$

$= 1.326z + \{0 - 1.326 - 2.898(0.08878)\} = 1.326z - 1.5833$

(9-66) $D_{ce}(z) = 2.898 + \dfrac{1.476(1.326z - 1.5833)}{z - 0.6478} = \dfrac{4.855z - 4.214}{z - 0.6478}$

(d) $1 + D_{ce}(z)G(z) = 1 + \left(\dfrac{4.855z - 4.214}{z - 0.6478}\right)\left(\dfrac{0.00484z + 0.00468}{z^2 - 1.905z + 0.905}\right) = 0$

$= z^3 - 2.530z^2 + 2.137z - 0.606 = 0$

$\alpha_c(z)\alpha_e(z) = (z^2 - 1.7512z + 0.7788)(z - 0.7788)$

$= z^3 - 2.53z^2 + 2.143z - 0.6065 = 0$ ✓

9-5. (a) $\alpha_e(z) = z^2 - 1.5576z + 0.6065$

$\alpha_e(A) = \begin{bmatrix} 0.0489 & 0.0328 \\ 0 & 0.0159 \end{bmatrix}$ } From Problem 9-3(a)

$G = \alpha_e(A)\begin{bmatrix} CA \\ CA^2 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.3297 \\ 0.1841 \end{bmatrix}$, by computer

(b) $GCA = \begin{bmatrix} 0.3297 \\ 0.1841 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} = \begin{bmatrix} 0.3297 & 0.03139 \\ 0.1841 & 0.01727 \end{bmatrix}$

$|zI - A + GCA| = \begin{vmatrix} z - 0.6703 & -0.06381 \\ 0.1841 & z - 0.8877 \end{vmatrix} = \underline{z^2 - 1.558z + 0.6066} = \alpha_e(z)$ ✓

(c) By computer: $D_{ce}(z) = \dfrac{1.226z^2 - 1.085z}{z^2 - 1.410z + 0.5220}$

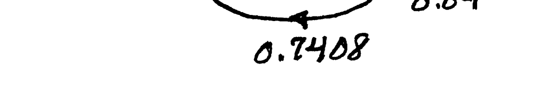
$1 + D_{ce}(z)G(z) = 0 = 1 + \left(\dfrac{1.226z^2 - 1.085z}{z^2 - 1.410z + 0.5220}\right)\left(\dfrac{0.00484z + 0.00468}{z^2 - 1.905z + 0.905}\right)$

$= z^4 - 3.309z^3 + 4.1127z^2 - 2.2756z + 0.4724$

$\alpha_c(z)\alpha_e(z) = (z^2 - 1.7512z + 0.7788)(z^2 - 1.5576z + 0.6065)$

$= z^4 - 3.309z^3 + 4.113z^2 - 2.2746z - 0.4723 = 0$ ✓

9-6. (a) $G(z)H(z) = \dfrac{1.0368}{z - 0.7408}(0.04)$



$x(k+1) = 0.7408x(k) + 1.0368u(k)$

$y(k) = 0.04x(k)$

9-6. (b) $1+G(z)H = 1 + \dfrac{0.04147}{z-0.7408} = 0 \Rightarrow z - 0.6993 = 0 \Rightarrow z = 0.6993$

$\therefore \tau = \dfrac{-T}{\ln r} = \dfrac{-(0.6)}{\ln 0.6993} = \underline{1.677s}$

(c) $z = \epsilon^{-T/\tau} = \epsilon^{-0.6/1} = 0.5488, \therefore \alpha_c(z) = \underline{z - 0.5488}$

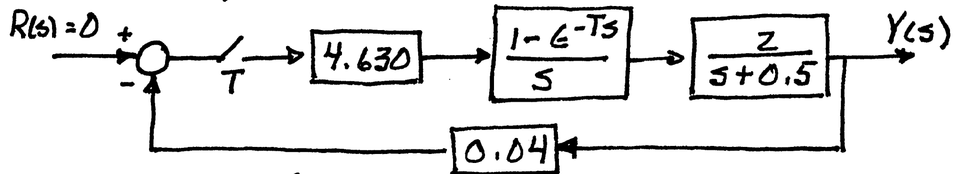
(9-25) $K = (1)(B)^{-1}[\alpha_c(A)] = (1.0368)^{-1}(0.7408 - 0.5488)$

$= \underline{0.1852}$

(d) (9-15) $\alpha_c(z) = |zI - (A - BK)| = z - [0.7048 - (1.0368)(0.1852)]$

$= \underline{z - 0.5488}$ ✓

(e) $K_1 = \dfrac{0.1852}{0.04} = 4.630$



(f) $G(z) = \dfrac{1.0368}{z - 0.7408}$

$1 + KG(z)H = 1 + \dfrac{(4.630)(1.0368)(0.04)}{z - 0.7408} = 1 + \dfrac{0.1920}{z - 0.7408} = 0$

$\therefore z - 0.7408 + 0.1920 = z - 0.5488 = \alpha_c(z)$ ✓

9-7. From Problem 9-6(a): $x(k+1) = \overset{A}{0.7408}x(k) + \overset{B}{1.0368}u(k)$

$y(k) = \overset{C}{0.04}x(k)$

(a) $\tau = 0.5s,\ z = \epsilon^{-T/\tau} = \epsilon^{-0.6/0.5} = 0.3012, \therefore \alpha_e(z) = \underline{z - 0.3012}$

(9-48) $G = \alpha_e(A)(C)^{-1}(1) = (0.7408 - 0.3012)(\frac{1}{0.04})(1) = \underline{10.99}$

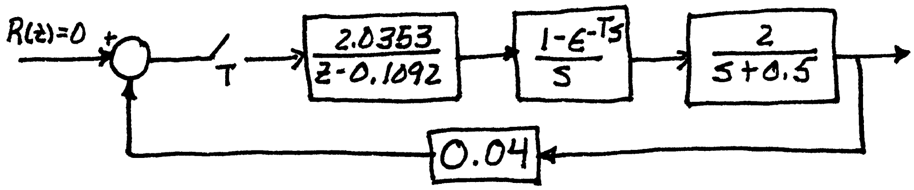
(b) $zI - A + GC = z - 0.7408 + (10.99)(0.04) = z - 0.3012$ ✓

(c) $D_{ce}(z) = K(zI - A + BK + GC)^{-1}G$

$= 0.1852\left(z - 0.7408 + (1.0368)(0.1852) + (10.99)(0.04)\right)^{-1}(10.99)$

$= \underline{\dfrac{2.0353}{z - 0.1092}}$

(d)



9-7.(e) $1 + D_{ce}(z)\,G(z)\,H = 1 + \left(\frac{2.0353}{z-0.1092}\right)\left(\frac{1.0368}{z-0.7408}\right)(0.04) = 0$

$\therefore\ z^2 - 0.8500z + 0.08090 + 0.08440$

$= z^2 - 0.8500z + 0.1653 = (z-0.5488)(z-0.3012) = 0$

$= \alpha_c(z)\,\alpha_e(z)$

(f) Since the system is first order, a reduced-order estimator is zeroth order. Hence no dynamic reduced-order estimator can be designed.

9-8.(a) From Problem 9-7(a), $x(k+1) = \overset{A}{0.7408}\,z + \overset{B}{1.0368}\,u(k)$

$y(k) = \overset{C}{0.04}\,x(k)$

$\alpha_e(z) = z - 0.312$

(9-72) $G = \alpha_e(A)\,[CA]^{-1}(1)$

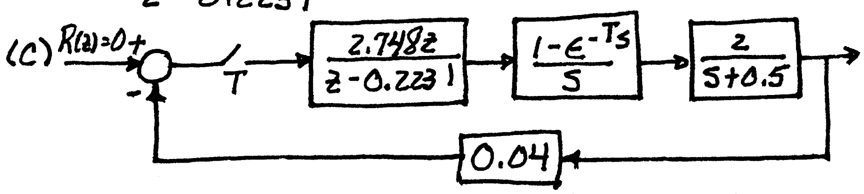
$= (0.7408 - 0.3012)\dfrac{1}{(0.04)(0.7408)} = \underline{14.84}$

(b) $D_{ce}(z) = Kz\left[zI - A + GCA + BK - GCBK\right]^{-1}G$

$= (0.1852)z\left[z - 0.7408 + (14.84)(0.04)(0.7408) + (1.0368)(0.1852) - (14.84)(0.04)(1.0368)(0.1852)\right]^{-1}(14.84)$

$= \dfrac{2.748z}{z - 0.2231}$

(c)



(d) $1 + D_{ce}(z)\,G(z)\,H = 1 + \left(\frac{2.748z}{z-0.2231}\right)\left(\frac{1.0368}{z-0.7408}\right)(0.04) = 0$

$\Rightarrow\ z^2 - 0.9639z + 0.1653 + 0.1140z$

$= z^2 - 0.8499z + 0.1653 = (z-0.5488)(z-0.3012) = \alpha_c(z)\,\alpha_e(z) = 0$

(e) See Problem 9-7(f).

9-9.(a) $\begin{bmatrix} x(k+1) \\ q(k+1) \end{bmatrix} = \begin{bmatrix} A & -BK \\ GC & A-GC-BK \end{bmatrix}\begin{bmatrix} x(k) \\ q(k) \end{bmatrix}$

9-9(a)

$$= \begin{bmatrix} 0.7408 & -(1.0368)(0.1852) \\ (10.99)(0.04) & 0.7408-0.4396-0.1920 \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix}$$

$$= \begin{bmatrix} 0.7408 & -0.1920 \\ 0.4396 & 0.1092 \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix}$$

(b) $$|zI-A_f| = \begin{vmatrix} z-0.7408 & 0.1920 \\ -0.4396 & z-0.1092 \end{vmatrix} = \underline{z^2-0.8500z+0.1653=0}$$

(Checks Prob. 9-7(d))

(c) $$\begin{bmatrix} x(k+1) \\ q(k+1) \end{bmatrix} = \begin{bmatrix} A & -BK \\ GCA & A-GCA-BK \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix}$$

$$= \begin{bmatrix} 0.7408 & -(1.0368)(0.1852) \\ (14.84)(0.04)(0.7408) & 0.7408-0.4397-.1920 \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix}$$

$$= \begin{bmatrix} 0.7408 & -0.1920 \\ 0.4397 & 0.1091 \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix}$$

$$\therefore \quad |zI-A_f| = \begin{vmatrix} z-0.7408 & 0.1920 \\ -0.4397 & z-0.1091 \end{vmatrix} = \underline{z^2-0.8499z+0.1652=0}$$

(Checks Prob. 9-8(d))

9-10. (a) See Problem 9-6 solution.

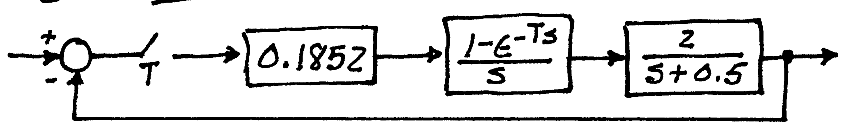
(a) $x(k+1) = 0.7408\,x(k) + 1.0368\,u(k)$

$y(k) = x(k)$

(b) $\tau = \underline{1.677s}$

(c) $K = (4.632)(0.04) = \underline{0.1852}$; $\alpha_c(z) = z - 0.5488$

(d) $\alpha_c(z) = \underline{z-0.5488}$

(e) 

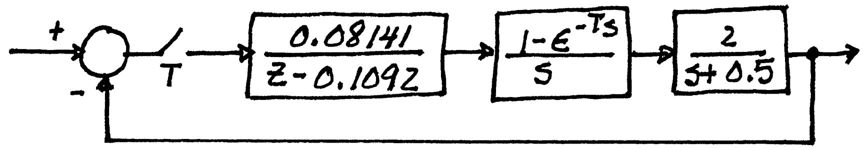
(f) $1 + KG(z) = 0 = z - 0.5488 = \alpha_c(z)$

(b) See Problem 9-7 solution.

(a) $G = (10.99)(0.04) = \underline{0.4264}$, $\alpha_e(z) = z - 0.3012$

(b) $$D_{ce} = \left(\frac{2.0353}{z-0.1092}\right)(0.04) = \underline{\frac{0.08141}{z-0.1092}}$$

9-10. (b) (c)



(d) $z^2 - 0.8500z + 0.1653 = (z-0.5488)(z-0.3012) = \alpha_c(z)\alpha_e(z)$

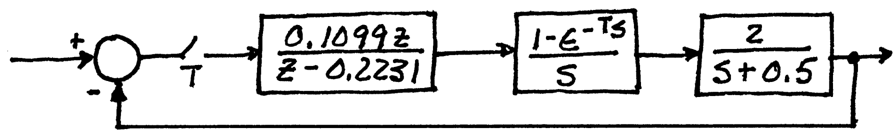
(e) See Problem 9-7(e).

(c) See Problem 9-8 solution.

(a) $\alpha_e(z) = z - 0.3012$, $G = (14.84)(0.04) = \underline{0.5936}$

(b) $D_{ce} = \frac{2.748z}{z-0.2231}(0.04) = \underline{\frac{0.1099z}{z-0.2231}}$

(c)



(d) $z^2 - 0.8499z + 0.1653 = (z-0.5488)(z-0.3012) = \alpha_c(z)\alpha_e(z)$

(d) $(GC)_{new} = G_{old}(0.04)\frac{C_{old}}{0.04} = (GC)_{old}$; $K_{new} = K_{old}$;

$(BK)_{new} = (BK)_{old}$

$\therefore$ Problem solution unchanged from Problem 9-9.

