# Algorithm Analysis: Time Complexity

A Comprehensive Guide for Technical Excellence in Data Structures and Algorithms.

## 1. Understanding Asymptotic Notations

Time complexity measures how the runtime of an algorithm grows as the size of the input (`n`) grows. We use **Asymptotic Notations** to describe this growth rate by ignoring constant factors and lower-order terms. This allows us to predict performance for large datasets.

### Big O Notation (`O`)

**Worst-Case Time Complexity (Upper Bound).** It defines the maximum time an algorithm will take for a given input size `n`. It's the most critical metric for performance guarantees.

**Example:**
Sequential Search is `O(n)`.

### Big Omega Notation (`Ω`)

**Best-Case Time Complexity (Lower Bound).** It defines the minimum time an algorithm will take for any given input size `n`.

**Example:**
Optimized Bubble Sort is `Ω(n)` (when the array is already sorted).

### Big Theta Notation (`Θ`)

**Tight Bound (Average Case).** It is used when the best-case and worst-case complexities are the same, providing a tight constraint on the function's growth rate.

**Example:**
Merge Sort is always `Θ(n \log n)`.

## 2. Time Complexity of Searching Algorithms

| ALGORITHM | CONDITION | BEST CASE (Ω) | AVERAGE CASE (Θ) | WORST CASE (O) |
|---|---|---|---|---|
| Sequential (Linear) Search | Unsorted/Sorted Array | $O(1)$ (Key at first index) | $O(n)$ | $O(n)$ (Key at last or not present) |
| Binary Search (Iterative/Recursive) | **Requires Sorted Array** | $O(1)$ (Key at middle index) | $O(\log n)$ | $O(\log n)$ |

**Note:** Binary Search is significantly faster than Sequential Search for large, sorted inputs because it eliminates half the remaining elements in each step, reducing the search space logarithmically.

## 3. Time Complexity of Sorting Algorithms

| ALGORITHM | BEST CASE (Ω) | AVERAGE CASE (Θ) | WORST CASE (O) | SPACE COMPLEXITY (AUXILIARY) |
|---|---|---|---|---|
| Optimized Bubble Sort | $O(n)$ (Already Sorted) | $O(n^2)$ | $O(n^2)$ | $O(1)$ |
| Selection Sort | $O(n^2)$ (Constant comparisons always) | $O(n^2)$ | $O(n^2)$ | $O(1)$ |
| Insertion Sort | $O(n)$ (Already Sorted) | $O(n^2)$ | $O(n^2)$ | $O(1)$ |
| Quick Sort | $O(n \log n)$ (Optimal Pivot) | $O(n \log n)$ | $O(n^2)$ (Worst Pivot selection) | $O(\log n)$ (Avg) / $O(n)$ (Worst) |

| ALGORITHM | BEST CASE (Ω) | AVERAGE CASE (Θ) | WORST CASE (O) | SPACE COMPLEXITY (AUXILIARY) |
|---|---|---|---|---|
| Merge Sort | $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ | $O(n)$ (Requires temporary array) |
| Heap Sort | $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ | $O(1)$ |

**Takeaway:** Algorithms with $O(n \log n)$ average time complexity (Merge Sort, Quick Sort, Heap Sort) are generally considered the most efficient for large datasets.

## 4. Time Complexity of Data Structure Operations

| DATA STRUCTURE | ACCESS/INDEXING | SEARCH | INSERTION | DELETION |
|---|---|---|---|---|
| Array (Basic) | $O(1)$ | $O(n)$ (Sequential) | $O(n)$ (Insert in middle/beginning) | $O(n)$ (Delete from middle/beginning) |
| Linked List (Singly) | $O(n)$ (Requires traversal) | $O(n)$ | $O(1)$ (At head) / $O(n)$ (At end/position) | $O(1)$ (At head) / $O(n)$ (At end/position) |
| Stack / Queue (Array based) | $O(n)$ | $O(n)$ | $O(1)$ (Push/Enqueue) | $O(1)$ (Pop/Dequeue) |
| Hash Map / Hash Table | N/A | $O(1)$ (Average) / $O(n)$ (Worst Case Collision) | $O(1)$ (Average) / $O(n)$ (Worst Case Collision) | $O(1)$ (Average) / $O(n)$ (Worst Case Collision) |
| Balanced Binary Search Tree (AVL/Red-Black) | $O(\log n)$ | $O(\log n)$ | $O(\log n)$ | $O(\log n)$ |

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