UNIVERSIDADE DA MADEIRA
Faculdade de Ciências Exatas e da Engenharia

1º SEMESTRE 2021/2022
Cálculo I (Parte I)

Folhas de exercícios: 1 / 2 / 3 / 4 / 5 / 6 Folhas de apoio: 1, 2, 3, 4

1ª Frequência: | **17 de novembro 2021 (14:00-16:00)**
10 valores (mínimo 3,0)

FOLHA APOIO 1 (págs.1 - 4)

Revisão

$$n,k\in\mathbb{Z}\rightarrow\left|\begin{array}{l} a^n a^k = a^{n+k} \\ a^n b^n = (ab)^n \\ \dfrac{a^n}{a^k} = a^{n-k} \\ \dfrac{a^n}{b^n} = \left(\dfrac{a}{b}\right)^n \\ a^{-n} = \dfrac{1}{a^n} \\ (a^n)^k = a^{nk} \end{array}\right. \qquad n>1,k\in\mathbb{Z}\rightarrow\left|\begin{array}{l} \sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab} \\ \sqrt[n]{a^k} = a^{\frac{k}{n}} \\ \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}} \\ a\sqrt[n]{b} = \sqrt[n]{ba^n} \\ \left(\sqrt[n]{a^k}\right)^t = \sqrt[n]{a^{kt}} \end{array}\right.$$

$$(a+b)^n = \sum_{k=1}^{n}\binom{n}{k}a^{n-k}b^k$$

$$(a+b)^2 = a^2+2ab+b^2 \qquad (a-b)^2 = a^2-2ab+b^2$$

$$a^2-b^2 = (a+b)(a-b)$$

$$ab=0 \Leftrightarrow a=0\vee b=0 \qquad ab\neq 0 \Leftrightarrow a\neq 0\wedge b\neq 0$$

$$\sqrt{k}=a \Rightarrow k=a^2$$

$$\sqrt{k^2}=|k|\,,\quad k\in\mathbb{R}$$

$$|x|\leq a \Leftrightarrow x\leq a\wedge x\geq -a \qquad |x|\geq a \Leftrightarrow x\geq a\vee x\leq -a$$

FOLHA APOIO 1 (págs.2 - 4)

Logaritmos - *propriedades:*

$$
\begin{array}{|l}
x, y \in \mathbb{R}^+,\ a,\ b \in \mathbb{R}^+ \setminus \{1\} \\
\log_a x = y \Leftrightarrow x = a^y \\
\log_a (xy) = \log_a x + \log_a y \\
\log_a \left(\dfrac{x}{y}\right) = \log_a x - \log_a y \\
\log_a x^y = y \log_a x \\
\log_b x = \dfrac{\log_a x}{\log_a b} \\
a^{\log_a x} = x \\
\log_a a^y = y \\
\log_a 1 = 0
\end{array}
$$

Fórmulas trigonométricas *correntes:*

$$
\begin{array}{|l}
\sin^2 \beta + \cos^2 \beta = 1 \\
\operatorname{tg}^2 \beta + 1 = \sec^2 \beta \\
\operatorname{cotg}^2 \beta + 1 = \operatorname{cosec}^2 \beta \\
\operatorname{tg} \beta = \dfrac{\sin \beta}{\cos \beta} \\
\operatorname{cotg} \beta = \dfrac{1}{\operatorname{tg} \beta} = \dfrac{\cos \beta}{\sin \beta} \\
\sec \beta = \dfrac{1}{\cos \beta} \\
\operatorname{cosec} \beta = \dfrac{1}{\sin \beta}
\end{array}
$$

Fórmulas de adição:

$$
\begin{array}{|l}
\sin (\alpha \pm \beta) = \sin \alpha \cos \beta \pm \sin \beta \cos \alpha \\
\cos (\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\
\operatorname{tg} (\alpha \pm \beta) = \dfrac{\operatorname{tg} \alpha \pm \operatorname{tg} \beta}{1 \mp \operatorname{tg} \alpha \operatorname{tg} \beta} \\
\operatorname{cotg} (\alpha \pm \beta) = \dfrac{\operatorname{cotg} \alpha \operatorname{cotg} \beta \mp 1}{\operatorname{cotg} \alpha \pm \operatorname{cotg} \beta}
\end{array}
$$

FOLHA APOIO 1 (págs.3 - 4)

Fórmulas de duplicação:

$$\left|\begin{array}{l}
\sin(2\beta) = 2\sin\beta\cos\beta = \dfrac{2\,\mathrm{tg}\,\beta}{1+\mathrm{tg}^2\,\beta} \\
\cos(2\beta) = \cos^2\beta - \sin^2\beta = \dfrac{1-\mathrm{tg}^2\,\beta}{1+\mathrm{tg}^2\,\beta} \\
\mathrm{tg}\,(2\beta) = \dfrac{2\,\mathrm{tg}\,\beta}{1-\mathrm{tg}^2\,\beta} \\
\mathrm{cotg}\,(2\beta) = \dfrac{\mathrm{cotg}^2\,\beta - 1}{2\,\mathrm{cotg}\,\beta} \\
\sin^2\beta = \dfrac{1-\cos(2\beta)}{2} \\
\cos^2\beta = \dfrac{1+\cos(2\beta)}{2}
\end{array}\right.$$

Fórmulas de bissecção:

$$\left|\begin{array}{l}
\sin\dfrac{\beta}{2} = \pm\sqrt{\dfrac{1-\cos\beta}{2}} \\
\cos\dfrac{\beta}{2} = \pm\sqrt{\dfrac{1+\cos\beta}{2}} \\
\mathrm{tg}\,\dfrac{\beta}{2} = \pm\sqrt{\dfrac{1-\cos\beta}{1+\cos\beta}} = \dfrac{\sin\beta}{1+\cos\beta} = \dfrac{1-\cos\beta}{\sin\beta} \\
\mathrm{cotg}\,\dfrac{\beta}{2} = \pm\sqrt{\dfrac{1+\cos\beta}{1-\cos\beta}} = \dfrac{\sin\beta}{1-\cos\beta} = \dfrac{1+\cos\beta}{\sin\beta}
\end{array}\right.$$

Fórmulas de transf. logarítmica:

$$\left|\begin{array}{l}
\sin\alpha\cos\beta = \dfrac{1}{2}\left(\sin(\alpha+\beta) + \sin(\alpha-\beta)\right) \\
\sin\alpha\sin\beta = -\dfrac{1}{2}\left(\cos(\alpha+\beta) - \cos(\alpha-\beta)\right) \\
\cos\alpha\cos\beta = \dfrac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right) \\
\sin\alpha \pm \sin\beta = 2\sin\dfrac{\alpha\pm\beta}{2}.\cos\dfrac{\alpha\mp\beta}{2} \\
\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}.\cos\dfrac{\alpha-\beta}{2} \\
\cos\alpha - \cos\beta = -2\sin\dfrac{\alpha+\beta}{2}.\sin\dfrac{\alpha-\beta}{2} \\
\mathrm{tg}\,\alpha \pm \mathrm{tg}\,\beta = \dfrac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta}
\end{array}\right.$$

FOLHA APOIO 1 (págs.4 - 4)

Tabela de valores:

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$
	0^o	30^o	45^o	60^o	90^o	180^o	270^o
seno	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
coseno	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
tangente	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$*$	0	$*$
cotangente	$*$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$*$	0

$*$ expressões sem significado em $\mathbb{R}$