9-11. (a) $1 + KG(z)H_k = 0 = 1 + (1)\frac{0.125(z-1)}{z^2-2z+1}(1) = 0$

$\therefore z^2 - 1.875z + 1.125 = 0$

zeros: $z = 0.9375 \pm j0.4961 = \underline{1.061} \; \underline{/\pm 27.9^\circ}$, $\therefore$ $\underline{\text{unstable}}$

(b) $r = \epsilon^{-T/\tau} = \epsilon^{-1/4} = 0.7788$

$\theta = -\frac{\ln r}{\zeta}\sqrt{1-\zeta^2} = \frac{-(-0.25)}{0.707}(0.707) = 0.25 \Rightarrow \underline{14.32^\circ}$

$\therefore$ zeros: $z = 0.7788 \underline{/\pm 14.32^\circ} = 0.7546 \pm j0.1926$

$\therefore \alpha_c(z) = (z - 0.7546 + j0.1926)(z - 0.7546 - j0.1926)$

$= \underline{z^2 - 1.5092z + 0.6065}$

$$\alpha_c(A) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - 1.5092\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + 0.6065\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.0973 & 0.4908 \\ 0 & 0.0973 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0.125 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.375 \\ 0.25 \end{bmatrix}$$

9-11. (b) $[B \;\; AB]^{-1} = \begin{bmatrix} 0.125 & 0.375 \\ 0.25 & 0.25 \end{bmatrix}^{-1} = \begin{bmatrix} -4 & 6 \\ 4 & -2 \end{bmatrix}$

$$\therefore \; K = [0 \;\; 1][B \;\; AB]^{-1}\alpha_c(A) = [0 \;\; 1]\begin{bmatrix} -4 & 6 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 0.0973 & 0.4908 \\ 0 & 0.0973 \end{bmatrix}$$

$$= \underline{[0.3893 \;\; 1.769]}$$

(c) $zI - A + BK = \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.04866 & 0.2211 \\ 0.09732 & 0.4422 \end{bmatrix}$

$$= \begin{bmatrix} z - 0.9513 & -0.7789 \\ 0.09732 & z - 0.5578 \end{bmatrix}$$

$$\therefore \; |zI - A + BK| = z^2 - 1.5091z + 0.6064 = \alpha_c(z)$$

9-12. (a) $\tau = 2s$; $z = \epsilon^{-T/\tau} = \epsilon^{-1/2} = 0.6065$

$$\therefore \; \alpha_e(a) = (z - 0.6065)^2 = z^2 - 1.213z + 0.3678$$

$$\alpha_e(A) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1.213 & -1.213 \\ 0 & -1.213 \end{bmatrix} + \begin{bmatrix} 0.3678 & 0 \\ 0 & 0.3678 \end{bmatrix} = \begin{bmatrix} 0.1548 & 0.7870 \\ 0 & 0.1548 \end{bmatrix}$$

$$CA = [1 \;\; 0]\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = [1 \;\; 1]$$

$$\begin{bmatrix} C \\ CA \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\therefore \; G = \alpha_e(A)\begin{bmatrix} C \\ CA \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.1548 & 0.7870 \\ 0 & 0.1548 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underline{\begin{bmatrix} 0.7870 \\ 0.1548 \end{bmatrix}}$$

(b) $\alpha_e(z) = |zI - (A - GC)|$

$$GC = \begin{bmatrix} 0.7870 \\ 0.1548 \end{bmatrix}[1 \;\; 0] = \begin{bmatrix} 0.7870 & 0 \\ 0.1548 & 0 \end{bmatrix}$$

$$|zI - A + GC| = \left| \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.7870 & 0 \\ 0.1548 & 0 \end{bmatrix} \right| = \begin{vmatrix} z - 0.213 & -1 \\ 0.1548 & z - 1 \end{vmatrix}$$

$$= z^2 - 1.213z + 0.3678 = \alpha_c(z)$$

(c) $D_{ce} = K[zI - A + BK + GC]^{-1}G$

$$BK = \begin{bmatrix} 0.125 \\ 0.25 \end{bmatrix}[0.3893 \;\; 1.769] = \begin{bmatrix} 0.04866 & 0.2211 \\ 0.09732 & 0.4423 \end{bmatrix}$$

$$\therefore \; zI - A + BK + GC = \begin{bmatrix} z-1 & -1 \\ 0 & z-1 \end{bmatrix} + \begin{bmatrix} 0.04866 & 0.2211 \\ 0.09732 & 0.4423 \end{bmatrix} + \begin{bmatrix} 0.7870 & 0 \\ 0.1548 & 0 \end{bmatrix}$$

9-12.(c) $= \begin{bmatrix} z-0.1643 & -0.7289 \\ 0.2521 & z-0.5577 \end{bmatrix}$

$\therefore |zI-A+BK+GC| = \underline{z^2-0.7220z+0.2880} = \Delta$

$\therefore D_{ce}(z) = \begin{bmatrix} 0.3893 & 1.267 \end{bmatrix} \frac{1}{\Delta} \begin{bmatrix} z-0.5577 & 0.7289 \\ -0.2521 & z-0.1643 \end{bmatrix} \begin{bmatrix} 0.7870 \\ 0.1548 \end{bmatrix}$

$= \dfrac{0.5784z-0.5197}{z^2-0.7220z+0.2880}$

(d) $1+D_{ce}(z)G(z) = 0 = 1 + \left(\dfrac{0.5784z-0.5197}{z^2-0.7220z+0.2880}\right)\left(\dfrac{0.125(z+1)}{z^2-2z+1}\right)$

$\Rightarrow \underline{z^4-2.722z^3+2.797z^2-1.290z+0.2232=0}$

$\alpha_c(z)\alpha_e(z) = (z^2-1.213z+0.3678)(z^2-1.509z+0.6065)$

$= z^4-2.722z^3+2.805z^2-1.291z+0.2231$ ✓

9-13. (a) From Problem 9-12, $\alpha_e(z) = z-0.6065$

From (9-57), $A_{aa}=1$ $\quad A_{ab}=1$ $\quad B_a=0.125$

$A_{ba}=0$ $\quad A_{bb}=1$ $\quad B_b=0.25$

(9-64) $G = \alpha_e(A_{bb})(A_{ab})^{-1}(1) = \dfrac{1-0.6065}{1} = \underline{0.3935}$

(b) $\alpha_e(z) = |zI-A_{bb}+GA_{ab}| = z-1+0.3935 = \underline{z-0.6065}$

(c) $[zI-A_{bb}+GA_{ab}+(B_b-GB_a)K_b]^{-1}$

$= (z-0.6065+[0.25-(0.3935)(0.125)]1.769)^{-1} = (z-0.2513)^{-1}$

$Gz + \{A_{ba}-GA_{aa}-K_1(B_b-GB_a)\}$

$= 0.3935z + \{0-0.3935-0.3893(0.2008)\} = 0.3935z-0.4717$

$\therefore$ (9-66): $D_{ce}(z) = 0.3893 + \dfrac{1.769(0.3935z-0.4717)}{z-0.2513} = \underline{\dfrac{1.085z-0.9323}{z-0.2513}}$

(d) $1+D_{ce}(z)G(z) = 0 = \left(\dfrac{1.085z-0.9323}{z-0.2513}\right)\left(\dfrac{0.125(z+1)}{z^2-2z+1}\right)$

$\Rightarrow \underline{z^3-2.116z^2+1.5217z-0.3678=0}$

$\alpha_c(z)\alpha_e(z) = (z^2-1.5092z+0.6065)(z-0.6065)$

$= z^3-2.116z^2+1.5218z-0.3678 = 0$ ✓

9-14.(a) From Problem 9-12: $\alpha_e(z) = z^2 - 1.213z + 0.3678$

By computer: $G = \begin{bmatrix} 0.6322 \\ 0.1548 \end{bmatrix}$

(b) $GCA = \begin{bmatrix} 0.6322 \\ 0.1548 \end{bmatrix} [1 \quad 0] \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.6322 & 0.6322 \\ 0.1548 & 0.1548 \end{bmatrix}$

$\therefore |zI - A + GCA| = \left| \begin{bmatrix} z-1 & -1 \\ 0 & z-1 \end{bmatrix} + \begin{bmatrix} 0.6322 & 0.6322 \\ 0.1548 & 0.1548 \end{bmatrix} \right| = \begin{vmatrix} z-0.3678 & -0.3678 \\ 0.1548 & z-0.8452 \end{vmatrix}$

$= z^2 - 1.213z + 0.6378 = \alpha_e(z)$

(c) By computer: $D_{ce}(z) = \dfrac{0.5198z^2 - 0.4596z}{z^2 - 0.7872z + 0.2231}$

(d) $1 + D \quad (z)G(z) = 1 + \left(\dfrac{0.5198z^2 - 0.4596z}{z^2 - 0.7872z + 0.2231}\right)\left(\dfrac{0.125(z+1)}{z^2 - 2z + 1}\right) = 0$

$\Rightarrow \underline{z^4 - 2.722z^3 + 2.805z^2 - 1.291z + 0.2231 = 0}$

This result checks $\alpha_c(z)\alpha_e(z)$ calculated in Problem 9-12(d).

9-15. (a) $q(k+1) = (A_{bb} - GA_{ab})q(k) + Gy(k+1) + (A_{ba} - GA_{aa})y(k)$ (9-62)

$= [1 - (0.3935)(1)]q(k) + 0.3935y(k+1) + [0 - 0.3935]y(k)$

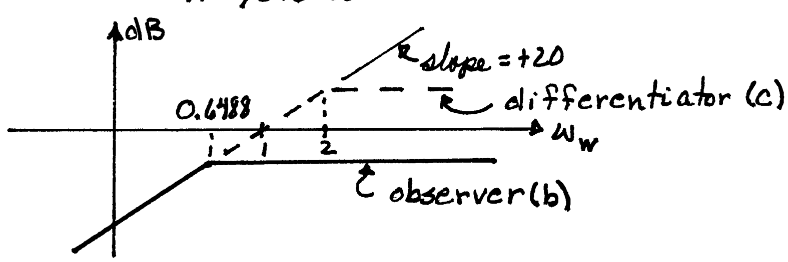
$= 0.6065q(k) + 0.3935[y(k+1) - y(k)]$

$\therefore \dfrac{Q(z)}{Y(z)} = \dfrac{0.3935(z-1)}{z - 0.6065} = D_d(z)$

(b) $z = \dfrac{1 + \frac{T}{2}w}{1 - \frac{T}{2}w} = \dfrac{1 + 0.5w}{1 - 0.5w}$

$\therefore D_d(z) = \dfrac{0.3935(1 + 0.5w \quad 1 + 0.5w)}{1 + 0.5w - 0.6065(1 - 0.5w)} = \dfrac{0.3935w}{0.3935 + 0.6065w}$

$= \dfrac{w}{1 + w/0.6488}$



9-15. (c) $D_d(z) = \frac{z-1}{z} \Rightarrow D_d(w) = \frac{1+0.5w-1+0.5w}{1+0.5w} = \underline{\frac{w}{1+w/2}}$

(d) The difference is in the high-frequency gain. The observer rejects high-frequency noise more than the differentiator.

9-16. (9-62) $q_b(k+1) = (A_{bb} - GA_{ab})q_b(k) + Gy(k+1) + (A_{ba} - GA_{aa})y(k)$

$$+ (B_b - GB_a)(-K_1 y(k) - K_b q_b(k))$$

$$= [A_{bb} - GA_{ab} - (B_b - GB_a)K_b]q_b(k) + G\,y(k+1)$$

$$+ [A_{ba} - GA_{aa} - K_1(B_b - GB_a)]\,y(k)$$

$$\therefore Q_b(z) = [zI - A_{bb} + GA_{ab} + (B_b - GB_a)K_b]^{-1}[Gz$$

$$+ \{A_{ba} - GA_{aa} - K_1(B_b - GB_a)\}]\,Y(z)$$

$$\therefore D_{ce}(z) = -\frac{U(z)}{Y(z)} = \frac{K_1 Y(z) + K_b Q_b(z)}{Y(z)}$$

$$= K_1 + K_b[zI - A_{bb} + GA_{ab} + (B_b - GB_a)K_b]^{-1}[Gz$$

$$+ \{A_{ba} - GA_{aa} - K_1(B_b - GB_a)\}]$$

9-17. (a) $\bar{q}(k+1) = Aq(k) + Bu(k)$

$$q(k+1) = \bar{q}(k+1) + G[y(k+1) - C\bar{q}(k+1)]$$

$$\therefore q(k+1) = Aq(k) + Bu(k) + Gy(k+1) - GC[Aq(k) + Bu(k)]$$

$$= [A - GCA]q(k) + [B - GCB]u(k) + Gy(k+1)$$

$$\therefore zQ(z) = [A - GCA]Q(z) + [B - GCB]U(z) + zGY(z)$$

Thus $Q(z) = [zI - A + GCA]^{-1}[(B - GCB) + zGC(zI - A)^{-1}B]U(z)$

$$= (zI - A)^{-1}BU(z)$$

$$\therefore (zI - \ )^{-1}B = [zI - A + GCA]^{-1}[(B - GCB) + zGC(zI - A)^{-1}B]$$

$$[I - \{zI - A + GCA\}^{-1}zGC](zI - A)^{-1}B = [zI - A + GCA]^{-1}(B - GCB)$$

$$[zI - A + GCA]^{-1}[zI - A + GCA - zGC](zI - A)^{-1}B$$

$$= [zI - A + GCA]^{-1}(B - GCB)$$

$$\therefore [(I - GC)z - (I - GC)A](zI - A)^{-1}B = (I - GC)B$$

9-17. (a) $(I-GC)(zI-A)(zI-A)^{-1}B=(I-GC)B$

$$(I-GC)B=(I-GC)B$$

(b) $\bar{q}(k+1)=Aq(k)+Bu(k)$, $\quad u(k)=-Kq(k)$

$$q(k+1)=\bar{q}(k+1)+G[y(k+1)-C\bar{q}(k+1)]$$
$$=Aq(k)+Bu(k)+Gy(k+1)-GCAq(k)-GCBu(k)$$
$$=[A-GCA]q(k)+[B-GCB]u(k)+Gy(k+1)$$
$$zQ(z)=[A-GCA]Q(z)+[B-GCB][-KQ(z)]+GzY(z)$$
$$\therefore\ [zI-A+GCA+BK-GCBK]Q(z)=GzY(z)$$
$$Q(z)=[zI-A+GCA+BK-GCBK]^{-1}GzY(z)$$
$$U(z)=-KQ(z)=-D_{ce}(z)Y(z)$$
$$\therefore\ D_{ce}(z)=zK[zI-A+GCA+BK-GCBK]^{-1}G$$

9-18. $\underline{x}(k+1)=A\underline{x}(k)+B\underline{u}(k)$, $\quad \underline{y}(k)=C\underline{x}(k)$

$$\underline{y}(0)=C\underline{x}(0)$$
$$\underline{y}(1)=C\underline{x}(1)=CA\underline{x}(0)+CB\underline{u}(0)$$
$$\underline{x}(2)=A\underline{x}(1)+B\underline{u}(1)=A^2\underline{x}(0)+AB\underline{u}(0)+B\underline{u}(1)$$
$$\underline{y}(2)=C\underline{x}(2)=CA^2\underline{x}(0)+CAB\underline{u}(0)+CB\underline{u}(1)$$
$$\cdots$$
$$\underline{y}(N-1)=CA^{N-1}\underline{x}(0)+CA^{N-2}B\underline{u}(0)+\cdots+CB\underline{u}(N-2)$$

$$\therefore\ \begin{bmatrix}\underline{y}(0)\\ \underline{y}(1)-CB\underline{u}(0)\\ \cdots\\ \underline{y}(N-1)-CA^{N-2}B\underline{u}(0)-\cdots-CB\underline{u}(N-2)\end{bmatrix}=\begin{bmatrix}C\\ CA\\ \cdots\\ CA^{N-1}\end{bmatrix}\underline{x}(0)$$

Since the left side is known, the argument is then the same as for (9-80).

9-19. (a) $A=\begin{bmatrix}1 & 0.0952\\ 0 & 0.905\end{bmatrix}$, $B=\begin{bmatrix}0.00484\\ 0.0952\end{bmatrix}$; $C=[0\quad 1]$

$$CA=[0\quad 0.905]$$

$$\begin{bmatrix}C\\ CA\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0.905\end{bmatrix}\qquad \therefore \text{ unobservable}$$

9-19. (b) Position cannot be determined from only velocity measurements,

9-20. (a) $\underline{x}(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0.125 \\ 0.25 \end{bmatrix} u(k)$

$y(k) = [0 \quad 1] \underline{x}(k)$

$CA = [0 \quad 1] \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = [0 \quad 1]$

$\begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ Inverse doesn't exist, $\therefore$ the system is <u>not</u> observable.

(b) $x(t)$ = position, $\dot{x}(t) = v(t)$ = velocity

$\therefore v(t) = \dfrac{dx}{dt} = x(t) = \int_{t_1}^{t_2} v(t)dt + x(t_1)$

<u>$x(t_1)$</u> cannot be determined from $v(t)$.

