System Analysis

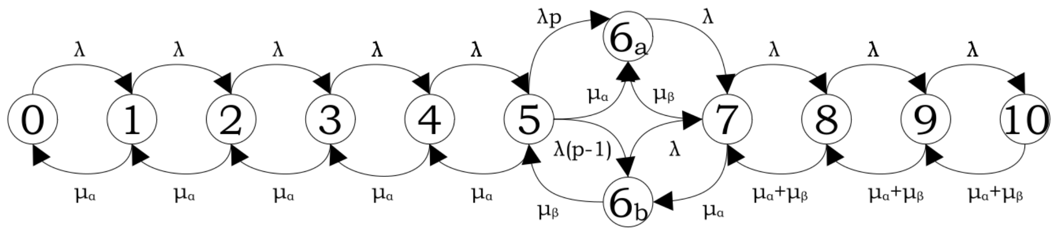

Figure 1: Diagram of Transitions

From Local Balance Equations:

$$\lambda P_0 = \mu_\alpha P_1 \Leftrightarrow P_1 = \frac{\lambda}{\mu_\alpha} P_0$$

$$\lambda P_1 = \mu_\alpha P_2 \Leftrightarrow P_2 = \left(\frac{\lambda}{\mu_\alpha}\right)^2 P_0$$

In general:

$$P_i = \left(\frac{\lambda}{\mu_\alpha}\right)^i P_0, \quad i = 1, 2, 3, 4, 5 \tag{1}$$

For the states 6_a and 6_b, a general state 6 is defined, where $P_6 = P_{6a} + P_{6b}$.

$$\lambda P_5 = (\mu_\alpha + \mu_\beta) P_6 \Leftrightarrow P_6 = \left(\frac{\lambda}{\mu_a + \mu_b}\right) P_5 \Leftrightarrow$$

$$\Leftrightarrow P_6 = \frac{\lambda}{\mu_a + \mu_b} \left(\frac{\lambda}{\mu_a}\right)^5 P_0 \tag{2}$$

$$2\lambda P_6 = (\mu_\alpha + \mu_\beta) P_7 \Leftrightarrow P_7 = \left(\frac{2\lambda}{\mu_a + \mu_b}\right) P_6 \Leftrightarrow P_7 = 2\left(\frac{\lambda}{\mu_a + \mu_b}\right)^2 \left(\frac{\lambda}{\mu_a}\right)^5 P_0$$

In general:

$$P_i = 2\left(\frac{\lambda}{\mu_a + \mu_b}\right)^{i-5} \left(\frac{\lambda}{\mu_a}\right)^5 P_0, \quad i = 7, 8, 9, 10 \tag{3}$$

From normalisation and 1, 2, 3,:

$$\sum_{i=0}^{10} P_i = 1 \Leftrightarrow$$

$$P_0 = \left[1 + \frac{\lambda}{\mu_a} + \left(\frac{\lambda}{\mu_a}\right)^2 + \cdots + \frac{\lambda}{\mu_a + \mu_b}\left(\frac{\lambda}{\mu_a}\right)^5 + 2\left(\frac{\lambda}{\mu_a + \mu_b}\right)^2\left(\frac{\lambda}{\mu_a}\right)^5 + \cdots\right]^{-1}$$

The mean number of clients is derived from the formula:

$$E[n(t)] = \sum_{i=0}^{10} iP_i =$$

$$= \left[\frac{\lambda}{\mu_a} + 2\left(\frac{\lambda}{\mu_a}\right)^2 + \cdots + 6\frac{\lambda}{\mu_a + \mu_b}\left(\frac{\lambda}{\mu_a}\right)^5 + 7 \times 2\left(\frac{\lambda}{\mu_a + \mu_b}\right)^2\left(\frac{\lambda}{\mu_a}\right)^5 + \cdots\right]P_0$$

Finally, throughput is calculated from the formula:

$$\gamma = \lambda(1 - P_{blocking}) = \lambda(1 - P_{10})$$