9-21. (a) From Problem 9-6: $x(k+1) = \overset{A}{0.7408}\, x(k) + \overset{B}{1.0368}\, u(k)$

$y(k) = \underset{C}{0.04}\, x(k)$

(9-81): $[B \quad AB \cdots A^{n-1}B] = B = 1.0368$

$B^{-1} = \dfrac{1}{1.0368}$, <u>$\therefore$ controllable</u>

(b) $x(k+1) = 0.7408\, x(k) + 1.0368\, u(k)$

$q(k+1) = 0.3012\, q(k) + 10.99[0.04 x(k)] + 1.0368\, u(k)$

$= 0.4396\, x(k) + 0.3012\, q(k) + 1.0368\, u(k)$

$\therefore \begin{bmatrix} x(k+1) \\ q(k+1) \end{bmatrix} = \begin{bmatrix} 0.7408 & 0 \\ 0.4396 & 0.3012 \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix} + \begin{bmatrix} 1.0368 \\ 1.0368 \end{bmatrix} u(k)$

$AB = \begin{bmatrix} 0.7408 & 0 \\ 0.4396 & 0.3012 \end{bmatrix} \begin{bmatrix} 1.0368 \\ 1.0368 \end{bmatrix} = \begin{bmatrix} 0.7681 \\ 0.7681 \end{bmatrix}$

$\therefore [B \quad AB] = \begin{bmatrix} 1.0368 & 0.7681 \\ 1.0368 & 0.7681 \end{bmatrix}$ Inverse doesn't exist. Hence the system is <u>not</u> controllable.

(c) In (a), $\underline{x}(k)$ is controllable. Hence $q(k)$ is not controllable.

9-22. (a) $x(k+1) = Ax(k) + Bu(k)$; $y(k) = Cx(k)$

$q(k+1) = (A-GC)q(k) + GCx(k) + Bu(k)$

$$\therefore \begin{bmatrix} x(k+1) \\ q(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ GC & A-GC \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} u(k)$$

(b) $$A_t B_t = \begin{bmatrix} A & 0 \\ GC & A-GC \end{bmatrix} \begin{bmatrix} B \\ B \end{bmatrix} = \begin{bmatrix} AB \\ (GC+A-GC)B \end{bmatrix} = \begin{bmatrix} AB \\ AB \end{bmatrix}$$

$$\therefore [B_t \quad A_t B_t] = \begin{bmatrix} B & AB \\ B & AB \end{bmatrix}$$ $\therefore$ Uncontrollable, since the inverse doesn't exist.

9-23. (a) $x(k+1) = Ax(k) + Bu(k)$; $y(k) = Cx(k)$

(9-68) $q(k+1) = (A-GCA)q(k) + [B - GCB]u(k) + Gy(k+1)$

$\therefore \quad Gy(k+1) = GCx(k+1) = GCAx(k) + GCBu(k)$

$\therefore \quad q(k+1) = (A-GCA)q(k) + (B-\cancel{GC}B)u(k) + GCAx(k) - \cancel{GCBu(k)}$

$$\therefore \begin{bmatrix} x(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ GCA & A-GCA \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} u(k)$$

$$\therefore A_t B_t = \begin{bmatrix} A & 0 \\ GCA & A-GCA \end{bmatrix} \begin{bmatrix} B \\ B \end{bmatrix} = \begin{bmatrix} AB \\ AB \end{bmatrix}$$

$$\therefore [B_t \quad A_t B_t] = \begin{bmatrix} B & AB \\ B & AB \end{bmatrix}$$ $\therefore$ Uncontrollable, since the inverse doesn't exist.

9-24. $Y(z) = C(zI-A)^{-1}BU(z) = \dfrac{CB}{z-A}U(z)$, for a 1st order system.

From (9-38): $zQ(z) = (A-GC)Q(z) + GY(z) + BU(z)$

$$\therefore (z-A+GC)Q(z) = \left[G\frac{CB}{z-A} + B\right]U(z) = \left(\frac{B(z-A+GC)}{z-A}\right)U(z)$$

(1) $\therefore Q(z) = \dfrac{B}{z-A}U(z)$

From (9-38), the mode of $q(k)$ is $(A-GC)^k$. Hence, from (1), this mode is not excited by $u(k)$, since $(z-A+GC)$ doesn't appear in the denominator.

9-25. $Y(z) = C(zI-A)^{-1}BU(z) = \dfrac{CB}{z-A}U(z)$, for a 1st order system.

From Problem 9-23:

$$zQ(z) = (A-GCA)Q(z) + (B-GCB)U(z) + GzY(z)$$

$$(z-A+GCA)Q(z) = \left[Gz\left(\frac{CB}{z-A}\right) + B - GCB\right]U(z)$$

$$= \left[\frac{GCBz + Bz - AB - GCBz + GCBA}{z-A}\right]U(z)$$

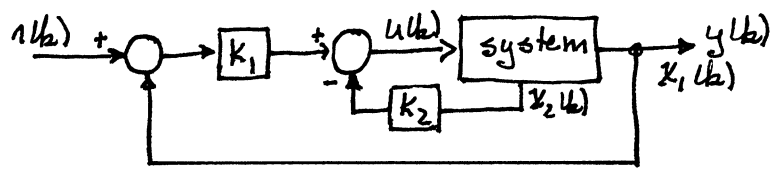
$$= \frac{B(z-A+GCA)}{z-A}U(z)$$

(1) $\therefore Q(z) = \dfrac{B}{z-A}U(z)$

From (9-68), the mode of $q(k)$ is $(A-GCA)^k$. Hence, from (1), this mode is not excited by $u(k)$, since $(z-A+GCA)$ doesn't appear in the denominator.

9-26. $\underline{x}(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0.125 \\ 0.25 \end{bmatrix}u(k)$

$y(k) = [1 \quad 0]\underline{x}(k)$



$\therefore \underline{x}(k+1) = A\underline{x}(k) + B[K_1(r(k) - x_1(k)) - K_2x_2(k)]$

$$= (A-BK)\underline{x}(k) + K_1Br(k)$$

From Problem 9-11, $K = [0.3893 \quad 1.769]$, and

$$\underline{x}(k+1) = \begin{bmatrix} 0.9513 & 0.7789 \\ -0.09732 & 0.5578 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0.04866 \\ 0.09733 \end{bmatrix}r(k)$$

9-27. From Problems 9-6 and 9-26,

$$x(k+1) = (A-BK)x(k) + K_1Br(k)$$

$$= [0.7408 - (1.0368)(0.1852)]x(k) + (1.0368)(0.1852)r(k)$$

$$= -0.5488x(k) + 0.1920r(k)$$

9-28. In (9-15), $\alpha_c(z) = |zI - A + BK|$

Thus, $\alpha_c(z) = |zI - (A - BK)^T)| = |zI - A^T - K^T B^T|$

From (9-25), $K^T = \{[0\,0 \cdots 1][B \;\; AB \cdots A^{n-1}B]^{-1}\alpha_c(A)\}^T$

Comparing this equation with (9-46),

$\alpha_e(z) = |zI - A + GC|$, then $\alpha_c(z) \to \alpha_e(z)$ if $A \to A^T$, $G \to K^T$, and $C \to B^T$

$$\therefore\; G = \{[0\;0\;0 \ldots 1][C^T \; A^T C^T \ldots A^{T(n-1)} C^T]^{-1}\alpha_e(A^T)\}^T$$

$$= \alpha_e(A)\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$, which is (9-48).

9-29. (9-56) $\begin{bmatrix} \underline{x}(k+1) \\ \underline{q}(k+1) \end{bmatrix} = \begin{bmatrix} A & -BK \\ GC & A - GC - BK \end{bmatrix}\begin{bmatrix} \underline{x}(k) \\ \underline{q}(k) \end{bmatrix}$

(9-44) $\underline{x}(k+1) = A\underline{x}(k) + Bu(k) + B_1 w(k)$

$y(k) = C\underline{x}(k) + \underline{v}(k)$

Below (9-38), $\underline{q}(k+1) = (A - GC)\underline{q}(k) + Gy(k) + Bu(k)$

$= (A - GC)\underline{q}(k) + GC\underline{x}(k) + G\underline{v}(k) - BK\underline{q}(k)$

$\therefore\; \underline{q}(k+1) = (A - GC - BK)\underline{q}(k) + GC\,\underline{x}(k) + G\underline{v}(k)$

$\underline{x}(k+1) = A\underline{x}(k) - BK\underline{q}(k) + B_1 w(k)$

$$\therefore\; \begin{bmatrix} \underline{x}(k+1) \\ \underline{q}(k+1) \end{bmatrix} = \begin{bmatrix} A & -BK \\ GC & A - BK - GC \end{bmatrix}\begin{bmatrix} \underline{x}(k) \\ \underline{q}(k) \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & G \end{bmatrix}\begin{bmatrix} \underline{w}(k) \\ \underline{v}(k) \end{bmatrix}$$

9-30. (9-26) $\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k)$

$y(k) = C\underline{x}(k)$

(9-68) $\underline{q}(k+1) = (A - GCA)\underline{q}(k) + [B - GCB]\underline{u}(k) + Gy(k+1)$

$\therefore\; y(k+1) = C\underline{x}(k+1) = CA\underline{x}(k) + CB\underline{u}(k)$

$\underline{u}(k) = -K\underline{q}(k)$

$\therefore\; \underline{x}(k+1) = A\underline{x}(k) - BK\underline{q}(k)$

$\underline{q}(k+1) = (A - GCA)\underline{q}(k) - (BK - GCBK)\underline{q}(k)$

$+ GCA\,\underline{x}(k) - GCBK\underline{q}(k)$

$= (A - GCA - BK)\underline{q}(k) + GCA\,\underline{x}(k)$

9-30. $\therefore \begin{bmatrix} \underline{x}(k+1) \\ \underline{q}(k+1) \end{bmatrix} = \begin{bmatrix} A & -BK \\ GCA & A-GCA-BK \end{bmatrix} \begin{bmatrix} \underline{x}(k) \\ \underline{q}(k) \end{bmatrix}$

CHAPTER 10

10-1. (N-1) dependence omitted. (10-22) into (10-21) yields

$$S_2 = [A\underline{x} + B\underline{u}]^T Q(N)[A\underline{x}+B\underline{u}] + \underline{x}^T Q(N-1)\underline{x} + \underline{u}^T R(N-1)\underline{u}$$

Now

$$\frac{\partial S_2}{\partial \underline{u}} = B^T Q(N)[A\underline{x}+B\underline{u}] + B^T Q(N)[A\underline{x}+B\underline{u}] + 2R(N-1)\underline{u} = 0$$

$$2B^T Q(N) A\underline{x} + 2[B^T Q(N) B + R(N-1)]\underline{u} = \underline{0}$$

$$\therefore \underline{u}(N-1) = -[B^T Q(N) B + R(N-1)]^{-1} B^T Q(N) A\underline{x}(N-1) = -K(N-1)\underline{x}(N-1)$$

10-2. show that, with $D = [B^T P B + R]^{-1}$,

$$[A-BK]^T P[A-BK] + Q + K^T R K = A^T[P - PBDB^T P]A + Q$$

$$\cancel{Q} + \cancel{A^T P A} + K^T B^T PBK - A^T PBK - K^T B^T PA + K^T RK = \cancel{A^T P A} - A^T PBDB^T A + \cancel{Q}$$

$$K^T[B^T PB + R]K = A^T PB[K - DB^T PA] + K^T B^T PA$$

$$K^T D^{-1} K = A^T PB\,[K - DB^T PA] + K^T B^T PA$$

$$K^T D^{-1}[K - DB^T PA] = A^T PB[K - DB^T PA]$$

$$K^T D^{-1} = A^T PB \qquad \text{true from (10-30)}$$

10-3. $J_N = \sum_{k=0}^{N} \underline{x}^T(k) Q\,\underline{x}(k) + \underline{u}^T(k) R\,\underline{u}(k)$

If each element of Q and R are multiplied by a positive constant β,

$$J_N' = \beta\left[\sum \underline{x}^T(k) Q\,\underline{x}(k) + \underline{u}^T(k) R\,\underline{u}(k)\right] = \beta J_N$$

Multiplying a function by a positive constant does not change the positions or relative values of its minima.

10-4. $\mathcal{H} = \begin{bmatrix} D & E \\ F & G \end{bmatrix} = \begin{bmatrix} A^{-1} & A^{-1}R_C \\ QA^{-1} & A^T + QA^{-1}R_C \end{bmatrix}$, $R_C = BR^{-1}B^T$

(1) $|\mathcal{H}| = |D||G - FD^{-1}E| = |A^{-1}||A^T + QA^{-1}R_C - QA^{-1}AA^{-1}R_C|$

$= |A^{-1}||A^T| = |A^{-1}||A| = 1$

(2) $|\mathcal{H}| = |G||D - EG^{-1}F| = |A^T + QA^{-1}R_C||A^{-1} - A^{-1}R_C(A^T + QA^{-1}R_C)^{-1}QA^{-1}|$

now

$$|A^{-1} - A^{-1}R_C(A^T+QA^{-1}R_C)^{-1}QA^{-1}| = |A^{-1}R_C||R_C^{-1} - (A^T + QA^{-1}R_C)^{-1}QA^{-1}|$$

10-4. $= |A^{-1}R_c|\,|I-(A^T+QA^{-1}R_c)^{-1}QA^{-1}R_c|\,|R_c^{-1}|$

$= |A^{-1}R_c|\,|(A^T+QA^{-1}R_c)^{-1}|\,|A^T|\,|R_c^{-1}| = |(A^T+QA^{-1}R_c)^{-1}|$

$\therefore\ |\mathcal{H}| = 1$ = product of eigenvalues

10-5. 1^{st} order system, $J_N = \sum_{k=0}^{N}(Qx^2(k) + Ru^2(k))$

$\alpha = Q/R$, $\therefore J_N = R[\sum(\alpha x^2(k) + u^2(k))]$

Location of the minimum is a function only of α, and is independent of the positive constant R.

10-6. $A=0.9,\ B=0.1,\ Q=1,\ R=5,\ N=3$

(a) $S_1 = x^2(3) + 5u^2(3)$

$\frac{\partial S_1}{\partial u(3)} = 10u(3) = 0$, $\therefore u(3) = \underline{0}$; $S_1^0 = x^2(3)$

$S_2 = S_1^0 + F_1 = [0.9x(2) + 0.1u(2)]^2 + x^2(2) + 5u^2(2)$

$\frac{\partial S_2}{\partial u(2)} = 2[0.9x(2) + 0.1u(2)]0.1 + 10u(2)$

$0.18x(2) + 10.02u(2) = 0$; $\therefore u(2) = \underline{-0.01796x(2)}$

$S_2^0 = [0.9x(2) - 0.001796x(2)]^2 + x^2(2) + 5[-0.01796x(2)]^2$

$= 1.8102x^2(2)$

$\therefore S_1 = S_2^0 + F_2 = 1.8102[0.9x(1) + 0.1u(1)]^2 + x^2(1) + 5u^2(1)$

$\frac{\partial S_1}{\partial u(1)} = 0.3258x(1) + 10.0362u(1) = 0$; $\therefore u(1) = \underline{-0.03246x(1)}$

$S_1 = 2.461x^2(1)$

$S_0 = 2(2.461)[0.9x(0) + 0.1u(0)]0.1 + 10u(0) \Rightarrow u(0) = \underline{-0.0440x(0)}$

$\therefore K(0) = 0.0440$ $\quad K(2) = 0.01796$

$K(1) = 0.0325$ $\quad K(3) = 0$

(b) $P(3) = 1$, $K(3) = 0$

$K(2) = \frac{(0.1)(1)(0.9)}{0.01(1)+5} = \underline{0.01796}$

$P(2) = (0.9)(1)[0.9 - 0.1(0.01796)] + 1 = 1.8084$

$K_1 = \frac{(0.1)(1.8084)(0.9)}{0.01(1.8084)+5} = \underline{0.03243}$

10-6.(b) $P(1) = (0.9)(1.8084)[0.9-0.1(0.03243)]+1 = 2.460$

$$K(\;) = \frac{(0.1)(2.460)(0.9)}{0.01(2.460)+5} = \underline{0.0441} \qquad \therefore \underline{\text{Checks (a)}}$$

$K(0)$

(c) $|u(0)|$ is maximum, with $|u(0)| = |-0.0441\,x(0)|$

10-7. $A=0.9$; $B=0.1$; $Q=1$; $R=0$

(a) $P(3)=1$, $K(3)=\underline{0}$

$K(2) = [(0.1)(1)(0.1)+0]^{-1}(0.1)(1)(0.9) = \frac{0.09}{0.01} = \underline{9}$

$P(2) = 0.9(1)[0.9-(0.1)(9)]+1 = \underline{1}$

$\therefore K(1) = \underline{9}$

$P(0) = 0.9(1)[0.9-(0.1)(9)]+1 = \underline{1}$

$\therefore K(0) = \underline{9}$

(b) $|u(0)| = 9\,|x(0)|$

(c) In Problem 10-6, $K(0) = 0.0441$

In Problem 10-7(a), $K(0) = 9$

The magnitude of $u(k)$ does not affect the cost, for this problem. Thus $|u(k)|$ is large to minimize $|x(k)|$.

10-8 (a) $A=0.9$, $B=0.1$, $Q=1$, $R=5$; $R_c = \frac{(0.1)(0.1)}{5} = \underline{0.002}$

$$\therefore \mathcal{H} = \begin{bmatrix} 1.1111 & 0.002222 \\ 1.1111 & 0.902222 \end{bmatrix}$$

$|\lambda I - \mathcal{H}| = (\lambda-1.1)(\lambda-0.902222) - (-1.1111)(-0.002222)$

$= \lambda^2 - 2.01333\lambda + 1 = (\lambda - 1.1223)(\lambda - 0.8910)$

$$\therefore \begin{bmatrix} 1.1111 & 0.002222 \\ 1.1111 & 0.902222 \end{bmatrix}\begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} = 1.1223\begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix}$$

$\therefore 0.0111\,h_{11} = 0.002222\,h_{21} \Rightarrow h_{21} = \underline{1},\ h_{11} = \underline{0.02}$

$\mathcal{H}\,\underline{h}_2 = 0.8910\,\underline{h}_2$

$\therefore 0.2201\,h_{12} = 0.002222\,h_{22} \Rightarrow h_{22} = \underline{1},\ h_{12} = \underline{0.01010}$

$$\therefore W = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} = \begin{bmatrix} 0.2 & 0.01010 \\ 1 & 1 \end{bmatrix}$$

10-8. (a) $\therefore \hat{P} = w_{21}/w_{11} = \underline{5}$

$$\hat{K} = [5 + (0.1)^2(5)]^{-1}(0.1)(5)(0.9) = \frac{0.45}{5.05} = \underline{0.0891}$$

computer solution: $\hat{K} = 0.0899$

(b) $z - (A - B\hat{K}) = z - (0.9 - 0.00891) = \underline{z - 0.891}$

(c) $\tau = \dfrac{-T}{\ln(0.891)} = 8.664T$

10-9. (a) Since $R=0$, the gain $K(k)$ doesn't vary with time. Thus, from Problem 10-7(a), $\hat{K} = \underline{9}$.

(b) $z - (A - B\hat{K}) = z - (0.9 - (0.1)(9)) = \underline{z}$

Thus the system is an ideal delay of $\underline{T}$ seconds.

(c) The output is exactly equal to the input after one sample period.

10-10. (a) $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = P(1)$, $R = 2$, $\underline{K}(1) = \underline{[0 \quad 0]}$

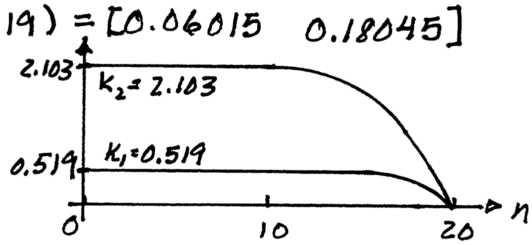
$$B^T P(1) B + R = B^T B + R = [0.125 \quad 0.25]\begin{bmatrix} 0.125 \\ 0.25 \end{bmatrix} + 2 = \underline{2.078125}$$

$$B^T P(1) A = B^T A = [0.125 \quad 0.25]\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = [0.125 \quad 0.375]$$

$$\therefore \underline{K}(0) = \frac{1}{2.078125}[0.125 \quad 0.375] = [0.06015 \quad 0.18045]$$

(b) $\underline{K}(19) = [0.06015 \quad 0.18045]$

(c)



10-11. (a) $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = P(1)$, $R = 20$, $\underline{K}(1) = \underline{[0 \quad 0]}$

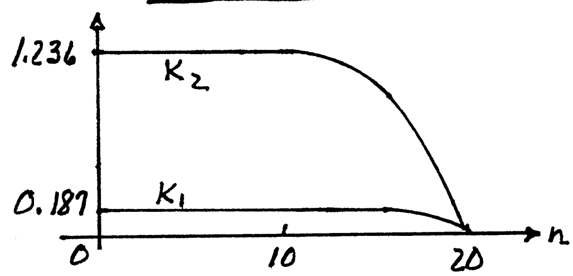
$$B^T P(1) B + R = B^T B + R = \underline{20.07815}$$

$$B^T P(1) A = B^T A = [0.125 \quad 0.25]\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = [0.125 \quad 0.375]$$

$$\therefore \underline{K}(0) = \frac{1}{20.07815}[0.125 \quad 0.375] = \underline{[0.00623 \quad 0.01868]}$$

10-11.(b) $\underline{K}(19) = \underline{[0.00623 \quad 0.01868]}$

(c)



(b) The inputs are weighted more heavily in this problem. Thus the gains are lower.

10-12. (a) By computer, $\hat{\underline{K}} = [0.519 \quad 2.103]$

(b) $\therefore A - B\hat{K} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.125 \\ 0.25 \end{bmatrix} [0.519 \quad 2.103]$

$$= \begin{bmatrix} 0.9351 & 0.7371 \\ -0.1298 & 0.4743 \end{bmatrix}$$

$\therefore |zI - A| = \begin{vmatrix} z-0.9351 & -0.7371 \\ 0.1298 & z-0.4743 \end{vmatrix} = z^2 - 1.4094 + 0.5392 = 0$

$\underline{\text{roots}}$: $z = 0.7047 \pm j0.2064 = \underline{0.7343} \underline{\angle \pm 16.32^\circ}$

(c) $\tau = \dfrac{-T}{\ln(0.7343)} = \underline{3.238T}$

10-13.(a) By computer, $\hat{\underline{K}} = [0.1874 \quad 1.2357]$

(b) $\therefore A - B\hat{K} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.125 \\ 0.25 \end{bmatrix} [0.1874 \quad 1.2357]$

$$= \begin{bmatrix} 0.9766 & 0.8455 \\ -0.04685 & 0.6911 \end{bmatrix}$$

$\therefore |zI - A| = \begin{vmatrix} z-0.9766 & -0.8455 \\ 0.04685 & z-0.6911 \end{vmatrix} = z^2 - 1.6677z + 0.7145 = 0$

$\underline{\text{roots}}$: $z = 0.8339 \pm j0.1385 = \underline{0.8453} \underline{\angle \pm 9.43^\circ}$

(c) $\tau = \dfrac{-T}{\ln(0.8453)} = \underline{5.95T}$

10-13.(d) The time constant in this problem is larger, since R is large. With R larger, the inputs will be forced to be smaller.

10-14. (a) $A = 0.7408$; $B = 1.0368$; $Q = 2$; $R = 1$, $N = 3$

$S_1 = 2x^2(3) + u^2(3)$

$\frac{\partial S_1}{\partial u(3)} = 0 \Rightarrow \underline{u^o(3) = 0}$; $S_1 = \underline{2x^2(3)}$

$S_2 = S_1^o + 2x^2(2) + u^2(2) = 2[0.7408x(2) + 1.0368u(2)]^2 + 2x^2(2) + u^2(2)$

$\frac{\partial S_2}{\partial u(2)} = 4[0.7408x(2) + 1.0368u(2)](1.0368) + 2u(2) = 0$

$= 3.0722x(2) + 6.2998u(2) \Rightarrow u^o(2) = \underline{-0.4877x(2)}$

$S_2^o = 2.3484x^2(2)$

$S_3 = 2.3484[0.7408x(1) + 1.0368u(1)]^2 + 2x^2(1) + u^2(1)$

$\frac{\partial S_3}{\partial u(1)} = 3.6074x(1) + 5.0488u(1) + 2u(1) = 0$

$\therefore u^o(1) = \underline{-0.5118x(1)}$

By computer: $u^o(0) = \underline{-0.5124x(0)}$

(b) $P(3) = 2$, $K(3) = \underline{0}$

$K(2) = \frac{(1.0368)(2)(0.7408)}{(1.0368)^2(2) + 1} = \underline{0.4877}$

$P(2) = (0.7408)(2)[0.7408 - (1.0368)(0.4877)] + 2 = 2.3484$

$K(1) = \frac{(1.0368)(2.3484)(0.7408)}{(1.0368)^2(2.3484) + 1} = \underline{0.5118}$

$P(0) = 2.3653 \Rightarrow K(0) = \underline{0.5128}$

(c) $|u(0)| = 0.5128\,|x(0)|$

10-15(a) $A = 0.7408$; $B = 1.0368$; $Q = 10$; $R = 1$; $N = 3$

$P(3) = 10$, $K(3) = \underline{0}$

$K(2) = \frac{(1.0368)(10)(0.7408)}{(1.0368)^2(10) + 1} = 0.6537$

$P(2) = (0.7408)(10)[0.7408 - (1.0368)(0.6537)] + 10 = 10.4670$

10-15.(a) $K(1) = \dfrac{(1.0368)(10.4670)(0.7408)}{(1.0368)^2(10.4670)+1} = \underline{0.6562}$

$P(1) = 10.9357$; $K(0) = \underline{0.6585}$

(b) $|u(0)| = \underline{0.6585\,|x(0)|}$

(c) From Problem 10-14(c): $|u(0)| = 0.5128\,|x(0)|$

J in this problem emphasizes $|x(k)|$ more. Thus $|u(k)|$ is larger to reduce $|x(k)|$ faster.

10-16.(a) $A = 0.7408 \qquad Q = 2 \qquad \therefore R_c = \dfrac{(1.0368)^2}{1} = 1.0750$

$B = 1.0368 \qquad R = 1$

$$\mathcal{H} = \begin{bmatrix} 1.3499 & 1.4511 \\ 2.700 & 3.643 \end{bmatrix}$$

$$\therefore |\lambda I - \mathcal{H}| = (\lambda - 1.3499)(\lambda - 3.643) - (2.670)(1.4511)$$
$$= \lambda^2 - 4.993\lambda + 1 = 0$$

$\underline{\text{roots}}$: $\lambda = 4.784\,,\ 0.210$

$\underline{\text{eigenvector}}$: $\mathcal{H}\underline{h}_1 = \lambda_1 \underline{h}_1 \Rightarrow \begin{bmatrix} 1.3499 & 1.4511 \\ 2.700 & 3.643 \end{bmatrix}\begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} = 4.784 \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix}$

$1.3499\,h_{11} + 1.4511\,h_{21} = 4.784\,h_{11}$: let $h_{11} = 1$, $\therefore h_{21} = 2.367$

$$\therefore \hat{P} = \frac{w_{21}}{w_{11}} = \frac{h_{21}}{h_{11}} = 2.367$$

$$\therefore \hat{K} = \frac{(1.0368)(2.367)(0.7408)}{1 + (1.0368)^2(2.367)} = \underline{0.513}$$

(b) $z - A + B\hat{K} = z - 0.7408 + (1.0368)(0.513) = \underline{z - 0.2089} = 0$

(c) $\tau = \dfrac{-T}{\ln a} = \dfrac{-0.6}{\ln(0.2089)} = \underline{0.3835}$

10-17.(a) $A = 0.7408 \qquad Q = 10 \qquad R_c = \dfrac{(1.0368)^2}{1} = 1.075$

$B = 1.0368 \qquad R = 1$

$$\mathcal{H} = \begin{bmatrix} 1.3499 & 1.4511 \\ 13.499 & 15.252 \end{bmatrix}$$

10-17(a) $|\lambda I - \mathcal{H}| = \begin{vmatrix} \lambda - 1.3499 & -1.4511 \\ -13.499 & \lambda - 15.252 \end{vmatrix} = \lambda^2 - 16.602\lambda + 1 = 0$

roots: $\lambda = 16.542,\ 0.0605$

$\therefore \mathcal{H}\underline{h}_1 = \lambda_1 \underline{h}_1 \Rightarrow \begin{bmatrix} 1.3499 & 1.4511 \\ 13.499 & 15.252 \end{bmatrix}\begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} = 16.542\begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix}$

$\therefore 15.192\, h_{11} = 1.4511\, h_{21} \Rightarrow$ Let $h_{11} = 1$; $h_{21} = 10.469$

$\therefore \hat{P} = \dfrac{w_{21}}{w_{11}} = \dfrac{h_{21}}{h_{11}} = 10.469$

$\therefore \hat{K} = \dfrac{(1.0368)(10.469)(0.7408)}{1 + (1.0368)^2(10.469)} = \underline{0.656}$

(b) $z - A + B\hat{K} = z - 0.7408 + (1.0368)(0.656) = \underline{z - 0.06065 = 0}$

(c) $\tau = \dfrac{-T}{\ln r} = \dfrac{-0.6}{\ln(0.06065)} = \underline{0.2141\,s}$

10-18.(a) (10-38) $K(k) = [B^T P(k+1) B + R]^{-1} B^T P(k+1) A$

$\therefore K(N-1) = [B^T Q B]^{-1} B^T Q A$

(10-39) $P(k) = A^T P(k+1)[A - BK(k)] + Q$

$\therefore P(N-1) = A^T Q [A - BK(N-1)] + Q$

$= A^T Q [A - B(B^T Q B)^{-1} B^T Q A] + Q$

$= AQ\left[A - \dfrac{BBQA}{BQB}\right] + Q = \underline{Q}$

$\therefore P(k)$ is constant, $\therefore K(k)$ is constant, from (10-38).

(b) (10-38) $K = \dfrac{BQA}{BQB + R} = \dfrac{A}{B}$; $u(k) = -Kx(k)$

$\therefore u(0) = -Kx(0) = -\dfrac{A}{B}x(0)$

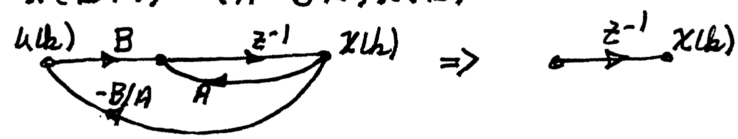
$x(1) = Ax(0) + Bu(0) = Ax(0) - Ax(0) = 0$

$u(1) = 0$

$x(2) = Ax(1) + Bu(1) = 0$

$\therefore u(0) = -\dfrac{A}{B}x(0)$; $u(k) = 0$, $k \geq 1$.

(c) $x(k+1) = (A - BK)x(k)$



10-18. (d) $x(k+1) = Ax(k) + Bu(k) = (A-BK)x(k)$

$\therefore$ char. eq.: $zI-(A-BK) = z - A + B(\frac{A}{B}) = \underline{z = 0}$

(e) From (b), <u>one sample instant</u>

10-19. (a) $y_i = Kx_i^2 + e_i$ $\quad \underline{f} = \begin{bmatrix} x_1^2 \\ x_2^2 \\ \cdots \\ x_N^2 \end{bmatrix}, \quad \underline{e} = \begin{bmatrix} e_1 \\ e_2 \\ \cdots \\ e_N \end{bmatrix}$

$\underline{y} = K\underline{f} + \underline{e}$

$$\underline{e}^T\underline{e} = [\underline{y} - K\underline{f}]^T[\underline{y} - K\underline{f}] = \underline{y}\,\underline{y}^T - 2K\underline{f}^T\underline{y} + K^2\underline{f}^T\underline{f}$$

$$\frac{\partial(\underline{e}^T\underline{e})}{\partial K} = -2\underline{f}^T\underline{y} + 2K\underline{f}^T\underline{f} = 0$$

$$\therefore K = \frac{\underline{f}^T\underline{y}}{\underline{f}^T\underline{f}} = \frac{x_1^2 y_1 + x_2^2 y_2 + \cdots + x_N^2 y_N}{x_1^4 + x_2^4 + \cdots + x_N^4}$$

(b) $$K = \frac{(0)^2(0.01) + (1)^2(1.01) + (2)^2(3.98)}{(0)^4 + (1)^4 + (2)^4} = \frac{16.930}{17} = \underline{0.9959}$$

(c) $$K = \frac{16.930 + (1.5)^2 3.30}{17 + (1.5)^4} = \frac{24.355}{22.063} = \underline{1.1034}$$

$$\% \text{ error} = \frac{|0.9959 - 1.1034|}{0.9959} = 10.8\,\%$$

10-20. (a) $\underline{f}(k) = [y(k-1) \quad u(k-1)]$

$\underline{f}(1) = [0 \quad 10]$

$$F(3) = \begin{bmatrix} 0 & 10 \\ 12.2 & 10 \\ 20.1 & 10 \end{bmatrix}$$

$$F^TF = \begin{bmatrix} 0 & 12.2 & 20.1 \\ 10 & 10 & 10 \end{bmatrix}\begin{bmatrix} 0 & 10 \\ 12.2 & 10 \\ 20.1 & 10 \end{bmatrix} = \begin{bmatrix} 552.85 & 323 \\ 323 & 300 \end{bmatrix}; \ |F^TF| = 61{,}526$$

$$\therefore (F^TF)^{-1} = \frac{1}{61{,}526}\begin{bmatrix} 300 & -323 \\ -323 & 552.85 \end{bmatrix}; \ F^TY = \begin{bmatrix} 0 & 12.2 & 20.1 \\ 10 & 10 & 10 \end{bmatrix}\begin{bmatrix} 12.2 \\ 20.1 \\ 31.8 \end{bmatrix} = \begin{bmatrix} 884.4 \\ 641 \end{bmatrix}$$

$$\hat{\Theta} = (F^TF)^{-1}F^TY = \frac{1}{61{,}526}\begin{bmatrix} 300 & -323 \\ -323 & 552.85 \end{bmatrix}\begin{bmatrix} 884.4 \\ 641 \end{bmatrix} = \begin{bmatrix} 0.947 \\ 1.12 \end{bmatrix} = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$$\therefore G(z) = \frac{b_1}{z - a_1} = \underline{\frac{1.12}{z - 0.947}}$$

(b) From (a): $$P(3) = [F^T(3)F(3)]^{-1} = \frac{1}{61{,}526}\begin{bmatrix} 300 & -323 \\ -323 & 552.85 \end{bmatrix}$$

10-20.(b) $\underline{f}^T(4) = [y(3) \quad u(3)] = [31.8 \quad 0]$

In (10-118): $1 + [31.8 \quad 0]\frac{1}{61{,}526}\begin{bmatrix} 300 & -323 \\ -323 & 552.85 \end{bmatrix}\begin{bmatrix} 31.8 \\ 0 \end{bmatrix} = 5.931$

$\therefore$ (10-118): $L(4) = \frac{1}{61{,}526}\begin{bmatrix} 300 & -323 \\ -323 & 552.85 \end{bmatrix}\begin{bmatrix} 31.8 \\ 0 \end{bmatrix}\frac{1}{5.931} = \begin{bmatrix} 0.02614 \\ -0.02815 \end{bmatrix}$

$\therefore$ (10-119):

$$\hat{\theta}(4) = \begin{bmatrix} 0.947 \\ 1.12 \end{bmatrix} + \begin{bmatrix} 0.02614 \\ -0.02815 \end{bmatrix}\left[30.0 - [31.8 \quad 0]\begin{bmatrix} 0.947 \\ 1.12 \end{bmatrix}\right] = \begin{bmatrix} 0.944 \\ 1.12 \end{bmatrix}$$

$$\therefore G(z) = \frac{1.12}{z - 0.944}$$

10-21. $$F^T(5)\,Y(5) = \begin{bmatrix} y(2) & y(1) & y(0) & u(2) & u(1) & u(0) \\ y(3) & y(2) & y(1) & u(3) & u(2) & u(1) \\ y(4) & y(3) & y(2) & u(4) & u(3) & u(2) \end{bmatrix}^T \begin{bmatrix} y(3) \\ y(4) \\ y(5) \end{bmatrix}$$

10-22.(a) $R_w = 2 \qquad A = 0.8 \qquad C = 1 \qquad M(0) = 2$

$R_v = 1 \qquad B = 0.2 \qquad B_1 = 1$

$G(k) = \dfrac{M(k)}{M(k) + R_v}$; $P(k) = M(k)[1 - G(k)]$; $M(k+1) = 0.64\,P(k) + R_w$

$G(0) = 2/3 = 0.667$

$P(0) = 2(1 - 2/3) = 2/3$

$M(1) = 0.64(2/3) + 2 = 2.4267$

$G(1) = 2.4267/3.4267 = 0.7082$

$P(1) = 2.4267(1 - 0.7082) = 0.7081$

$M(2) = 0.64(0.7081) + 2 = 2.4532$

$G(2) = 2.4532/3.4532 = 0.7104$

$P(2) = 2.4532\,(1 - 0.7104) = 0.7104$

$M(3) = 0.64\,(0.7104) + 2 = 2.4547$

$G(3) = 2.4547/3.4547 = 0.7105$

$\therefore G_{ss} \approx \underline{0.7105}$

(b) $M(0)$ = covariance of errors in $\underline{q}(0)$

$\therefore \sigma_{q(0)} = \sqrt{2} = \underline{1.414}$

10-22.(c) $\bar{q}(k) = \bar{q}(k) + 0.7105[y(k) - \bar{q}(k)]$ (10-127)

$\bar{q}(k+1) = 0.8q(k) + 0.2u(k)$

(d) $D_{ce}(z) = zK[zI - A + GCA + BK - GCBK]^{-1}G$

$$= \frac{(0.2179)(0.7105)z}{z - 0.8 + (0.7105)(0.8) + (0.2179)(0.02) - (0.7105)(0.2179)(0.02)}$$

$$= \frac{0.1548z}{z - 0.2190}$$

(e) $1 + D_{ce}(z)G(z) = 0 = 1 + \left(\frac{0.2}{z-0.8}\right)\left(\frac{0.1548z}{z-0.2190}\right)$

$= 1 + \therefore z^2 - 0.9880z + 0.1752 = 0 = (z - 0.756)(z - 0.232)$

(f) $\epsilon^{-T/\tau_1} = 0.756 \Rightarrow \tau_1 = \underline{0.715s}$

$\epsilon^{-T/\tau_2} = 0.232 \Rightarrow \tau_2 = \underline{0.137s}$

(g) $P_{ss} \approx 0.71 = \sigma_e^2$, $\therefore \sigma_e = 0.843 \Rightarrow 3\sigma_e = 2.53$

$$\left.\begin{aligned} 90.1 - 2.5 &= 87.6 \\ 90.1 + 2.5 &= 92.6 \end{aligned}\right\} \therefore \text{range: } 87.6 < x(k) < 92.6$$

(h) By computer: For ϕ_m, gain is always less than 1, gain margin $\approx$ 37dB

10-23. Results of Problem 10-22 are used.

(a) $G(z) = \frac{0.2}{z-0.8}$, $G(1) = \frac{0.2}{1-0.1} = \underline{1}$

$\therefore x_{ss} = y_{ss} = (1)(10) = \underline{10}$

(b) $q(k) = 0.8q(k-1) + 0.2u(k-1) + 0.7150y(k)$

$- 0.7105[0.8q(k-1) + 0.2u(k-1)]$

$= 0.2316q(k-1) + 0.0579u(k-1) + 0.7105y(k)$

In steady state: $q(k) = q(k-1) = q_{ss}$

$\therefore (1 - 0.2316)q_{ss} = 0.0579\overset{u_{ss}}{(10)} + 0.7105\overset{y_{ss}}{(10)}$

$\therefore q_{ss} = \underline{10}$

(c) yes

(d) $w(k) = 5 =$ constant: In steady-state, $x(k+1) = x(k) = x_{ss}$

$\therefore x_{ss} = 0.8x_{ss} + 2 + 5 \Rightarrow x_{ss} = 35 = y_{ss}$

10-23.(d) From (b): $\hat{q}_{ss} = 0.07535 \overset{(10)}{u_{ss}} + 0.9246 \overset{(35)}{y_{ss}} = 33.11$

$\therefore$ error $= \left|\frac{35-33.11}{35}\right|(100) = \underline{5.4\%}$

10-24. (a) $G(k) = \frac{M(k)}{M(k)+r_v}$; $P(k) = M(k)[1-G(k)]$; $M(k+1) = 0.64P(k) + \overset{0}{\cancel{r_v}}$

$G(0) = 2/3$

$P(3) = 2/3$

$M(1) = 0.64(2/3) = 0.4267$

$G(1) = 0.4267/1.4267 = 0.299$

$P(1) = 0.299$

$M(2) = 0.191$

$P(2) = 0.160$

$M(3) = 0.103$

$\vdots$

$\therefore M \to 0$, $\therefore G \to 0$

by computer;

k	G	P
0	0.667	0.667
1	0.299	0.299
2	0.161	0.161
3	0.093	0.093
4	0.056	0.056
5	0.035	0.035
10	0.00354	0.00354
20	0.00006	0.00006

(b) with $G=0$: $\underset{\sim}{q}(k) = \bar{\underset{\sim}{q}}(k)$

$\bar{\underset{\sim}{q}}(k+1) = A\underset{\sim}{q}(k) + Bu(k) \Rightarrow \underset{\sim}{q}(k+1) = A\underset{\sim}{q}(k) + Bu(k)$

(c) The measurements are ignored [$G=0$]. The input an the system dynamics are known perfectly, $\therefore$ no errors after the initial errors die out.

(d) $G(k) = \frac{M(k)}{M(k)+0} = 1$; $P(k) = 0$; $M(k+1) = 0.64P(k) + 2 = 2$

In the steady-state, $P(k+1) = P(k) = \frac{2}{1-0.64} = 5.55$

$\underset{\sim}{q}(k) = \bar{\underset{\sim}{q}}(k) + y(k) - \bar{\underset{\sim}{q}}(k) = y(k)$

$\bar{\underset{\sim}{q}}(k+1) = A\underset{\sim}{q}(k) + Bu(k)$

$\therefore$ $u(k)$ has no effect on the estimate, and the system dynamics [A+B matrices] do not directly affect the estimate. The estimation errors are zero, since $P=0$. Since $\underset{\sim}{q}(k) = y(k) = x(k)$, there is no estimation error.

10-25. (a) $\sigma_v = 0.1°$, ∴ measurements with 0.1° accuracy 68% of time.
measurements with 0.3° accuracy almost always.

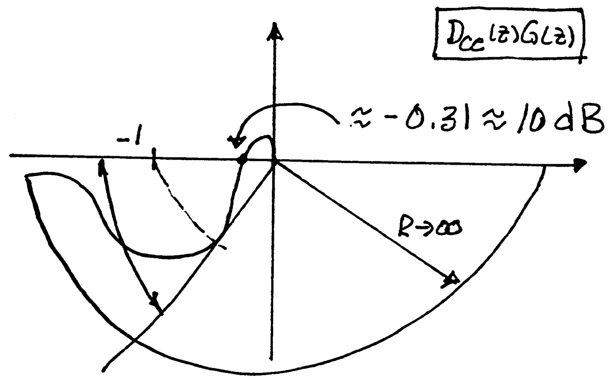
(b) $K(0) = [0.9 \quad 0]$
$K(1) = [0.990 \quad 0.981]$
$K(2) = [0.815 \quad 0.572]$
$K(3) = [0.781 \quad 0.490]$
$K(4) = [0.764 \quad 0.468]$
$K(5) = [0.763 \quad 0.488]$; steady state

(c) $\sigma_{e1} = [0.00763]^{1/2} = 0.0873°$ for q_1 (b)
$\sigma_{e2} = [0.0147]^{1/2} = 0.121°$ for q_2 (b)

(d) By computer: $D_{ce}(z) = \dfrac{1.421z^2 - 1.168z}{z^2 - 0.3371z + 0.1280}$

(e) $G(z) = \dfrac{0.125(z+1)}{z^2 - 2z + 1}$

$\Phi_m \approx 38°$
gain margin ≈ 10dB



10-26. (a) $z = \epsilon^{sT} = \epsilon^{-7T}\epsilon^{\pm j2T}$, $T = 0.006$

$d_c(z) = z^2 - 2\epsilon^{-0.042}\cos(0.012) + \epsilon^{-0.084}$
$= z^2 - 1.9176z + 0.91943$

(b) $K = [48.418 \quad 9.7480]$

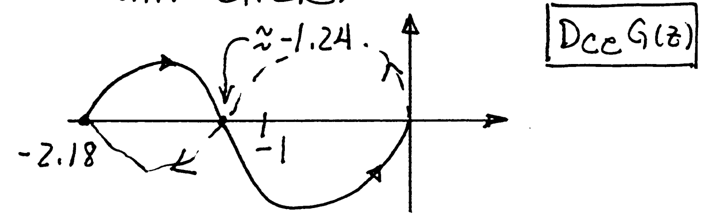
(c) $G = \begin{bmatrix} 0.17465 \\ -0.29085 \end{bmatrix}$

(d) $G(z) = \dfrac{0.005964z - 0.005893}{z^2 - 1.97618z + 0.976286}$

$D_{ce}(z) = \dfrac{5.6250z^2 - 5.5379}{z^2 - 1.89235z + 0.86568}$

10-26. (d) $D_{ce}(z)$ is unstable, with one pole outside the unit circle.

(e)



$Z = N + P = -1 + 1 = 0$; $P = 1$ since D_{ce} has one unstable pole.

$\therefore$ system is stable.

(f) $\phi_m \approx 15^0$, gain margin ≈ 1.87 dB

10-27. $\dfrac{\partial^2(\underline{e}^T\underline{e})}{\partial k^2} = 2\,\underline{x}^T\underline{x} = 2\,[x_1^2 + x_2^2 + \cdots + x_n^2] > 0$

$\therefore$ point is a minimum.

10-28. $J(\theta) = \underline{e}^T W \underline{e} = [y - F\theta]^T W [y - F\theta]$

$= y^T W y - \theta^T F^T W y - y^T W F \theta + \theta^T F^T W F \theta$

$= y^T W y - 2\theta^T F^T W y + \theta^T F^T W F \theta$, since $W = W^T$

$\therefore \dfrac{\partial J}{\partial \theta} = -2F^T W y + 2F^T W F \hat{\theta}_{WLS} = 0$

$\therefore \hat{\theta}_{WLS} = [F^T(N) W(N) F(N)]^{-1} F^T(N) W(N) Y(N)$

10-29. $P_1 = P(N+1)$, $f_1 = f(N+1)$

$P = P(N)$, $f = f(N)$; etc.

$P_1 = \frac{1}{\gamma}P - \frac{1}{\gamma}Pf_1[\frac{1}{a} + f_1^T \frac{1}{\gamma} P f_1]^{-1} f_1^T \frac{1}{\gamma} P$ (10-115)

$F_1^T W_1 y_1 = \gamma F^T W y + f_1 a y_1$ (10-116)

$\hat{\theta}_1 = [F_1 W_1 F_1]^{-1} F_1^T W_1 y_1$ (10-106)

$P = [F^T W F]^{-1}$ (10-111)

10-29. $\therefore\ \hat{\theta}_1 = \left\{\underset{\hat{\theta}}{\frac{1}{\gamma}P} - \frac{1}{\gamma}Pf_1\left[\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right]^{-1}f_1^T\frac{1}{\gamma}P\left[\gamma F^TWy + f_1 a y_1\right]\right.$

$$= \underset{①}{\overbrace{PF^TWy}} - Pf_1\left[\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right]^{-1}\underset{②}{f_1^T PF^TWy} + \underset{③}{\frac{1}{\gamma}Pf_1 a y_1}$$

$$-\frac{1}{\gamma}Pf_1\left[\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right]^{-1}\underset{④}{f_1^T\frac{1}{\gamma}Pf_1 a y_1} \qquad (a)$$

First two terms of (a)

$$\underset{①}{\hat{\theta}} - Pf_1\left[\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right]^{-1}\underset{②}{f_1^T\hat{\theta}}$$

Now: $I = \left(\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right)^{-1}\left(\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right)$

$\therefore$ third term of (a):

$$\frac{1}{\gamma}Pf_1 a y_1 = \frac{1}{\gamma}Pf_1\left(\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right)^{-1}\left(\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right)a y_1$$

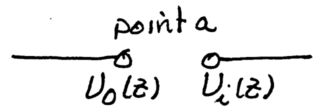
$\therefore$ ③ and ④ of (a):

$$\underset{③}{\frac{1}{\gamma}Pf_1\left(\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right)^{-1}\left(\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right)a y_1} - \underset{④}{\frac{1}{\gamma}Pf_1\left[\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right]^{-1}f_1^T\frac{1}{\gamma}Pf_1 a y_1}$$

$$= \frac{1}{\gamma}Pf_1\left(\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right)^{-1}\left[\underbrace{y_1 + f_1^T\frac{1}{\gamma}Pf_1 a y - f_1^T\frac{1}{\gamma}Pf_1 a y}_{y_1}\right]$$

$\therefore$ (a) becomes:

$$\hat{\theta}_1 = \hat{\theta} + \underbrace{\frac{1}{\gamma}Pf_1\left(\frac{1}{a}+f_1^T\frac{1}{\gamma}Pf_1\right)^{-1}}_{L_1}\left[y_1 - f_1^T\hat{\theta}\right]$$

10-30. (a) 

From Figure 10-7, $Q(z) = (zI-A)^{-1}(B-GCB)U_i(z)$

$$-(zI-A)^{-1}GCAQ(z) + (zI-A)^{-1}Gz\,C(zI-A)^{-1}BU_i(z)$$

$$\therefore [I+(zI-A)^{-1}GCA]\,Q(z) = (zI-A)^{-1}[B-GCB+zGC(zI-A)^{-1}B]\,U_i(z)$$

$$\therefore (zI-A)^{-1}[zI-A+GCA]\,Q(z) = (zI-A)^{-1}[B-GCB+zGC(zI-A)^{-1}B]\,U_i(z)$$

$$\therefore Q(z) = [zI-A+GCA]^{-1}[B-GCB+zGC(zI-A)^{-1}B]\,U_i(z)$$

$$\therefore \frac{U_o(z)}{U_i(z)} = -K[zI-A+GCA]^{-1}[B-GCB+zGC(zI-A)^{-1}B]$$

10-30. (b) This is of the form of Figure 9.8,

$$\therefore \frac{V_o(z)}{V_i(z)} = -D_{ce}(z)G(z)$$

$$= -zK[zI - A + GCA + BK - GCBK]^{-1}GC[zI - A]^{-1}B$$

(c) Plot the negative of the transfer function derived,

CHAPTER 11

11.1. $G(s) = \dfrac{s+2}{(s+3)(s+1)}$, $T = 1\,s.$

(a) standard z-transform

$$G(s) = \frac{1/2}{(s+1)} + \frac{1/2}{(s+3)}$$

$$D(z) = G(z) = \frac{1/2}{1-\epsilon^{-1}z^{-1}} + \frac{1/2}{1-\epsilon^{-3}z^{-1}} = \frac{1-\frac{1}{2}(\epsilon^{-1}+\epsilon^{-3})z^{-1}}{1-(\epsilon^{-1}+\epsilon^{-3})z^{-1}+\epsilon^{-4}z^{-2}}$$

(b) Bilinear z-transform , $s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$, $T = 1\,s.$

$$D(z) = G(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}\cdot\frac{2}{T}} = \frac{\left(2\,\frac{1-z^{-1}}{1+z^{-1}} + 2\right)}{\left(2\,\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 4\left(2\,\frac{1-z^{-1}}{1+z^{-1}}\right) + 3}$$

$$= \frac{4+4z^{-1}}{15-2z^{-1}-z^{-2}}$$

(c) Matched z-transform

$$D(z) = G(s)\Big|_{s+\alpha = 1-\epsilon^{-\alpha}z^{-1}} \quad , \; T=1$$

$$= \frac{1-\epsilon^{-2}z^{-1}}{1-(\epsilon^{-1}+\epsilon^{-3})z^{-1}+\epsilon^{-4}z^{-2}}$$

11.2. $G(s) = \dfrac{(s+1)(s+20)}{(s+2)(s+10)}$, $T = 1\,s.$ $T = .01\,s.$

(a)

$$G(z) = G(s)\Big|_{s=\frac{1-z^{-1}}{T}} = \frac{\left(\frac{1-z^{-1}}{T}+1\right)\left(\frac{1-z^{-1}}{T}+20\right)}{\left(\frac{1-z^{-1}}{T}+2\right)\left(\frac{1-z^{-1}}{T}+10\right)}$$

$$= \frac{(1+T)(1+20T)}{(1+2T)(1+10T)}\;\frac{\left(z-\frac{1}{1+T}\right)\left(z-\frac{1}{1+20T}\right)}{\left(z-\frac{1}{1+2T}\right)\left(z-\frac{1}{1+10T}\right)}$$

(b)

$$G(z) = G(s)\Big|_{s=\frac{z-1}{T}} = \frac{(z-1+T)(z-1+20T)}{(z-1+2T)(z-1+10T)}$$

11.2. (c) $G(z) = G(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$

$$= \frac{(1+\frac{T}{2})(1+\frac{20T}{2})}{(1+\frac{2T}{2})(1+\frac{10T}{2})} \frac{\left(z-\frac{1-T/2}{1+T/2}\right)\left(z-\frac{1-10T}{1+10T}\right)}{\left(z-\frac{1-T}{1+T}\right)\left(z-\frac{1-5T}{1+5T}\right)}$$

(d) $G(z) = G(s)\Big|_{s=\epsilon^{Ts}}$

(e) $G(z) = G(s)\Big|_{s+\alpha=1-z^{-1}\epsilon^{-\alpha T}} = 1 - \frac{9/4}{1-z^{-1}\epsilon^{-2T}} + \frac{45/4}{1-z^{-1}\epsilon^{-10T}}$

11.3. Ans. $G(s) = \frac{1}{(s+.494)[(s+.245)^2+(.964)^2]}$

11.4. Ans. $D(z) = (1.49\times10^{-5})\frac{(z+1)^3}{(z^2-1.966z+.9697)(z-.9694)}$

11.5(a)

S-FREQUENCY	MAGNITUDE,dB	PHASE	W-FREQUENCY
0	-271.07	0	0
50	-97.4202	85.4321	2.50001E-03
100	-66.2523	80.4359	5.00004E-03
150	-46.5956	74.3301	7.50014E-03
200	-30.7376	65.784	1.00003E-02
250	-15.6925	50.8372	1.25007E-02
300	1.10882	6.3139	1.50011E-02
350	7.57222	-93.2477	1.75018E-02
400	9.80014	-140.009	2.00027E-02
450	12.4175	-185.591	2.25038E-02
500	13.4815	-225.002	2.50052E-02
550	15.0422	-256.814	2.75069E-02
600	17.4484	-300.293	.030009
650	15.5558	-358.766	3.25114E-02
700	10.643	-29.8907	3.50143E-02
750	6.71248	-44.1172	3.75176E-02
800	3.75077	-52.1593	4.00214E-02
850	1.43021	-57.431	4.25256E-02
900	-.448515	-61.2057	4.50304E-02
950	-2.01575	-64.0754	4.75358E-02
1000	-3.35195	-66.3499	5.00417E-02

11.5.(b) $D(z) = \dfrac{.99481z^2 - 1.98962z + .99481}{z^2 - 1.98265z + .984603} \cdot \dfrac{.0001z^2 + .0002z + .0001}{z^2 - 1.98603z + .98987}$

$\cdot \dfrac{.01558z^2 - .01558}{z^2 - 1.98265z + .984603}$

S-FREQUENCY	MAGNITUDE, dB	PHASE	W-FREQUENCY
0	-206.795	-180	0
50	-73.5303	-94.5063	2.50001E-03
100	-54.3812	-99.4406	5.00004E-03
150	-41.7625	-105.459	7.50014E-03
200	-30.9235	-113.873	1.00003E-02
250	-19.7316	-128.606	1.25007E-02
300	-6.18925	-172.73	1.50011E-02
350	-2.44455	-271.472	1.75018E-02
400	-2.4749	-316.683	2.00027E-02
450	-1.55526	-1.45474	2.25038E-02
500	-2.09172	-42.3845	2.50052E-02
550	-2.19485	-75.4335	2.75069E-02
600	-1.34363	-119.51	.030009
650	-4.6741	-178.278	3.25114E-02
700	-10.9067	-209.565	3.50143E-02
750	-16.059	-223.885	3.75176E-02
800	-20.1602	-231.988	4.00214E-02
850	-23.5491	-237.299	4.25256E-02
900	-26.4324	-241.103	4.50304E-02

(c) $D(z) = \dfrac{.098596z^2 - .197192z + .098596}{z^2 - 1.9938z + .99482} \cdot \dfrac{z^2}{z^2 - 1.986z + .9898}$

$\cdot \dfrac{.0314z^2 - .0314}{z^2 - 1.98265z + .9846}$

S-FREQUENCY	MAGNITUDE, dB	PHASE	W-FREQUENCY
0	31.8421	-180	0
50	-65.3579	-94.1284	2.50001E-03
100	-46.1854	-98.7001	5.00004E-03
150	-33.5678	-104.358	7.50014E-03
200	-22.7287	-112.411	1.00003E-02
250	-11.6091	-126.669	1.25007E-02
300	1.97877	-168.837	1.50011E-02
350	6.28466	-269.21	1.75018E-02
400	6.09738	-316.763	2.00027E-02
450	6.62627	-2.27035	2.25038E-02
500	5.86552	-41.5093	2.50052E-02
550	5.8058	-73.6367	2.75069E-02
600	6.56895	-113.556	.030009
650	2.86936	-175.608	3.25114E-02
700	-3.30432	-204.707	3.50143E-02
750	-8.36398	-218.002	3.75176E-02
800	-12.403	-225.437	4.00214E-02
850	-15.7473	-230.199	4.25256E-02
900	-18.5994	-233.506	4.50304E-02
950	-21.0908	-235.924	4.75358E-02
1000	-23.3066	-237.756	5.00417E-02

11-6. (a) $D(z) = \dfrac{.97988z^2 - 1.95977z + .97988}{z^2 - 1.9828z + .9938} \cdot \dfrac{10^{-8}z^2}{z^2 - 1.9823z + .986}$

$$\cdot \frac{.0314z^2 - .0314}{z^2 - 1.9809z + .9828}$$

S-FREQUENCY	MAGNITUDE, dB	PHASE	W-FREQUENCY
0	63.3848	-180	0
50	-61.94	-95.1107	2.50001E-03
100	-44.7546	-100.765	5.00004E-03
150	-32.573	-107.749	7.50014E-03
200	-21.8766	-117.[illegible]	1.00003E-02
250	-10.9571	-135.249	1.25007E-02
300	1.3132	-184.133	1.50011E-02
350	5.38402	-275.[illegible]	1.75018E-02
400	5.39977	-324.259	2.00027E-02
450	5.55017	-9.12092	2.25038E-02
500	4.31353	-47.7542	2.50052E-02
550	4.45408	-32.7355	2.75069E-02
600	3.69813	-126.391	.030009
650	-6.10325E-02	-169.757	3.25114E-02
700	-5.07844	-195.392	3.50143E-02
750	-9.51153	-239.343	3.75176E-02
800	-13.2228	-217.739	4.00214E-02
850	-16.3756	-223.417	4.25256E-02
900	-19.1036	-227.418	4.50304E-02
950	-21.5099	-230.3[illegible]6	4.75358E-02
1000	-23.6628	-232.6[illegible]7	5.00417E-02

(b) $D(z) = \dfrac{.98596z^2 - 1.97192z + .98596}{z^2 - 1.9948z + .9958} \cdot \dfrac{10^{-8}}{z^2 - 1.989776z + .9936}$

$$\cdot \frac{.0314z - .0314}{z^2 - 1.9844z + .98646}$$

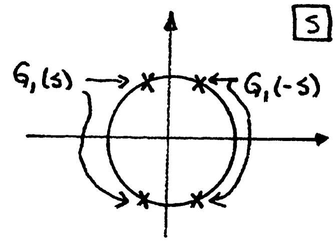
S-FREQUENCY	MAGNITUDE, dB	PHASE	W-FREQUENCY
0	-190.298	-180	0
50	-65.3849	-94.1265	2.50001E-03
100	-46.2414	-98.6163	5.00004E-03
150	-33.5824	-104.014	7.50014E-03
200	-22.6763	-111.441	1.00003E-02
250	-11.3295	-124.299	1.25007E-02
300	3.4093	-165.947	1.50011E-02
350	6.76675	-275.634	1.75018E-02
400	6.69926	-318.614	2.00027E-02
450	7.76643	-5.69963	2.25038E-02
500	6.90267	-46.5817	2.50052E-02
550	7.06844	-75.6437	2.75069E-02
600	9.99685	-120.372	.030009
650	5.85685	-201.571	3.25114E-02
700	-2.11056	-230.511	3.50143E-02
750	-7.73369	-241.722	3.75176E-02
800	-12.0033	-248.[illegible]73	4.00214E-02

11-7. See the problems above.

11-8.

$$\omega^4 - \omega^2 + 1.25 \Big|_{\omega = s/j} = (s^2 + \tfrac{1}{2} + j1)(s^2 + \tfrac{1}{2} - j1)$$

$$\therefore \quad s_{1,2} = 1.057 \underline{/ \pm 58.2825^\circ}, \quad s_{3,4} = 1.057 \underline{/ \pm 121.7175^\circ}$$



11-9.

$$D(z) = K_P + \frac{K_I T z}{z-1} + \frac{K_D}{T}\left(\frac{z-1}{z}\right) = \frac{(K_P + K_I + K_D) + (-K_P - 2K_D)z^{-1} + K_D z^{-2}}{1 - z^{-1}}$$

$$\therefore \quad a_0 = K_P + K_I T + K_D/T \qquad b_1 = -1$$

$$a_1 = -K_P - 2K_D/T \qquad b_2 = 0$$

$$a_2 = K_D/2$$

11-10.

$$D(z) = K_P + \frac{K_I T}{2}\left(\frac{z+1}{z-1}\right) + \frac{2K_D}{T}\left(\frac{z-1}{z+1}\right)$$

$$= \frac{(K_P + K_I T/2 + 2K_D/T)z^2 + (K_I T - 4K_D/T)z + (-K_P + K_I T/2 + 2K_D/T)}{z^2 - 1}$$

$$a_0 = K_P + K_I T/2 + 2K_D/T \qquad b_1 = 0$$

$$a_1 = K_I T - 4K_D/T \qquad b_2 = -1$$

$$a_2 = -K_P + K_I T/2 + 2K_P/T$$

11-11.

(8-52) $\quad D(z) = K_P + \frac{K_I T}{2}\left(\frac{z+1}{z-1}\right) + \frac{K_D}{T}\left(\frac{z-1}{z}\right) \qquad \left\{ \begin{array}{l} s = \frac{2}{T}\left(\frac{z-1}{z+1}\right) \\ s = \frac{z-1}{Tz} \end{array} \right.$

11-12. Use the MATLAB program in Example 11.1 and

(a) Increase the order of the Butterworth prototype, n;
(n=16 yields an attenuation at 500 Hz of 31.0 dB)

(b) Decrease the cutoff frequency to say 250 Hz before increasing the order of the Butterworth prototype.
(n=5 yields an attenuation at 500 Hz of 30.1 dB)

(c) Many other combinations are also valid, for example at a cutoff of 300 Hz.
(n=7 yields an attenuation at 500 Hz of 31.1 dB)

11-13. No. The frequency response has a peak between 0.1 and 0.2 Hz, making it unstable for this application.

11-14. Change the MATLAB program in example 11.3 by substituting 60 Hz for each occurrence of 50 Hz. Due to numerical difficulties, the notch attenuation values will be inaccurate:

```
»z1,p1,k1]=buttap(3); [num1,den1]=zp2tf(z1,p1,k1);
[num2,den2]=lp2bs(num1,den1,2*pi*59.99999,2*pi*2); f=[10 58 59
60 61 62 100]; w=2*pi*f; h=freqs(num2,den2,w); mag=abs(h);
loglog(f,mag); fs=1000;
[numd,dend]=bilinear(num2,den2,fs,59.99999);
hd=freqz(numd,dend,w/fs); magd=abs(hd), db=20*log10(magd),
loglog(f,magd)

Warning: Matrix is close to singular or badly scaled.
        Results may be inaccurate. RCOND = 5.341927e-25

magd =

    1.0000    0.9939    0.7400    0.0000    0.7238    0.9937    1.0000


db =

    0.0000   -0.0530   -2.6153  -92.5967   -2.8071   -0.0549   -0.0000
```

11-14

```
»numd, dend

numd =

    0.9878   -5.5106   13.2107  -17.3730   13.2107   -5.5106    0.9878


dend =

    1.0000   -5.5558   13.2647  -17.3728   13.1566   -5.4657    0.9758
```

Note that the attenuation at 58 and 62 Hz is about .7%, and at 59 and 61 Hz is about 2.7 dB. The attenuation is calculated to be about 92 dB, slightly less than the desired specification.

CHAPTER 12

12-1.

$$D(z) = \alpha_0 + \frac{\alpha_1 + \alpha_2 z^{-1} + \alpha_3 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}} = \frac{\alpha_0 + \alpha_1 + (\alpha_0 b_1 + \alpha_2) z^{-1} + (\alpha_0 b_2 + \alpha_3) z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}$$

$\therefore\ a_0 = \alpha_0 + \alpha_1$; $a_1 = \alpha_0 b_1 + \alpha_2$; $a_3 = \alpha_0 b_2 + \alpha_3$

with $\alpha_1 = 0$,

$\alpha_0 = a_0$; $\alpha_2 = a_1 - a_0 b_1$; $\alpha_3 = a_3 - a_0 b_2$

12-2.

From prob. 12-1, with $\alpha_2 = 0$,

$\alpha_0 = a_1/b_1$; $\alpha_1 = a_0 - a_1/b_1$; $\alpha_3 = a_3 - a_1 b_2/b_1$

12-3.

From prob. 12-1, with $\alpha_3 = 0$,

$\alpha_0 = a_3/b_2$; $\alpha_1 = a_0 - a_3/b_2$; $\alpha_2 = a_1 - a_3 b_1/b_2$

12-4.

$$D(z) = \alpha_0 + \frac{\alpha_1 z^{-1}(1 + b_1 z^{-1}) + \alpha_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}} = \frac{\alpha_0 + (\alpha_0 b_1 + \alpha_1) z^{-1} + (\alpha_0 b_2 + \alpha_1 b_1 + \alpha_2) z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}$$

$\therefore\ a_0 = \alpha_0$; $a_1 = \alpha_0 b_1 + \alpha_1$; $a_2 = \alpha_0 b_2 + \alpha_1 b_1 + \alpha_2$

$\therefore\ \alpha_0 = a_0$; $\alpha_1 = a_1 - a_0 b_1$; $\alpha_2 = a_2 - a_0 b_2 - a_1 b_1 + a_0 b_1^2$

12-5.

Any direct structure with

$a_0 = K_P + K_I T + K_D/T$ $\qquad b_1 = -1$

$a_1 = -K_P - 2K_D/T$ $\qquad b_2 = 0$

$a_2 = K_D/T$

12-6. Any direct structure with

$$a_0 = K_P + \frac{K_I T}{2} + \frac{2K_D}{T} \qquad b_1 = 0$$

$$a_1 = K_I T - \frac{4K_D}{T} \qquad b_2 = -1$$

$$a_2 = -K_P + \frac{K_I T}{2} + \frac{2K_D}{T}$$

12-7. No. The 1X structure requires complex pole pairs.

CHAPTER 13

13-1. $a_0 = \lfloor 1.0 \times 2^{14} + .5 \rfloor = 16\,384$

$a_1 = a_0$

$Q[b_1] = -\lfloor .97337 \times 2^{14} + .5 \rfloor = -\lfloor 15948.19 \rfloor = -15948$ $\left(\begin{smallmatrix}\text{stored}\\ \text{value}\end{smallmatrix}\right)$

$Q[b_1] = -15948/2^{14} = -.9733\underline{9}$

Last digit is slightly larger.

13-2. $a_0 = a_2 = 16\,384$

$Q[a_1] = -\lfloor 1.9661 \times 2^{14} + .5 \rfloor = -32213$ (stored value)

$Q[a_1] = -32213/16384 = -1.9661$

$Q[b_1] = -\lfloor 1.9654 \times 2^{14} + .5 \rfloor = -32201$ (stored)

$\Rightarrow -32201/16\,384 = -1.9654$

$Q[b_2] = \lfloor .99930 \times 2^{14} + .5 \rfloor = 16\,373$ (stored)

$\Rightarrow 16373/16\,384 = .9993\underline{3}$

13-3. $a_0 = a_2 = 16\,384$

$Q[a_1] = -32213/2^{14} = -1.9661$

$Q[b_1] = -\lfloor (.22433 + .52749) \times 10^{14} + .5 \rfloor = -12318$ (stored)

$\Rightarrow -.75183$

$Q[b_2] = \lfloor (.22433) * (.52749) \times 2^{14} + .5 \rfloor = 1939$ (stored)

$\Rightarrow .11835$

13-4. The quantized version is:

$$D(z) = \left(\frac{1 + z^{-1}}{1 - .9733\underline{9}z^{-1}}\right)\left(\frac{1 - 1.9661z^{-1} + z^{-2}}{1 - 1.9654z^{-1} + .9993\underline{3}z^{-2}}\right)\left(\frac{1 - 1.9661z^{-1} + z^{-2}}{1 - .7518\underline{3}z^{-1} + .1183\underline{5}z^{-2}}\right)$$

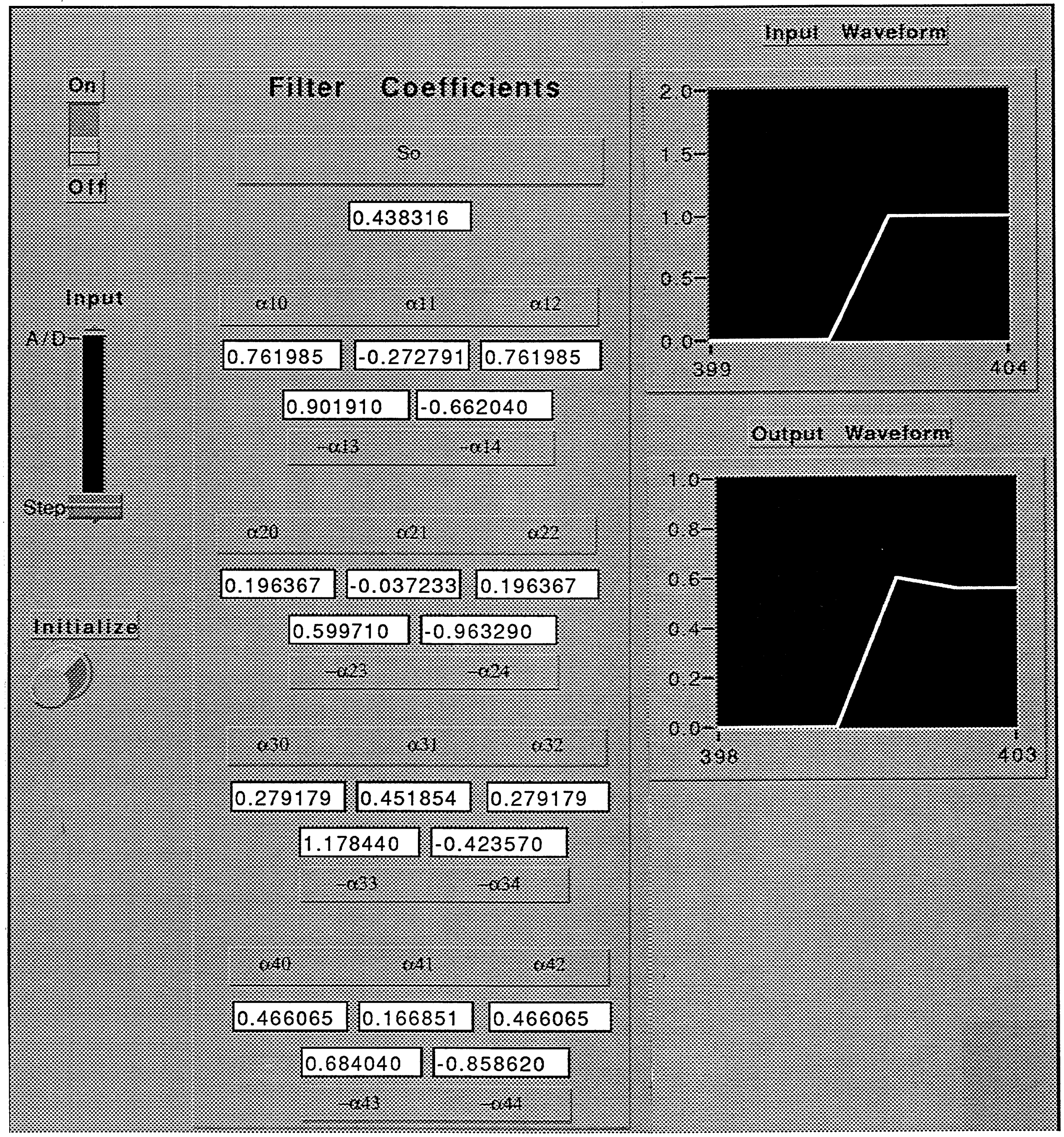
The underlined digits are changed by quantizing. There is no difference visually in the frequency plots.

13-5. INTEL 8086 : $n=16$

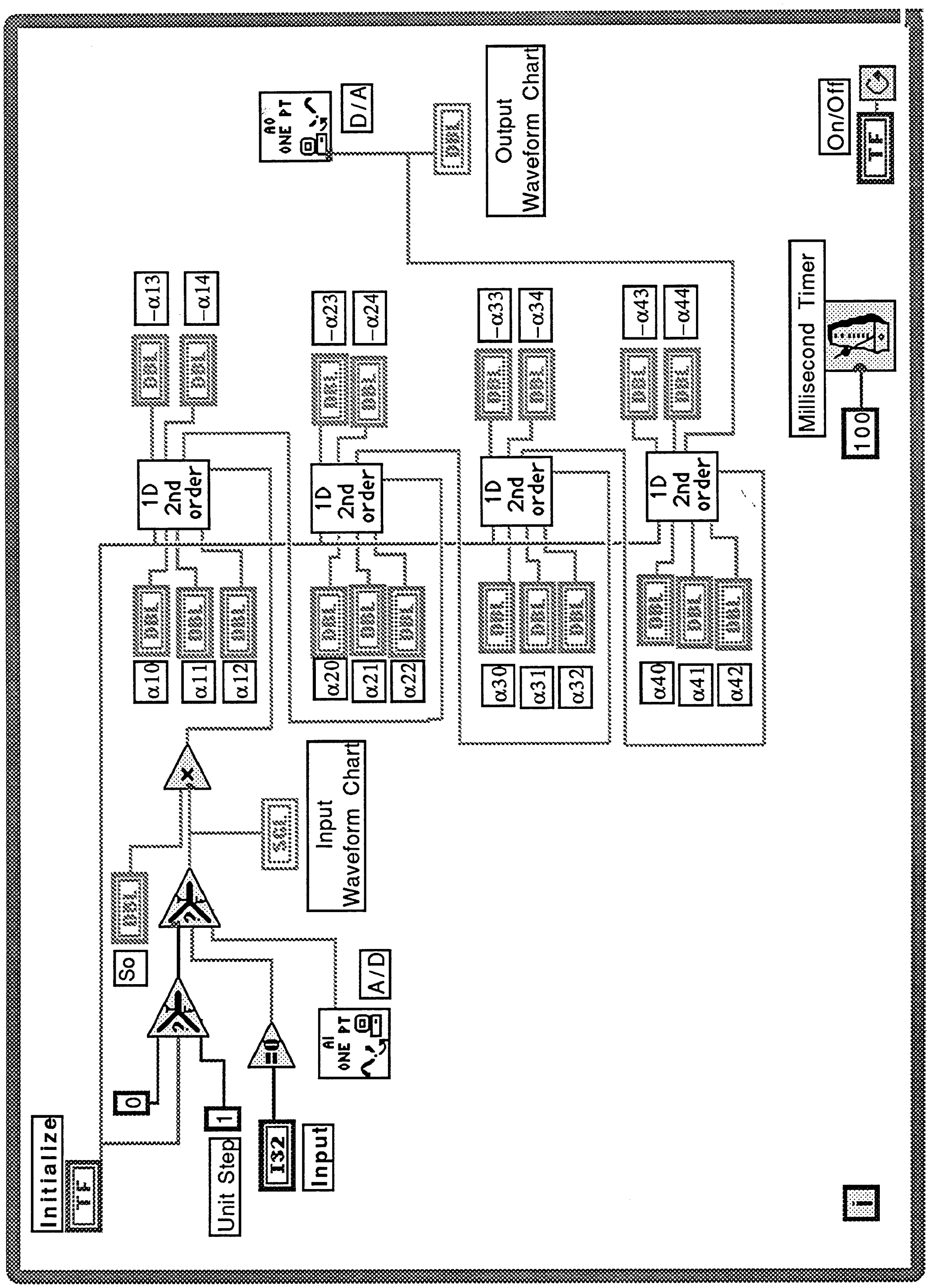
$$\text{Result} = \lfloor (aX/4) * 2^{n-1} \rfloor * 4 = \lfloor (.435102)(.713001) \times 2^{n-3} \rfloor \times 4$$

$$= \lfloor .3102281611 \times 2^{13} \rfloor \times 4 = 10\,164$$

13-6. Modify the LabVIEW Program of Fig. 13-20 to implement four cascaded, second-order, 1D Modules as shown below:

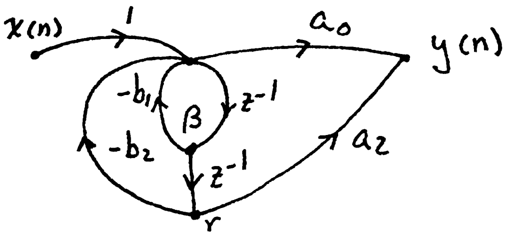


On
Off
Input
A/D
Step
Initialize
Filter Coefficients
So
0.438316
α10 α11 α12
0.761985 -0.272791 0.761985
0.901910 -0.662040
-α13 -α14
α20 α21 α22
0.196367 -0.037233 0.196367
0.599710 -0.963290
-α23 -α24
α30 α31 α32
0.279179 0.451854 0.279179
1.178440 -0.423570
-α33 -α34
α40 α41 α42
0.466065 0.166851 0.466065
0.684040 -0.858620
-α43 -α44
Input Waveform
2.0
1.5
1.0
0.5
0.0
399
404
Output Waveform
1.0
0.8
0.6
0.4
0.2
0.0
398
403


Initialize
TF
0
1
Unit Step
I32
Input
So
DBL
SGL
Input Waveform Chart
A/D
AI ONE PT
1D 2nd order
α10
α11
α12
−α13
−α14
α20
α21
α22
−α23
−α24
α30
α31
α32
−α33
−α34
α40
α41
α42
−α43
−α44
AO ONE PT
D/A
Output Waveform Chart
Millisecond Timer
100
On/Off

CHAPTER 14

14-1



$$D(z) = \frac{Y(z)}{X(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}$$

(a) $$\frac{\partial D(z)}{\partial a_0} = \frac{1}{1+b_1 z^{-1} + b_2 z^{-2}} \qquad \frac{\partial D(z)}{\partial b_1} = \frac{-(a_0 + a_1 z^{-1} + a_2 z^{-2}) z^{-1}}{(1+b_1 z^{-1} + b_2 z^{-2})^2}$$

$$\frac{\partial D(z)}{\partial a_1} = \frac{z^{-1}}{1+b_1 z^{-1} + b_2 z^{-2}} \qquad \frac{\partial D(z)}{\partial b_2} = \frac{-(a_0 + a_1 z^{-1} + a_2 z^{-2}) z^{-2}}{(1+b_1 z^{-1} + b_2 z^{-2})^2}$$

$$\frac{\partial D(z)}{\partial a_2} = \frac{z^{-2}}{1+b_1 z^{-1} + b_2 z^{-2}}$$

(b) $D(z) = T_{ab}(z)$; $\dfrac{\partial T_{ab}(z)}{\partial F_{nm}(z)} = T_{an}(z)\, T_{mb}(z)$

$$\therefore \frac{\partial D(z)}{\partial a_0} = \frac{\partial T_{ab}(z)}{\partial F_{\alpha b}(z)} = T_{a\alpha}(z)\, T_{bb}(z)$$

$$\frac{\partial D(z)}{\partial a_1} = \frac{\partial T_{ab}(z)}{\partial F_{\beta b}(z)} = T_{a\beta}(z)\, T_{bb}(z) = T_{a\beta}(z)$$

$$\frac{\partial D(z)}{\partial a_2} = \frac{\partial T_{ab}(z)}{\partial F_{rb}(z)} = T_{ar}(z)\, T_{bb}(z) = T_{ar}(z)$$

$$\frac{\partial D(z)}{\partial(-b_1)} = \frac{\partial T_{ab}(z)}{\partial F_{\beta\alpha}(z)} = T_{a\beta}(z)\, T_{\alpha b}(z)$$

$$\frac{\partial D(z)}{\partial(-b_2)} = \frac{\partial T_{ab}(z)}{\partial F_{r\alpha}(z)} = T_{ar}(z)\, T_{\alpha b}(z)$$

Results same as in part a).

(c) $$\Delta D(z) = \frac{\partial D(z)}{\partial a_0}\Delta a_0 + \frac{\partial D(z)}{\partial a_1}\Delta a_1 + \frac{\partial D(z)}{\partial a_2}\Delta a_2 + \frac{\partial D(z)}{\partial b_1}\Delta b_1 + \frac{\partial D(z)}{\partial b_2}\Delta b_2$$

$$= \frac{(1+b_1 z^{-1} + b_2 z^{-2})(\Delta a_0 + z^{-1}\Delta a_1 + z^{-2}\Delta a_2) - (z^{-1}\Delta b_1 + z^{-2}\Delta b_2)(a_0 + a_1 z^{-1} + a_2 z^{-2})}{(1+b_1 z^{-1} + b_2 z^{-2})}$$

14-2. (a) $a_0 = 1$

$a_1 = 1.858741$ $\qquad$ $b_1 = -.748386$

$a_2 = 1$ $\qquad$ $b_2 = .213374$

$a_0 = 1 \Rightarrow 01.000\,000\,000\ldots$

$Q_\gamma(a_0) = 1 \Rightarrow 01.000\,000$ $\qquad \therefore \Delta a_0 = Q_\gamma(a_0) - a_0 = 0$

$a_1 = 1.858741 \Rightarrow 01.110\,110\,111\cdots$

$Q_\gamma(a_1) = 01.110111 \Rightarrow 1.859375$ $\quad \therefore \Delta a_1 = .000634$

$a_2 = 1$ $\qquad \Delta a_2 = 0$

$b_1 = -.748386 \Rightarrow (11.010\,000\,000\,110\cdots)_{2cms}$

$Q_\gamma(b_1) = (11.010\,000) \Rightarrow -.75$ $\qquad \Delta b_1 = -.001614$

$b_2 = .213374 \Rightarrow 00.001\,101\,101\cdots$

$Q_\gamma(b_2) = 00.001\,110 \Rightarrow .21875$ $\qquad \Delta b_2 = .005376$

(b) $$\Delta D(z) = \frac{.002248z^3 - .00285047z^2 - .00824331z - .005376}{(z^2 - .748386z + .213374)^2}$$

(c) See next page

14-3. (a) $\Delta a_0 = 0$

$Q_\gamma(a_1) = 1.85876465$ $\qquad \Delta a_1 = .00002365$

$\Delta a_2 = 0$

$Q_\gamma(b_1) = -0.74835205$ $\qquad \Delta b_1 = .00003395$

$Q_\gamma(b_2) = .21337891$ $\qquad \Delta b_2 = .00000491$

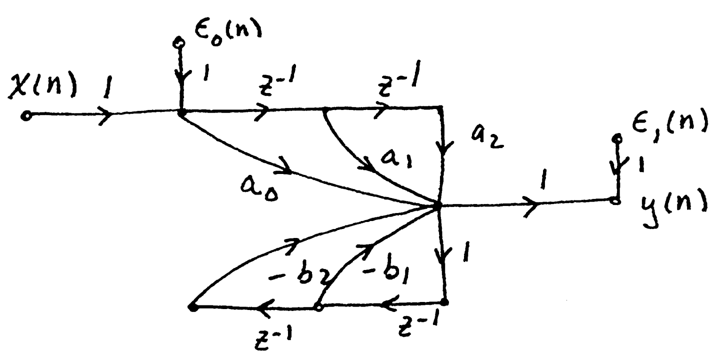
(b) $$\Delta D(z) = \frac{-.00031585z^3 - .00065365z^2 - .00034358z - .00000491}{(z^2 - .748386z + .213374)^2}$$

(c) See page 139.

14-2.(c)

S-FREQUENCY	MAGNITUDE	PHASE	W-FREQUENCY
.1	6.57751E-02	-182.794	5.00004E-03
1.1	6.56822E-02	-210.763	5.50555E-02
2.1	6.53319E-02	-238.842	.105388
3.1	6.44642E-02	-267.082	.156253
4.1	6.27306E-02	-295.442	.207921
5.1	5.98133E-02	-323.753	.260675
6.1	5.55713E-02	-351.714	.314823
7.1	.050145	-18.9322	.370705
8.1	.043941	-45.0021	.428699
9.1	3.74933E-02	-69.5884	.489237
10.1	3.12923E-02	-92.4832	.552813
11.1	2.56748E-02	-113.614	.620006
12.1	2.08033E-02	-133.02	.691502
13.1	1.67046E-02	-150.812	.768124
14.1	1.33233E-02	-167.134	.850872
15.1	.010567	-182.139	.94098
16.1	8.33445E-03	-195.972	1.03999
17.1	6.53118E-03	-208.754	1.14988
18.1	5.07526E-03	-220.579	1.27318
19.1	3.89884E-03	-231.504	1.41326
20.1	2.94710E-03	-241.531	1.57467
21.1	2.17653E-03	109.421	1.76369
22.1	1.55313E-03	101.605	1.9893
23.1	1.05095E-03	95.6722	2.2648
24.1	6.52149E-04	93.4688	2.61073
25.1	3.52915E-04	101.496	3.06063
26.1	2.03309E-04	140.588	3.67326
27.1	2.68779E-04	-178.46	4.56184
28.1	3.88756E-04	-168.875	5.97613
29.1	4.86047E-04	-169.495	8.59723
30.1	5.49579E-04	-173.342	15.1765
31.1	5.78099E-04	-178.34	63.2843
32.1	5.71670E-04	-183.565	-29.2239
33.1	5.30254E-04	-188.277	-11.8478
34.1	4.53965E-04	-191.298	-7.40652
35.1	3.45785E-04	-189.514	-5.36723

14-4.



$a_0 = 1$ $b_1 = -.75$

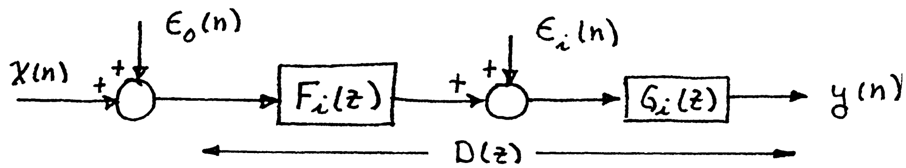
$a_1 = 1.859375$ $b_2 = .21875$

$a_2 = 1$

14-3.(c)

S-FREQUENCY	MAGNITUDE	PHASE	W-FREQUENCY
.1	6.10730E-03	-181.955	5.00004E-02
1.1	6.06929E-03	-201.548	5.50555E-02
2.1	5.96138E-03	-221.36	.105388
3.1	5.76794E-03	-241.527	.156253
4.1	5.47113E-03	-262.077	.207921
5.1	5.06113E-03	-282.887	.260675
6.1	4.54646E-03	-303.685	.314823
7.1	3.95808E-03	-324.085	.370705
8.1	3.34279E-03	-343.679	.428699
9.1	2.74885E-03	-2.11776	.489237
10.1	2.21277E-03	-19.1753	.552813
11.1	1.75373E-03	-34.76	.620006
12.1	1.37555E-03	-48.8921	.691502
13.1	1.07216E-03	-61.6658	.768124
14.1	8.32835E-04	-73.2144	.850872
15.1	6.45858E-04	-83.6849	.94098
16.1	5.00435E-04	-453.22	1.03999
17.1	3.87446E-04	-461.957	1.14988
18.1	2.99548E-04	-470.014	1.27318
19.1	2.30994E-04	-477.498	1.41326
20.1	1.77361E-04	-484.505	1.57467
21.1	1.35270E-04	-131.119	1.76369
22.1	1.02155E-04	-137.42	1.9893
23.1	7.60676E-05	-143.483	2.2648
24.1	5.55287E-05	-149.387	2.61073
25.1	3.94173E-05	-155.225	3.06063
26.1	2.68848E-05	-161.114	3.67326
27.1	1.72927E-05	-167.235	4.56184
28.1	1.01661E-05	-173.902	5.97613
29.1	5.15894E-06	-181.741	8.59723
30.1	2.02491E-06	-191.726	15.1765
31.1	5.77709E-07	-193.299	63.2843
32.1	9.23109E-07	-162.916	-29.2239
33.1	2.97342E-06	-172.224	-11.8478
34.1	6.77310E-06	-181.342	-7.40652
35.1	1.25273E-05	-188.646	-5.36723
36.1	2.05154E-05	-195.061	-4.19142
37.1	3.11268E-05	-201.069	-3.42335
38.1	4.48927E-05	-206.923	-2.87995

14-4.
(cont.)



$$Y(z) = X(z)D(z) + E_0(z)D(z) + E_1(z) + E_2(z)G_2(z)$$

$$E_i(z) = \frac{L_i\, z^{-b/2}}{1-z^{-1}}$$

$$E_n(z) = \sum_{i=0}^{2} E_i(z)\,G_i(z) \quad , \; G_0(z) = D(z)$$

$$\epsilon_{n_i}(\infty) = \lim_{z\to 1}(1-z^{-1})E_{n_i}(z) = \frac{L_i}{2}\, z^{-b}\, G_i(z)\Big|_{z=1}$$

14.4. $\epsilon_n(\infty) \doteq \sum_{i=0}^{2} \left(\frac{2^{-b}}{2}\right) L_i |G_i(1)|$

(a) $G_0(1) = D(1) = 8.2333$, $G_1(1) = 1$

$$G_2(1) = \left.\frac{-b_1 z^{-1} - b_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}\right|_{z=1} = 1.13333$$

$$\therefore \epsilon_n(\infty) \doteq \frac{2^{-7}}{2}[8.2333 + 1 + 1.13333] = 0.0404948$$

(b) $G_0(z) = G_2(z) = D(z)$, $G_1(z) = 1$

$D(1) = 8.2333$

$$\therefore \epsilon_n(\infty) \doteq \frac{2^{-7}}{2}[8.2333 + 8.2333 + 1] = 0.0682292$$

(c) $G_0(z) = D(z)$, $G_0(1) = 8.2333$

$G_1(z) = 1$

$G_2(z) = \dfrac{-b_1 z^{-1} - b_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}$, $G_2(1) = 1.1333$

$G_3(z) = \dfrac{z^{-1}}{1 + b_1 z^{-1} + b_2 z^{-2}}$, $G_3(1) = 2.1333$

$G_4(z) = \dfrac{z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}$, $G_4(1) = 2.1333$

$\therefore \epsilon_n(\infty) \doteq .0571615$

$G_0(1) = 8.2333$; $G_1(z) = 1$

$G_2(z) = D(z)$, $G_2(1) = 8.2333$

$G_3(z) = z^{-1}$; $G_3(1) = 1$

$G_4(z) = z^{-1} D(z)$; $G_4(1) = 8.2333$

$\therefore \epsilon_n(\infty) \leq .104297$

14-5.
(a) $b_1 + b_2 - \frac{1}{2} = .98381 + .09443 - \frac{1}{2} > 0$ $\therefore$ limit cycles

$2\sqrt{b_2} \leq |b_1|$

$$\therefore |y^g(k)| \doteq \frac{2^{-b}}{1 - .98381 + .09443} = 9.03996 \times 2^{-b}$$

(b) No overflow, no limit cycle.

(C) Overflow oscillations exist, limit cycles are possible

$$|y^{g}(b)| \doteq \frac{2^{-b}}{1-|b_1|+b_2} = 14.74 \times 2^{-b}$$

14-6.

step	pole-zero pairs
1	α_1/β_1
2	α_2/β_2
3	α_3/β_3

14-7.

step	pole-zero pairs
1	α_5/β_5
2	α_3/β_1
3	α_2/β_2
4	α_4/β_3
5	α_1/β_4
